The Timing of Tax Collections and the

Structure of "Irrelevance" Theorems in a Cash-in-Advance Model:

Vintage Article

by

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Abstract: A standard timing protocol allows in a cash-in-advance model allows the government to elude the inflation tax. That matters. Altering the timing of tax collections to make the government hold cash overnight disables some classical propositions but enables others. The altered timing protocol loses a Ricardian proposition and also the proposition that open market operations, accompanied by tax adjustments needed to finance the change in interest on bonds due the public, are equivalent with pure units changes. The altered timing enables a Modigliani-Miller equivalence proposition that does not otherwise prevail.

KEY WORDS: Ricardian equivalence, quantity theory of money, Modigliani-Miller theorem.

Prologue

In 2008 and 2009, central banks have engaged in unusual open market operations, unusual in terms of the types and quality of collateral they are accepting. They evidently hope that the conditions underlying irrelevance propositions for open market operations do not prevail. But perhaps these are not pure open market operations but instead the harbingers of future fiscal adjustments that will cost tax payers.

This seems to be a good time to dust off classical results about the fiscal conditions for open market operations to be neutral, Modigliani-Miller theorems about equivalent fiscal and monetary policies, and other irrelevance theorems about government finance, like Ricardian equivalence theorems. Some of these theorems have been posed most sharply in the context of overlapping generations models with fiat money, but versions of them also prevail within models with an infinitely lived representative consumer who pays taxes, holds government securities, and faces a cash-in-advance constraint. In 1988, Bruce Smith and I wrote this paper to evaluate the robustness of some classic quantity theoretic predictions associated with a cash-in-advance to a plausible alteration in the timing of government transactions. Much of the monetary theory within a standard cash-in-advance model is about who gets stuck holding currency overnight. Important results about monetary and fiscal policy hinge on timing protocols that allow the government to carry no cash overnight, thereby evading the inflation tax. When Bruce and I alter the timing protocol to expose the government to the inflation tax, some neat classical propositions disappear, but there materializes another classical proposition that could not otherwise be obtained.

Introduction

Cash-in-advance models in the style of Lucas (1980, 1982) possess many attractive features, not the least of which is that, under appropriate assumptions about the timing of transactions, they can be used to express several "classical" propositions in monetary theory. Pure exchange versions of these models deliver a constant (and in fact, unit) velocity "quantity theory" equation and some classical "neutrality" propositions. For example, it is possible to show that a one-time exchange of money for bonds, via an open market operation, is equivalent in its effects to simply changing the units of currency. Cash-in-advance models also are associated with "Ricardian equivalence" theorems that assert the irrelevance of the timing of (lump-sum) taxation.

It is known that some of the important monetary and real aspects of these models hinge on the assumed timing of transactions. Several papers study consequences of varying such assumptions. For instance, Svensson (1985), Lucas (1984), and Lucas and Stokey (1987) consider the impact of varying the timing of transactions relative to when various elements of the state of nature are revealed to agents. One such variation induces a "precautionary demand" for money to emerge. Helpman and Razin (1985) subject some asset purchases to a cash-in-advance constraint by changing the timing of securities trading relative to transactions in goods markets.

This paper investigates consequences of changing the timing of another set transactions, namely government revenue collection. The standard assumption is that the government collects its revenue *before* goods market trades occur but that households and firms collect their period t revenue *after* goods trading has ended at date t. Hence for these private agents, cash revenues received at t cannot be spent until $t+1$.[1] The usual timing convention gives the government an advantage in revenue collection relative to the private sector.

We examine the consequences of placing the government and the private sector on an equal footing with respect to revenue collection. Specifically, we assume that government tax revenues are not collected until after goods trading has concluded at t. Then the government must, just like private agents, carry cash revenues from time t into time $t+1$. This subjects government tax revenues, as well as the revenues of firms, to the inflation tax. This apparently minor change in the timing of transactions has a significant impact on the whether the model generates some "classical" propositions about money and monetary policy.

