Analytics of a Gold Exchange Standard

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May 5, 2026

Abstract

In a two-country commodity money model, country 1 issues paper pounds convertible into gold; country 2 issues paper dollars pegged to the pound; depending on the regime it may additionally hold gold coins or paper sterling as monetary reserves. We study four monetary regimes, each defined by a different combination of gold and paper monies. A relative-price mechanism simultaneously governs price levels and determines equilibrium non-monetary uses of gold. More deflationary regimes divert more gold from private consumption into monetary reserves. A pound-reserve standard transfers resources to country 1: country 2 exports goods to acquire paper pounds, reducing its consumption, while the resulting seigniorage raises country 1's consumption and hence its monetary gold demand. The model formalises Hawtrey's argument that the gold exchange standard economises on gold and thereby sustains a higher world price level, which explains why France's conversion of sterling balances into gold in the late 1920s was deflationary.

Keywords: Gold standard, gold exchange standard, token money, gold-coin fractions, two-country model, pyramiding, quantity theory

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> The introduction of the precious metals for the purposes of money may with truth be considered as one of the most important steps towards the improvement of commerce, and the arts of civilised life; but it is no less true that, with the advancement of knowledge and science, we discover that it would be another improvement to banish them again from the employment to which, during a less enlightened period, they had been so advantageously applied. Ricardo (1816, p. 65)

1 Introduction

During World War I, all belligerent nations except the United States suspended convertibility and allowed their currencies to depreciate relative to gold. Returning to gold at prewar parities while maintaining the same fraction of gold coins in the money supply as before the war threatened severe deflation: much of the world's monetary gold had migrated to the United States during the war. Hawtrey (1919) warned that if the major countries returned to the gold standard simultaneously, each requiring full gold backing for its currency, the resulting worldwide competition for a limited supply of gold would drive up gold's value relative to goods and drive down the world price level.[1]

In Hawtrey's view, a remedy was a *gold exchange standard*:[2] peripheral countries should hold their monetary reserves in sterling (or dollars) rather than in gold, allowing the major economies to maintain gold convertibility without triggering excess demand for the metal. The Genoa Conference of 1922 endorsed this recommendation, advising central banks to hold interest-bearing foreign exchange reserves in lieu of gold. Britain returned to gold in 1925 under the Gold Standard Act.[3] Following the Genoa arrangements, the Central European countries—Germany, Austria, Hungary, and others—and France held large sterling balances rather than acquiring gold directly (see Eichengreen (1992) and Rothbard (2002)). The model is designed to determine whether this arrangement was deflationary and, if so,

[1] Hawtrey described an earlier episode: France's return to specie in 1797 pulled gold from Britain and contributed to the deflation that accompanied Britain's own return to gold after the Napoleonic Wars in 1821. See Sargent and Velde (1995) for more details about that episode. Sargent (2019, Section 3.2) formalises this "Hawtrey effect": issues of token money that displace gold from monetary use raise the price level by a relative-price mechanism rather than a pure quantity-theory effect.

[2] Keynes (1913) advocated a gold-exchange standard for India. Rothbard (2002) had many interesting things to say about a gold or silver exchange standard; see especially his account of how Charles A. Conant designed and oversaw implementation of a version of the system in US protectorates during the early 1900s (p. 218, "Conant, Monetary Imperialism, and the Gold-Exchange Standard").

[3] Gold Standard Act 1925, 15 & 16 Geo. 5 c. 29, receiving Royal Assent on 13 May 1925.

relative to what baseline.

This paper extends the commodity money model of Sargent (2019) to two countries.[4] Country 1 (the United Kingdom analogue) issues pounds with a gold-coin fraction $\lambda_1 \geq 0$, so that this fraction of its currency stock consists of gold coins or paper warehouse certificates for gold held as reserves. Country 2 (the rest of the world) pegs its dollar to the pound at a fixed exchange rate $\bar{\epsilon}$. In the central regime—the *pound-reserve standard*—country 2 holds a fraction μ_2 of its dollar currency backed by paper pounds rather than gold, reproducing the arrangement endorsed by the Genoa Conference. These exported paper pounds carry no gold-backing obligation for country 1, so country 1 earns a pure seigniorage transfer when country 2 accumulates them as reserves.

The model is governed by two structural equations inherited from Sargent (2019): a quantity-theory equation and a marginal-utility condition equating the price of gold in terms of goods to the ratio of the marginal utility of goods to the marginal utility of gold. Together with trade balance conditions, these equations determine the world relative price ρ of the consumption good in terms of gold, and hence the nominal price levels $p_i = e_i\rho$. A key insight is that in a commodity money system the price *level* is a *relative* price. Adding gold to monetary uses reduces its non-monetary uses, lowering ρ and hence the price level; replacing gold coins with token money releases gold and raises the price level—Adam Smith's "consumption boon".

Section 3 introduces a general gold-coin-fraction framework and derives, for any target relative price $\bar{\rho} \in (0, \rho_R)$, the complete *iso-$\bar{\rho}$ locus*: the set of gold-coin-fraction pairs $(\lambda_1, \lambda_2) \in [0,1]^2$ consistent with equilibrium at $\rho = \bar{\rho}$. The locus is strictly decreasing—a higher fraction in one country must be offset by a lower fraction in the other to hold the world price level fixed—and three of the four classical regimes appear as special points on the locus: the Ricardo maximum ($\lambda_1 = \lambda_2 = 0$), the asymmetric gold exchange standard ($\lambda_1 > 0$, $\lambda_2 = 0$), and the symmetric gold standard ($\lambda_1 = \lambda_2 = \lambda$). The pound-reserve regime, which replaces country 2's gold coins with paper sterling, is analysed in Section 4, which establishes the strict four-way ordering of world price levels. Section 5 develops the corresponding ordering for non-monetary gold and consumption allocations, and Section 6

[4] An appendix to Sargent (2019) provides an excuse for using a *static* model to analyze doctrines that hinge on specifications of demand functions for money, objects whose micro-foundations really ought to be modeled within a coherent *dynamic* model. Dynamic macro-money models without stocks of money, like the models analyzed by Lagos (2025), do not provide a useful framework within which to pose the questions addressed in this paper. Other dynamic models in the cash-in-advance tradition such as Velde and Weber (2000) do provide a useful starting point. Sargent (2019) argued that the operating characteristics of his static model faithful mimic ones that would emerge from a model cast in the Velde and Weber mold.

connects the results to the history of the 1920s. Section 7 analyses country 2 leaving the gold standard altogether. Section 8 concludes.

