Price-Level Uncertainty and Instability in the United Kingdom

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Abstract

Was UK inflation was more stable and/or less uncertain before 1914 or after 1945? We address these questions by estimating a statistical model with changing volatilities in transient and persistent components of inflation. Three conclusions emerge. First, since periods of high and low volatility occur in both eras, neither features uniformly greater stability or lower uncertainty. When comparing peaks with peaks and troughs with troughs, however, we find clear evidence that the price level was more stable before World War I. We also find some evidence for lower uncertainty at pre-1914 troughs, but its statistical significance is borderline.

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1 Introduction

Figure 1 portrays annual data on the consumer price level and inflation rate in the United Kingdom for the period 1791-2011. To us, the figure shows contending patterns of price stability and predictability. Data on the logarithm of the price level, shown in the left panel, convey the impression that the gold and silver commodity standards that prevailed before 1914 had produced a century of long-term predictability of price levels, while the years after 1914 witnessed a struggle to prove that a well-managed fiat standard could deliver as much price stability as had been achieved under the gold standard. But data on inflation, exhibited in the right panel, suggest that volatility peaked early in the 1800s and declined for much of the following two centuries, albeit with momentous interruptions during and after World War I and amidst the Great Inflation of the 1970s.

These countervailing visual impressions are associated with different features of the inflation process. The left panel is dominated by variation in a stochastic trend for inflation, which was close to zero for much of the period between 1791 and 1913 and then turned positive around the time of World War I. Variation in the right panel is dominated by transient volatility, especially in the first-half of the 19th century. Both sources of variation make future price levels difficult to predict and contribute to price level instability. Because transient inflation volatility seems to have fallen while persistent inflation variation seems to have increased, it is not obvious whether future price levels were more predictable before World War I or after World War II.[1]

In addition, because persistent inflation variation gives rise to predictable movements, the two sources of variation have different implications about nominal *stability* as opposed to nominal *uncertainty*. To the extent that trend inflation is forecastable, the positive trend that appears after 1914 matters less for price-level uncertainty than for price-level instability.

[1]It is important to account for the decline in transient volatility when comparing the two eras, which is part of our interpretation of some of the comments by Christopher Sims that set us off on this paper.

In principle, the price level could have been less stable after World War II even if it were more predictable. The two features also operate differently at different forecast horizons, with persistent inflation variation mattering more in the long run. Thus, price levels could have been more predictable over short horizons after World War II even they were less predictable over long horizons.[2]

In this paper, we roll up our sleeves and estimate a statistical model simple and flexible enough to let us evaluate evidence about movements in conditional volatilities of future price levels. At a minimum, we think that four features are required for an adequate statistical representation. First, to fit both the 19th century, when average inflation was close to zero, and the 1970s, when average inflation was in double digits, the model must include a stochastic trend in inflation. Second, to fit the short-term volatility seen near the beginning of the sample, the model must also include a transient component. Third, so that volatility can change, innovations to the two components must have time-varying variances. These three features are captured in a simple and elegant model developed by Stock and Watson (2007) that we adapt to include a fourth feature.

The fourth feature concerns measurement error in older data. Christina Romer (1986a,b) warns about hazards involved in comparing data from before and after World War II. She points out that pre-war data were constructed differently and measured less accurately, and she contends that much of the apparent decline in volatility in several important macroeconomic variables that followed the war could be due to improved measurements. To respect this possibility, we append a measurement equation to Stock and Watson's model and allow pre-World War II data to be measured with noise. To estimate the magnitude of measurement errors and to purge older data of noise, we solve a nonlinear signal-extraction problem.[3]

[2] The terms *transitory* and *persistent* refer to unobserved components in a statistical model of inflation, while *uncertainty* and *instability* are about conditional moments of the log price level.

[3] Hendry (2001) also studies UK inflation data over the period 1875-1991 but does not address measurement error in historical data.

After estimating the model using Bayesian methods, we use it to quantify price-level uncertainty and instability. We measure uncertainty and instability, respectively, by the conditional variance and second moment of cumulative inflation. By comparing conditional variances and second moments at various dates over various forecast horizons, we trace the rise and fall of price stability and predictability in the U.K.[4]

Three main conclusions emerge. First, we find periods of high and low volatility both before 1914 and after 1945, with transient volatility dominating in the first period and persistent variation becoming important in the second. As a consequence, neither period features uniformly greater stability or lower uncertainty. This leads us to compare peaks with peaks and troughs with troughs across eras. Second, we find clear evidence that the price level was more *stable* at pre-1914 peaks and troughs, respectively, than at those that occurred after 1945. And third, from the same for measures of uncertainty, we find mixed evidence for lower uncertainty during the period before World War I. There is no evidence that price-level uncertainty was lower at the pre-1914 peak (1800) than at the post-1945 peak (1976). On the contrary, measures of uncertainty at the two peaks are quite similar. There is some evidence that price-level uncertainty was lower at the pre-1914 trough in 1890 than at the post-1945 troughs in 1955 and 2005, but its statistical significance is borderline.

The remainder of the discussion is organized as follows. Section 2 describes the data, while section 3 discusses the statistical model and our priors. Various features of the posterior

[4]Several aspects of British economic history make the UK a particularly interesting case for our purposes. First, for much of our long historical sample, the British empire played a very important role in the global economy, as exemplified by the central role of the British pound during the gold standard. Second, the establishment of economic and statistical institutions at the national level in the UK preceded by many decades similar institutions in other advanced economies, meaning that the coverage and quality of British historical data is probably better than in the rest of the industrialized world. Third, British monetary history has been characterized by a number of sharp changes in policy regimes, ranging from the silver and gold standard, to Bretton Woods, to money and exchange-rate targeting, and, finally, to inflation targeting.

are presented in section 4. Section 5 concludes with a summary and a few interpretative remarks.

2 Data

The consumer price data shown in figure 1 are spliced together from four sources. The U.K. Ministry of Labor launched the first official consumer price index at the beginning of World War I, and data from this source measures inflation between 1915 and 1947 (see B.R. Mitchell (1988), table 10, p. 738-39). Bowley (1937) contributed two extensions, first going back to 1880 using "virtually similar material" as the Ministry of Labor and then proceeding "more tentatively" with rougher raw material back to 1846 (Mitchell, p. 717). We refer to these periods as Bowley I and II, respectively (Mitchell, table 9, p. 738). Lindert and Williamson (1983) contributed an even more tentative backward extension, constructing a 'best guess' consumer price index going back to 1783 (Mitchell, table 8, p. 737). Two other sources complete the data set. The sample was extended from 1947 through 2011 by appending data from the Global Financial Database. Last but not least, wholesale price data for 1721-1790 taken from Phelps-Brown and Hopkins (1956) are used as a training sample to calibrate aspects of the prior. The sample used for estimation covers the period 1791-2011.

3 An unobserved-components, stochastic-volatility model for inflation

Our statistical representation extends Stock and Watson's (2007) unobserved components model for inflation:

$$\begin{aligned}
\pi_t &= \mu_t + \sqrt{r_t}\varepsilon_{\pi t}, \qquad (1)\\
\mu_t &= \mu_{t-1} + \sqrt{q_t}\varepsilon_{\mu t},\\
\ln r_t &= \ln r_{t-1} + \sigma_r \eta_{rt},\\
\ln q_t &= \ln q_{t-1} + \sigma_q \eta_{qt},
\end{aligned}$$

where π_t is inflation, μ_t is trend inflation, and r_t and q_t are stochastic volatilities that evolve as geometric random walks. The innovations $\varepsilon_{\pi t}, \varepsilon_{\mu t}$, are standard normal, serially uncorrelated, and independent of all the other shocks in the model. Following Shephard (2013), we assume that the log volatility innovations η_{rt} and η_{qt} are iid normal with mean zero and covariance matrix

$$W = \begin{bmatrix} \sigma_r^2 & \sigma_{rq} \\ \sigma_{rq} & \sigma_q^2 \end{bmatrix}, \tag{2}$$

thus allowing the log volatility innovations to be correlated.

Our main extension of the Stock-Watson-Shephard model confronts the measurement issues raised by Romer. To address her concern, we regard (1) as transition equations for a nonlinear state-space model, and we add measurement error to π_t,

$$y_t = \pi_t + m_t. \tag{3}$$

Our first challenge is to specify a plausible form for the measurement error process m_t.