Under our timing, which creates symmetry between the government and the private sector, a version of a unit velocity quantity theory equation is still valid. Furthermore, it turns out that a fixed tax strategy supports an identical equilibrium allocation and price process under both Lucas's timing and our own. However, it is generally not possible to obtain the result that open market operations are equivalent to pure units changes for the currency stock. Indeed, we construct an open market operation that leaves the entire price level process unaltered. That establishes a Modigliani-Miller theorem for government finance under our timing assumption. No such theorem holds under the timing restrictions imposed by Lucas. Finally, under our alternative timing assumptions, "Ricardian equivalence" theorems are impossible to obtain, because the timing of (lump-sum) taxation must affect an equilibrium.

Before giving the details, we want to explain why we have chosen to investigate the consequences of changing this particular timing assumption. To us, symmetry in transactions timing between government and private agents seems an attractive feature in and of itself. In addition, there are historical episodes that suggest that it is important to consider the consequences of subjecting government tax collections to the inflation tax. Governments that have made liberal use of the printing press have sometimes claimed that they were obligated to do so because other sources of government revenue were being eroded by inflation. This issue is discussed in the context of the German hyperinflation by Bresciani-Turroni (1937) and in the context of the American Revolution by Ferguson (1961).

The Model

A single nonstorable good is produced in per capita amount ξ_t at time t by some "trees" that neither grow nor depreciate. There is a competitive market in shares of trees that entitle an owner of shares to a pro-rata share of the "dividend" ξ_t. The output ξ_t may be consumed either by a representative private agent in amount c_t or by the government in amount g_t. We let s_t denote the shares chosen to be held by the representative agent at t, there being one share in the aggregate. Let x_t denote the state of the economy at t, and let $r(x_t)$ denote the price of a share in units of the time t consumption good as a function of the state. Households pay a lump sum tax of τ_t to the government at t, denominated in units of the time t consumption good.

A government purchases g_t units of output at t, issues unbacked currency in amount $M_{t+1}-M_t$ at t, levies taxes τ_t at t, borrows or lends, and engages in open market operations.[2] Let M_t denote the stock of currency carried over by agents from period $(t-1)$ to period t. Let $\ell_{t+1}(x_{t+1})$ denote government-issued state contingent claims to currency at the beginning of time $t+1$ contingent on the state of the economy being x_{t+1} at $t+1$. These claims are issued at time t. Let $n(x_{t+1}, x_t)$ be the kernel for pricing nominal state contingent one-period claims. That is, $n(x_{t+1}, x_t)$ is the price in terms of dollars at time t when the time t state is x_t of a claim to one dollar in time $t+1$ contingent on the time $t+1$ state being x_{t+1}. We let $\ell^p_{t+1}(x_{t+1})$ be the desired holdings by private agents of one period state contingent nominal claims. For convenience, we assume that the government does not own shares in trees.

Let the state be $x_t = (\xi_t, g_t)$. Assume that x_t is an exogenous stochastic process that obeys $0 \leq g_t \leq \xi_t$. Assume that x_t is Markov with transition density $f(x', x)$, where

$$\text{Prob}\,\{x_{t+1} \leq x' \mid x_t = x\} = \int_0^{x'} f(s, x) d\, s.$$

The Timing of Trades

The monetary theory of this model hinges on a set of restrictions on the timing of trades. We first describe the restrictions imposed by Lucas.

Lucas's Timing

Each integer period $t \geq 0$ is divided into three successive stages, a "securities trading session", a "shopping session", and a "dividend collection session."

Securities trading session

Private agents enter the securities trading session at date t with their previously acquired shares s_{t-1}, which are now worth $r(x_t)s_{t-1}$, in real terms, and with the *entire* stock of currency M_t. We shall soon identify the restriction that under Lucas's timing forces only private agents (and not the government) to hold currency between periods. Private agents receive (or pay, if negative) the dollar amount $\ell^p_t(x_t)$ from their previous state-contingent nominal one-period loans. During the securities trading session at t, private agents purchase s_t shares, a function $\ell^p_{t+1}(x_{t+1})$ of state contingent claims on dollars at the beginning of period $t+1$, and currency in the amount m^p_t. Private agents also pay taxes τ_t to the government during the securities trading period.