Our analysis relates to several strands of the literature. Barro (1979) analyses price-level determination under the gold standard in a dynamic model in which the monetary authority chooses how much gold to absorb as reserves; his price-level formulas are close in spirit to those derived here. Lucas (1982) constructs a two-country monetary model with endogenous exchange rates; his focus, however, is on interest-rate differentials and portfolio equilibrium rather than commodity money. Sargent (2019) analyses the Smith and Ricardo optimality results and the Hawtrey price-level mechanism in a single-country setting; the present paper extends those results to a two-country world. Velde and Weber (2000) construct a one-country model with two commodity monies (gold and silver) and use it to study bimetallism; their framework shares the commodity-money structure of Sargent (2019) but focuses on the ratio of the two metals rather than on international reserve arrangements. Eichengreen (1992) describes the interwar gold exchange standard and documents the mechanisms—large sterling balances, their concentration, and their sudden conversion—that our model formalises.[5]

2 Setup

2.1 Preferences, endowments, and currency

There are two countries indexed by $i \in \{1, 2\}$, sharing common preferences and structural parameters (α, A, k) but potentially differing in endowments $(\check{c}_i, \check{g}_i)$. In each country a representative consumer enjoys a standard consumption good c_i and gold g_i (ounces). Gold has two uses: an amount $\bar{g}_i - g_i \geq 0$ is used as commodity money; the remainder g_i is consumed (e.g., as jewellery). Preferences are Cobb–Douglas:

$$u(c_i, g_i) = A\, c_i^{\alpha} g_i^{1-\alpha}, \qquad \alpha \in (0, 1),\ A > 0, \tag{1}$$

with the ratio of marginal utilities $u_{c_i}/u_{g_i} = (\alpha/1{-}\alpha)(g_i/c_i)$. The parameters α, A, and the Cambridge $k > 0$ are the same in both countries.

Country 1's government defines one pound as $1/e_1$ ounces of gold ($e_1 > 0$ is the pound

[5] Keynes (1923) advocated a managed gold exchange standard, arguing that a pure gold standard was both wasteful of real resources and unnecessarily rigid. He was thus advocating what we call the Ricardo maximum: token money should replace commodity money to the extent compatible with convertibility.

price of gold); country 2's government defines one dollar as $1/e_2$ ounces. The international exchange rate is $\epsilon = e_2/e_1$ (dollars per pound). Nominal price levels are p_1 (pounds per unit) and p_2 (dollars per unit), so the common world relative price of the consumption good in terms of gold is $\rho = p_i/e_i$.

Each country i has an exogenous endowment $\check{c}_i > 0$ of the standard good and $\check{g}_i > 0$ of gold, with world totals $\check{C} \equiv \check{c}_1 + \check{c}_2$ and $\check{G} \equiv \check{g}_1 + \check{g}_2$. In addition to gold coins, the government may issue token money $m_i \geq 0$, intrinsically useless but a perfect substitute for gold coins as a component of the stock of currency.[6] The total currency stock is

$$M_i = e_i\big(\bar{g}_i - g_i\big) + m_i. \tag{2}$$

Table 1 summarises notation.

Table 1: Notation. Subscript $i \in \{1, 2\}$ denotes the country.

Symbol	Description
c_i	Consumption of the standard good in country i
g_i	Gold consumed (non-monetary use) in country i
$\bar{g}_i$	Total gold in country i (monetary + non-monetary)
$\check{c}_i$, $\check{g}_i$	Endowments of the standard good and gold in country i
m_i	Token money issued in country i
e_i	Price of gold in country i's currency (currency units per ounce)
$\epsilon = e_2/e_1$	Exchange rate: dollars per pound
p_i	Nominal price level in country i
$\rho = p_i/e_i$	World relative price: ounces of gold per unit of consumption
α	Common utility exponent on the standard good, $\alpha \in (0, 1)$
k	Common Cambridge k, $k > 0$; desired ratio of real money balances to consumption income
$\bar{\epsilon}$	Fixed dollar/pound exchange rate peg
λ_i	Gold-coin fraction of the money supply in country i, $\lambda_i \in [0, 1]$
μ_2	Fraction of country 2's money supply backed by paper pounds, $\mu_2 \in [0, 1)$

[6]In the spirit of Sargent and Wallace (1982), we can also think of token money m_i as being backed by safe evidences of private indebtedness, i.e., Adam Smith's "real bills", and what we call gold coins as being paper bank notes that are backed by banks' holdings of gold bullion reserves.

2.2 Structural equations

Each country i is governed by two equations. The *quantity-theory* equation equates real money supply to real money demand:

$$\frac{e_i\big(\bar{g}_i - g_i\big) + m_i}{p_i} = k\, c_i. \tag{3}$$

The *marginal-utility* condition equates the relative price of gold to the marginal rate of substitution:

$$\frac{p_i}{e_i} = \frac{\alpha}{1-\alpha}\,\frac{g_i}{c_i}. \tag{4}$$

Under autarky, consumption equals the endowment and all gold is allocated domestically:

$$c_i = \check{c}_i, \qquad \bar{g}_i = \check{g}_i. \tag{5}$$

Conditions (3)–(5) give two independent copies of the Sargent (2019) closed-economy model. The autarky price level is

$$p_i = \frac{\alpha}{1-\alpha+k\alpha}\left(\frac{e_i\check{g}_i + m_i}{\check{c}_i}\right),$$

which is increasing in token money m_i. The Ricardo maximum—the largest token-money stock consistent with convertibility—releases all gold from monetary use: $m_i^{\max} = k\frac{\alpha}{1-\alpha}\check{g}_i e_i$. Setting m_i above this bound places country i in a pure quantity-theory regime in which gold no longer circulates as money. Appendix B derives the closed-form autarky equilibrium, proves the feasibility bounds on token money (Proposition B.1), and establishes the welfare-maximising role of the Ricardo maximum. We focus on the open economy for the rest of the paper.

3 Ricardo Maxima and Gold-Coin Fractions

This section constructs the open-economy framework. We begin with the Ricardo maximum as the natural reference point—it is the frontier of achievable price levels, the highest relative price ρ consistent with equilibrium—and then generalise to arbitrary gold-coin fractions λ_i. The central result is the *iso-ρ locus*: the complete set of (λ_1, λ_2) pairs consistent with a given world price level. Three classical monetary regimes appear as special points on this locus; the fourth—the pound-reserve standard—is studied in Section 4. To keep the notation manageable we write ρ_R for the Ricardo-maximum relative price derived below, ρ_A for the asymmetric-standard price, $\rho_S(\lambda)$ for the symmetric-standard price with common fraction λ, and ρ_P for the pound-reserve price of Section 4.

