Two Computations to Fund Social Security

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July 23, 1996

Note: For helpful criticism, we thank Kenneth Judd, Jean Tirole, and participants at seminars at the Federal Reserve Bank of Minneapolis, Federal Reserve Bank of St. Louis, Department of Finance and Business Economics at USC, Department of Economics at the University of California at Santa Barbara, Department of Economics at Arizona State University, and the Institute for Policy Reform in Washington D.C. An earlier version of this paper was titled "Transition between Social Security Systems." Sargent's research on this paper was financed by a grant from the National Science Foundation to the National Bureau of Economic Research.

ABSTRACT

We use a general equilibrium model to study the impact of fully funding social security on the distribution of consumption across cohorts and over time. In an initial stationary equilibrium with an unfunded social security system, the capital-output ratio, debt-output ratio, and rate of return to capital are 3.2, 0.6 and 6.8%, respectively. In our first experiment, we suddenly terminate social security payments but compensate 'entitled' generations by a massive one-time increase in government debt. Eventually the aggregate physical capital stock rises by 40%, the return on capital falls to 4.4% and the labor income tax rate falls from 33.9% to 14%. We estimate the size of the 'entitlement debt' to be 2.7 times real GDP, which is paid off by levying a 38% labor income tax rate during the first 40 years of the transition. In our second experiment, we leave social security benefits untouched but force the government temporarily to increase the tax on labor income so as gradually to accumulate private physical capital, from the proceeds of which it eventually finances social security payments. This particular 'government run' funding scheme delivers larger efficiency gains (in both the exogenous and endogenous price cases) than privatization, an outcome stemming from the scheme's public provision of insurance both against life span risk and labor income volatility.

1. Introduction

This paper evaluates two schemes for cushioning a transition from an unfunded to a more fully funded social security system within an economy in which a sudden and *uncompensated* installation of full funding would on average benefit young and unborn cohorts, but harm older cohorts.[1] We use a model in which uninsurable uncertainties about lifetimes and labor incomes create forces for 'capital overaccumulation,' which have been overcorrected by an initial unfunded social security system.[2] We study transition policies designed to redistribute enough of the permanent gains from future to current generations to induce a majority of each cohort to assent to the transition. [3]

We employ the risk-sensitive, linear-quadratic framework of Hansen and Sargent (1995), which permits us to extend the transition calculations pioneered by Auerbach and Kotlikoff (1987) to environments in which social security is partly a device for insuring risky incomes and life-times. Our model contains a theory of the distribution of consumption within and across cohorts, makes contact with the Deaton–Paxson (1994) observations, and lets us study the effects of a transition on the distribution of welfare across and within cohorts.[4]

Our economy consists of overlapping generations of 65-period lived consumers who face life span uncertainty, and whose incomes and preferences are subject to shocks. Individuals work until a mandatory retirement age of 45 and pay labor and capital income taxes. After retirement, they receive social security benefits in the benchmark economy. To finance its exogenous purchases, the government taxes labor and capital income, confiscates accidental bequests, and issues one-period risk-free debt which returns the same payoff as private physical capital. We start in an initial stationary equilibrium in which the debt/GDP ratio is 0.59, the social security replacement rate is 60%, the capital/GDP ratio is 3.2, the rate of return on capital is 6.8%, and the equilibrium labor income tax rate is 33.9%.

We consider two alternative schemes to fund social security. In the first experiment, the government surprises everybody by terminating social security, and a transition begins to a new stationary equilibrium in which the labor income tax rate drops to 14.2% in the case of endogenous factor prices. By issuing a huge amount of government debt, the government 'buys out' all of those cohorts who were alive at the time of the policy change and were entitled to retirement benefits under the old system. This 'entitlement debt' is about 2.7

times the initial GDP. To retire this entitlement debt, the government raises the labor income tax rate to 38.0% for 40 years. The capital stock rises by 40% and the rate of return on asset holdings falls to 4.44% across the stationary equilibria. Our 'buy out' scheme on average protects the consumption of the originally 'entitled' people, which indicates how it can contribute to making the transition to fully funded social security politically feasible. In the second experiment, we instruct the government to acquire claims on physical capital so that social security benefits can be financed by the returns from publicly held private capital. This scheme for fully funding social security creates similar effects on existing cohorts, but the benefits to later generations seem to be larger, an outcome that is linked to the differing motives for 'precautionary savings' with which the two funding calculations confront households.

The paper is organized as follows. Section 2 describes the model economy. Section 3 states the households' information, preferences and opportunities. Section 4 summarizes the government's role in the economy. Section 5 describes how we compute equilibrium transition paths. Section 6 contains our numerical findings from two alternative privatization schemes. Section 7 concludes.

2. The Model Economy

2.1. Preliminaries

The economy consists of overlapping generations of finitely lived individuals who may live up to $T_0 + 1$ years, and an infinitely lived government. Individuals and the government can invest at a constant risk-free gross rate of return. During the first $T_1 + 1$ periods of life, a consumer receives an exogenous labor income which he divides between consumption, taxes, and accumulation of assets. During the final $T_0 - T_1$ periods of life, a consumer receives social security retirement benefits and also consumes his accumulated assets. The government taxes income from capital and labor, issues debt, purchases goods, and pays retirement benefits. For any variable z, we use a subscript t to denote age, an argument s in parentheses to denote calendar time, and a superscript $s-t$ to denote date of birth. Thus, $z^{s-t}(s) \equiv z_t(s) \equiv z_t^{s-t}(s)$. The output of our analysis will be a mapping from parameters summarizing government tax and benefit policies to cross-section probability distributions over a time-and-age dependent vector $x_t(s)$ of state variables describing the situation of people across cohorts over time.

2.2. Demographics

At date s, a cohort of measure $N_0(s)$ consumers is born who live during periods $s, s+1, \ldots, s+T_0$. As a given cohort ages, its members face random survival, according to a life-table of age-to-age survival probabilities $\{\alpha_t\}_{t=0}^{T_0}$, where α_t is the probability of surviving from age t to $t+1$, conditional on having survived to t. Let $N_t(s)$ be the number of age t people alive at time s and n be the (constant) gross rate of population growth. It follows that $N_t(s) = \lambda_t N_0(s-t)$, where $\lambda_t = \Pi_{j=0}^{t-1} \alpha_j$ for $t = 0, \ldots T_0$, and $\lambda_0 = 1$. The law of motion for births is given by $N_0(s) = n^s N_0(0)$. The population fraction of cohort t at each time s is given by

$$f_t = \frac{\lambda_t n^{-t}}{\sum_{\tau=0}^{T_0} \lambda_\tau n^{-\tau}}. \tag{1}$$

Note that the age distribution $\{f_t\}$ is assumed to be independent of calendar time s.[5]

The total population alive at time s is

$$N(s) = N_0(0)n^s \sum_{t=0}^{T_0} n^{-t}\lambda_t. \tag{2}$$

2.3. Distributions of people and aggregate quantities

Individuals face different sequences of random labor income shocks, in addition to life span risk, which they cannot fully insure.[6] As a result, individuals within each cohort self-insure by accumulating two risk-free assets – government bonds and claims on physical capital – that bear the same rates of return, and combine these with social security retirement benefits to provide for old-age consumption. We let ϵ_0^t denote the history of a vector of random shocks $\epsilon_\tau, \tau = 0, \dots, t$, which an individual has received from the time of his birth to age t. For technical reasons described below, we shall assume that the shocks are Gaussian.

The state vector $x_t(s) = x_t(s; \epsilon_0^t, x_0)$ measures the stock of assets, and any variables that a consumer of age t at time s uses to forecast his future preferences or opportunities. We shall specify the structure of our model so that it delivers the consumption plan of a consumer as a time- and age-dependent *linear* function of his state vector

$$c_t(s; \epsilon_0^t, x_0) = \eta_{ct}(s) x_t(s; \epsilon_0^t, x_0), \tag{3}$$

where the state vector follows the linear law of motion

$$x_{t+1}(s+1; \epsilon_0^{t+1}, x_0) = A_t(s) x_t(s; \epsilon_0^t, x_0) + C_t(s)\epsilon_{t+1}, \tag{4}$$

where ϵ_{t+1} is a martingale difference sequence adapted to $J_t = (\epsilon_0^t, x_0)$, with $E(\epsilon_{t+1}|J_t) = 0$, $E(\epsilon_{t+1}\epsilon_{t+1}'|J_t) = I$.

Our *economic* model will impose restrictions on the vectors $\eta_{ct}(s)$ and the matrices $A_t(s)$. We assume that consumers have rational expectations, making a consumer's choices, and therefore both $\eta_{ct}(s)$ and $A_t(s)$, depend on the sequence of prices and government fiscal policies over the remainder of the consumer's potential life-span, namely, $s, s+1, \dots, s+T_0-t$.

Given our assumptions, we can analytically compute the probability distributions for the state vector and for linear functions of the state vector. Let $\mu_t(s) = Ex_t(s), \Sigma_t(s) = E(x_t(s) - \mu_t(s))(x_t(s) - \mu_t(s))'$. Given first and second moments for the state vector of the newborns $(\mu_0(s), \Sigma_0(s))$, the moments of the state vector for consumers follow the law of motion

$$\mu_{t+1}(s+1) = A_t(s)\mu_t(s) \tag{5}$$

$$\Sigma_{t+1}(s+1) = A_t(s)\Sigma_t(s)A_t(s)' + C_t(s)C_t(s)'. \tag{6}$$

Aggregate quantities of interest such as aggregate per capita consumption, aggregate per capita physical capital, etc., can be easily computed by obtaining weighted averages of features of the distributions of quantities across individuals.[7] For example, per capita aggregate consumption is

$$c(s)/N(s) = \sum_{t=0}^{T_0} \mu_{ct}(s) f_t. \tag{7}$$

Also, the distribution of consumption with mean $\mu_{ct}(s)$ and variance $\Sigma_{ct}(s)$ is given by

$$\mu_{ct}(s) = \eta_{ct}(s)\mu_t(s) \tag{8}$$

$$\Sigma_{ct}(s) = \eta_{ct}(s)\Sigma_t(s)\eta_{ct}(s)'. \tag{9}$$

Our theory of the distribution of consumption across time and cohorts is summarized by the formulas above, and is designed to accommodate "fanning out" and "fanning in" of consumption distributions observed by Deaton and Paxson (1994), given an appropriate initial distribution.[8]

2.4. The resource constraint

The economy-wide physical resource constraint is given by

$$g(s)N(s) + \sum_{t=0}^{T_0} c_t(s)N_t^{s-t} + K(s) = R(s-1)K(s-1) + w(s)\sum_{t=0}^{T_1} \varepsilon_t N_t^{s-t} + N_0(s)k_{-1}(s) \ , \tag{10}$$

where $R(s-1) = 1 + r(s-1) - \delta$ is the rate of return on asset holding, $K(s-1) = \sum_{t=0}^{T_0} k_t(s-1)N_t^{s-1-t}$ physical capital in the economy, and $N_0(s)k_{-1}(s)$ is the amount of physical capital, if any, that newborns bring into the economy. In equation (10), $g(s)$ is per capita government purchases of goods at time s, ε_t is an exogenous efficiency endowment of age-t people, δ is the rate of depreciation of capital, $r(s-1)$ is the gross-of-depreciation rate of return on physical capital from time $s-1$ to time s, and $w(s)$ is the base wage rate at time s. We set the 'efficiency' sequence $\{\varepsilon_t\}$ roughly to match an average age-wage rate profile.