The government arrives in the securities trading session at t holding no currency, and owing $\ell_t(x_t)$ dollars. During the securities trading session, the government collects taxes and prints (or retires, if negative) $M_{t+1} - M_t$ units of currency. It also issues a list $\ell_{t+1}(x_{t+1})$ of time $t+1$ state contingent one-period nominal one-period securities and acquires currency in the amount m_t^g in response to a cash-in-advance restriction on the government's purchases of goods at t.

Equilibrium in the securities trading session requires

$$m_t^p + m_t^g = M_{t+1} \tag{1}$$

$$s_t = 1 \tag{2}$$

$$\ell_{t+1}^p(x_{t+1}) = \ell_{t+1}(x_{t+1}) \tag{3}$$

Shopping session

During the shopping session, the household divides into a worker-shopper pair. The shopper acquires consumption goods for cash, subject to the restriction $p_t c_t \leq m_t^p$. The worker stays home and sells the proceeds ξ_t from its local tree to private and government shoppers. The government is subject to the cash-in-advance restriction $m_t^g = p_t g_t$.

Dividend collection session

During the dividend collection session, agents collect the dividends from their shares in trees. These dividends equal $p_t \xi_t$. Currency received as dividends at t cannot be used to purchase anything at date t. It must be carried from t to period $t+1$. Notice that had we permitted the government to purchase trees, then the government might have had to hold some currency between periods.

Taken as a whole, the restrictions embedded in this setup impel private agents to hold *all* of the currency stock between periods. What imposes this outcome is less the cash-in-advance restriction itself than the separation of the "shopping session" from the "dividend collection session." This separation prevents agents from economizing on currency by collecting at least some of their cash dividends prior to making at least some purchases at time t. This setup exposes private agents to an inflation tax and permits the price of shares to be influenced by the equilibrium inflation process. Notice, however, that the government's tax collections are exempt from the inflation tax.

This setup assigns an advantage in collecting payments to the government. While private agents have to wait until the dividend collection session to collect returns on shares in trees, the government collects taxes during the securities trading session, prior to shopping. We want to study some of the consequences of withdrawing this advantage from the government. In particular, we describe an alternative set of restrictions that make the government collect its taxes during the dividend collection session, just as private agents must.

Symmetric Timing

The setup is identical with the one described above, except that now the government collects taxes at the start of the dividend collection session, after the shopping session. Private agents are subject to the cash-in-advance restriction

$$m_t^p \geq p_t(c_t + \tau_t) \tag{4}$$

because they are required to pay taxes in the form of currency during the shopping session. We suppose that the government is divided into a worker-shopper pair. We imagine that the government shopper leaves home with m_t^g dollars and shops, not returning home with goods until the shopping session is over (too late, that is, to make use of any of the currency that the government "worker" or tax collector has collected). At the beginning of the dividend collection session, the government worker gives the tax collections, $p_t\,\tau_t$, to the government shopper. The government then carries currency in amount $p_t\,\tau_t$ over into period $t+1$. Thus, the government has $p_t\,\tau_t$ units of currency, worth $p_t\,\tau_t/p_{t+1}$ units of the time $t+1$ good, at the beginning of period $t+1$, to which it adds $M_{t+2} - M_{t+1}$ units of freshly created currency during the time $t+1$ securities trading session.

We now describe the choice problem facing the representative private agent, and the policy processes selected by the government under our timing conventions.

Private Agents' Choice Problem

Let $u(c)$ be an increasing, strictly concave, twice continuously differentiable one-period utility function. The infinitely lived representative private agent chooses state-contingent sequences $\{c_t\}_{t=0}^{\infty}$, $\{m_t^p\}_{t=0}^{\infty}$, $\{s_t\}_{t=0}^{\infty}$, $\{\ell_{t+1}^p\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{5}$$

subject to

$$\text{(6)} \qquad \frac{m_t^p}{p_t} + r(x_t)s_t + \frac{1}{p_t}\int \ell_{t+1}^p(x_{t+1})\, n\,(x_{t+1}, x_t)\, dx_{t+1} = \theta_t(x_t), \quad t \geq 0$$