In a companion paper on the United States (Cogley and Sargent 2014), we were able to exploit price-level data constructed by Christopher Hanes (1997) to identify measurement error. Following Romer, Hanes created a noisy postwar price-level series whose properties are consistent with prewar data. For the period 1948-1990, Hanes's noisy measure overlaps with the noise-free measure constructed by the US Bureau of Labor Statistics, and the difference between noisy and clean inflation measures sharply identifies measurement-error parameters. Alas, as far as we know, no one has created a consistent price-level series for the UK analogous to that of Hanes. Consequently, identification is weaker than for the US. In this paper, we simply assume that the measurement-error process has the same functional form as in the US. Although this assumption is debatable, it can be defended weakly by noting that the statisticians and economists who first constructed historical price indices for the two countries faced the same conceptual problems and had comparable data sources. Thus, it seems likely that they adopted kindred solutions and compromises. If that is so, the measurement-error processes are likely to be similar.

For the US, we found that measurement errors are well approximated by a mean-zero, first-order autoregressive process,

$$m_t = \rho_{mt} m_{t-1} + \sigma_{mt} \varepsilon_{mt}, \tag{4}$$

where the noise innovation ε_{mt} is iid standard normal and independent of the state innovations. We also adopt an $AR(1)$ form for the UK.

For the US, a single consistently noisy series was available for the period 1798-1990; hence no breaks in measurement-error parameters were necessary. For the UK, a long series can be compiled only by splicing a number of sources that B.R. Mitchell describes as having different degrees of noise. Hence breaks in measurement-error parameters are wanted. In particular, we assume that ρ_{mt} and σ_{mt} are constant within each subsample but vary across subsamples. We also assume that inflation is correctly measured after 1947. The measurement-error parameters therefore break at the following dates,

$$\begin{aligned}
\rho_{mt} &= \rho_{1m}, \quad \sigma_{mt} = \sigma_{1m} \quad t \leq 1846, \\
\rho_{mt} &= \rho_{2m}, \quad \sigma_{mt} = \sigma_{2m} \quad 1847 \leq t \leq 1879, \\
\rho_{mt} &= \rho_{3m}, \quad \sigma_{mt} = \sigma_{3m} \quad 1880 \leq t \leq 1914, \\
\rho_{mt} &= \rho_{4m}, \quad \sigma_{mt} = \sigma_{4m} \quad 1915 \leq t \leq 1947, \\
\rho_{mt} &= 0, \quad \sigma_{mt} = 0 \quad t \geq 1948.
\end{aligned} \tag{5}$$

Equations (4) and (5) define define additional transition equations for a nonlinear state-space model, while equation (3) defines the measurement equation. In contrast to the model for the US, only one observation on inflation is available in each year, noisy ones for 1791-1947 and clean ones for 1948-2011. The presence of overlapping clean and noisy measures in the postwar U.S. was important for identification. Unfortunately, as far as we know, consistently noisy postwar data are unavailable for the UK.

The distinction between the smooth transitions of r_t and q_t and the discrete variation in measure-error parameters helps to identify the model. In contrast, Cecchetti, et al. (2007)

represent log-volatility innovations as a two-state Markov process, thus allowing jumps in r_t and q_t as well. Exploring their fat-tailed specification would be an interesting extension, but identifying m_t would be more difficult if r_t could also jump. We suspect that other identifying information would be required in that case.

3.1 Priors

Our next task is to estimate the latent states π_t, μ_t, r_t, q_t, and m_t, the covariance matrix W for log-volatility innovations, and the measurement-error parameters ρ_{im}, σ_{im}, $i = 1, ...4$. We do this via Bayesian methods. Toward this end, we must specify priors for the initial states μ_0, π_0, r_0, q_0, and m_0 and the constant hyperparameters W, ρ_{im}, σ_{im}, $i = 1, ...4$. The transition equations then imply priors for the remaining states μ_t, π_t, q_t, r_t, and m_t. Following much of the literature, we assume independent priors for these elements, and we specify marginal priors for each. Many aspects of the priors are calibrated from the 1721-1790 training sample.

Starting with the initial states, the prior for (μ_0, π_0) is normal with mean equal to the training sample average (0.34 percent per annum) and variance

$$P_0 = \begin{bmatrix} 0.15^2 & 0 \\ 0 & 0.025^2 \end{bmatrix}. \tag{6}$$

Since prior credible sets for π_0 and μ_0 are roughly (-0.3,0.3) and (-0.05,0.05), respectively, the prior is weakly informative about initial inflation.

We also adopt normal priors for $\ln r_0$ and $\ln q_0$, the logs of the initial innovation variances for the transitory and persistent components of inflation, respectively. The prior median for $\ln r_0$ is the log of the training sample variance for inflation (-5.46), thereby equating the prior median for r_0 and q_0 with the training sample variance.[5] Similarly, the prior mean for $\ln q_0$ is the log of the training sample variance divided by 25 (-8.68). We set the prior standard deviation for both to 5, a value that is huge on a log scale. This makes the prior on $\ln r_0$ and $\ln q_0$ very weakly informative.

[5] The prior mean and mode are larger and smaller, respectively, than the prior median.

For the measurement error parameters ρ_{im} and σ_{im}, we adopt the same prior for all pre-World War II subsamples. The prior for ρ_m is normal with mean zero and standard deviation 0.45, thus centering on a white-noise specification and concentrating the preponderance of prior mass in the stationary region (see the solid line in the top-right panel of figure 2).[6] For σ_m, we adopt an inverse-gamma[7] prior whose mode equals 50 percent of the training sample standard deviation for inflation (70.71 percent of the variance), thereby expressing an initial belief that historical data are very noisy. Not wanting to hardwire this prior belief, however, we set the prior degrees of freedom to 2, so that a centered 95 credible set ranges from roughly 25 percent to more than 100 percent of the training sample standard deviation. The result, an $IG_1(0.04, 2)$ specification, is portrayed by a solid line in the top-left panel of figure 2. By combining the priors for ρ_m and σ_m, we can deduce the implied prior for the unconditional standard deviation of m_t. The solid line in the bottom-left panel depicts this prior.

A prior for W, the covariance matrix for log-volatility innovations, is harder to calibrate using the training sample. Instead, we adopt an informative inverse-Wishart prior that is inspired by Stock and Watson's calibration. We start with their parameter for the variance of log-volatility innovations in quarterly data, adjust for time aggregation to an annual sampling frequency, and then set the diagonal elements of the prior scale matrix so that the prior modes for σ_r^2 and σ_q^2 equal the adjusted value. Lacking a strong prior view about the covariance σ_{rq}, we set the off-diagonal elements of the prior scale matrix to zero, thus centering the prior covariance on zero. After centering the prior in this way, we set the degree of freedom parameter to deliver plausible prior credible sets. After some experimentation, we

[6]Truncating the prior at the boundary of the stationary region was unnecessary because only a tiny fractions of draws were in the non-stationary region.

[7]A standard deviation σ is an IG_1 random variable if and only if the variance σ^2 is an IG_2 random variable (Bauwens, et al., 1999, p. 292). We abuse terminology by referring to both as inverse-gamma random variables.

settled on an $IW(0.055 \cdot I_2, 10)$ prior for W. Solid lines in figure 3 portray prior histograms for the standard deviations σ_r and σ_q and the correlation $\sigma_{rq}/\sigma_r\sigma_q$. A centered prior 95 percent credible set for the standard deviations ranges from 0.17 to 0.45, while that for the correlation covers the interval ± 0.6.

4 Features of the posterior

This section records various features of the posterior probability distribution that conditions on our full sample 1791-2011. We approximate the posterior via a MCMC algorithm. Except for the structural breaks in measurement-error parameters, the model is very similar to that of Cogley and Sargent (2014). Our MCMC algorithm is therefore also very similar. Details can be found in their appendix A. Shephard (2013) describes an alternative approach based on particle filters.

4.1 Measurement error

Figure 2 depicts posterior distributions for ρ_{im} and σ_{im}. As expected, the measurement error parameters vary across subsamples, with accuracy improving throughout the 18th and 19th centuries. For the period before World War I, Lindert and Williamson's 'best guess' series for 1791-1846 has the highest measurement-error variance, followed by Bowley I (1847-1879), and then Bowley II (1880-1914). For these subsamples, measurement errors account for 34 percent, 56 percent, and 40 percent, respectively, of measured inflation variance.[8] Somewhat to our surprise, however, the trend toward improving accuracy does not continue after 1914. The unconditional measurement-error variance for the Ministry of Labor's first official index (1915-1947) is about the same as that for Lindert and Williamson's series, and measurement errors account for 61 percent of inflation variance in this period. The persistence of imputed measurement errors is also greatest for the 1915-1947 subperiod.

In the appendix, we report estimates for a more strongly informative prior based on

[8] These numbers are averages across posterior sample paths.

results for the U.S. to see whether evidence for a monotonic improvement in measurement can be found. The short answer is no; the results in the appendix are similar to those reported here.