3.1 The iso-ρ locus

We open trade between the two countries. Country 2 pegs its dollar to the pound at $\bar{\epsilon}$, fixing $e_2 = \bar{\epsilon} e_1$ and hence $p_2 = \bar{\epsilon} p_1$. Free trade in goods and gold equalises the price of consumption goods in ounces of gold across countries:

$$\frac{p_1}{e_1} = \frac{p_2}{e_2} = \rho. \tag{6}$$

When both countries are at the Ricardo maximum ($\bar{g}_i - g_i = 0$, all money is token), the marginal-utility condition and the trade balance give

$$c_i = \alpha \left(\check{c}_i + \frac{\check{g}_i}{\rho} \right). \tag{7}$$

Summing over i and imposing goods-market clearing $c_1 + c_2 = \check{C}$ yields the *Ricardo-maximum relative price*:

$$\rho_R = \frac{\alpha}{1-\alpha} \frac{\check{G}}{\check{C}}. \tag{8}$$

At ρ_R all gold goes to private consumption and the exchange rate peg $\bar{\epsilon}$ affects only nominal variables.[7] Substituting ρ_R back into (7) and into $g_i = \frac{1-\alpha}{\alpha} \rho c_i$ gives the explicit Ricardo-maximum allocation:

$$c_i = \alpha\, \check{c}_i + \frac{(1-\alpha)\, \check{C}}{\check{G}}\, \check{g}_i, \tag{9}$$

$$g_i = \frac{\alpha\, \check{G}}{\check{C}}\, \check{c}_i + (1-\alpha)\, \check{g}_i. \tag{10}$$

Country i is a net importer of the consumption good if and only if its gold endowment share exceeds its goods endowment share: $\check{g}_i / \check{G} > \check{c}_i / \check{C}$.

Now suppose country i holds a fraction $\lambda_i \in [0, 1]$ of its total currency in gold coins.

[7] Appendix C provides explicit price-level and token-money-supply formulas, the net-trade-flow expression, and a formal peg-neutrality proposition (Proposition C.1).

Since $M_i/p_i = kc_i$, the gold-coin requirement is

$$\bar{g}_i - g_i = \lambda_i \, k \, \rho \, c_i, \tag{11}$$

and the marginal-utility condition continues to give $g_i = \frac{1-\alpha}{\alpha}\rho c_i$. Substituting into the trade balance for country i yields the key consumption formula:

$$c_i = \frac{\alpha}{1+\alpha\lambda_i k}\left(\check{c}_i + \frac{\check{g}_i}{\rho}\right). \tag{12}$$

A higher gold-coin fraction λ_i reduces c_i, since more gold is diverted from private consumption into monetary reserves.

Summing (12) over i and imposing $c_1 + c_2 = \check{C}$ gives the general market-clearing condition:

$$\frac{\alpha}{1+\alpha\lambda_1 k}\left(\check{c}_1 + \frac{\check{g}_1}{\rho}\right) + \frac{\alpha}{1+\alpha\lambda_2 k}\left(\check{c}_2 + \frac{\check{g}_2}{\rho}\right) = \check{C}. \tag{13}$$

For a fixed target price $\bar{\rho} \in (0, \rho_R)$, equation (13) defines the *iso-$\bar{\rho}$ locus*: the set of pairs $(\lambda_1, \lambda_2) \in [0,1]^2$ consistent with equilibrium at $\rho = \bar{\rho}$. Writing the income factor $F_i(\bar{\rho}) \equiv \alpha(\check{c}_i + \check{g}_i/\bar{\rho})$, the locus satisfies

$$\frac{F_1(\bar{\rho})}{1+\alpha\lambda_1 k} + \frac{F_2(\bar{\rho})}{1+\alpha\lambda_2 k} = \check{C}. \tag{14}$$

Since $\bar{\rho} < \rho_R$ implies $F_1 + F_2 > \check{C}$, at least one λ_i must be positive to restore clearing. Differentiating (14) implicitly gives

$$\frac{d\lambda_2}{d\lambda_1} = -\frac{F_1(\bar{\rho})}{F_2(\bar{\rho})}\left(\frac{1+\alpha\lambda_2 k}{1+\alpha\lambda_1 k}\right)^2 < 0, \tag{15}$$

so the iso-$\bar{\rho}$ locus is *strictly decreasing*: a higher gold-coin fraction in one country must be offset by a lower fraction in the other to preserve the world price level. Solving (14) explicitly for λ_2 as a function of λ_1 gives

$$\lambda_2(\lambda_1; \bar{\rho}) = \frac{1}{\alpha k}\left[\frac{F_2(\bar{\rho})\,(1+\alpha\lambda_1 k)}{\check{C}\,(1+\alpha\lambda_1 k) - F_1(\bar{\rho})} - 1\right], \tag{16}$$

provided $\check{C}(1+\alpha\lambda_1 k) > F_1(\bar{\rho})$. The feasible locus connects the right endpoint $\lambda_2 = 0$ (the asymmetric case) to the left endpoint $\lambda_2 = 1$; their λ_1 values are

$$\lambda_1^r = \frac{F_1(\bar\rho) + F_2(\bar\rho) - \check{C}}{\alpha k\big[\check{C} - F_2(\bar\rho)\big]}, \qquad \lambda_1^\ell = \frac{1}{\alpha k}\left[\frac{F_1(\bar\rho)\,(1+\alpha k)}{(1+\alpha k)\,\check{C} - F_2(\bar\rho)} - 1\right]. \tag{17}$$

The symmetric point $(\bar\lambda, \bar\lambda)$ with $\bar\lambda = \frac{1-\alpha}{\alpha k}(\rho_R/\bar\rho - 1)$ lies on the locus whenever $\bar\lambda \in [0,1]$. Every pair on the locus yields the same price level but a different consumption distribution, since moving along the locus shifts the real reserve burden between countries.

Proposition 3.1. *For every $\bar\rho \in (0, \rho_R)$, the iso-$\bar\rho$ locus* (14) *is a strictly decreasing curve in $[0,1]^2$. All pairs (λ_1, λ_2) on the locus deliver the same world price level $p_i = e_i\bar\rho$, but different distributions of consumption: moving along the locus shifts the real reserve burden between countries without altering the world price level. Three special cases of equation* (13) *determine the world price for the classical regimes:*

(i) Ricardo maximum *($\lambda_1 = \lambda_2 = 0$): $\rho = \rho_R$ (highest).*

(ii) Asymmetric gold exchange standard *($\lambda_1 > 0$, $\lambda_2 = 0$):*

$$\rho_A = \frac{\alpha\left(\dfrac{\check{g}_1}{1+\alpha\lambda_1 k} + \check{g}_2\right)}{\dfrac{(1-\alpha+\alpha\lambda_1 k)\,\check{c}_1}{1+\alpha\lambda_1 k} + (1-\alpha)\,\check{c}_2} \;<\; \rho_R. \tag{18}$$

Country 2 free-rides on country 1's gold reserves; its consumption formula (12) *carries no reserve penalty.*

(iii) Symmetric gold standard *($\lambda_1 = \lambda_2 = \lambda > 0$, with λ equal to the value of λ_1 in case (ii)):*

$$\rho_S(\lambda) = \frac{\alpha}{1-\alpha+\alpha\lambda k}\,\frac{\check{G}}{\check{C}} \;<\; \rho_A \;<\; \rho_R. \tag{19}$$

Both countries draw on a fixed gold supply for reserves, giving the lowest price level.[8]