2.5. Two assumptions about factor prices

We perform our computations under two alternative assumptions about the rate of return $r(s-1)$ on assets and the wage rate $w(s)$ at date s. First, we make a small, open economy assumption by specifying that $r(s-1) \equiv r$ and $w(s) \equiv w$ are independent of the aggregate capital stock and constant over time. Second, we allow $r(s-1)$ and $w(s)$ to be determined from the marginal productivity conditions for a constant returns to scale Cobb–Douglas aggregate production function:

$$r(s-1) = r\left(\frac{K(s-1)}{\widetilde{N}(s)}\right) = \tilde{\alpha} A \left(\frac{K(s-1)}{\widetilde{N}(s)}\right)^{\tilde{\alpha}-1}, \tag{11}$$

$$w(s) = w\left(\frac{K(s-1)}{\widetilde{N}(s)}\right) = (1-\tilde{\alpha}) A \left(\frac{K(s-1)}{\widetilde{N}(s)}\right)^{\tilde{\alpha}}, \tag{12}$$

where $\widetilde{N}(s) = \sum_{t=0}^{T_1} \varepsilon_t N_t^{s-t}$ is aggregate labor input in efficiency units, and where $\tilde{\alpha} \in (0,1)$ is the income share of capital.

These two alternative assumptions imply different lengths for the transition between stationary equilibria. The endogeneity of factor prices allows us to incorporate the general equilibrium effects of fully funding social security at some computational cost.

2.6. Government policy and its transition

The government's time-varying deterministic fiscal policy parameter vector sequence is given by $\{g(s), \tau_a(s), \tau_\ell(s), S_t(s), b(s)\}$, where $g(s)$ is per-capita government expenditures net of social security payments; $\tau_a(s)$ and $\tau_\ell(s)$ are flat tax rates on asset income and labor income, respectively, levied on consumers at time s; $S_t(s)$ is the social security benefit for an age-t individual at time s, and $b(s)$ is per-capita one-period interest bearing government debt issued at time s.

We use a four-tuple of dates $0 \leq s_1 < s_2 < s_3$ to describe the government's policy transition. Before date $s=0$, the economy is assumed to be in an initial stationary equilibrium determined by a stationary demographic structure, fiscal policy and unfunded social security system with replacement rate $\theta(s) = \theta_1$. At date $s=0$ the government announces that from date $s_1 \geq 0$ until date s_2, fiscal parameters and social security system will change. From date s_2 on, government policy parameters will remain constant through date $s = s_3$ on forever. However, the economy continues to adjust to the change in government policy from date $s = s_2$ to $s = s_3$ as cohorts who started their lives between s_1 and s_2 (policy transition years) work their ways through the system and, in the case of endogenous factor prices, as the price vector converges to its final stationary equilibrium value. The system will converge to a final stationary equilibrium at time s_3.

2.7. The length of the transition period: s_3

Under the small country assumption, we set $s_3 = T_0 + s_2$, since it takes exactly a full lifetime for the transition generations to work their way through the system. Under the closed economy assumption, the transition period s_3 in principle is infinity since time variation in the wage rate and the return on asset holdings induces time variation in households' dynamic programming problems which lasts long after government policy parameters cease to vary. The capital-labor ratio and factor prices continue to move over time forever. We follow Auerbach and Kotlikoff (1987) and assume that we are making only a small approximation error by forcing convergence to a final stationary equilibrium after a long but finite transition period.

2.8. The transition

We start the transition from a distribution of assets and information $(\mu_t(s), \Sigma_t(s))$ that would have been appropriate had the initial policy settings continued forever. At the date of the announcement $s = 0$, anyone born after date $s_1 - T_0$ – anyone with a chance of being alive at date s_1 or after – will re-compute his optimal consumption-saving plan for the remainder of his life in light of the altered tax rates and social security benefit rates that he faces. Cohorts born between $s_1 - T_0$ and s_2 will face a time-varying sequence of tax rates and benefit rates. Those born at s_2 and after face constant benefit and tax rates. However, in the case of endogenous factor prices, these cohorts still face changing factor prices and therefore their dynamic programs will differ relative to cohorts who are born into the final stationary equilibrium.

Figure 1.1, a version of a standard text book image of the demography of an overlapping generations model, will occur as the 'floor' of some three dimensional graphs in which we shall record consumption distributions during a transition. Figure 1.1 records time along the horizontal axis, and birth date along the vertical axis. A cohort survives from its birthdate to T_0 plus its birthdate, depicted as a *horizontal* line. At a given date, a vertical line traces out those cohorts alive. There is a distribution across individuals, say of consumption or asset holdings, above each (birthdate, time) pair in this figure; in a third dimension above this floor, we shall plot means and standard deviations of some of these distributions (for example, see figures 1.3a and 1.4a). Time s budget constraints involve sums of distributions at a given point in time, i.e., along vertical lines in Figure 1.1.

Under an assumption that $r(s-1) \equiv r$ and $w(s) \equiv w$ we have to compute fewer dynamic programs than with time-varying r and w, because there are fewer constellations of rates of return, wage rates, taxes rates, and benefit rates encountered by different cohorts. In particular, it is enough for us to compute $\mu^{s-t}(s), \Sigma^{s-t}(s), \eta^{s-t}(s)$ for $s-t = -T_0 + s_1 - 1, \ldots, s_2$ and $t = 0, \ldots, T_0$. We can use the decision rules for those born at $s = -T_0 + s_1 - 1$ to determine the initial stationary distribution of consumers' state vector. We use the relevant 'tails' of the decision rules to predict the behavior of anyone who lives through any part of the transition.

Figure 1.2 shows four mean consumption profiles for an experiment in which social security is eliminated at time $s = 0$ but those 'entitled' are given a 'compensation', an experiment to be described in detail below. We set $T_0 = 65, s_1 = 0, s_2 = 40$. The figure plots $\{\mu_{ct}(s-t)\}_{t=0}^{T_0}$ for four distinct birthdays $s = -66, s = -45, s = 1, s = 39$, corresponding to one cohort unaffected because it dies out before the transition at $s = 0$, a cohort that is 20 years old when the transition begins, one that is born the period after the transition begins, and one that is born after the transition has been completed.

Figure 1.3a shows shows mean consumption profiles for *all* of the cohorts affected by the transition. Figure 1.2 just records four 'slices' of Figure 1.3a. Notice how a version of Figure 1.1 is laid out on the 'floor' of Figure 1.3a. Aggregate per capita consumption at a point in time s is a weighted-by-f_t average of $\mu_{ct}(s)$, which corresponds to summing elements corresponding to those associated with a fixed date in figure 1.3 (a line parallel to the date born axis in figure 1.3a, and a line on the diagonal of the floor of Figure 1.3b). Figure 1.4a shows how the (cross sectional) standard deviations of consumption are affected by the transition.

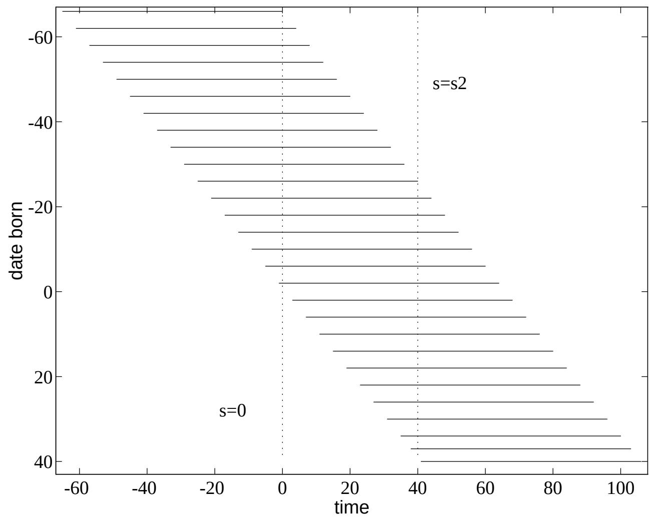

Figure 1.1. The 'floor' of some 3-D diagrams showing consumption or asset profiles of generations alive during a transition for the 'open economy case.' Time is on the x axis, date of birth is on the y axis. A vertical line picks off cohorts of increasing age at a particular date; a horizontal line describes the life-time profile of a given cohort. Everyone born before $-T_0 + s_1$ faces time-invariant tax and benefit rates associated with an initial stationary equilibrium. Everyone born after s_2 faces time-invariant tax and benefit rates associated with a terminal stationary equilibrium. People born between $-T_0 + s_1$ and 0 are 'surprised' at date $s = 0$ by being informed of the transition from s_1 to s_2. They resolve their problems with the announced tax and benefit rates, starting from the asset levels determined by their old saving programs. To analyze the transition with fixed factor prices requires that we compute distinct optimal consumption-saving plans for cohorts born from $-T_0 + s_1$ to s_2, because each of these cohorts faces a different sequence of life-time tax and benefit rates. The system settles down to a new stationary equilibrium only after all of the people who have lived through the transition have died off. With endogenous factor prices, the transition takes longer.

Figures 1.3b and 1.4b represent the same information in figures 1.3a and 1.4a, respectively, in a more compact but harder to read way, by plotting a mean or standard deviation against *date born* on the x-axis and *age* on the y-axis. With these axis, a given calendar date is a diagonal along the 'floor' of the graph, and economy wide averages must be calculated by computing weighted sums along this diagonal.

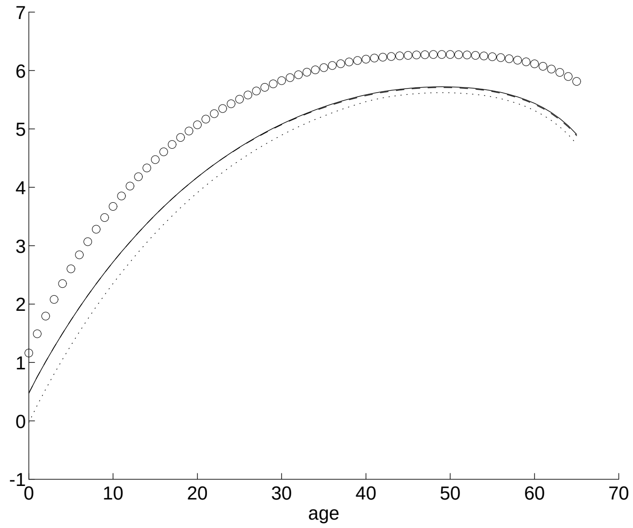

Figure 1.2. Mean consumption profiles μ_{ct} for four cohorts born before, during, and after a transition. Profiles for cohorts born at $s = -66$, in the initial stationary equilibrium, (solid line), one period after the transition starts (the dotted line), born at $s = -45$ – who are 20 when the transition starts– (dashed line), and in the new stationary state (circles). The transition is a 'compensated' removal of social security, which explains why the two profiles for the fully compensated generations lie on top of one another.