That measurement error accounts for so much of interwar inflation variation seems more a puzzle to be explained than a firm conclusion. One possibility is that this is a symptom of autocorrelation in the inflation gap $\pi_t - \mu_t$. Since our model assumes that $\pi_t - \mu_t$ is a martingale difference and that m_t is $AR(1)$, weakly autocorrelated variation in y_t would be attributed to measurement error. More work is needed to get to the bottom of this.[9]

4.2 Variance of log-volatility innovations

Figure 3 portrays posterior distributions for elements of W, the innovation variance for the log volatilities. Overall, the figure suggests that the log-volatility innovation variances are weakly identified.[10] Posteriors for σ_r and σ_q overlap substantially with the prior but are shifted slightly to the right. Hence the data want a bit more time variation in log volatilities than is encoded in the prior. Moreover, the posterior for σ_r is shifted further to the right than that for σ_q, suggesting that the innovation variance for the transient component of inflation varies more than that of the permanent component. The posterior for the correlation coefficient $\sigma_{rq}/\sigma_r\sigma_q$ also overlaps substantially with its prior. As in Cogley and Sargent (2014), weak evidence is found of positive correlation between innovations to $\ln r_t$ and $\ln q_t$.[11]

[9] Mertens (2012) extends the Stock-Watson model by allowing serial correlation in the the inflation gap. For cleanly measured post-World War II data, this extension is straightforward. Matters are more complicated with noisy data because allowing $AR(1)$ components in both $\pi_t - \mu_t$ and m_t could weaken identification.

[10] The more strongly informative prior discussed in the apppendix helps a little in this respect, but not much.

[11] Shephard (2013) finds a stronger positive correlation in quarterly post-World War II U.S. data.

4.3 Hidden states

Figure 4 portrays posterior median and interquartile ranges for the latent states μ_t (the stochastic trend in inflation), $\pi_t - \mu_t$ (the transitory component of inflation), and $\sqrt{q_t}$ and $\sqrt{r_t}$ (the standard deviations of the log-volatility innovations). A number of salient points emerge.

The median estimate of trend inflation μ_t hovered around zero throughout the 19th century and increased gradually to about 2 percent by 1930. Trend inflation rose sharply during the Great Inflation, with the median estimate exceeding 10 percent during 1973-1981 and peaking at 16 percent in 1975. The median estimate of μ_t declined throughout the 1980s and 1990s and settled between 2.5 and 3 percent after the Bank of England achieved operational independence in 1997.[12,13]

Second, the transient component $\pi_t - \mu_t$ is centered near zero (by design) throughout the sample. It was enormously volatile during the Napoleonic wars, highly volatile after World War I, and less volatile after World War II. Because measurement error has been purged, the model asserts that this decline in volatility is genuine.

Third, much of the long-term decline in inflation volatility is due to a decrease in the variance r_t of transient shocks. Transient volatility was highest during the Napoleonic Wars

[12]While our postwar estimates are broadly consistent with those of Cecchetti et al. (2007), the precise timing of shifts differ. In particular, their estimates of μ_t have more spikes, especially in the mid-1970s when trend inflation rose and fell more quickly than our estimates. The chief difference between their specification and ours is that by assuming that r_t and q_t follow a two-state variance process they allow fat-tailed shocks to stochastic volatility. Their specification attempts to capture abrupt shifts in regime, while ours assumes a gradual drift in volatilities.

[13]Chan, et al. (2013) introduce a priori bounds on the random walk component μ_t. Our estimates of μ_t seem plausible, making explicit bounds unnecessary. In addition, bounds that are reasonable for periods in which a commodity standard or inflation-targeting policy was operative would be inappropriate for other eras, and vice versa.

when convertibility to gold was suspended.[14] Over the next 75 years, however, the median estimate of $\sqrt{r_t}$ declined by about 90 percent, reaching about 1.25 percent per annum by 1890. Convertibility to gold was again suspended during and after World War I, and transient volatility increased sharply at that time. But it declined throughout much of the remainder of the sample and reached its lowest point after 1997. By 2011, the median estimate of $\sqrt{r_t}$ was about 1.2 percent per annum, just a bit lower than in 1890. Thus, transient volatility was tamed twice, once in the 19th century and again after World War II.

Fourth, prior to World War II, most bursts of inflation were transient. Before 1950, the median estimate of the standard deviation of shocks to the permanent component μ_t was an order of magnitude smaller than that of shocks to the transient component $\pi_t - \mu_t$. After the Second World War, however, the relative importance of permanent shocks increased, as q_t rose and r_t fell. The biggest change came during the Great Inflation of the 1970s, when q_t increased sharply and began to approach r_t. Both volatilities declined after Thatcher's disinflation, but r_t fell faster. At the end of the sample, both q_t and q_t/r_t remained high by historical standards. Thus, while transient volatility has been tamed, the conquest of persistent volatility remains a work in progress.

As we shall see, the conditional variance for the log price level depends on both q_t and r_t, and both contribute terms that make the conditional variance increase with the forecast horizon. That q_t and q_t/r_t remain high will be important.

4.4 Orders of integration

Our model implies that inflation is $I(1)$ and that the log price level is $I(2)$, a common specification for post-World War II data. However, whether these orders of integration are consistent with pre-World War I data is not obvious. To examine this issue, we estimate an

[14]Convertibility was suspended between 1797 and 1821, during which time the Bank of England effectively operated a fiat regime.

augmented Dickey-Fuller regression,

$$y_t = \mu + \rho y_{t-1} + \sum\nolimits_{j=1}^{2} \zeta_j \Delta y_{t-j} + u_t, \tag{7}$$

where y_t is measured inflation for the period 1791-1913, and we calculate the t-statistic for $\rho - 1$. This augmented Dickey-Fuller statistic is -7.1, and its 1-percent asymptotic critical value is -3.43. The test therefore seems very strongly to reject a unit root in inflation.

However, the estimates shown in figure 4 suggest that the random-walk component of inflation was small prior to World War I and that the transient component plus measurement error was large. It is tenuous whether an augmented Dickey-Fuller test can detect a small random-walk component hidden under substantial noise. To check the size of the test for a specification like ours, we simulate our state-space model, generating artificial data on measured inflation by drawing from the posterior for the hidden state π_t and the measurement-error parameters (ρ_{im}, σ_{im}) and then calculating the implied distribution for the augmented Dickey-Fuller statistic. It turns out that the null distribution is shifted well to the left of the asymptotic distribution and that the correct 10 percent critical value is -8.61, implying that a unit root in inflation is not rejected. Indeed, the p-value for a sample statistic of -8.7 is 0.52.

None of this proves that inflation was $I(1)$, but it does establish that our $I(1)$ representation is not grossly at odds with the data. We are sufficiently reassured that our representation for inflation is good enough for pre-World War I data to allow us to proceed.

4.5 Price-level uncertainty

As in Cogley and Sargent (2014), we measure price-level uncertainty by the conditional standard deviation of cumulative h-year inflation,

$$\sigma(p_{t+h} - p_t | \omega^t, W) = \sqrt{q_t \sum\nolimits_{j=1}^{h} (h-j+1)^2 \exp(j\sigma_q^2/2) + r_t \sum\nolimits_{j=1}^{h} \exp(j\sigma_r^2/2)}, \tag{8}$$

where W is the log-volatility innovation variance and $\omega^t = (\pi^t, \mu^t, r^t, q^t)$ represents histories of time-varying states.[15] This is the square root of the prediction-error variance for a hypothetical forecaster who knew the structure of the model and its parameters and had experienced a history of realizations ω^t. We sample from its posterior distribution by calculating $\sigma(p_{t+h} - p_t|\omega^t, W)$ for every (ω^t, W) pair in our MCMC sample. We call the resulting values *smoothed* conditional volatilities because they are derived from a posterior for (ω^t, W) that conditions on the full sample y^T.

For forecast horizons of 5 and 10 years, figure 5 portrays the posterior median and interquartile range for $\sigma(p_{t+h} - p_t|\omega^t, W)$. Both the decline in transient volatility and the flareup of persistent volatility shape long run trends in price-level uncertainty, with the first mattering more before 1914 and the second becoming influential after 1945.