Proof. The strict ordering $\rho_S(\lambda) < \rho_A < \rho_R$ follows from evaluating (13) at each special case. For the right inequality, evaluating (13) at $\lambda_2 = 0$ and comparing to the Ricardo-maximum clearing condition shows $\rho_A < \rho_R$. For the left inequality, fix any $\rho > 0$: adding country 2's reserve requirement ($\lambda_2 = \lambda > 0$) strictly reduces the left-hand side of (13)

[8]Appendix D.1 gives the explicit consumption formula (36) for the symmetric case and the expression (37) for the fraction of world gold absorbed as monetary reserves.

relative to the asymmetric case ($\lambda_2 = 0$), so goods-market clearing requires a lower ρ, giving $\rho_S(\lambda) < \rho_A$. □

The asymmetric standard is the right endpoint $\left(\lambda_1^r, 0\right)$ of the iso-ρ_A locus (Figure 1), while the symmetric standard achieving the same price level ρ_A lies at the midpoint $(\bar{\lambda}, \bar{\lambda})$ on the 45° diagonal: any redistribution of the reserve burden between countries that keeps the total demand for monetary gold constant preserves the world price level.

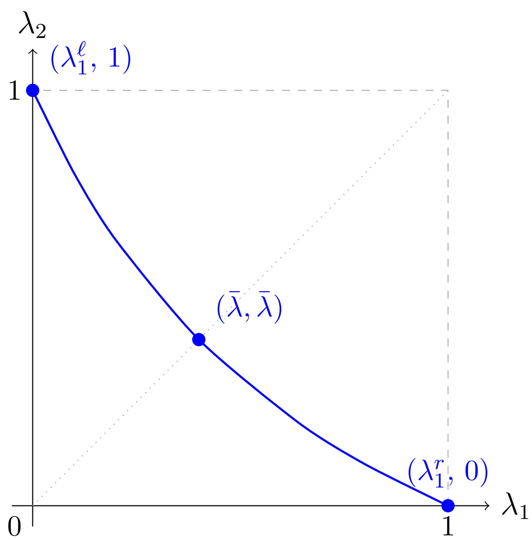

Figure 1: Iso-ρ_A locus in (λ_1, λ_2) space. The curve is the set of gold-fraction pairs that deliver a common world price level ρ_A; it is strictly decreasing (strict convexity follows from differentiating (16) twice). The right endpoint $(\lambda_1^r, 0)$ is the asymmetric gold-exchange standard (country 2 holds no monetary gold); the left endpoint $(\lambda_1^\ell, 1)$ is constrained by country 2's gold-fraction ceiling. The midpoint $(\bar{\lambda}, \bar{\lambda})$ on the 45° diagonal is the symmetric gold standard. Moving south-east along the locus shifts the reserve burden from country 2 to country 1 without changing the world price level. The figure is drawn for the symmetric-country case ($\alpha = \frac{1}{2}$, $k = 1$, $F_1 = F_2$), in which $\lambda_1^r = 1$ and $\lambda_1^\ell = 0$ exactly; for asymmetric countries both endpoints lie in the interior of the unit-square boundary.

4 The Pound-Reserve Standard

4.1 Structure and equilibrium

We now introduce the arrangement endorsed by the Genoa Conference: country 2 holds *paper pounds*—token liabilities of country 1—as reserves for a fraction $\mu_2 \in [0, 1)$ of its dollar currency. The equilibrium mechanism works in three steps. First, country 2 must export real resources to acquire paper pounds, which reduces its consumption c_2. Second, country 1 receives the corresponding seigniorage inflow, which raises its consumption c_1. Third, because country 1 backs fraction λ_1 of its now-larger domestic currency with gold, its monetary gold demand $\lambda_1 k\rho c_1$ rises relative to the asymmetric standard, depressing ρ below ρ_A. Country 2, however, holds only paper and contributes no direct gold demand, so total monetary gold remains below the symmetric level and $\rho_P > \rho_S(\lambda_1)$. The formal analysis follows.

Let $m_{1,2}$ denote the pounds exported to country 2 as reserves. Country 2 holds no gold ($\bar{g}_2 - g_2 = 0$). Country 1 backs fraction λ_1 of its *domestically circulating* pound supply with gold; the exported pounds $m_{1,2}$ carry no gold obligation.[9] The quantity-theory conditions are

$$M_1 - m_{1,2} = k\, p_1\, c_1, \qquad M_2 = k\, p_2\, c_2, \tag{20}$$

so gold backing in country 1 satisfies $\bar{g}_1 - g_1 = \lambda_1 k\rho c_1$, identical in form to (11). Letting $\mu_2 \in [0, 1)$ be the fraction of country 2's money stock backed by pound reserves, we have $m_{1,2} = \mu_2 k p_1 c_2$.

Country 2 must export real resources to acquire its pound reserves; its trade balance gives

$$c_2 = \frac{\alpha}{1 + \alpha\mu_2 k}\left(\check{c}_2 + \frac{\check{g}_2}{\rho}\right). \tag{21}$$

Country 1 receives the seigniorage inflow $m_{1,2} = \mu_2 k p_1 c_2$; its trade balance gives

$$c_1 = \frac{\alpha}{1 + \alpha\lambda_1 k}\left(\check{c}_1 + \frac{\check{g}_1}{\rho} + \mu_2 k c_2\right). \tag{22}$$

Country 1's consumption is *raised* by the seigniorage term $\mu_2 k c_2$: by issuing costless paper pounds that country 2 holds as reserves, country 1 captures a share of country 2's real income.[10] Substituting (21) into (22) gives the reduced form

$$c_1 = \frac{\alpha}{1 + \alpha\lambda_1 k}\left(\check{c}_1 + \frac{\check{g}_1}{\rho} + \frac{\alpha\mu_2 k}{1 + \alpha\mu_2 k}\left(\check{c}_2 + \frac{\check{g}_2}{\rho}\right)\right), \tag{23}$$

in which the factor $\alpha\mu_2 k/(1 + \alpha\mu_2 k) \in (0, 1)$ is the fraction of country 2's resources trans-

[9] Appendix D.2 details the split of country 1's token pound supply between domestic circulation and exports, and derives the money-supply identities that underpin equations (20)–(22).