$$\text{(7)} \qquad \Big[\frac{p_t \xi_t}{p_{t+1}} + r(x_{t+1})\Big] s_t + \frac{1}{p_{t+1}}\ell_{t+1}^p(x_{t+1}) + \frac{m_t^p - p_t(c_t + \tau_t)}{p_{t+1}} = \theta_{t+1}\,(x_{t+1}), \quad t \geq 0$$

$$\text{(4)} \qquad m_t^p \geq p_t(c_t + \tau_t) \quad , \quad t \geq 0$$

$\theta_0\,(x_0)$ given. Here $\theta_t\,(x_t)$ is the private agent's real wealth at t, denominated in units of time t consumption goods. Later, we shall give an expression for $\theta_0(x_0)$ in equilibrium. The private agent solves this problem taking as given stochastic processes for $\{p_t\}_{t=0}^{\infty}$ and $\{\tau_t\}_{t=0}^{\infty}$, and the pricing functions $n(x_{t+1}, x_t)$ and $r(x_t)$.

Government

The government selects stochastic processes for $\{\tau_t\}_{t=0}^{\infty}$, $\{\ell_{t+1}(x_{t+1})\}_{t=0}^{\infty}$, $\{M_{t+1}\}_{t=0}^{\infty}$ that satisfy the budget constraints

$$\text{(8)} \qquad g_0 = \frac{1}{p_0}\int \ell_1(x_1) n(x_1, x_0)\, dx_1 \;+\; \frac{M_1 - M_0}{p_0}$$

$$\text{(9)} \qquad g_t = \tau_{t-1}\frac{p_{t-1}}{p_t} + \frac{1}{p_t}\int \ell_{t+1}(x_{t+1})\, n(x_{t+1}, x_t) dx_{t+1} - \frac{\ell_t(x_t)}{p_t} + \frac{M_{t+1} - M_t}{p_t}, \quad t \geq 1.$$

Market Clearing

The market clearing conditions continue to be given by (1), (2), and (3). We shall use the following:

Definitions:

A *feasible allocation* is a pair of stochastic processes $\{c_t, g_t\}_{t=0}^{\infty}$ that satisfy $c_t + g_t = \xi_t$ for all t.

A *price system* is a nonnegative stochastic process $\{p_t\}_{t=0}^{\infty}$ and a triple of nonnegative valued functions $r(x_t), n(x_{t+1}, x_t)$, and $q(x_{t+1}, x_t)$, where $q(x_{t+1}, x_t)$ is the price at t of a claim to one unit of the good if state x_{t+1} occurs at $t+1$.

A *government policy* is a collection of stochastic processes $\{\tau_t\}_{t=0}^{\infty}$, $\{M_{t+1}\}_{t=0}^{\infty}$, $\{\ell_{t+1}(x_{t+1})\}_{t=0}^{\infty}$ that satisfy (8)-(9) for a given price system.

A *competitive equilibrium* is a feasible allocation, a price system, a government policy, and a sequence of private asset holdings s_t, M_{t+1}, $\ell_{t+1}^p(x_{t+1})$ for which (i) the household's problem is solved, and (ii) the market clearing conditions (1)-(3) are satisfied.

Integrated Budget Constraints

By an arbitrage argument (see Sargent (1987, pp. 160-161)), the pricing kernel $q(x_{t+1}, x_t)$ for one-step ahead *real* contingent claims is related to $n(x_{t+1}, x_t)$ by

$$q\left(x_{t+1}, x_t\right) = \frac{p_{t+1}}{p_t}\, n\left(x_{t+1}, x_t\right). \tag{10}$$

Multiply both sides of (7) by $q\left(x_{t+1}, x_t\right)$, integrate with respect to x_{t+1}, and use the result to eliminate $\int \ell_{t+1}(x_{t+1}) n(x_{t+1}, x_t) dx_{t+1}$ from (6). We obtain

$$\begin{aligned} \theta_t(x_t) = {} & \frac{m_t^p}{p_t}\left[1 - \int n(x_{t+1}, x_t) dx_{t+1}\right] \\ & + s_t\Big[r(x_t) - \int \Big[r(x_{t+1}) + \frac{p_t \xi_t}{p_{t+1}}\Big] q(x_{t+1}, x_t) dx_{t+1}\Big] \\ & + (c_t + \tau_t) \int n(x_{t+1}, x_t)\, dx_{t+1} + \int \theta_{t+1}(x_{t+1})\, q\left(x_{t+1}, x_t\right) dx_{t+1} \end{aligned} \tag{11}$$