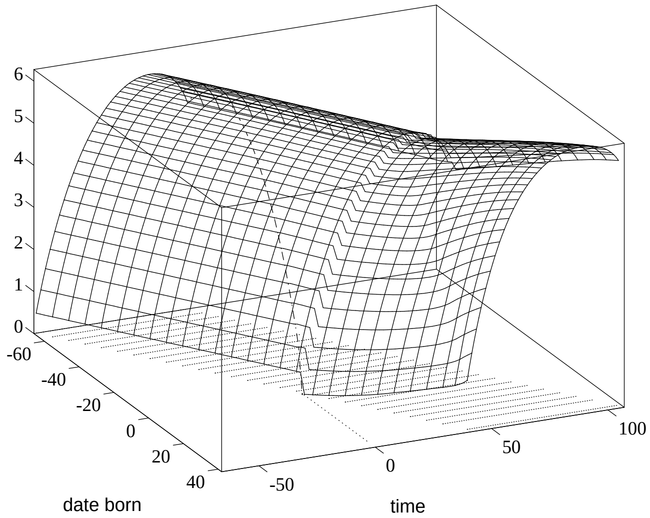

Figure 1.3a. Mean consumption profiles for all cohorts during a transition for experiment 1, a bond compensated, tax-financed removal of social security; 'textbook' figure on the floor.

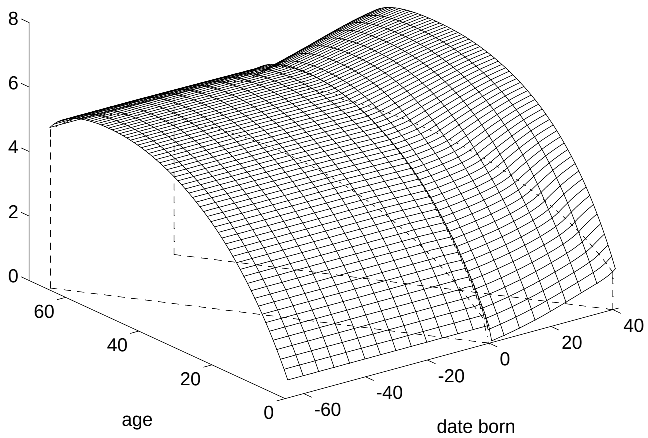

Figure 1.3b. Mean consumption profiles for all cohorts during the transition for experiment 1. Experiment 1 is a bond-compensated, tax-financed removal of social security.

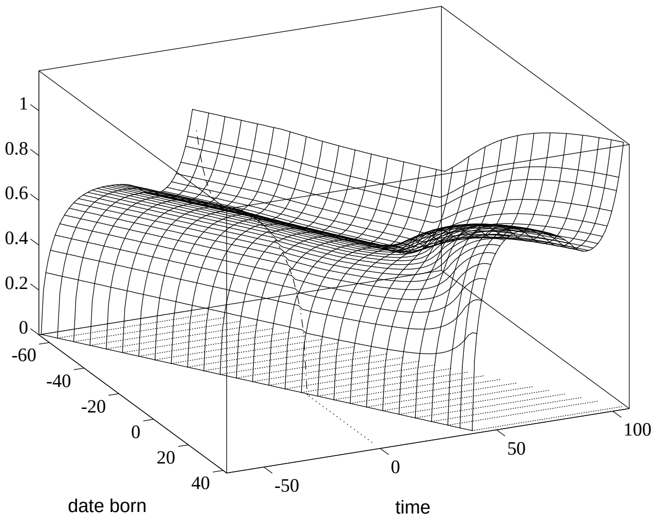

Figure 1.4a. Standard deviations of consumption for all cohorts during the transition for experiment 1; 'textbook' figure on the floor.

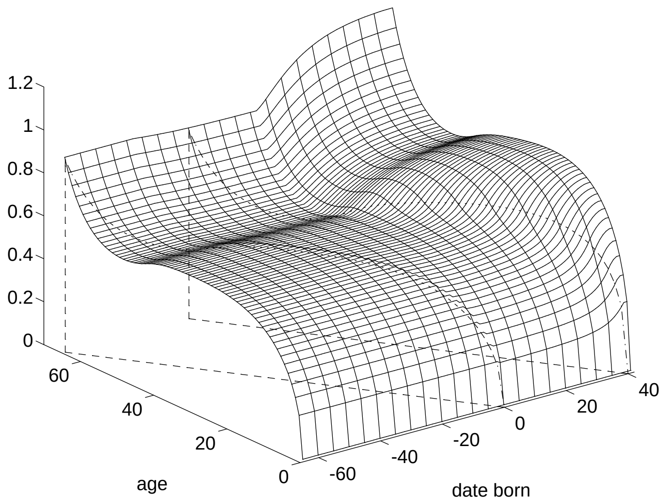

Figure 1.4b. Standard deviations of consumption for all cohorts during the transition for experiment 1.

From the decision rules for the transition cohorts, we shall build up the first two moments of the distributions of consumption and asset-holding profiles for each cohort, and use them to define and compute an equilibrium. In the next section, we say more about the economic model that generates the distributions of aggregate quantities across cohorts and over time.

3. Households

To exploit linear optimal control theory and obtain linear decision rules, we adopt the linear Gaussian quadratic formulation of Hansen and Sargent (1995). This incorporates a specification of preferences that modifies standard quadratic preferences with a single parameter designed to induce 'risk-sensitivity' to permit a deviation from state-separable preferences over uncertainty. When this parameter is less than zero, it inspires an additional source of 'precautionary savings' in our model stemming from preferences, over and above the precautionary savings emanating from the incompleteness of markets. See Hansen, Sargent, and Tallarini (1994) for analysis of the precautionary savings induced by these preferences in an infinite horizon setting.

3.1. Information

The consumer's preference shock γ_t and the idiosyncratic portion of his income d_t are driven by a vector information process z_t which follows

$$z_{t+1} = A_{22} z_t + C_2 \,\epsilon_{t+1} \quad , \tag{13}$$

where z_t is an $(n_z \times 1)$ vector and ϵ_{t+1} is a Gaussian vector martingale difference sequence that satisfies $E_t \epsilon_{t+1} \epsilon'_{t+1} = I$, where E_t is the conditional expectations operator. We assume that $\gamma_t = U_\gamma z_t$ and $d_t = U_{dt} z_t$, where U_γ, U_{dt} are nonstochastic selector matrices, and (γ_t, d_t) are preference and endowment shock processes to be described more below. To capture the ending of labor income shocks after retirement, we allow U_{dt} to be an age-varying sequence of selector matrices.

3.2. Opportunities

Following the tradition of 'permanent income' models of consumption, we restrict the set of assets with which individuals can smooth consumption over time and states of nature. At age $t \leq T_1$, the consumer receives labor income of $w(s)\varepsilon_t + d_t$, where d_t is an endowment shock process, T_1 is the exogenous retirement age, $w(s)$ is a real wage at time s, and $\{\varepsilon_t\}_{t=0}^{T_1}$ is a sequence of exogenous mean labor incomes which we design to mimic a typical age-earnings profile. In the computations below, we make the endowment shock d_t a first-order autoregression

$$d_t = \rho_d d_{t-1} + \epsilon_t,$$

where we shall make ϵ_t a homoskedastic Gaussian white noise with variance σ_d^2.

We allow for a single market in claims on physical capital and one-period government bonds that yield a common risk-free rate of return $R(s-1)-1$. We let $a_{t-1}(s-1)$ denote the consumer's holdings of assets at the beginning of age t at time s. Since physical capital and government bonds yield the same risk-free return and since they are taxed equally, the consumer's optimal portfolio weights are indeterminate. The household's budget constraint at age t and time s is

$$c_t(s) + a_t(s) = R(s-1)a_{t-1}(s-1) + w(s)\varepsilon_t + S_t(s) - \Upsilon_t(s) + d_t, \tag{14}$$

where $S_t(s)$ denotes social security payments, where $\varepsilon_t = 0$ for $t > T_1$, and tax payments are given by[9]

$$\Upsilon_t(s) = \tau_0(s) + \tau_\ell(s)(w(s)\varepsilon_t + d_t) + \tau_a(s)(R(s-1)-1)a_{t-1}(s-1). \tag{15}$$

We assume that

$$S_t(s) = \begin{cases} 0, & \text{if } t \leq T_1 \text{ (while working);} \\ S(s), & \text{if } t > T_1 \text{ (when retired).} \end{cases} \tag{16}$$

In (15), $\tau_0(s)$ denotes lump sum taxes, which are ordinarily set to zero, the sole exception being the one-time lump-sum transfers that we shall use to 'buy-out' cohorts who are alive and entitled to social security under the initial tax and benefit settings.

3.3. Inheritance

Total assets per capita owned by people who die between time s and $s+1$ are[10]

$$\mathcal{D}(s) = \sum_{t=0}^{T_0} (1-\alpha_t) f_t \eta_{at}(s) \mu_t(s). \tag{17}$$

3.4. Preferences

Following Hansen and Sargent (1995), we adopt a preference specification which delivers linear decision rules but at the same time allows for a form of 'risk-sensitivity'. Preferences over stochastic processes for consumption are defined recursively via

$$U_t = [-(\pi c_t - \gamma_t)^2/2 + \beta_t \mathcal{R}_t(U_{t+1})], \tag{18}$$

where

$$\mathcal{R}_t(U_{t+1}) = (2/\sigma)\log\; E[\exp(\sigma U_{t+1}/2) \mid J_t], \tag{19}$$

and where J_t is the information available to the consumer at age t. Here π is a preference parameter, and β_t is a survival-corrected discount factor given by

$$\beta_t = \widetilde{\beta} \cdot \alpha_t, \quad 0 \le t < T_0, \tag{20}$$

$$= \beta_T \in (0,1), \quad t = T_0, \tag{21}$$

where $\widetilde{\beta} \ge 0$.