[10] In this way, elements of a "fiscal theory of the price level" appear in our static model. See Sims (2013) for an introduction to a class of such theories in explicitly dynamic models.

ferred to country 1 through the pound-reserve mechanism; c_1 is strictly increasing in μ_2. Since $c_1 + c_2 = \check{C}$, the rise in c_1 is matched one-for-one by a fall in c_2: the pound-reserve arrangement redistributes consumption from country 2 to country 1 without affecting the world total. Setting $D_1 \equiv 1 + \alpha\lambda_1 k$ and $D_2 \equiv 1 + \alpha\mu_2 k$, summing (21) and (22) and imposing $c_1 + c_2 = \check{C}$ yields the pound-reserve equilibrium relative price:

$$\rho_P = \frac{\alpha[D_2\,\check{g}_1 + (D_1 + \alpha\mu_2 k)\,\check{g}_2]}{D_2\,(1-\alpha+\alpha\lambda_1 k)\,\check{c}_1 + [(1-\alpha)\,D_1 + \alpha\mu_2 k\,(1-\alpha+\alpha\lambda_1 k)]\,\check{c}_2}. \tag{24}$$

When $\mu_2 = 0$, (24) reduces to the asymmetric case ρ_A; when $\lambda_1 = \mu_2 = 0$, it reduces to ρ_R.

Proposition 4.1. *For any $\lambda_1 > 0$ and $\mu_2 \in (0, 1)$,*

$$\rho_S(\lambda_1) \;<\; \rho_P(\lambda_1, \mu_2) \;<\; \rho_A(\lambda_1) \;<\; \rho_R, \tag{25}$$

and ρ_P is strictly decreasing in μ_2. Price levels satisfy the same ordering: $p_i^S(\lambda_1) < p_i^P < p_i^A < p_i^R$.

Proof. $\rho_P > \rho_S(\lambda_1)$: Rearranging gold-market clearing ($g_i = \frac{1-\alpha}{\alpha}\rho c_i$, $\sum_i \bar{g}_i = \check{G}$, $\bar{g}_2 - g_2 = 0$) gives the implicit characterisation

$$\rho_P = \frac{\check{G}}{\lambda_1 k c_1 + \frac{1-\alpha}{\alpha}\check{C}}.$$

This is an *implicit* equation for ρ_P (since c_1 depends on ρ_P through (23)), but it suffices to establish the sign of the inequality. The analogous formula for the symmetric standard is $\rho_S(\lambda_1) = \check{G}/[(\lambda_1 k + \frac{1-\alpha}{\alpha})\check{C}]$. Since $c_1 < \check{C}$ (country 1 does not produce the entire world output), the denominator in the expression for ρ_P is smaller than that for $\rho_S(\lambda_1)$, so $\rho_P > \rho_S(\lambda_1)$.

ρ_P decreasing in μ_2: Differentiating (24) with respect to μ_2 (applying the quotient rule and collecting terms) shows that $d\rho_P/d\mu_2$ has the sign of $(1-\alpha)\rho_R - (1-\alpha+\alpha\lambda_1 k)\rho_P$. This is negative because $\rho_P > \rho_S(\lambda_1) = \frac{1-\alpha}{1-\alpha+\alpha\lambda_1 k}\rho_R$ implies $(1-\alpha+\alpha\lambda_1 k)\rho_P > (1-\alpha)\rho_R$.

$\rho_P < \rho_A$: At $\mu_2 = 0$, $\rho_P = \rho_A$; since ρ_P is decreasing in μ_2, it is strictly below ρ_A for $\mu_2 > 0$.

$\rho_A < \rho_R$: Follows from Proposition 3.1(ii). □

The mechanism is seigniorage compounded with gold demand. The paper pounds exported to country 2 carry no gold obligation, so country 1 earns a real transfer that raises c_1 (and correspondingly depresses c_2, as noted above). But since country 1 backs fraction λ_1 of its now-larger domestic circulation with gold, monetary gold demand $\lambda_1 k\rho c_1$ rises, depressing ρ below ρ_A. Country 2, however, holds only paper and contributes no direct gold demand, so ρ_P remains above $\rho_S(\lambda_1)$: the pound-reserve standard is less deflationary than requiring both countries to hold gold, confirming Hawtrey's argument.

Remark 4.2. *Country 1 earns seigniorage under the pound-reserve arrangement and enjoys higher c_1 than under the asymmetric standard; country 2 correspondingly suffers lower c_2. World welfare is lower (since $\rho_P < \rho_A$ implies less total non-monetary gold, by Corollary 5.1 below), so there is a conflict between country 1's national interest and world welfare. This conflict made the 1920s arrangement attractive to Britain but imposed a real cost on country 2 (France in particular), rendering the arrangement fragile.*

5 Regime Comparison: Prices, Gold, and Consumption

5.1 The four-way ordering

In every regime the marginal-utility condition holds at the common world price: $g_i = \frac{1-\alpha}{\alpha}\rho c_i$. Summing over countries gives

$$g_1 + g_2 = \frac{\rho}{\rho_R}\check{G}. \tag{26}$$

Total non-monetary gold is proportional to ρ: more deflationary regimes (lower ρ) divert more gold into monetary reserves and reduce aggregate gold consumption. The four-way ordering of Propositions 3.1–4.1 therefore translates directly into a four-way ordering of total non-monetary gold and of world welfare.

Corollary 5.1. *World welfare $W \equiv u_1 + u_2$ equals $A\left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}\rho^{1-\alpha}\check{C}$, which is strictly increasing in ρ. Hence $W^S(\lambda_1) < W^P(\lambda_1, \mu_2) < W^A(\lambda_1) < W^R$ for $\mu_2 \in (0, 1)$, where superscripts S, P, A, R denote the symmetric, pound-reserve, asymmetric, and Ricardo-maximum regimes respectively. World welfare is maximised at the Ricardo maximum and minimised at the symmetric gold standard.*

Table 2 collects the world price, total non-monetary gold, and monetary gold held by each country for all four regimes.[11]

Table 2: Prices, non-monetary gold, and monetary gold across the four regimes. Goods clearing: $c_1 + c_2 = \check{C}$; gold clearing: $\bar{g}_1 + \bar{g}_2 = \check{G}$.

Regime	ρ	$g_1 + g_2$	$\bar{g}_1 - g_1$	$\bar{g}_2 - g_2$
Ricardo max ($\lambda_i = 0$)	ρ_R (highest)	$\check{G}$	0	0
Asymmetric ($\lambda_1 > 0$, $\lambda_2 = 0$)	ρ_A	$\frac{\rho_A}{\rho_R}\check{G}$	$\lambda_1 k \rho_A c_1$	0
Pound-reserve ($\lambda_1 > 0$, $\mu_2 > 0$)	ρ_P	$\frac{\rho_P}{\rho_R}\check{G}$	$\lambda_1 k \rho_P c_1$	0
Symmetric ($\lambda_i = \lambda$)	$\rho_S(\lambda)$ (lowest)	$\frac{1-\alpha}{1-\alpha+\alpha\lambda k}\check{G}$	$\lambda k \rho_S(\lambda)\, c_1$	$\lambda k \rho_S(\lambda)\, c_2$

5.2 Distribution of consumption across countries

The distribution of consumption depends on who bears the reserve burden. Three features stand out.