We restrict ourselves to studying equilibria for which for all x_t, $1 - \int n(x_{t+1}, x_t) dx_{t+1} > 0$, so that the sure nominal interest rate is always positive. Under this condition, (11) indicates that private agents have an incentive to minimize the level of currency holdings.[3] Thus, when $1 - \int n(x_{t+1}, x_t) dx_{t+1} > 0$, the cash-in-advance restriction is always binding so that $m_t^p = p_t(c_t + \tau_t)$. Since the absence of arbitrage profits requires that the coefficient on s_t in (11) equal zero, (11) becomes

$$\theta_t(x_t) = (c_t + \tau_t) + \int \theta_{t+1}(x_{t+1}) q(x_{t+1}, x_t) dx_{t+1}. \tag{12}$$

Solving this equation forward gives for $t = 0$

$$\theta_0(x_0) = c_0 + \tau_0 + \sum_{t=1}^{\infty} \int (c_t + \tau_t) q^{(t)}(x_t, x_0) dx_t \tag{13}$$

where $q^{(t)}(x_t, x_0)$ is the pricing kernel for t-step ahead real contingent claims,[4] and where $\theta_0(x_0) = M_0/p_0 + r(x_0)$. Equation (13) is the intertemporal budget constraint for the representative private agent. It is identical with the corresponding intertemporal budget constraint under Lucas's timing.

By iterating on the sequence government budget constraints (8) and (9), we obtain

$$
\begin{aligned}
g_0 + \sum_{t=1}^{\infty} \int g_t q^{(t)}(x_t, x_0) dx_t &= \frac{M_1 - M_0}{p_0} + \sum_{t=1}^{\infty} \int \Big(\frac{M_{t+1} - M_t}{p_t}\Big) q^{(t)}(x_t, x_0) dx_t \\
&+ \tau_0 \int n(x_1, x_0) dx_1 + \sum_{t=1}^{\infty} \int \tau_t \Big[\int n(x_{t+1}, x_t) dx_{t+1}\Big] q^{(t)}(x_t, x_0) dx_t.
\end{aligned} \tag{14}
$$

This constraint differs from the corresponding constraint under Lucas's timing, namely,[5]

$$
\begin{aligned}
&g_0 + \sum_{t=1}^{\infty} q^{(t)}(x_t, x_0) g_t dx_t + b_0(x_0) \\
&= \tau_0 + \frac{M_1 - M_0}{p_0} + \sum_{t=1}^{\infty} \int q^{(t)}(x_t, x_0) \Big(\tau_t + \frac{M_{t+1} - M_t}{p_t}\Big) dx_t,
\end{aligned}
$$

in a way that reflects the requirement that the government must collect its tax receipts in the same way that private agents collect dividends. In particular, at time t the government levies taxes in the nominal amount $p_t \tau_t$. The proceeds of this tax are not available to be spent until $t+1$, so the discounted present value of this dollar amount $p_t \tau_t \int n(x_{t+1}, x_t) dx_{t+1}$ at t enters the government's present value budget constraint as a revenue source. Under Lucas's timing these tax revenues can be spent immediately, and hence are worth $p_t \tau_t$ dollars at t. Phrased differently, under our timing convention the government is subject to an inflation tax on its tax receipts, just as share holders suffer an inflation tax on dividends. This occurs because the government is now obligated to carry currency between periods.

Consequences of Altered Timing

The Quantity Theory Equation

Equilibrium condition (1), the cash-in-advance restrictions for the government and private agents (at equality), and the feasibility condition $c_t + g_t = \xi_t$ imply

$$
\frac{M_{t+1}}{p_t} = \xi_t + \tau_t \tag{15}
$$

or

$$
p_t = \frac{M_{t+1}}{\xi_t + \tau_t} \tag{16}
$$

The corresponding equation under Lucas's timing convention is $p_t = M_{t+1}/\xi_t$, which delivers unit velocity.