The parameter σ is the 'risk-sensitivity' parameter of Hansen and Sargent (1995). When $\sigma = 0$, $\mathcal{R}_t(U_{t+1}) \equiv E_t U_{t+1}$, so in this case preferences are quadratic. When $\sigma < 0$ ($\sigma > 0$), the consumer prefers early (late) resolution of uncertainty, and decision rules depend partly on noise statistics of transition laws.[11,12]

This preference specification leaves us with linear decision rules, but the standard version of 'certainty equivalence' fails to hold. In our application, the decision rules are influenced by an interaction of σ and the innovation variance of households' random labor income process. These parameters influence both the average level of savings over the life cycle, and the Deaton–Paxson 'fanning out' patterns. When $\sigma < 0$, this preference specification induces a 'precautionary motive' for saving in a sense distinct from that stemming from the incomplete insurance motive. [13]

Specialization

We shall assume that $\gamma_t = \bar{\gamma}$, so that preference shocks are absent. The parameters $\pi, \bar{\gamma}$, and σ govern the household's taste for consumption smoothing. Elementary calculations show that, *ceteris paribus*, the household's desired consumption path is *flatter*: (a) the *lower* is $\bar{\gamma}/\pi$, (b) the *larger* is its non-capital 'permanent income' as determined by the labor income process, and (c) the *larger* is $-\sigma$.

3.5. Optimal rules

The solution of the consumer's problem is a law of motion for the state and a set of age- and time-dependent decision rules

$$y_t(s; \epsilon_0^t, x_0) = \eta_{yt}(s) x_t(s; \epsilon_0^t, x_0), \tag{22}$$

where $y_t(s)$ is the consumer's optimum choice for some age t, time s variable y. We apply standard formulas to obtain the decision rules.[14]

4. The Government

Each period, the government purchases goods in the amount $g(s)$ per capita and pays $S(s)$ in per capita social security retirement benefits. It finances these expenditures in part by taxing capital and labor income at flat rates and putting a 100% tax on accidental bequests. Any gross-of-interest deficit is covered by issues of one-period risk free bonds.

4.1. The Government's Budget Constraint

Private asset holdings $a_t(s)$ of an age-t individual at time s are divided between government bonds and private capital: $a_t(s) = b_t(s) + k_t(s)$, where $b_t(s)$ is the time s holding of government debt by an age-t individual. Consumers are indifferent between physical capital and government bonds since the two assets have a common return and tax treatment. The government's budget constraint at s is:

$$
\begin{aligned}
&g(s)N(s) + \sum_{t=T_1+1}^{T_0} S_t(s)N_t^{s-t} + R(s-1)\sum_{t=1}^{T_0} b_{t-1}(s-1)N_t^{s-t} \\
&= \sum_{t=0}^{T_0} N_t^{s-t}\Big\{\tau_a(s)[R(s-1)-1]a_{t-1}(s-1) + \tau_\ell(s)w(s)\varepsilon_t\Big\} + \tau_0 N(s) \\
&\quad + \sum_{t=0}^{T_0} b_t(s)N_t^{s-t} + R(s-1)\sum_{t=0}^{T_0}(1-\alpha_t)k_t(s-1)N_t^{s-t-1}.
\end{aligned}
\tag{23}
$$

The second term on the left hand side of (23) is the total social security payment in period s. The last term on the right hand side of (23) is the bequest tax collected by the government.[15]

For an individual, asset holdings at the end of time $s-1$ equal those as at the beginning of time s (if the agent survives), but deaths make things different for the aggregate of assets held by a cohort. It is useful to write (23) using population shares of cohorts and end-of-period asset holdings:

$$
\begin{aligned}
&g(s) + \sum_{t=T_1+1}^{T_0} S_t(s)f_t + \frac{R(s-1)}{n}\sum_{t=1}^{T_0} b_t(s-1)f_t \\
&= \tau_a(s)[R(s-1)-1]\sum_{t=0}^{T_0} a_{t-1}(s-1)f_t + \tau_\ell(s)\sum_{t=0}^{T_1} w(s)\varepsilon_t f_t \\
&\quad + \tau_0 + \sum_{t=0}^{T_0} b_t(s)f_t + \frac{R(s-1)}{n}\sum_{t=0}^{T_0}(1-\alpha_t)a_t(s-1)f_t.
\end{aligned}
\tag{24}
$$

4.2. Stationary budget constraint

In a stationary equilibrium, all variables are independent of calendar time s, giving us the following version of the government budget:

$$\begin{aligned} & g + \sum_{t=T_1+1}^{T_0} S_t f_t + \left[\frac{R}{n} - 1\right]\bar{b} \\ & = \tau_a(R-1)\sum_{t=0}^{T_0} a_{t-1} f_t + \tau_\ell \sum_{t=0}^{T_1} w\varepsilon_t f_t + \frac{R}{n}\sum_{t=0}^{T_0}(1-\alpha_t)a_t f_t \ , \end{aligned} \tag{25}$$

where $\bar{b}$ represents the outstanding stock of per capita government bonds in steady-state.

5. Equilibrium

5.1. Definition of Equilibrium

An *allocation* is a stochastic process for $\{c_t(s), a_t(s)\}_{s=0}^{s3}$ for $t = 0, \ldots, T_0$, and a sequence $\{K(s)\}_{s=0}^{s3}$. A *government policy* is a sequence $\{b(s), g(s), \tau_\ell(s), S(s), \tau_a(s)\}_{s=0}^{s3}$. A *price system* is a sequence $\{w(s), r(s-1)\}_{s=0}^{s3}$. An *equilibrium* is an allocation, a price system, and a government policy such that:

i) given the price sequence and the government policy, the allocation solves the optimum problem for each household;

ii) the allocation and government policy satisfy the government budget constraint at each date s.

5.2. Stationary Equilibria

To compute a stationary equilibrium, we guess at a set of government expenditures, social security benefit levels, government debt levels and tax rates, solve the household's optimum problem, and check whether it implies that the stationary government budget constraint is satisfied. If not, we alter a subset of the government's policy parameters in a direction designed to bring the budget more closely into balance. We iterate to convergence.

5.3. Transition Dynamics

5.3.1. Case 1: Small, open economy

Tax rates and social security benefit rates are constant before $s = s_1$ and after $s = s_2$, so households born *before* $s = s_1 - T_0$ or *after* $s = s_2$, respectively, face identical parameters in their dynamic programming problems. However, people born in the interval $s \in (s_1 - T_0, s_2)$ face cohort-specific tax and benefit rates over at least parts of their life-times. Thus, to compute equilibria for transitions requires that we formulate the dynamic programming problems for households born in periods $-T_0 + s_1 + 1, -T_0 + s_1 + 2, \ldots, s_2$.[16] For given policy settings, these dynamic programming problems imply decision rules for variables $v^{s-t}(s)$ for $t = 0, \ldots, T_0$, $s - t = s_1 - T_0 + 1, \ldots, s_2$. We can compile the means of each variable in a matrix, with t in columns and $s - t$ in rows. Equilibrium conditions restrict sums across diagonals of this matrix.[17]

During a transition, tax and benefit rates are constant for $s < s_1$ and $s > s_2$. However, aggregate capital and government debt vary over the entire time interval $s \in [0, s_3 = T_0 + s_2]$; they can vary in the 'announcement period' $s \in [0, s_1]$ because people alive at $s = 0$ who might survive at s_1 and beyond alter their savings behavior before s_1; they vary in the 'post-policy-change period' $s \in [s_2, s_3]$ because at those dates there are still people alive who spent the early parts of their lives during the policy transition period $s \in [s_1, s_2 - 1]$.

We compute equilibrium transitions by specifying the ratios of government debt to GDP for the initial and final stationary equilibria. We use a secant algorithm and iterate on $\tau_{\ell 1}$ and $\tau_{\ell 2}$ to obtain the initial and final stationary equilibria, respectively. To find an equilibrium transition path for the economy, we first specify a transition path for social security benefit levels and tax rates. For each set of fixed tax rates, from periods $s = 0$ to $s = s_3$, we determine government debt per capita recursively from equation (24). We use a secant algorithm and iterate on the (scalar) labor income tax rate for the policy transition period $[s_1, s_2 - 1]$ to make debt to GDP ratio at s_3 equal to the prescribed level.

5.3.2. Case 2: Closed economy

Computing an equilibrium transition under endogenous factor prices involves three separate modules.

1. *Initial stationary equilibrium:* As an 'inner loop,' for fixed r and w, we use a secant algorithm and iterate on $\tau_{\ell 1}$ so that the stationary government budget equation is

satisfied with the pre-specified government debt/GNP ratio. This inner loop resides within an 'outer loop' in which we use a secant algorithm to search for the equilibrium r that satisfies $r = r(K/\tilde{N})$.[18]

2. *Final stationary equilibrium:* The two nested loops to compute an initial stationary equilibrium are embedded within a bigger outer loop to search for a debt/GDP ratio equal a prescribed level.
3. *Transition period:* As an 'inner loop,' for a fixed sequence $\{r(s-1)\}_{s=0}^{s_3}$, using a secant algorithm, we search for a tax rate on labor τ_ℓ over the time interval $[s_1, s_2]$ which sets government debt equal to the value prescribed in the terminal steady state, starting from conditions determined by the initial steady state. In the 'outer loop,' we use a relaxation algorithm on the sequence $\{r(s-1)\}_{s=0}^{s_3}$ during the transition to impose $r(s-1) = r\left(\frac{K(s-1)}{\tilde{N}(s)}\right)$.

6. Two Computations

We report the results of two computations that describe how the economy moves to a fully funded social security system. The first experiment eliminates social security but 'buys out' affected generations by a one-time increase in government debt. The second experiment forces the government to acquire claims on private physical capital to finance its social security benefit payments. These two computations share a common set of parameters and an initial stationary equilibrium which are described below.

6.1. Parameter Values

We calibrated the parameters of preferences, technology and information to make various simulated ratios of aggregate variables resemble corresponding ones for the U.S. economy. These values are shown in Tables 1 and 2.

Table 1. Preference Parameters

$\{\alpha_t\}_{t=0}^{T_0}$	π	σ	$\bar{\gamma}$	$\widetilde{\beta}$	T_0	T_1	n
Faber (1982)	1.0	–0.05	7.0	0.986	65	45	1.012

Note that we set $\sigma = -0.05$, so these computations assume risk-sensitive preferences. Our choice of the risk-sensitivity parameter σ, the innovation variance of the endowment shock process d_t, and the other preference parameters $\pi, \bar{\gamma}, \tilde{\beta}$ and the initial endowment k_{-1} (in Table 2), was guided by our desire to have our economic model match two objects in the U.S. economy: (a) a realistic mean age-consumption profile such as that depicted in Figure 1.5, and (b) an empirically plausible capital-output ratio. The variance of the innovation of the endowment process is also set with an eye to generate realistic 'fanning out' of the within-cohort distribution of consumption as a cohort ages (see Deaton and Paxson (1994)).