(i) *Symmetric requirements tilt consumption toward gold-rich countries.* Under symmetric gold-coin fractions (equation (12) with $\lambda_1 = \lambda_2 = \lambda$),

$$c_i = \frac{\alpha}{1+\alpha\lambda k}\,\check{c}_i + \frac{1-\alpha+\alpha\lambda k}{1+\alpha\lambda k}\,\frac{\check{C}}{\check{G}}\,\check{g}_i.$$

As λ rises, the weight on the gold endowment $\check{g}_i$ increases relative to the weight on $\check{c}_i$, so a country with a high gold share suffers a smaller proportional consumption loss from a larger common gold-coin fraction.

(ii) *Asymmetric requirements impose a one-sided burden on country 1.* Under the asymmetric standard ($\lambda_1 > 0$, $\lambda_2 = 0$), country 1's consumption carries the penalty factor $1/(1 + \alpha\lambda_1 k)$; country 2's formula is the Ricardo-maximum formula evaluated at the lower price ρ_A, with no reserve cost. Country 2 free-rides on country 1's gold commitment.

[11] When country 2 leaves the gold standard entirely (Section 7), the real allocation is identical to the Ricardo maximum—$\rho = \rho_R$ and both monetary gold stocks equal zero—even though nominal price levels and the exchange rate differ.

(iii) *Pound reserves redistribute the burden: country 2 pays seigniorage, country 1 benefits.* In the pound-reserve arrangement country 1 backs only its domestically circulating currency with gold; the exported pounds are pure token liabilities. Country 2 surrenders real goods to acquire them, reducing c_2 by the factor $1/(1 + \alpha\mu_2 k)$, while country 1's c_1 is raised by the seigniorage term $+\mu_2 k c_2$ in (22). Because country 2 holds paper rather than gold, total monetary gold commitment is $\lambda_1 k\rho c_1 < \lambda_1 k\rho\check{C}$, so $\rho_P > \rho_S(\lambda_1)$: the world price level is higher than under the symmetric gold standard. But the seigniorage-driven rise in c_1 increases monetary gold demand $\lambda_1 k\rho c_1$ relative to the asymmetric standard, so $\rho_P < \rho_A$.

In summary, the monetary regime simultaneously determines the world price level, total non-monetary uses of gold, and its distribution across countries. Regimes that economise on gold (keeping ρ close to ρ_R) raise price levels and free more gold for private use; asymmetric arrangements redistribute the real burden of the monetary system between countries.

6 The 1920s Gold Exchange Standard

The four regimes can be interpreted as stylised versions of the monetary arrangements of the 1920s. Country 1 is the United Kingdom; country 2 represents the Central European countries together with France, which collectively held large sterling balances after the Genoa Conference of 1922.

Under the *Gold Standard Act* of 1925,[12] gold coins no longer circulated domestically in the UK; instead the Bank of England stood ready to sell gold bullion against sterling in large minimum lots (the "gold bullion standard"). This maps to $\lambda_1 > 0$: a positive fraction of the domestically circulating pound supply was backed by gold. After the Genoa Conference, the Central European countries and others were encouraged to hold sterling rather than gold as their monetary reserves, giving $\mu_2 > 0$ with $\bar{g}_2 - g_2 = 0$.

The regime ordering in Propositions 3.1 and 4.1 can now be read as follows:

- *Ricardo maximum* ($\lambda_1 = \mu_2 = 0$): all gold freed from monetary use; highest world price level. A theoretical ceiling.
- *Asymmetric exchange standard* ($\lambda_1 > 0$, $\mu_2 = 0$): country 2 uses only its own token dollars with no pound reserves. Price level lower than Ricardo maximum.

[12] Gold Standard Act 1925, 15 & 16 Geo. 5 c. 29.

- *Pound-reserve standard* ($\lambda_1 > 0$, $\mu_2 > 0$, the actual 1920s arrangement): country 2 holds paper sterling; country 1 earns seigniorage, raising c_1 and hence monetary gold demand. Price levels lie strictly between the asymmetric and symmetric standards; the larger μ_2, the more deflationary.

- *Symmetric gold standard* ($\lambda_1 = \lambda_2 = \lambda > 0$): both countries hold gold coins; lowest price levels—the deflationary scenario Hawtrey feared.

Proposition 4.1 confirms Hawtrey's central claim: the gold exchange standard economises on gold relative to a symmetric gold standard. Because country 2 holds paper pounds rather than gold, total monetary gold equals only $\lambda_1 k\rho c_1$ rather than the symmetric standard's $\lambda_1 k\rho\check{C}$, and since $c_1 < \check{C}$ this implies $\rho_P > \rho_S(\lambda_1)$.

One reason the 1920s arrangement eventually came under strain was that country 2 (France in particular) bore a real cost proportional to μ_2 and therefore had an ongoing incentive to convert sterling reserves into gold. In the model, this conversion corresponds to a *regime switch*: replacing the pound-reserve equilibrium (Section 4, with $\mu_2 > 0$ and $\bar{g}_2 - g_2 = 0$) by the symmetric gold-standard equilibrium (Section 3(iii), with $\lambda_2 > 0$ and $\mu_2 = 0$). As the regime shifted from pound-reserve toward symmetric gold standard, ρ dropped from ρ_P toward $\rho_S(\lambda_1)$, contributing to the deflationary pressures of 1929–1931 that Hawtrey (1919) had warned against.

7 Leaving the Gold Standard

We now consider country 2 abandoning gold convertibility altogether by printing paper dollars in excess of the level at which all gold is released from monetary use.[13] Throughout this section we also assume country 1 is at its Ricardo maximum ($\lambda_1 = 0$, $\bar{g}_1 - g_1 = 0$): it has converted all gold coins into token pounds. This assumption is important: if $\lambda_1 > 0$, country 1's binding reserve requirement would remain operative even after country 2 departs, and the world relative price would satisfy $\rho < \rho_R$ rather than $\rho = \rho_R$. Setting $\lambda_1 = 0$ isolates the purely nominal consequences of country 2's departure, with country 1's real allocation undisturbed. This differs from Sections 3–6, where $\lambda_1 > 0$. Country 1 continues to convert paper pounds into gold at e_1. The exchange rate $\epsilon = e_2/e_1$ is now market-determined.

[13] The relevant threshold is the autarky Ricardo maximum $\bar{m}_2 = k\frac{\alpha}{1-\alpha}\check{g}_2 e_2$; in the open economy the binding level differs, but the condition $\bar{g}_2 - g_2 = 0$ is what matters here.

Because country 2 holds no gold as monetary reserves ($\bar{g}_2 - g_2 = 0$) and its quantity theory reduces to $m_2/p_2 = kc_2$, while country 1 remains at the Ricardo maximum ($\bar{g}_1 - g_1 = 0$), the trade balance for each country gives $c_i = \alpha(\check{c}_i + \check{g}_i/\rho)$—exactly the Ricardo-maximum formulas (7). World goods-market clearing therefore yields

$$\rho_{\text{off}} = \rho_R = \frac{\alpha}{1-\alpha}\frac{\check{G}}{\check{C}}, \tag{27}$$

and the real allocation (c_1, c_2, g_1, g_2) is identical to the Ricardo maximum. Country 2's departure from gold has *no effect on the real equilibrium*.