As mentioned above, transient volatility dominated during the 19th century. Consequently, paths for price-level uncertainty over the first half of our sample largely reflect the fall in r_t shown in figure 4. Shortly after the suspension of convertibility in 1797, median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ 5 and 10 years ahead peaked at 0.357 and 0.496, respectively, but then they fell as r_t declined, sharply in the first half of the century and more gradually in the second. Key steps in this process were the resumption of convertibility to gold in 1821; the Bank Charter Act of 1844, which progressively extinguished the right of country banks to issue paper money; and the gradual transformation of the Bank of England from a private enterprize to a central bank.[16] That the U.K. fought no extraordinarily costly war between 1815 and 1914 also played a big part, for war finance is the Achilles heel of a commodity standard.[17] By the end of the 19th century, through some combination of luck and practice, monetary and fiscal authorities had somehow reduced price-level uncertainty by remarkable

[15]Because the current log price level p_t is in the conditioning set, $\sigma(p_{t+h} - p_t|\omega^t, W) = \sigma(p_{t+h}|\omega^t, W)$, the conditional standard deviation for the future log price level p_{t+h}.

[16]See Feavearyear and Morgan (1963), chs. 8-12, for a narrative history of this period.

[17]War finance is a major challenge for any monetary regime, and a commodity standard is no exception.

amounts. In 1890, median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ 5 and 10 years ahead reached troughs of 0.061 and 0.077, respectively, and neither has returned to that level since.[18]

The first World War profoundly altered monetary arrangements, and price-level uncertainty increased as war finance became a paramount concern of economic policy. Authorities struggled to reestablish the gold standard after the war but failed to restore the degree of price-level predictability that had prevailed before 1914. Median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ 5 and 10 years ahead surged to 0.179 and 0.239 in 1920 before stabilizing in the 1930s at levels approximately 70 percent above those of 1890. Once shattered, the economic conditions on which prewar price predictability were based proved difficult to reconstitute.

Progress toward reducing price-level uncertainty stalled after World War II, with median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ changing only slightly in the 1950s. In the 1960s, as monetary and fiscal pressures in the U.S. and other countries began to undermine the Bretton Woods system, price-level uncertainty began to rise, eventually spiking during the Great Inflation of the 1970s. At the peak in 1976, median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ 5 and 10 years ahead were 0.402 and 0.451, respectively, levels not seen since the Napoleonic era. This time, the spike was mainly due to an increase in the variance q_t of persistent shocks to inflation.

Regaining control over trend inflation took the better parts of two decades. Trend inflation μ_t and its innovation variance q_t both declined after the Bank of England adopted an inflation-targeting framework and achieved operational independence. But because q_t remained relatively high by historical standards, so did $\sigma(p_{t+h} - p_t|\omega^t, W)$. A local trough in uncertainty was reached in 2005, at which time median estimates of $\sigma(p_{t+h} - p_t|\omega^t, W)$ were 0.149 and 0.162, respectively, at forecast horizons of 5 and 10 years. These estimates are about one-third those of 1976 but twice those of 1890.

Thus we find three peaks in price-level uncertainty – in 1800, 1920, and 1976 – as well as three troughs – in 1890, 1955, and 2005. Table 1 compares these peaks and troughs, recording

[18] More will be said about the posterior distribution below.

the ratio of mean smoothed volatilities as well as the proportion of posterior sample paths on which uncertainty was lower after the Second World War.

The first fact that emerges is that peaks in uncertainty in one era exceed troughs in the other, implying that neither period dominates. Mean estimates of uncertainty at the peak in 1800 exceed those at the postwar troughs (1955 and 2005) by factors of 2 or 3, and more than 95 percent of sample paths exhibit less uncertainty at the postwar troughs (see column 2, rows 3 and 5). Likewise, mean estimates of uncertainty at the peak in 1976 are more than 6 times greater those for 1890, and only a tiny fraction of sample paths exhibit less uncertainty at the postwar peak (see column 3, row 4).

A second fact is that levels of uncertainty at the two big peaks are comparable. Mean smoothed conditional standard deviations were about the same in 1800 and 1976, and the proportion of sample paths featuring lower volatility in 1976 was not far from 50 percent (see column 2, row 4). The forces driving inflation in the two periods were very different, but the amount of price-level uncertainty they engendered was similar. For uncertainty at the two big peaks, the decline in r_t offset the rise in q_t.

In these respects, our results are much like those for the United States (Cogley and Sargent 2014). One dimension in which results for the U.K. differ concerns trough-to-trough comparisons of uncertainty. For the U.S., we found comparable degrees of uncertainty at pre- and postwar troughs. For the U.K., uncertainty seems to be lower at the 1890 trough. Estimates of mean smoothed volatilities for 1955 and 2005 are double those for 1890, and only about 10 percent of posterior sample paths exhibited less uncertainty at the postwar troughs (see column 3, rows 3 and 5). We interpret this as persuasive but not decisive evidence that uncertainty was lower at the prewar trough.

4.6 Price-level instability

Conditional variances are suitable for measuring uncertainty, but the concept of price stability seems different. 'Stability' describes *total* variation, not just *unpredictable* variation.

For assessing price stability, we therefore compare (the square root of) conditional second moments across dates. Since the conditional second moment is the conditional variance plus the square of the conditional mean, we just need to add the latter to the numbers already reported. Conditional on the log-volatility innovation variance W and history of states ω^t, the conditional mean of cumulative h-year inflation is

$$E(p_{t+h} - p_t|\omega^t, W) = h\mu_t. \tag{9}$$

Therefore the conditional root mean square is

$$\text{cmrs}(p_{t+h} - p_t|\omega^t, W) = \sqrt{\sigma^2(p_{t+h} - p_t|\omega^t, W) + h^2\mu_t^2}. \tag{10}$$

As before, we sample from its posterior distribution by calculating cmrs for every draw of ω^t, W in our MCMC sample. Table 2 and figure 6 summarize the results.

Penalizing the conditional mean as well as the conditional variance shifts the balance of evidence in favor of the period before 1914. The Great Inflation of the 1970s now emerges as the time of greatest price-level instability, with the Napoleonic era coming in a distant second. At the peak in 1976, mean cmrs statistics are more than double those for 1800, and roughly 95 percent of posterior sample paths exhibit greater instability at the postwar peak (see column 2, row 4). Thus, while there was about as much price-level uncertainty at the two big peaks, the price level was much less stable during the Great Inflation.

Similarly, when comparing troughs with troughs, strong evidence emerges that the price level was more stable in 1890. Mean cmrs statistics for 1955 and 2005 are 2 to 3.5 times greater than those for 1890, and the postwar troughs exhibit less stability on all but a handful of posterior sample paths (see column 3, rows 3 and 5). Borderline evidence of greater uncertainty at postwar troughs becomes decisive evidence of greater instability.

4.7 Deflation risk

Another difference between pre- and postwar inflation dynamics concerns the risk of deflation. Following Cogley and Sargent (2014), we develop evidence about this feature of the data by calculating *smoothed conditional deflation probabilities*,

$$\mathrm{dpr}(\omega^t, W) \equiv pr(p_{t+h} < p_t | \omega^t, W) = \int I_{deflation}\Big(p_{t+h}(\omega^t, W, \xi_t^h), p_t(\omega^t, W)\Big) p_N(\xi_t^h) d\xi_t^h,$$

where $I_{deflation}\Big(p_{t+h}(\omega^t, W, \xi_t^h), p_t(\omega^t, W)\Big)$ is an indicator variable that records whether cumulative inflation going forward from a current log price level $p_t(\omega^t, W)$ is positive or negative,

$$\begin{aligned} I_{deflation}\Big(p_{t+h}(\omega^t, W, \xi_t^h), p_t(\omega^t, W)\Big) &= 1 \text{ if } p_{t+h} < p_t, \\ &= 0 \text{ otherwise.} \end{aligned} \tag{11}$$

The random vector ξ_t^h is a sequence of potential future shocks $\{\varepsilon_{\pi s}, \varepsilon_{\mu s}, \eta_{qs}, \eta_{rs}\}_{s=t+1}^{t+h}$ which, according to the model, has a normal unconditional distribution that we denote $p_N(\xi_t^h)$. The function dpr therefore represents the probability of cumulative deflation going forward from a given (ω^t, W) pair.

We approximate the posterior distribution for dpr by evaluating the second line of the right side of equation (11) for every (ω^t, W) pair in the posterior sample.[19] As before, we call the resulting values *smoothed* conditional deflation probabilities because they are derived from a posterior for ω^t and W that conditions on the full sample y^T. The results are summarized in table 3 and figure 7.

Deflation risk is decreasing in μ_t and either increasing or decreasing in σ_t, depending on whether μ_t is positive or negative. As shown in figure 7, changes in μ_t seem to have been quantitatively more important. Median estimates peaked above 75 percent in the 1880s when μ_t was slightly negative. A secondary peak of about 60 percent appeared in the 1820s, another period when μ_t was negative. After μ_t became positive around the turn of the 20th century, deflation risk fell sharply, eventually reaching a global trough below 1 percent in the 1970s when μ_t was highest.