We calibrated $\bar{\gamma}, \pi$ and the initial stock of capital to make the household consumption profile resemble an empirical profile from the Consumer Expenditure Survey data. Figure 1.5 displays the mean consumption profile generated by our calibrated economy and an actual consumption profile obtained using the Consumer Expenditure Survey data.[19]

Table 2. Technology Parameters

k_{-1}	4.0
σ_d	0.85
ρ_d	0.8
δ	0.06
$\{\varepsilon_t\}_{t=0}^{T_1}$	Hansen (1991)
Exogenous Factor Prices	
w	5.0147
$r-\delta$	0.0675
Endogenous Factor Prices	
A	2.2625
$\tilde{\alpha}$	0.40

For both computations, we specify levels of per capita government purchases and debt so that their ratios to output are 21% and 59%, respectively. Our choice of preference parameters, especially the risk-sensitivity parameter σ, is guided mainly by our wish to generate an empirically plausible capital-output ratio and age-consumption profile.[20] The capital income tax rate τ_a is set at 30% for both computations. The equilibrium labor income tax rate is 33.9% in the initial stationary equilibrium.[21] For both computations, we set $(s_1, s_2) = (0, 40)$, so that a transition in tax and benefit rates is *announced* at $s = 0$ to occur between $s = 0$ and $s = 40$. Under the small, open economy assumption, the economy's transition is completed exactly at date $s_3 = s_2 + T_0 = 105$. When factor prices are allowed to depend on aggregate physical capital, we require the economy's transition to take $s_3 = s_2 + 2T_0 = 170$ years.[22]

6.2. A buyout

The first experiment suddenly terminates *all* social security retirement benefits, but simultaneously compensates all of the cohorts who had been expecting to receive retirement benefits. The compensation is tailored cohort-by-cohort to leave unaltered the value of the value function of the mean person in that cohort. At time $s = s_1 = 0$, people of age $t = 1, \ldots, T_0$ are surprised by losing the discounted present value of their social security retirement benefits. A person of age t loses benefits valued at

$$\mathrm{ben}_t = S \sum_{j=\max(T_1-t,0)}^{T_0-t} \prod_{i=0}^{j} \widetilde{R}^{-1}(i),$$

where $\widetilde{R}(s) \equiv R(s)[1 - \tau_a(s)] + \tau_a(s)$ is the after-tax rate of return on assets. To insulate the expected utility of the mean person in each cohort, we make a lump sum transfer to each person of this present value *plus* a sum (necessitated by risk aversion and the change in after-tax earnings other than social security). Thus, this experiment has the following features:

1. The government compensates each person of age $t \geq 1$ in the amount comp_t, where $\mathrm{comp}_t = \mathrm{ben}_t$ + another term to account for risk aversion, and pays for it by issuing government bonds. These payments add to the government budget a one-time per-capita expenditure of $\sum f_t \mathrm{comp}_t$ at time $s = s_1$.
2. The government sets a tax rate on labor of $\tau_{\ell 1}$ from $s = s_1$ to $s = s_2 - 1$, after which it stabilizes the tax rate to $\tau_{\ell 2}$.
3. We choose $\tau_{\ell 1}, \tau_{\ell 2}$ to set the debt/GDP ratio in the terminal stationary equilibrium equal to its value in the initial stationary equilibrium.

6.2.1. Small, open economy

We calibrated the initial stationary equilibrium under fixed factor prices so that the income share of capital matched that under endogenous factor prices. This procedure guarantees that the initial stationary equilibria under fixed and endogenous factor prices are identical. Table 3 summarizes our numerical findings under the small, open economy assumption.

Table 3. Experiment 1 with fixed factor prices

time	τ_ℓ	θ	$r(s-1)-\delta$	Capital/GDP	Debt/GDP
$s < 0$	0.3385	0.60	0.0675	3.1615	0.5899
$s \in [s_1, s_2)$	0.3597	0.00	0.0675	(see Figures 1.10 and 1.6)	
$s \in [s_2, s_3)$	0.0831	0.00	0.0675	(see Figures 1.10 and 1.6)	
$s \geq s_3$	0.0831	0.00	0.0675	4.1567	0.5899

We start in an equilibrium in which the debt/GDP ratio is 0.59, the social security replacement rate θ is 60%, and the equilibrium labor income tax rate is 33.9%. At $s = 0$, a transition begins to a new stationary equilibrium in which the government supplies zero social security benefits and the labor income tax rate eventually drops to 8.3%. The government 'buys out' all who were entitled to retirement benefits under the old system by issuing a huge amount of government debt at $s = 0$. Figure 1.6 shows the time path of government debt. The debt issued in the buyout equals about 2.67 times GDP in the initial stationary equilibrium.[23] The government raises the labor income tax rate to 36% during the following 40 year policy-transition period to retire most of this 'entitlement debt'. It brings the debt/GDP ratio back to its initial stationary equilibrium value. Figures 1.7 and 1.8 show the time path of aggregate consumption and aggregate asset holdings. Aggregate physical capital rises monotonically to its new stationary value, while aggregate consumption dips before it begins a long rise to its higher eventual stationary value. The capital stock rises 66% across the stationary equilibria, with three-quarters of this rise taking place in only 46 years. Figure 1.3 shows that the buyout protects the consumption of the originally 'entitled' people on average. At the same time, however, fully funding social security leads to a small increase in the *dispersion* of consumption as indicated by Figure 1.4. This increase occurs because those of the previously entitled who are still working are exposed to labor income risk during the remaining periods of their working lives, which they must self-insure by saving.

6.2.2. Closed economy

Table 4 shows our numerical results under the closed economy assumption. Starting from the same initial stationary equilibrium, the economy eventually converges to a final stationary equilibrium where social security is privatized. Under the closed economy assumption, however, the equilibrium rate of return on capital falls from 6.75% to 4.44% in response to the increase in aggregate physical capital. As a consequence, the increase in the aggregate capital stock across stationary equilibria is limited to 40% compared to the 66% increase with fixed factor prices. This implies that, with an unchanged tax on capital income and with our requirement that debt-to-output ratio return to its initial level, the government must use a higher labor income tax rate in the final steady-state in order to satisfy its steady-state budget constraint. The labor income tax rate in the final steady-state is 14.2% compared with 8.3% under the small economy assumption.

Table 4. Experiment 1 with endogenous factor prices

time	τ_ℓ	θ	$r(s-1)-\delta$	Capital/GDP	Debt/GDP
$s < 0$	0.3385	0.60	0.0675	3.1593	0.5903
$s \in [s_1, s_2)$	0.3797	0.00	(see Figures 1.9 and 1.11)		
$s \in [s_2, s_3)$	0.1419	0.00	(see Figures 1.9 and 1.11)		
$s \geq s_3$	0.1419	0.00	0.0444	3.8651	0.5900

However, the 'entitlement debt' is not affected at all: the government has to issue an additional debt in the amount 2.67 times (initial) GDP to buy out those cohorts that are alive at $s = 0$. Figures 1.9–1.12 show the time path of aggregate asset holdings, consumption, government debt, and return on asset holdings, respectively.

6.3. Fully funding via fiscal policies

The second experiment leaves social security benefits unaltered but changes how they are financed. At time $s = 0$, the government switches to a fiscal policy designed to lower its debt per capita from 4.03 in the initial stationary equilibrium to -10.16 in the terminal stationary equilibrium, which implies a reduction in the debt/GDP ratio from .59 to -1.1785. We chose -10.16 for the debt in the terminal steady-state in order to induce our agents to produce outcomes in terms of their behavior similar to that under no social security and a government debt of 4.03. The idea behind this massive debt-retirement is to allow the government to build up a stock of private physical capital sufficiently large to make the 'income from publicly held private capital' be enough to pay for social security retirement benefits. To finance this 'debt reduction-private asset purchase' policy, the tax rate on labor income between $s_1 = 0$ and s_2 is raised just enough to hit the target level. The tax rate on labor income in the terminal stationary equilibrium is set to satisfy the government's final steady-state budget equation.

6.3.1. Small, open economy

Figures 2.1–2.5 and Table 5 display the results of our second experiment under exogenous factor prices.

Table 5. Experiment 2 with fixed factor prices

time	τ_ℓ	θ	$r(s-1)-\delta$	Capital/GDP	Debt/GDP
$s < 0$	0.3382	0.60	0.0675	3.1615	0.5899
$s \in [s_1, s_2)$	0.3723	0.60	0.0675	(see Figures 2.3 and 2.2)	
$s \in [s_2, s_3)$	0.1390	0.60	0.0675	(see Figures 2.3 and 2.2)	
$s \geq s_3$	0.1390	0.60	0.0675	4.1492	-1.1785

Qualitatively, the time path of the tax on labor income resembles that for experiment 1. The higher labor tax rate during the policy-transition period (37.2%) is used to acquire claims on private physical capital for the purpose of 'funding' the social security system.

Also, the debt reduction in this experiment induces quantitatively similar effects on the consumption profile to those found in experiment 1. In both computations, the transition eventually supports higher mean consumption profiles by inducing society to accumulate more physical capital. 65.46% more across the stationary equilibria.

6.3.2. Closed economy

Table 6 shows the results of our experiment two under the closed economy assumption.

Table 6. Experiment 2 with endogenous factor prices

time	τ_ℓ	θ	$r(s-1)-\delta$	Capital/GDP	Debt/GDP
$s < 0$	0.3385	0.60	0.0675	3.1593	0.5903
$s \in [s_1, s_2)$	0.3897	0.60	(see Figures 2.6 and 2.8)		
$s \in [s_2, s_3)$	0.2497	0.60	(see Figures 2.6 and 2.8)		
$s \geq s_3$	0.2497	0.60	0.0471	3.7638	−1.9250

Starting from the same initial stationary equilibrium, the economy eventually converges to a final stationary equilibrium where social security is funded by the income flow from the public acquisition of private physical capital. Under the closed economy assumption, the equilibrium rate of return on capital falls from 6.75% to 4.71% in response to the increase in aggregate physical capital. The increase in the aggregate capital stock across stationary equilibria is limited to 33.8% compared to the 65.5% increase under fixed factor prices. This is in part due to the (general equilibrium) reduction in the rate of return on asset holdings. A quantitatively more important reason for a smaller rise in the aggregate physical capital stock appears to be the relatively high labor income tax rate in the final steady-state. This tax rate falls from 33.85% to only 24.97%. This is related to the revenue raising requirements that the government is facing given an unchanged tax rate on capital income.

Figures 2.6–2.9 display the time paths of key aggregate variables in the second experiment.