According to (16), velocity of the *total* stock of currency is a function of current real taxation. Under the special restriction that

$$\tau_t = k\xi_t \tag{17}$$

equation (16) becomes

$$p_t = M_{t+1}/(1+k)\xi_t \tag{18}$$

which implies constant velocity.

It is also possible to obtain a slightly different version of a constant velocity relation from (15). In particular, recalling that at the end of period t, $p_t\tau_t$ dollars have been paid as taxes and are held by "the Treasury," define "currency in the hands of the public" at t to be

$$M^a_{t+1} = M_{t+1} - p_t\tau_t.$$

Then from (15)

$$\frac{M^a_{t+1}}{p_t} = \xi_t. \tag{19}$$

Thus, when velocity is computed using currency in the hands of the public, both Lucas's timing assumption and ours deliver constant velocity relations.[6]

Equilibrium Prices, Borrowing, and Currency Creation with Tax Processes Fixed Across Lucas's Timing and Our Timing

Equation (19) indicates that under an appropriate interpretation of "currency in the hands of the public," the timing of government revenue collection does not matter for price level determination.

Proposition 1: *Consider an economy facing an exogenously given stochastic process for $\{g_t, \tau_t\}$, with $\xi_t > g_t \geq 0$, and $\xi_t > \tau_t \geq 0$. Let $b_t(x_t)$ be the real value of state-contingent claims issued by the government. Assume that $b_t(x_t)$ is given for all t and all x_t. Then equilibria that obtain under Lucas's timing and ours are identical.*

Proof: Consider first the situation under Lucas's timing. The determination of equilibrium pricing kernels is as discussed by Sargent (1987, chapter 5). The government budget constraint for period t is

$$g_t - \tau_t = \frac{M_{t+1} - M_t}{p_t} - b_t(x_t) + \int b_{t+1}(x_{t+1})\, q\,(x_{t+1}, x_t) dx_{t+1} \quad ; \quad t \geq 0, \tag{20}$$

with M_0 given and $b_0(x_0) \equiv 0$. Letting

$$D_t \equiv \int b_{t+1}(x_{t+1})\, q\,(x_{t+1}, x_t) dx_{t+1} - b_t(x_t),$$

and using the fact that in equilibrium $M_{t+1}/p_t = \xi_t$, (20) implies the following growth rate of the money supply

$$\frac{\overline{M}_{t+1}}{\overline{M}_t} = \frac{\xi_t}{\xi_t - (g_t - \tau_t - D_t)}. \tag{21}$$

The implied inflation rate is

$$\frac{\bar{p}_t}{\bar{p}_{t-1}} = \left(\frac{\bar{M}_{t+1}}{\bar{M}_t}\right) \left(\frac{\xi_{t-1}}{\xi_t}\right) = \frac{\xi_{t-1}}{\xi_t - (g_t - \tau_t - D_t)} \quad ; \quad t \geq 1 \tag{21}$$

Finally, using (20) for $t = 0$ along with $b_0(x_0) = 0$ implies that the initial price level obeys

$$\frac{M_0}{\bar{p}_0} = \xi_0 - (g_0 - \tau_0 - D_0). \tag{22}$$

Now reconsider the same situation under the alternative timing assumption introduced above. An equilibrium price system ($^-$) obtaining under Lucas's timing will also be an equilibrium under our alternative timing if

$$\frac{M_{t+1}}{\bar{p}_t} = \xi_t + \tau_t \qquad \forall \quad t \geq 0 \tag{23}$$

and if the government budget constraint is satisfied. The time t government budget constraint is

$$g_t = \tau_{t-1}\left(\frac{p_{t-1}}{p_t}\right) + \frac{M_{t+1} - M_t}{p_t} - b_t(x_t) + \int b_{t+1}(x_{t+1})\, q(x_{t+1}, x_t) dx_{t+1} \; ; \; t \geq 1 \tag{24}$$

At $t = 0$

$$g_0 = \frac{M_1 - M_0}{p_0} + \int b_1(x_1)\, q\,(x_1, x_0) dx_1 \tag{25}$$

most hold.