The effects of changing conditional variances are less easy to see, but some insight can be gained by comparing years with similar values of μ_t and different values of r_t and q_t. For

[19] This is done by Monte Carlo integration.

instance, estimates for the late 1940s suggest the modest rates of trend inflation are sufficient to drive deflation risk close to zero provided that q_t and r_t are held in check. Similar rates of trend inflation in the 2000s were associated with higher deflation risk because of higher values of q_t. Nevertheless, these influences are secondary to those of changing μ_t.

Table 3 compares peaks and troughs in deflation risk before and after the Second World War. Peaks were reached in 1884, 1922, and 2001, and troughs occurred in 1911, 1974, and 2006. We also include 2011 to measure deflation risk at the end of the sample. As before, the top row records the ratio of mean deflation risks before and after the war, and the bottom depicts the probability that dpr was lower after 1945.

Mean deflation risk at the 2001 postwar peak was roughly 80 percent lower than at the prewar peak in 1884, and conditional deflation probabilities were lower in 2001 along more than 90 percent of posterior sample paths (see column 2, row 3). Comparing 2011 with 1884 yields slightly stronger evidence of reduced deflation risk (see column 2, row 5). At the prewar trough in 1911, mean deflation risk was 20-30 times greater than that of the postwar trough in 1974, and conditional deflation risk was higher in 1911 on 95 percent of sample paths (see column 3, row 3). Thus, when comparing peaks with peaks and troughs with troughs, clear evidence emerges that deflation risk was lower after 1945. Indeed, preventing a recurrence of the 1930s deflation might have been one of the central aims of post-World War II monetary policy.

5 Concluding remarks

Our analysis of UK inflation data evinces recurring episodes of rising and falling price-level volatility. Big shocks such as the French revolution and Napoleonic wars, two World Wars and the Great Depression, and the Great Inflation disrupted monetary arrangements and created appreciable uncertainty about future price levels. Because periods of high and low volatility appear both before 1914 and after 1945, neither period dominates uniformly. When comparing peaks with peaks and troughs with troughs, we find clear evidence that the price level was more *stable* before 1914. Weaker evidence also emerges that *uncertainty* was lowest at the pre-1914 trough in 1890. But its statistical significance is borderline, and there is no evidence that price-level uncertainty was lower at pre-1914 peaks than at post-1945 peaks.

Although the classical gold standard delivered more stability and perhaps less uncertainty, there is little reason to believe that a commodity standard would permanently guarantee either. Big shocks threaten commodity as well as fiat regimes, and convertibility can and perhaps should be suspended in times of crisis. The classical gold standard and Bretton Woods system ended not because the authorities thought they had discovered better methods for maintaining price stability but because other economic objectives supervened. That the U.K. fought no great war in the second half of the 19th century is surely one factor behind the success of the classical gold standard. When a great war did break out in 1914, the gold standard shattered, never to be restored. The ability to maintain a commodity standard is at least partly a consequence of economic stability.

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We thank Christopher Sims for comments that motivated our interest in this problem and François Velde for an insightful discussion of an earlier draft. We are also grateful to Mike Clements, Michael McMahon, Vincent Sterk, Mark Watson, a referee, and seminar participants at the Bank of France, Boston College, Cambridge, Chicago Booth, Manchester, Stanford, the Summer 2014 Midwest Macroeconomic Meetings, and University College London.

Appendix: a more strongly informative prior for measurement error parameters

One puzzling aspect of our results is that measurement error is worse in the first official index produced by the Ministry of Labor than in the ex post measures for earlier periods constructed by Bowley (1937) and Lindert and Williamson (1983). We are concerned that this might be an artifact of weak identification. To check the robustness of this finding, we re-estimate the model using a more strongly informative prior on measurement-error parameters. All other aspects of the model and prior are the same as in the text.

To strengthen the measurement-error prior, we assume not only that the measurement error process has the same functional form as in the US but also that its parameters are similar. We formalize 'similarity' by assuming that the UK prior for measurement-error parameters is the same as the US posterior constructed by Cogley and Sargent (2014). Although this is a stronger assumption than that made in section 3.1, three remarks can be offered in defense. First, as noted above, the statisticians and economists who first constructed historical price indices for the two countries may have adopted similar approaches to similar problems. Second, because those working in the UK probably had *better* data sources, their historical indices might be less noise ridden than those for the US. If that is true, our prior would overstate the magnitude of UK measurement error. And last but not least, the assumption is encoded in the prior, not hardwired into the posterior, so the data can still influence our conclusions.

The US posterior for ρ_m is well approximated by a normal distribution with mean 0.325 and standard deviation 0.145, while that for σ_m is well approximated by an inverse-gammma density with scale parameter 0.0368 and degrees of freedom 27. Solid lines in figure 8 depict these densities, along with the implied prior for the unconditional measurement-error variance $\sigma_m^2/(1-\rho_m^2)$. The other curves portray posteriors for the various subsamples. Although differences across subsamples are less pronounced than for the baseline model, measurement error is still worse for the period 1915-1947. That this feature survives a strengthening of the prior means that it cannot be dismissed as an artifact of weak identification.

Figures 9-13 and tables 4-6 portray the rest of the results for this model. Since the stronger prior has little effect on measurement-error posteriors, the remainder of the results also resemble those for the baseline specification. One exception is that evidence that the peak in instability in 1976 is greater than that for 1800 is weaker. Mean RMS statistics for 1976 are still roughly double those for 1800, but statistical significance is a bit weaker (compare column 2, row 4 in tables 2 and 5). Another minor exception is the 1820s peak in deflation probabilities that surpasses that of the 1880s (compare figures 7 and 13). Otherwise the results are much the same as those discussed above.

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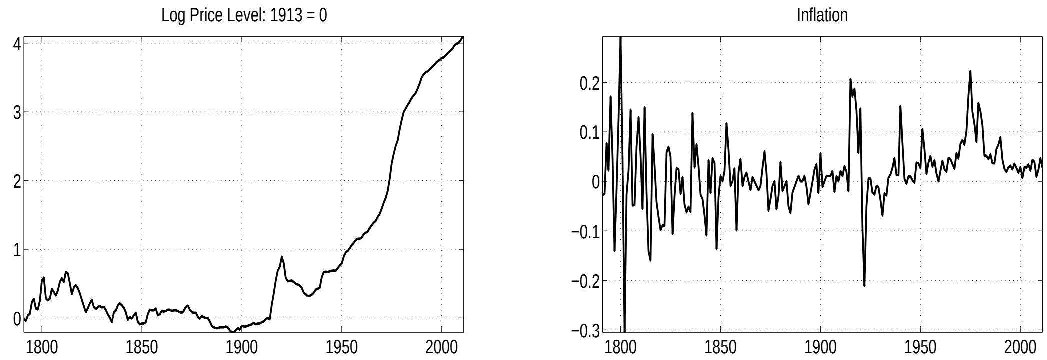