6.4. Interpretations

Table 7. Overall Efficiency Gains

Buy Out		Government Scheme	
Exogenous Prices	Endogenous Prices	Exogenous Prices	Endogenous Prices
1.28%	2.00%	2.12%	2.84%

Table 7 presents the overall efficiency gains from our two computations. Each entry in Table 7 represents an annuity computed as follows. First we compute the present value of additional wealth required to make the individuals along a transition path indifferent to remaining under the initial unfunded system.[24] Second, we compute the annuity implied by this stock measure by multiplying it with the interest rate in the initial stationary equilibrium. Finally, we express this as a fraction of real GDP at the initial stationary equilibrium.[25]

Table 7 indicates that 1) efficiency gains are larger with endogenous prices for a given experiment; and 2) efficiency gains are larger under the second experiment for given factor prices. The former is due largely to the implicit insurance higher labor income tax rates provide against earnings risk with endogenous factor prices. The labor income tax rate that balances the government budget under endogenous factor prices is higher than that under exogenous prices because the increase in aggregate capital is lower and the interest rate falls, requiring the government to keep labor income tax rates higher to make up for the smaller increase in capital income revenues compared to that under exogenous factor prices. The latter result is mostly due to the government scheme providing insurance against two risks: higher labor income taxes providing insurance against earnings risk and unchanged social security providing insurance against life span risk. However, the efficacy of the government run scheme depends very much on the return the government makes on its stock of private physical capital.

7. Concluding Remarks

We have extended the machinery of Auerbach and Kotlikoff to handle a model economy with a larger *state* than theirs, namely, the moments $(\mu_t(s), \Sigma_t(s))$ describing the distribution of wealth and information of age t people at date s. Our model is designed rapidly to compute how these moments would respond to various policy reforms. We used our model to study how alternative transitions between fiscal policies and social retirement arrangements affect the distribution of income and wealth within and across cohorts of people. In particular, we demonstrate the effects of two alternative schemes fully to fund social security. The first is a 'compensated buy out' which brings about a 40% rise in the capital stock and lowers the labor income tax rate to 14% in the final stationary equilibrium. The second is a funding scheme in which the government acquires claims on private physical capital to finance social security benefit payments. Although this scheme yields similar benefits to the immediate generations, later generations get larger benefits than under the first scheme.

It is feasible and worthwhile to extend this work in several directions. First, we are currently in the process of undertaking a systematic analysis of how the 'risk-sensitive' feature of our preference specification and the resulting precautionary saving affects our numerical findings. Second, because we use linear state-space methods, it is easy to extend our calculations to more complicated investment technologies, in particular to amend our specification of the production technology to incorporate human capital while retaining the mathematical structure of the model. Incorporating human capital promises to be an important innovation, because the basic overlapping generations structure leads us to expect important interactions between social retirement arrangements and the accumulation of *all* types of capital. Third, the basic calculations can be extended to incorporate settings in which the mortality tables $\{\alpha_t\}$ and the birth rate of population both vary over time. In development contexts, this extension is very important because the interactions between mortality tables, birth rates, and the productivity of capital are keys to how a social security arrangement impinges on capital accumulation. This will enable us to capture the influence of an 'aging population' on the desirability of privatizing social security. Finally, we are planning to examine other aspects of fiscal reform including the elimination of capital income taxation and the introduction of consumption taxation.

Appendix: Dynamic programming

We formulate the household's lifetime optimum problem as a discounted risk-sensitive linear control problem. Let $x_t = \begin{pmatrix} a_{t-1} \\ z_t \end{pmatrix}$. The linear quadratic Gaussian preference specification implies that the utility indexes take the form

$$U_t = x_t' \, P_t \, x_t + \xi_t. \tag{A1}$$

At age t, the household faces the problem

$$\max_{c_t, a_t} [-(\pi c_t - \gamma_t)^2/2 + \beta_t \, \mathcal{R}_t \, (U_{t+1})] \tag{A2}$$

subject to equations (13)-(16), (20) and (21).

This problem can be represented as a time varying linear quadratic exponential Gaussian control problem:

$$U_t = \max_{u_t, x_{t+1}} \left\{ u_t' Q_t \, u_t + x_t' R_t x_t + \frac{2\beta_t}{\sigma} \log E_t(\exp(\sigma U_{t+1}/2)) \right\} \tag{A3}$$

subject to

$$x_{t+1} = A_t x_t + B_t u_t + C_t \, w_{t+1}. \tag{A4}$$

This sequence of problems is to be solved by working backwards from $t = T_0$, beginning with terminal value function

$$U_{T_0+1} = x'_{T_0+1} P_{T_0+1} x_{T_0+1} + \xi_{T_0+1}. \tag{A5}$$

Terminal conditions

If we set $P_{T_0+1} = 0$, the solution of the consumer's problem is trivial and is to set c_t so that $c_t \equiv \gamma_t$ each period, and to 'borrow' whatever is required to support this pattern of consumption. To rule out this solution, we penalize the act of dying at $T_0 + 1$ with asset holdings that are large in absolute value, the intent being to penalize plans that imply holding *negative* assets at time $T_0 + 1$. We accomplish this by setting the $(1,1)$ element of P_{T_0+1}, i.e. the element corresponding to asset holdings, equal to a negative number.

Riccati equations

Associated with the household's dynamic programming problem are the operators

$$
\begin{aligned}
T_t(P) &= P + \sigma P C_t (I - \sigma C_t' P C_t)^{-1} C_t' P, \\
D_t(W) &= R_t + A_t'(\beta W - \beta^2 W B_t (Q_t + \beta B_t' W B_t)^{-1} B_t' W) A_t, \qquad (A7) \\
\mathcal{S}_t(k, P) &= \beta_t k - \left(\frac{\beta_t}{\sigma}\right) \log \det(I - \sigma C_t' P C_t).
\end{aligned}
$$

Hansen and Sargent (1995) show that the optimal value function is

$$
U_t = x_t' P_t x_t + \xi_t, \qquad (A7)
$$

where

$$
\begin{aligned}
P_t &= (D_t \circ T_t)\ P_{t+1}, \\
\xi_t &= \mathcal{S}_t(\xi_{t+1}, P_{t+1}).
\end{aligned} \qquad (A8)
$$

The optimal control is

$$
\begin{aligned}
u_t &= -F_t x_t, \\
F_t &= \beta[Q_t + \beta B_t' T_t(P_{t+1}) B_t]^{-1} B_t' T_t(P_{t+1}) A_t.
\end{aligned} \qquad (A9)
$$

Figures

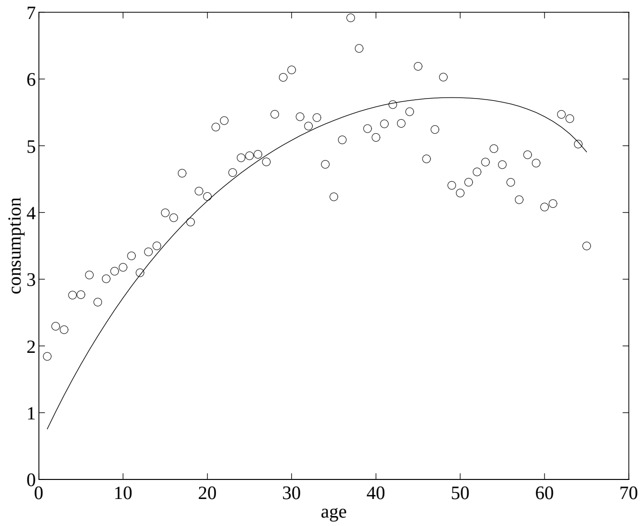

Figure 1.5. Simulated and actual consumption profiles.

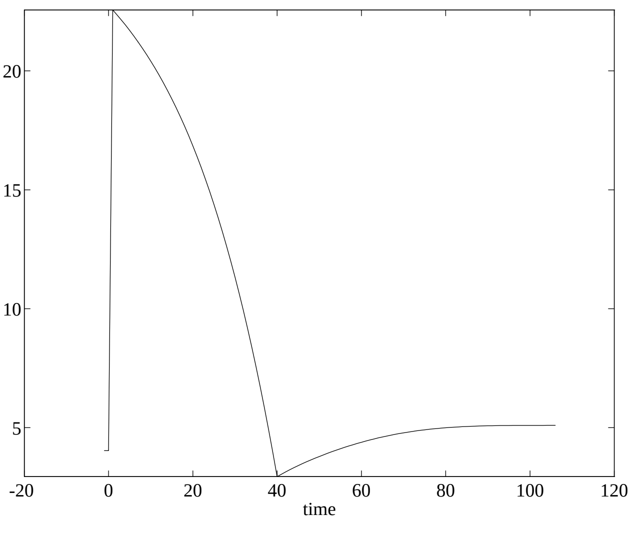

Figure 1.6. Government debt path.

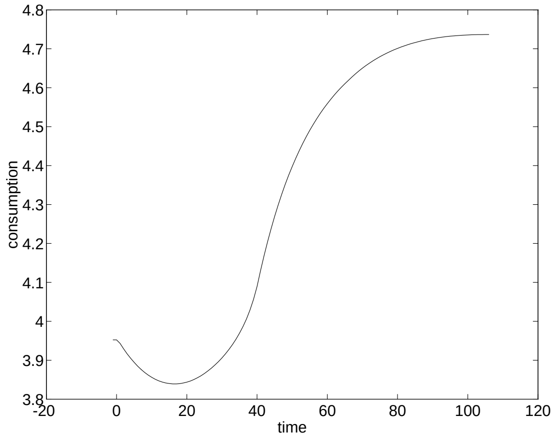

Figure 1.7. Aggregate consumption over time, experiment 1 (exogenegous factor prices).

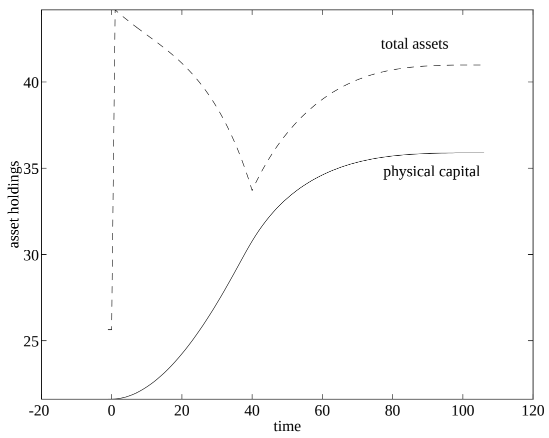

Figure 1.8. Aggregate asset holdings over time, experiment 1 (exogenous factor prices).

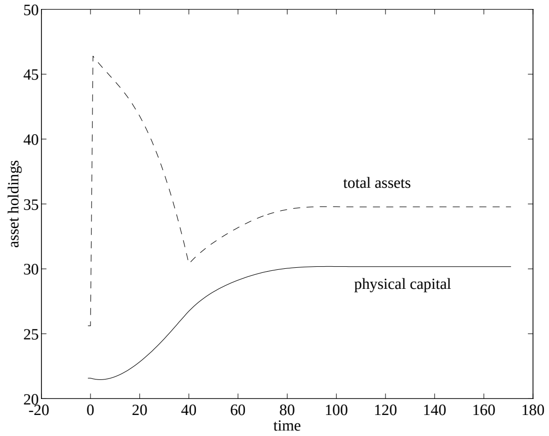

Figure 1.9. Aggregate asset holdings over time, experiment 1 (endogenous factor prices).