Imposing (23) and $p_{t-1}/p_t = \bar{p}_{t-1}/\bar{p}_t$ in (24) yields

$$\xi_{t-1}\left(\frac{\bar{p}_{t-1}}{\bar{p}_t}\right) = \xi_t - (g_t - \tau_t - D_t),$$

which is clearly satisfied. Similarly, setting $M_0/\bar{p}_0$ equal to its value in (22) implies satisfaction of (25). Thus the set of equilibrium price systems is the same under either timing convention. ∎

It is instructive to consider money growth rates under the two timing conventions as well. From (23), the currency creation process under our timing assumption, $\{\hat{M}_t\}$, must satisfy

(26) $$\frac{\hat{M}_{t+1}}{\hat{M}_t} = \left(\frac{\xi_t + \tau_t}{\xi_{t-1} + \tau_{t-1}}\right)\left(\frac{\bar{p}_t}{\bar{p}_{t-1}}\right)$$

Substituting (21) into (26) implies that

(27) $$\frac{\hat{M}_{t+1}}{\hat{M}_t} = \left(\frac{\xi_t + \tau_t}{\xi_{t-1} + \tau_{t-1}}\right)\left(\frac{\xi_{t-1}}{\xi_t}\right)\left(\frac{\bar{M}_{t+1}}{\bar{M}_t}\right).$$

Then if $\tau_t/\tau_{t-1} > \xi_t/\xi_{t-1}$, the rate of money growth will be higher under a symmetric timing assumption than it would be if Lucas's timing applied.

It is also of interest to consider the growth rate of "currency held by the public." As above, currency held by the public is $M^a_{t+1} = M_{t+1} - \bar{p}_t\tau_t$. Then

(28) $$\frac{M^a_{t+1}}{M^a_t} = \frac{\hat{M}_{t+1} - \bar{p}_t\tau_t}{\hat{M}_t - \bar{p}_{t-1}\tau_{t-1}} = \left(\frac{\xi_t}{\xi_{t-1}}\right)\left(\frac{\bar{p}_t}{\bar{p}_{t-1}}\right) = \frac{\bar{M}_{t+1}}{\bar{M}_t}.$$

Thus, currency held by the public grows at the same rate, independently of the assumed timing of government revenue collections.

Relations such as (19) and (28), along with Proposition 1, assert a sense in which the timing of government revenue collection has little impact. However, the analysis to this point has only considered equilibria for a *fixed* $\{\tau_t\}$ process. We now turn our attention to some experiments that involve altering the $\{\tau_t\}$ process. We shall see that the timing of taxation matters much more when government revenues are subject to the inflation tax, as under our timing, than is the case under Lucas's timing.

Do Open Market Operations Lead to Proportional Changes in Prices?

To study additional consequences of the altered timing, we reconsider an experiment that under Lucas's timing delivers the conclusion that open market operations are equivalent to pure currency units changes in their effects on the equilibrium price level process. In this experiment, the government buys government debt in exchange for currency, thus increasing the initial amount of currency and reducing the amount of government debt held by the public. If the government simultaneously reduces the present value of future taxes by just enough to compensate for the reduction in future debt service, then under Lucas's timing the entire price level path jumps proportionately to the increase in the currency stock.

Proposition 2: *Let $M_0 = \bar{M}_0$ be given. Similarly, let the stochastic process $x_t = (g_t, \xi_t)$ be given, and let an initial equilibrium be given by $\bar{q}\,(x_{t+1}, x_t)$, $\int \bar{b}\,(x_{t+1})\,\bar{q}\,(x_{t+1},\, x_t)\,dx_{t+1}$, $\bar{M}_{t+1}$, $\bar{p}_t$, and $\bar{\tau}_t$ for $t \geq 0$. Then consider an alternate government borrowing, taxation, and currency creation strategy that satisfies*

$$\hat{\tau}_0 = \bar{\tau}_0 \tag{29}$$

$$\hat{M}_1 > \bar{M}_1 \tag{30}$$

$$D \equiv -\Big(\frac{\hat{M}_1 - \bar{M}_0}{\hat{p}_0}\Big) + \Big(\frac{\bar{M}_1 - \bar{M}_0}{\bar{p}_0}\Big) = \int \Big[\hat{b}_1\,(x_1) - \bar{b}_1\,(x_1)\Big]\,\bar{q}(x_1, x_0)dx_1 \tag{31}$$

$$\sum_{j=1}^{\infty} \int (\hat{\tau}_j - \bar{\tau}_j)\,\bar{q}^{(j)}(x_j, x_0)dx_j = D. \tag{32}$$