Nominal variables, however, differ. Country 1's price level remains anchored to gold: $p_1 = e_1\rho_R$. Country 2's price level is governed by its money supply:

$$p_2 = \frac{m_2}{k\,\alpha(\check{c}_2 + \check{g}_2/\rho_R)}, \tag{28}$$

which exceeds the Ricardo-maximum price level since m_2 exceeds $\bar{m}_2$. The dollar–pound exchange rate is

$$\epsilon = \frac{p_2}{p_1}, \tag{29}$$

and depreciates in proportion to m_2—the "pure and simple" quantity theory of exchange rate determination. Country 1's gold parity insulates its price level from country 2's monetary expansion.[14]

8 Concluding Remarks

This paper extends the commodity money model of Sargent (2019) to a two-country open economy and uses it to study the monetary-policy choices of the 1920s. A useful analytical object is the *iso-ρ locus*: for any target world price $\bar{\rho} \in (0, \rho_R)$, equation (14) characterises the complete set of gold-coin-fraction pairs (λ_1, λ_2) that support that price level. The locus is strictly decreasing, so the reserve burden can be redistributed between countries while holding the world price level constant; and it nests the four classical regimes as special cases—the Ricardo maximum at one extreme, the symmetric gold standard at the other, and the asymmetric and pound-reserve arrangements in between.

The strict four-way ordering $\rho_S(\lambda_1) < \rho_P(\lambda_1, \mu_2) < \rho_A(\lambda_1) < \rho_R$ (Propositions 3.1–

[14] Appendix D.3 compares the leaving-gold regime more fully with the gold-standard alternatives and derives the endogenous dollar price of gold (40).

4.1) translates directly into corresponding orderings of total non-monetary gold and world welfare via $g_1+g_2 = (\rho/\rho_R)\check{G}$. The pound-reserve standard lies between the two asymmetric extremes because seigniorage raises country 1's consumption—and hence its monetary gold demand—above the purely asymmetric level, while country 2's paper holdings keep total monetary gold below the symmetric level.

France's conversion of sterling balances into gold in the late 1920s amounted to a regime switch from the pound-reserve equilibrium to the symmetric gold standard, driving ρ from ρ_P toward $\rho_S(\lambda_1)$ and contributing to the deflation of 1929–1931 that Hawtrey (1919) had warned against. When country 2 leaves gold entirely (with country 1 at its Ricardo maximum), the real allocation is unchanged—identical to the Ricardo maximum—but the dollar price level becomes proportional to the money supply and the exchange rate floats.

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A Derivations of Equations (18) and (24)

A.1 Derivation of (18): asymmetric case ($\lambda_1 > 0$, $\lambda_2 = 0$)

From (12), $c_1 = \frac{\alpha}{1+\alpha\lambda_1 k}(\check{c}_1 + \check{g}_1/\rho)$ and $c_2 = \alpha(\check{c}_2 + \check{g}_2/\rho)$. Setting $c_1 + c_2 = \check{C}$, multiplying through by ρ, and collecting terms:

$$\rho\left[\frac{(1-\alpha+\alpha\lambda_1 k)\,\check{c}_1}{1+\alpha\lambda_1 k} + (1-\alpha)\check{c}_2\right] = \alpha\left(\frac{\check{g}_1}{1+\alpha\lambda_1 k} + \check{g}_2\right),$$

which gives (18) upon dividing.

A.2 Derivation of (24): pound-reserve case ($\lambda_1 > 0$, $\mu_2 > 0$)

Write $D_1 \equiv 1 + \alpha\lambda_1 k$ and $D_2 \equiv 1 + \alpha\mu_2 k$. From (21)–(22),

$$c_1 = \frac{\alpha\check{c}_1}{D_1} + \frac{\alpha\check{g}_1}{D_1\rho} + \frac{\alpha^2\mu_2 k\,\check{c}_2}{D_1 D_2} + \frac{\alpha^2\mu_2 k\,\check{g}_2}{D_1 D_2\rho}.$$

Adding $c_2 = \frac{\alpha\check{c}_2}{D_2} + \frac{\alpha\check{g}_2}{D_2\rho}$, imposing $c_1 + c_2 = \check{C}$, and collecting terms in ρ and $1/\rho$:

$$\rho\,\frac{D_2(1-\alpha+\alpha\lambda_1 k)\check{c}_1 + [(1-\alpha)D_1 + \alpha\mu_2 k(1-\alpha+\alpha\lambda_1 k)]\check{c}_2}{D_1 D_2} = \frac{\alpha[D_2\check{g}_1 + (D_1 + \alpha\mu_2 k)\check{g}_2]}{D_1 D_2}.$$

Dividing gives (24).

B Autarky Equilibrium

B.1 Closed-form solution

Under autarky, each country i is governed by equations (3), (4), and (5). The two systems are entirely decoupled. Substituting (5) into (3) and (4) and solving for p_i and g_i gives

$$g_i = \frac{1-\alpha}{1-\alpha+k\alpha}\left(\check{g}_i + \frac{m_i}{e_i}\right), \tag{30}$$

$$p_i = \frac{\alpha}{1-\alpha+k\alpha}\left(\frac{e_i\check{g}_i + m_i}{\check{c}_i}\right). \tag{31}$$

The price level p_i is increasing in m_i: issuing tokens raises the price level by reducing the relative scarcity of gold in consumption. The amount of gold used as money is

$$\bar{g}_i - g_i = \frac{k\alpha}{1-\alpha+k\alpha}\check{g}_i - \frac{1-\alpha}{1-\alpha+k\alpha}\frac{m_i}{e_i},$$

so an increase in token money crowds out gold coin one-for-one in real terms.

B.2 Bounds on token money

Proposition B.1. *Under autarky, country $i \in \{1,2\}$ replicates a closed-economy instance of Sargent (2019):*

(i) Gold coins and token money coexist as money if and only if

$$0 \;\le\; m_i \;\le\; k\,\frac{\alpha}{1-\alpha}\,\check{g}_i\,e_i. \tag{32}$$

(ii) Setting m_i above the upper bound in (32) places country i in a pure quantity-theory regime in which no gold circulates as money: $m_i/p_i = k\check{c}_i$ and gold trades at the endogenous price $\tilde{e}_i = \frac{1-\alpha}{\alpha}\frac{\check{c}_i}{\check{g}_i}p_i > e_i$.

The upper bound is the Ricardo maximum discussed in Section 2: at this bound all gold is released from monetary use ($\bar{g}_i - g_i = 0$), confirming the formula $m_i^{\max} = k\frac{\alpha}{1-\alpha}\check{g}_i e_i$ given there.