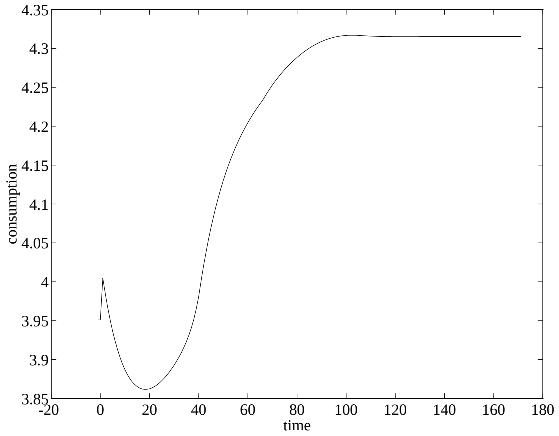

Figure 1.10. Aggregate consumption path over time, experiment 1 (endogenous factor prices).

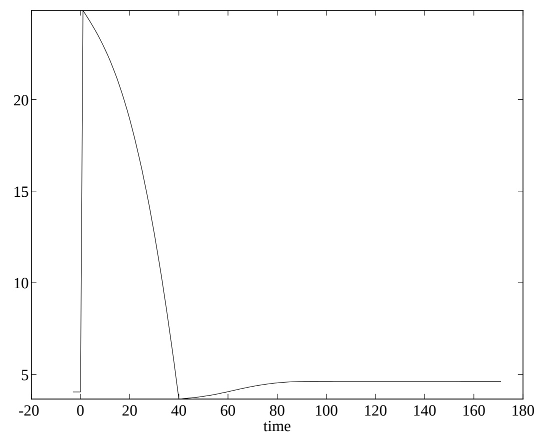

Figure 1.11. Government debt over time, experiment 1 (endogenous factor prices).

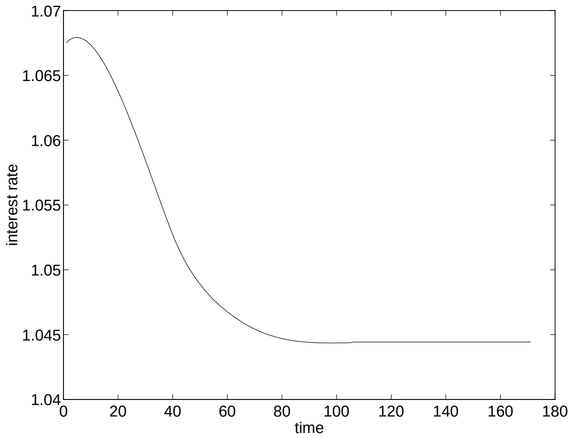

Figure 1.12. Rate of return on asset holdings over time, experiment 1 (endogenous factor prices).

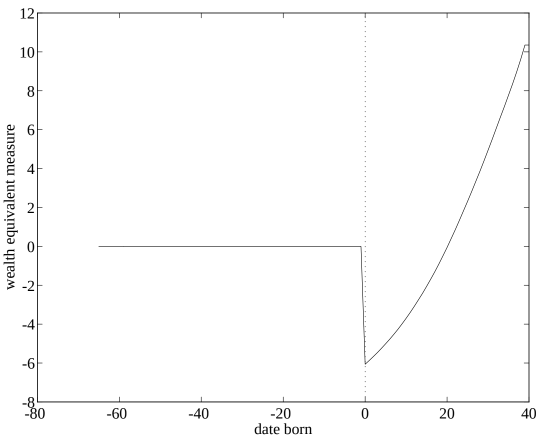

Figure 1.13. Generational distribution of welfare gains from social security reform. Birthdate is on the horizonal axis, wealth-equivalent measure of welfare gain (loss if negative) is on the vertical axis; experiment 1, exogenous prices.

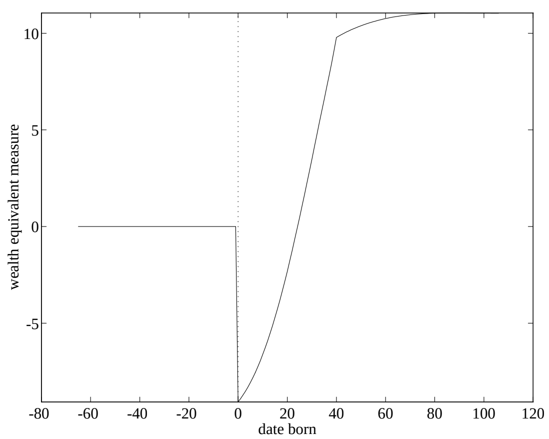

Figure 1.14. Generational distribution of welfare gains from social security reform. Birthdate is on the horizonal axis, wealth-equivalent measure of welfare gain (loss if negative) is on the vertical axis; experiment 1, endogenous prices.

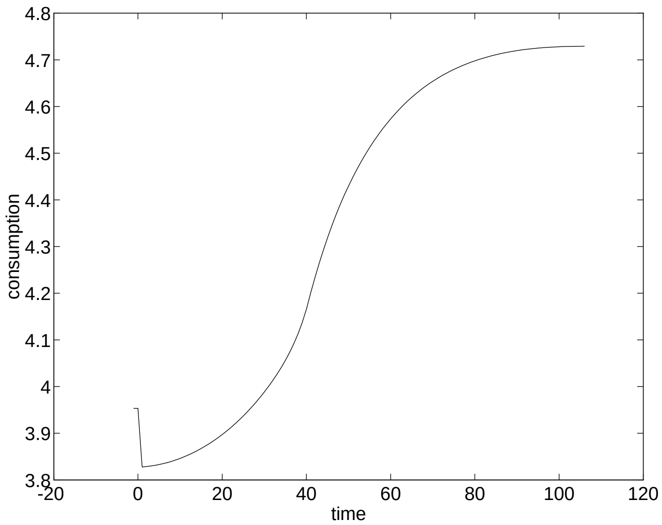

Figure 2.1. Aggregate consumption path.

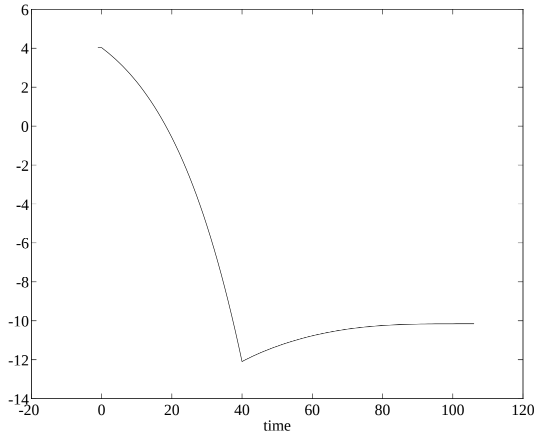

Figure 2.2. Government debt over time, experiment 2 (exogenous factor prices).

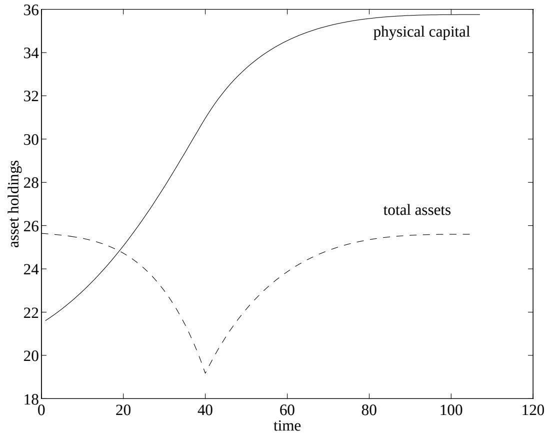

Figure 2.3. Aggregate asset holdings over time, experiment 2 (exogenous factor prices).

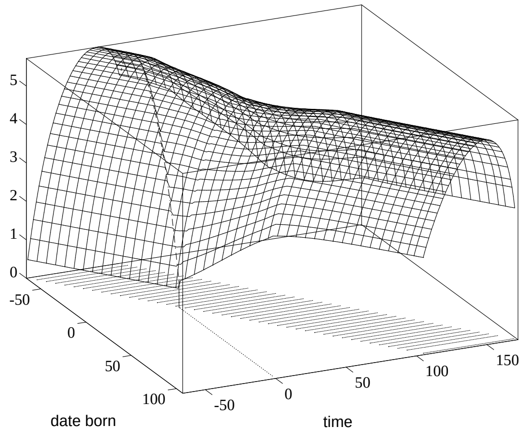

Figure 2.4a. Evolution of consumption profiles, experiment 2 (endogenous factor prices).

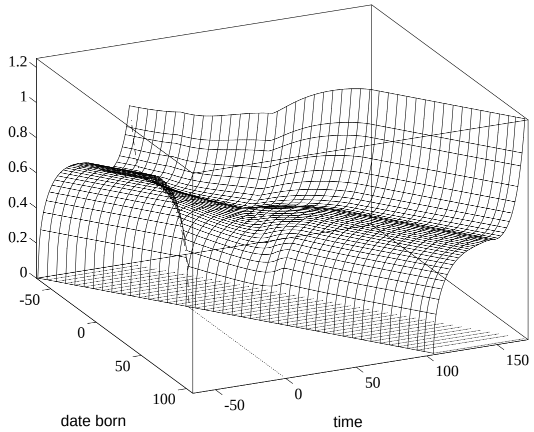

Figure 2.5a. Evolution of consumption spreads, experiment 2 (endogenous factor prices).

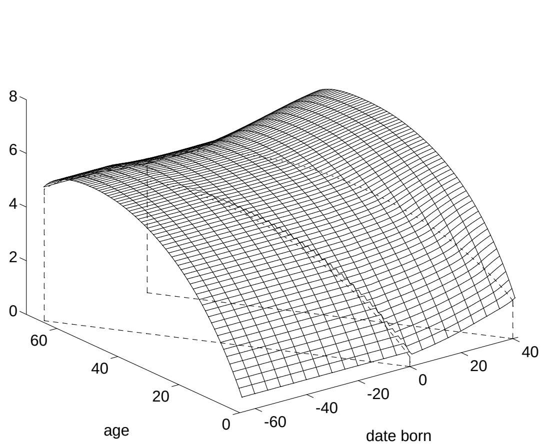

Figure 2.4b. Evolution of consumption profiles, experiment 2 (endogenous factor prices).