Figure 1: UK Consumer Price Level and Inflation

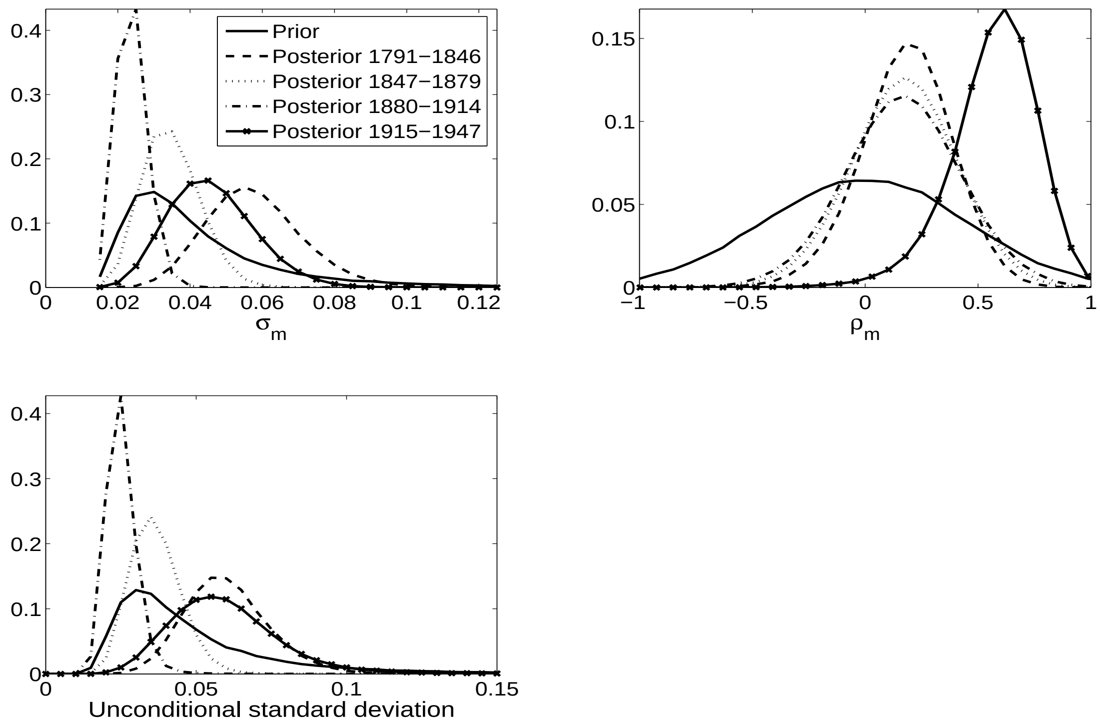

Figure 2: Priors and posteriors for measurement-error parameters

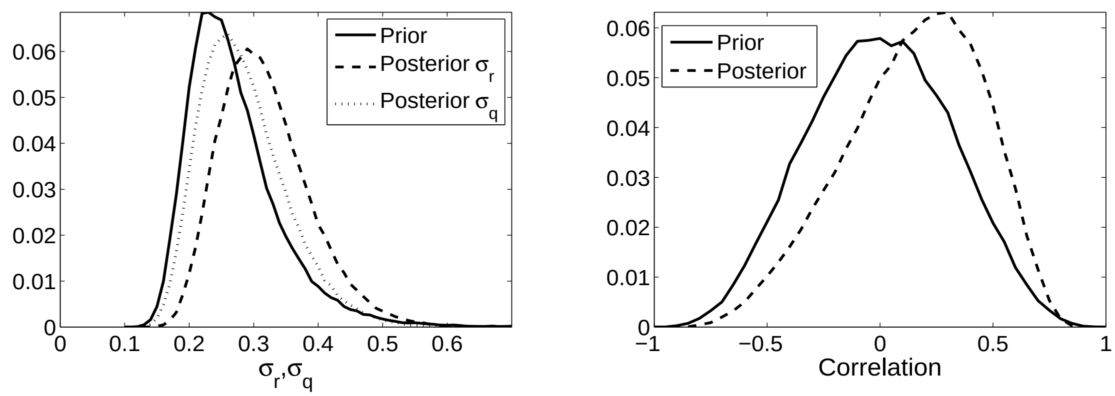

Figure 3: Priors and posteriors for variances and covariances of log-volatility innovations

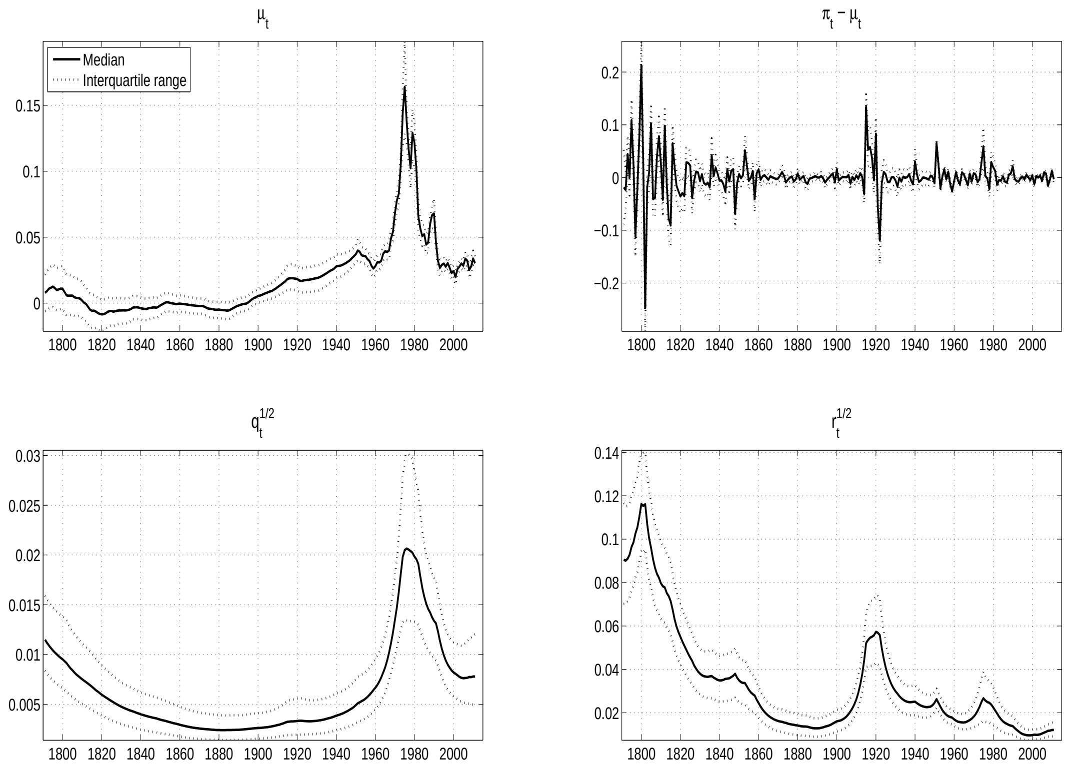

Figure 4: Posteriors of hidden states (conditioned on sample 1791-2011).

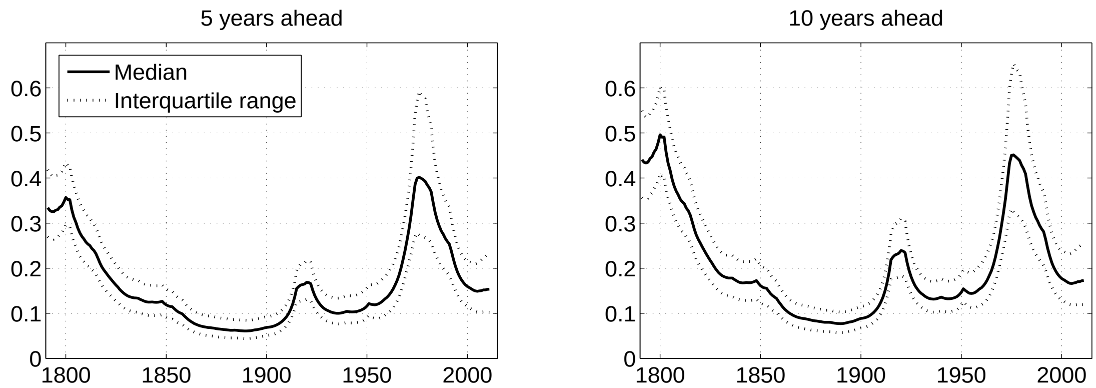

Figure 5: Posterior median and interquartile range for $\sigma(p_{t+h} - p_t | \omega^t, W)$