B.3 Welfare and decoupling

Since g_i in (30) is increasing in m_i on $[0,\, k\frac{\alpha}{1-\alpha}\breve{g}_i e_i]$, the government of country i maximises welfare $u(\breve{c}_i, g_i)$ by setting m_i at its Ricardo maximum. This is the autarky counterpart of the Ricardo–Smith optimality results in Sargent (2019, Propositions 2 and 3).

Under autarky the equilibrium (p_i, g_i) depends only on $(e_i, m_i, \breve{g}_i, \breve{c}_i)$ and the common parameters α, k. In particular, the two countries may set different gold prices $e_1 \neq e_2$, issue different amounts of token money, and have different endowment sizes without any cross-border spillover. This decoupling breaks down once goods and gold are traded, as Sections 3–7 show.

C Ricardo-Maximum Open Economy: Prices, Trade Flows, and Peg Neutrality

C.1 Price levels and token-money supplies

When both countries operate at the Ricardo maximum in the open economy ($\bar{g}_i - g_i = 0$, $e_2 = \bar{\epsilon} e_1$), the world relative price is ρ_R from (8). The corresponding nominal price levels are

$$p_1 = e_1\,\rho_R = \frac{\alpha}{1-\alpha}\,\frac{\breve{G}}{\breve{C}}\,e_1, \qquad p_2 = \bar{\epsilon}\,p_1, \tag{33}$$

and the token-money supplies that support these price levels are

$$m_i = k\,p_i\,c_i, \qquad i = 1, 2, \tag{34}$$

where c_i is given by (9).

C.2 Net trade flows

Country i's net imports of the consumption good are

$$c_i - \breve{c}_i = (1-\alpha)\,\breve{C}\left(\frac{\breve{g}_i}{\breve{G}} - \frac{\breve{c}_i}{\breve{C}}\right). \tag{35}$$

Country i is a net importer of the consumption good (and net exporter of gold) if and only if its gold endowment share exceeds its goods endowment share: $\breve{g}_i/\breve{G} > \breve{c}_i/\breve{C}$.

C.3 Peg neutrality

Proposition C.1. *The real allocation* (c_1, c_2, g_1, g_2) *in* (9)–(10) *and the world price* ρ_R *in* (8) *depend only on world endowments* $(\check{c}_1, \check{c}_2, \check{g}_1, \check{g}_2)$ *and* α*. They are independent of* e_1*,* $\bar{\epsilon}$*, and the token-money supplies* m_1*,* m_2*. The peg* $\bar{\epsilon}$ *affects only nominal variables: it pins down* $p_2/p_1 = \bar{\epsilon}$ *and hence the dollar value of country 2's token supply* $m_2 = k\bar{\epsilon}\, e_1\, \rho_R\, c_2$*.*

When both countries are at the Ricardo maximum, all gold flows to private consumption and the monetary system—whether coin, token, or pegged token—is a veil over the real economy. Real allocations are determined entirely by preferences and endowments.

D Additional Regime Details

D.1 Symmetric gold-coin fractions: explicit consumption and gold formulas

When both countries impose the same gold-coin fraction λ, substituting $\rho_S(\lambda)$ from (19) into the consumption formula (12) gives

$$c_i = \frac{\alpha}{1+\alpha\lambda k}\,\check{c}_i + \frac{1-\alpha+\alpha\lambda k}{1+\alpha\lambda k}\,\frac{\check{C}}{\check{G}}\,\check{g}_i. \tag{36}$$

As λ rises, the weight on the gold endowment $\check{g}_i$ increases relative to the weight on the goods endowment $\check{c}_i$. A country with a high gold share $\check{g}_i/\check{G}$ relative to its goods share $\check{c}_i/\check{C}$ therefore suffers a smaller proportional consumption loss when the common gold-coin fraction rises; as $\lambda \to 1^-$, consumption shares are tilted most strongly toward gold-endowment shares.

The world fraction of gold absorbed as monetary reserves is

$$\frac{\sum_i(\bar{g}_i - g_i)}{\check{G}} = \frac{\alpha\lambda k}{1-\alpha+\alpha\lambda k}, \tag{37}$$

which is strictly increasing in λ. A larger gold-coin fraction diverts more of the world gold stock into monetary use, lowering $\rho_S(\lambda)$ and the price level.

D.2 Pound-reserve structural setup

The total token pound supply m_1 issued by country 1 is split between pounds circulating domestically ($m_{1,1}$) and pounds exported to country 2 as monetary reserves ($m_{1,2}$):

$$m_1 = m_{1,1} + m_{1,2}. \tag{38}$$

The total money supplies in the two countries are

$$\begin{aligned} M_1 &= e_1(\bar{g}_1 - g_1) + m_1, \\ M_2 &= \bar{\epsilon}\, m_{1,2} + m_2, \end{aligned} \tag{39}$$

where m_2 is country 2's own token dollar supply and $\bar{\epsilon}\, m_{1,2}$ is the dollar value of the pound reserves held by country 2. Only the domestically circulating pounds $m_{1,1}$ require gold backing; the exported reserves $m_{1,2}$ are pure token claims that carry no gold-backing obligation for country 1. The quantity-theory conditions (20) follow from substituting (39) and noting that the domestic portion $M_1 - m_{1,2}$ equals kp_1c_1; the definition $\mu_2 \equiv m_{1,2}/(kp_1c_2)$ then yields the consumption formulas (21)–(22).

D.3 Leaving the gold standard: comparison with gold-standard regimes

When country 2 leaves gold (Section 7), the world relative price equals $\rho_{\text{off}} = \rho_R$—the highest value among all regimes studied. The real allocation is identical to the Ricardo maximum: both countries consume the same quantities of goods and non-monetary gold as when no gold is absorbed into monetary reserves.

Relative to the gold-standard configurations in Section 3, the departure from gold changes only nominal variables. Country 2's price level $p_2 = m_2/(kc_2)$ is proportional to its money supply, and the dollar–pound exchange rate floats freely. Changes in m_2 pass through one-for-one into p_2 and the exchange rate with no effect on ρ, c_i, or g_i, provided country 1 maintains its gold parity. Country 1's price level $p_1 = e_1\rho_R$ is fully insulated from country 2's monetary expansion.

The dollar price of gold adjusts endogenously to maintain purchasing-power parity:

$$e_2 = \frac{p_2}{\rho_R} = \frac{m_2}{k\,\alpha\left(\check{c}_2 + \dfrac{\breve{g}_2}{\rho_R}\right)\rho_R}, \tag{40}$$

so the exchange rate $\epsilon = e_2/e_1 = p_2/p_1$ depreciates in proportion to m_2, implementing the "pure and simple" quantity theory of exchange-rate determination of Sargent (2019, Section 4).

The clean separation between nominal and real variables breaks down if an expansion of m_2 triggers a run on country 1's gold reserves—a possibility that lies outside the present static model but is of obvious relevance to the interwar episodes studied in Eichengreen (1992).