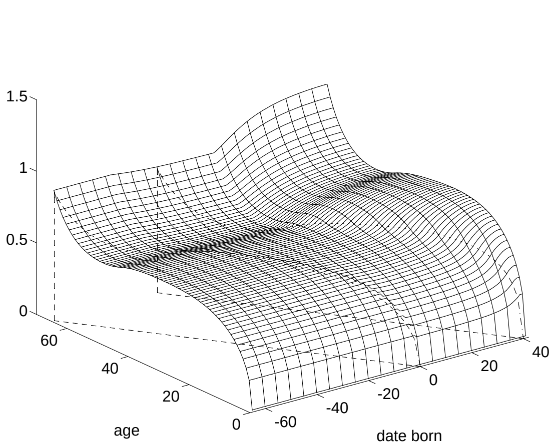

Figure 2.5b. Evolution of consumption spreads, experiment 2 (endogenous factor prices).

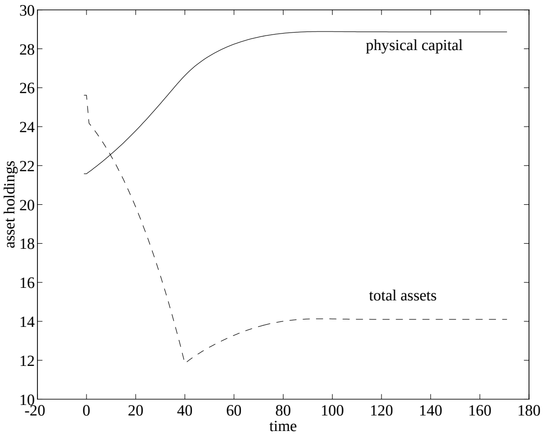

Figure 2.6. Aggregate asset holdings over time, experiment 2 (endogenous factor prices).

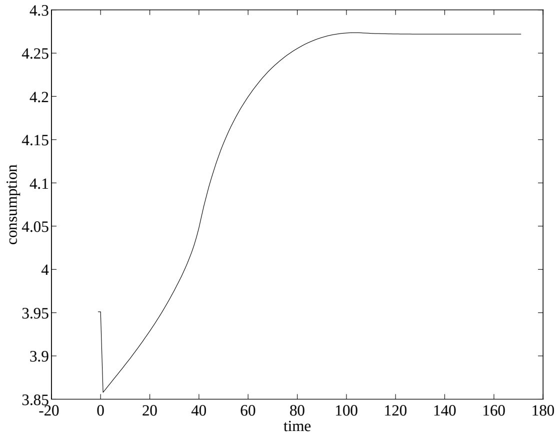

Figure 2.7. Aggregate consumption over time, experiment 2 (endogenous factor prices).

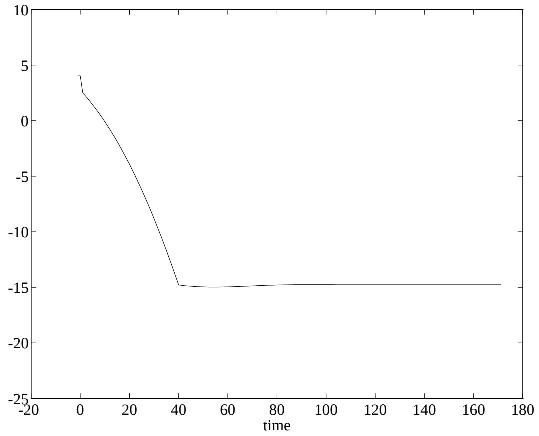

Figure 2.8. Government debt over time, experiment 2 (endogenous factor prices).

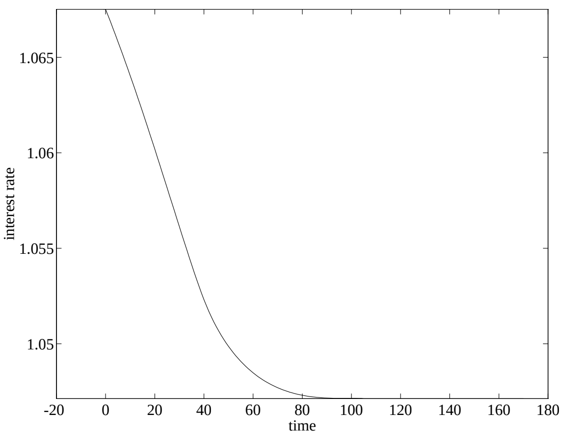

Figure 2.9. Rate of return on asset holdings over time, experiment 2 (endogenous factor prices).

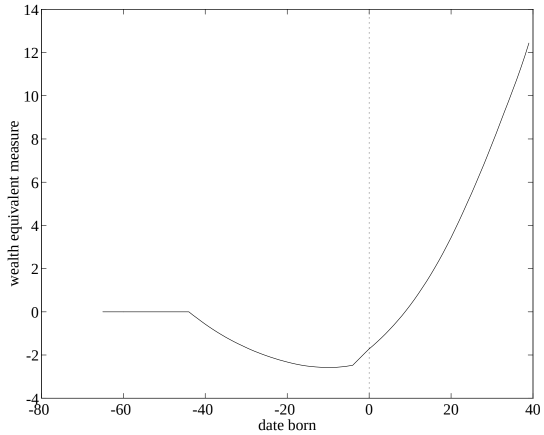

Figure 2.10. Generational distribution of welfare gains from social security reform. Birthdate is on the horizonal axis, wealth-equivalent measure of welfare gain (loss if negative) is on the vertical axis; experiment 2, exogenous prices.

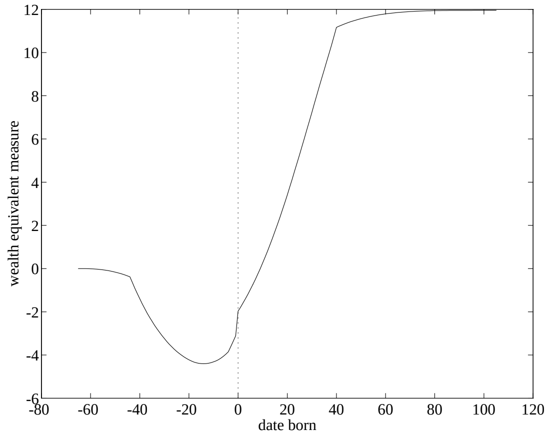

Figure 2.11. Generational distribution of welfare gains from social security reform. Birthdate is on the horizonal axis, wealth-equivalent measure of welfare gain (loss if negative) is on the vertical axis; experiment 2, endogenous prices.

Notes

1. The Chilean economy switched toward a fully funded social security system in 1981, and raised its national saving rate from 2.8% of GDP in 1980 to 14.3% in 1991. See Diamond and Valdés-Prieto (1994) for a detailed description of the Chilean social security reform.
2. Social security provides insurance against being born into a larger than expected cohort [Green (1988)], partial insurance against mortality risk in the absence of private annuity markets [İmrohoroğlu, İmrohoroğlu and Joines (1995) and especially Hubbard and Judd (1987)], or operates as a device to leave negative bequests in the presence of growing human capital [Abel (1988)]. These forces impinge on the dynamic efficiency [in the sense of Diamond (1965)] of the economy. Abel, Mankiw, Summers and Zeckhauser (1989) argue that the current U.S. economy is dynamically efficient.
3. Our alternative transition policies manipulate the "generational accounts" of different cohorts. See Kotlikoff (1992).
4. Among others, see Aiyagari (1993), Huggett (1994) and İmrohoroğlu and İmrohoroğlu (1995) on the impact of fiscal policies on wealth distribution.
5. We are currently working on a version of our model in which time-variation in cohort shares induces an economic transition to a new stationary equilibrium under a pay-as-you-go social security system. In such a setup we can address the impact of the 'aging of the population' and the feasibility and desirability of fully funding social security under demographic dynamics.
6. We formulate the model to permit preference shocks, though we don't 'turn them on' in the computations reported here.
7. Aggregate quantities are deterministic functions of time because all randomness averages out across a large number of individuals.
8. So far, we have restricted ourselves to single-consumer, Gaussian economies. Allowing for different types of consumers would help us obtain more leptokurtic distributions of income and wealth which are empirically more plausible.
9. Since capital income taxes are paid on the net return to capital, we are assuming that depreciation is deducted from taxable income.

10. We assume that people who die hold capital and government bonds in proportions equal to the economy-wide average.

11. When $\sigma \neq 0$, the assumption that ϵ_{t+1} is a Gaussian process is needed for us to invoke the risk-sensitive recursive control formulation of Hansen and Sargent (1995).

12. In the computations reported in this paper, we have set the preference shock process $\gamma_t \equiv 0$, but it would be cheap for us to activate preference shocks within our model.

13. It can be interpreted in terms of a 'pessimistic' type of behavior. See Hansen, Sargent, and Tallarini (1994).

14. In using dynamic programming to obtain decision rules, we are following Rust (1992), İmrohoroğlu, İmrohoroğlu and Joines (1995) and Rios-Rull (1994a,b). See the appendix for details about the backward recursions used in the dynamic program.

15. The bequest tax at calendar time s is collected at the end of $s-1$, and carried over into the beginning of s. The government collects all of the assets of people who die at the end of $s-1$. The government debt component of the bequest tax is cancelled; while the physical capital stock collected is carried over into s, and earns a gross return of $R(s-1)$ during the process. The bequest tax term in (23) is what the government has at its disposal in period s.

16. Allowing for endogenous factor prices requires us to solve dynamic programming problems for households born in periods $-T_0+s_1+1,\ldots,s_3$.

17. See Figure 1.1 for a visualization of how aggregate consumption for date s is a weighted average of $\mu_c^{s-t}(s)$ along an appropriate diagonal in the $(s-t,t)$ plane.

18. Since the technology is Cobb-Douglas in the case of endogenous factor prices, the equilibrium wage rate w is easily computed from the equilibrium rate of return on capital.

19. The actual consumption profile in Figure 1.5 is obtained from the Consumption Expenditure Survey data from 1987. See İmrohoroğlu, İmrohoroğlu and Joines (1995) for a description of how this profile was estimated.

20. Kydland and Prescott (1994) argue that the wealth-output ratio for United States is about 3.2 and that the real interest rate is 6.9%.

21. A recent paper by Mendoza, Razin and Tesar (1995) uses national income accounts and government revenue statistics to construct time-series of tax rates for several industrialized countries. Our choice of 30% for the capital income tax rate is motivated in part by their estimates and in part by the current capital gains tax rate of 28%. Our equilibrium labor income tax rate is very close to the 31% figure that Lucas (1990) uses.
22. We experimented with longer transitions to see how the assumed length of the transition affects the numerical findings. Taking the transition to be 235 years ($s_3 = s_2 + 3T_0$) or more had almost no effect.
23. This is roughly 4.5 times the current government debt.
24. This is simply computing the present value of the wealth-equivalent welfare gains for all cohorts, some of which are negative as Figures 1.13–1.14 and 2.10–2.11 show.
25. Note that this overall efficiency gain measure subtracts the welfare losses to certain cohorts, typically those born during the early years of the transition to the final stationary equilibrium.

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