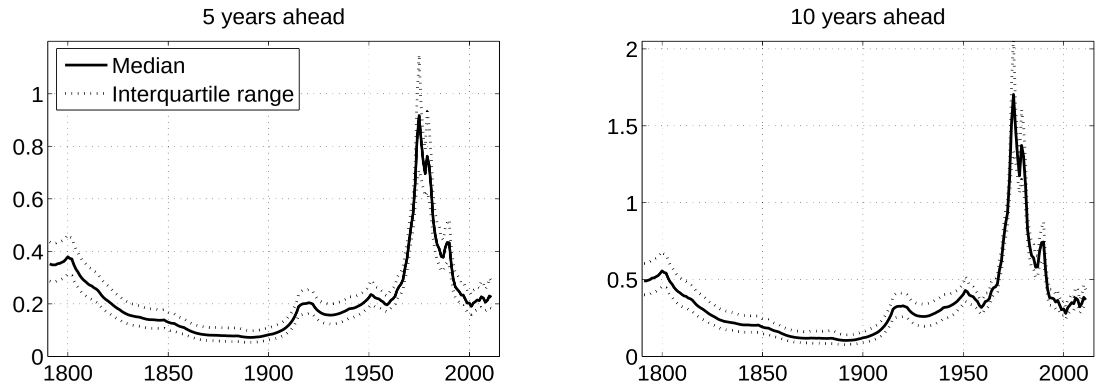

Figure 6: Posterior median and interquartile range for cmrs

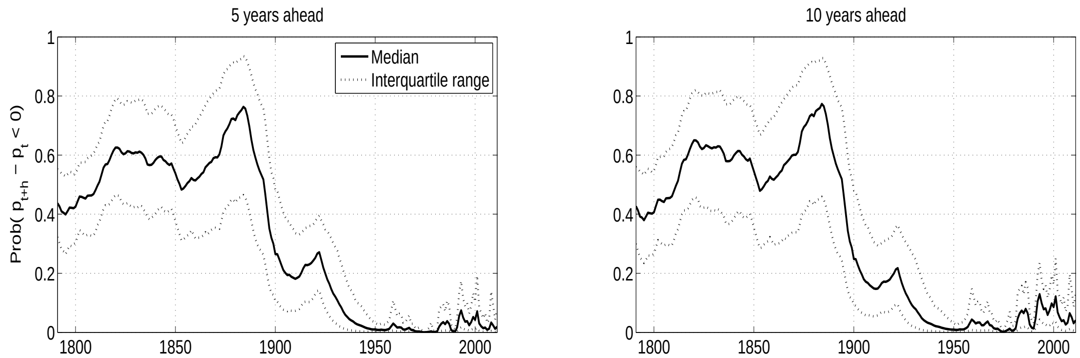

Figure 7: Smoothed deflation probabilities 5 and 10 years ahead

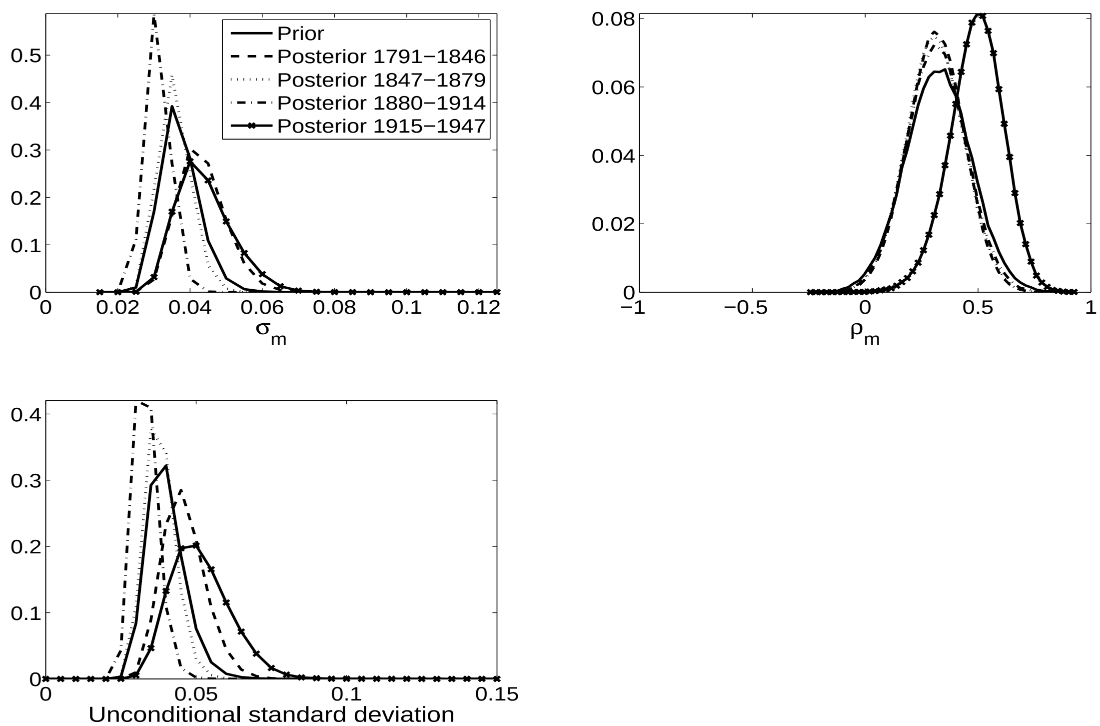

Figure 8: Priors and posteriors for measurement-error parameters (strongly informative prior)

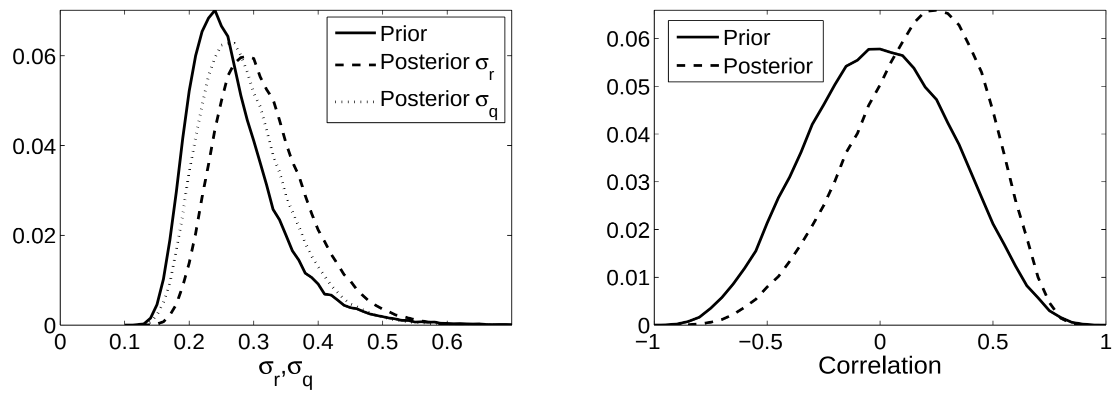

Figure 9: Priors and posteriors for variances and covariances of log-volatility innovations (strongly informative prior)

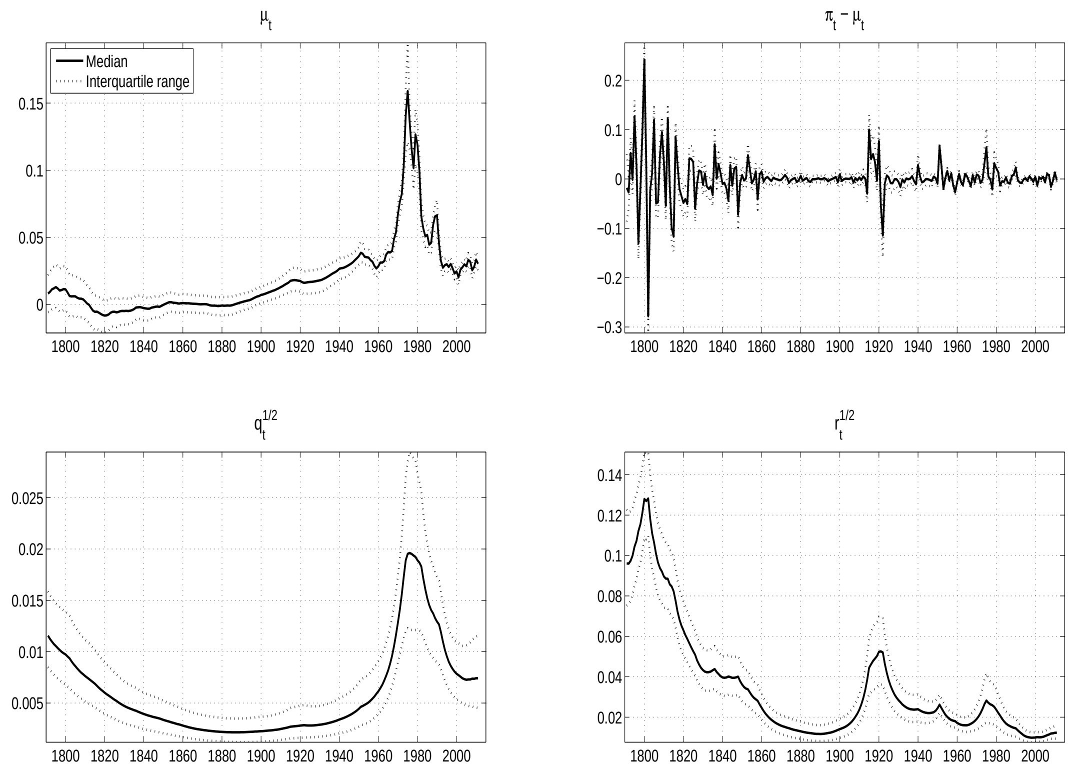

Figure 10: Posteriors of hidden states (conditioned on sample 1791-2011) (strongly informative prior)

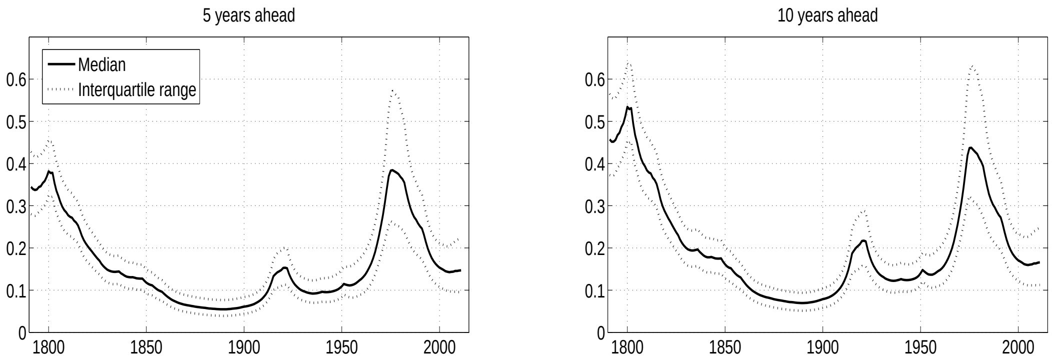

Figure 11: Posterior median and interquartile range for $\sigma(p_{t+h} - p_t|\omega^t, W)$ (strongly informative prior)

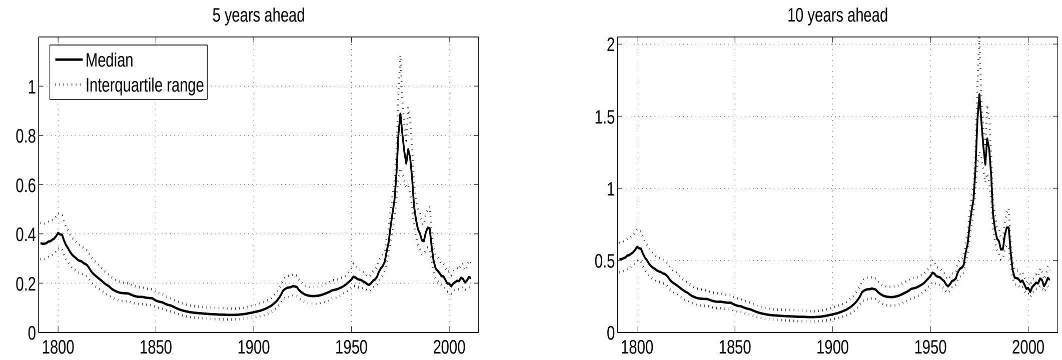

Figure 12: Posterior median and interquartile range for crms (strongly informative prior)

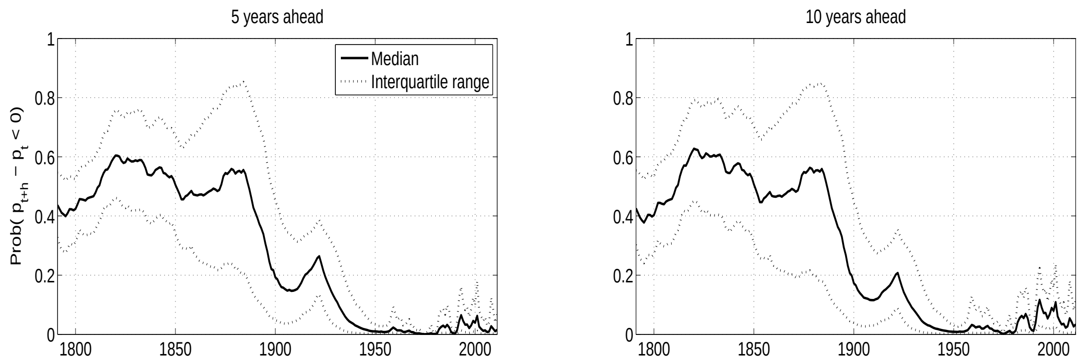

Figure 13: Smoothed deflation probabilities 5 and 10 years ahead (strongly informative prior)

Table 1: Relative Conditional Standard Deviations

	1800		1890		1920	
	5 years	10 years	5 years	10 years	5 years	10 years
1955	0.365	0.313	1.983	1.909	0.740	0.622
	(0.971)	(0.987)	(0.117)	(0.107)	(0.727)	(0.819)
1976	1.211	0.993	6.576	6.060	2.453	1.974
	(0.432)	(0.555)	(0.002)	(0.001)	(0.090)	(0.147)
2005	0.442	0.358	2.400	2.183	0.895	0.711
	(0.935)	(0.974)	(0.099)	(0.110)	(0.578)	(0.716)
2011	0.482	0.393	2.617	2.397	0.976	0.781
	(0.899)	(0.950)	(0.104)	(0.110)	(0.553)	(0.677)

Note: The top entry in each panel is the ratio of the postwar mean smoothed conditional standard deviation relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which conditional standard deviations are lower in the postwar year.

Table 2: Relative Conditional Root-Mean-Square Statistics

	1800		1890		1920	
	5 years	10 years	5 years	10 years	5 years	10 years
1955	0.575	0.676	2.846	3.405	1.036	1.119
	(0.915)	(0.832)	(0.011)	(0.005)	(0.433)	(0.351)
1976	2.028	2.424	10.03	12.20	3.652	4.009
	(0.066)	(0.034)	(0.000)	(0.000)	(0.009)	(0.005)
2005	0.565	0.595	2.796	2.997	1.018	0.985
	(0.910)	(0.908)	(0.018)	(0.009)	(0.453)	(0.458)
2011	0.616	0.646	3.046	3.234	1.109	1.069
	(0.862)	(0.852)	(0.018)	(0.009)	(0.407)	(0.400)

Note: The top entry in each panel is the ratio of mean smoothed cmrs relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which cmrs statistics are lower in the postwar year.

Table 3: Relative Deflation Probabilities

	1884		1911		1922	
	5 years	10 years	5 years	10 years	5 years	10 years
1974	0.010	0.015	0.031	0.051	0.025	0.042
	(0.989)	(0.982)	(0.968)	(0.938)	(0.982)	(0.960)
2001	0.175	0.228	0.531	0.760	0.421	0.616
	(0.941)	(0.916)	(0.694)	(0.567)	(0.784)	(0.656)
2006	0.035	0.069	0.107	0.229	0.085	0.186
	(0.981)	(0.967)	(0.910)	(0.817)	(0.947)	(0.875)
2011	0.079	0.118	0.240	0.392	0.190	0.318
	(0.970)	(0.951)	(0.839)	(0.744)	(0.894)	(0.808)

Note: The top entry in each panel is the ratio of the postwar mean probability of deflation relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which the probability of deflation was lower in the postwar year.

Table 4: Relative Conditional Standard Deviations (Strongly Informative Prior)

	1800		1890		1921	
	5 years	10 years	5 years	10 years	5 years	10 years
1955	0.324 (0.982)	0.277 (0.994)	2.093 (0.109)	2.024 (0.094)	0.789 (0.671)	0.667 (0.749)
1976	1.091 (0.508)	0.893 (0.634)	7.039 (0.005)	6.520 (0.001)	2.655 (0.078)	2.149 (0.123)
2005	0.396 (0.963)	0.318 (0.989)	2.557 (0.100)	2.325 (0.105)	0.965 (0.529)	0.766 (0.654)
2011	0.432 (0.933)	0.349 (0.972)	2.787 (0.101)	2.552 (0.100)	1.051 (0.508)	0.841 (0.619)

Note: The top entry in each panel is the ratio of the postwar mean smoothed conditional standard deviation relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which conditional standard deviations are lower in the postwar year.

Table 5: Relative Conditional Root-Mean-Square Statistics (Strongly Informative Prior)

	1800		1890		1921	
	5 years	10 years	5 years	10 years	5 years	10 years
1955	0.528	0.623	2.862	3.298	1.133	1.235
	(0.953)	(0.901)	(0.010)	(0.005)	(0.364)	(0.283)
1976	1.845	2.204	10.01	11.66	3.960	4.366
	(0.110)	(0.065)	(0.000)	(0.000)	(0.008)	(0.005)
2005	0.512	0.554	2.829	2.931	1.120	1.098
	(0.950)	(0.956)	(0.015)	(0.010)	(0.381)	(0.375)
2011	0.567	0.600	3.076	3.174	1.218	1.189
	(0.910)	(0.914)	(0.015)	(0.010)	(0.344)	(0.326)

Note: The top entry in each panel is the ratio of mean smoothed crms relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which crms statistics are lower in the postwar year.

Table 6: Relative Deflation Probabilities (Strongly Informative Prior)

	1820		1910		1922	
	5 years	10 years	5 years	10 years	5 years	10 years
1975	0.021 (0.998)	0.019 (0.998)	0.061 (0.918)	0.061 (0.893)	0.046 (0.948)	0.047 (0.935)
2001	0.186 (0.981)	0.238 (0.963)	0.548 (0.649)	0.782 (0.533)	0.407 (0.771)	0.600 (0.651)
2011	0.084 (0.996)	0.122 (0.987)	0.247 (0.785)	0.401 (0.689)	0.183 (0.883)	0.308 (0.795)

Note: The top entry in each panel is the ratio of the postwar mean probability of deflation relative to that in the prewar base year. Entries in parentheses record the proportion of sample paths on which the probability of deflation was lower in the postwar year.