Under Lucas's timing the $(\hat{\ })$ policy supports the same allocation and state contingent claims prices as an equilibrium $(^{-})$. Moreover, the price level and currency stock obey

$$\frac{\hat{M}_{t+1}}{\hat{p}_t} = \frac{\bar{M}_{t+1}}{\bar{p}_t} \tag{33}$$

$$\frac{\hat{p}_{t+1}}{\hat{p}_t} = \frac{\bar{p}_{t+1}}{\bar{p}_t}. \tag{34}$$

Proof: This is Proposition 5.4 of Sargent (1987). Conditions (29)-(32) assert that *vis-a-vis* the $(^{-})$ policy, the $(\hat{\ })$ policy amounts to an open market operation at $t = 0$, accompanied by an alteration in the present value of future tax collections sufficient to offset the government's altered debt service. Equations (33) and (34) assert that this open market operation produces a once-and-for-all change in prices that is proportional to the change in the currency stock. Moreover, an immediate implication of (33) and (34) is that

$$\frac{\hat{M}_{t+1}-\hat{M}_t}{\hat{p}_t} = \frac{\bar{M}_{t+1}-\bar{M}_t}{\bar{p}_t} \qquad \forall \quad t \geq 1.$$

Thus

$$\sum_{t=1}^{\infty}\int\Big(\frac{\hat{M}_{t+1}-\hat{M}_t}{\hat{p}_t}\Big)\,\bar{q}^{(t)}(x_t,x_0)dx_t = \sum_{t=1}^{\infty}\int\Big(\frac{\bar{M}_{t+1}-\bar{M}_t}{\bar{p}_t}\Big)\,\bar{q}^{(t)}(x_t,x_0)dx_t. \tag{35}$$

▮

Proposition 2 fails when the government collects revenue symmetrically with private agents.

Proposition 3: *The equivalence class of government policies delineated in proposition 2 fail to support the same equilibria under our alternative timing protocol.*

Proof: We verify this claim by considering the policy change described by (29)-(32), and by seeking additional restrictions implied by the outcomes (34) and (35) (but not necessarily (33)). To begin, the government budget constraint (14) implies the following restriction across the ($\hat{}$) and the ($^-$) equilibria:

$$\begin{aligned}
&\sum_{t=1}^{\infty}\int\Big(\frac{\hat{M}_{t+1}-\hat{M}_t}{\hat{p}_t}\Big)\,\bar{q}^{(t)}(x_t,x_0)dx_t + \frac{\hat{M}_1-\bar{M}_0}{\hat{p}_0} + \hat{\tau}_0\int \bar{n}(x_1,x_0)dx_1 + \\
&\sum_{t=1}^{\infty}\int\hat{\tau}_t\Big[\int \bar{n}(x_{t+1},x_t)dx_t\Big]\,\bar{q}^{(t)}(x_t,x_0)dx_t = \\
&\sum_{t=1}^{\infty}\int\Big(\frac{\bar{M}_{t+1}-\bar{M}_t}{\bar{p}_t}\Big)\,\bar{q}^{(t)}(x_t,x_0)dx_t + \frac{\bar{M}_1-\bar{M}_0}{\bar{p}_0} + \bar{\tau}_0\int \bar{n}(x_1,x_0)dx_1 + \\
&\sum_{t=1}^{\infty}\int\bar{\tau}_t\Big[\int \bar{n}(x_{t+1},x_t)dx_t\Big]\,\bar{q}^{(t)}(x_t,x_0)dx_t.
\end{aligned} \tag{36}$$

Imposing (29) and the desired outcomes (34) and (35) on (36), and rearranging gives

$$\begin{aligned}
&\sum_{t=1}^{\infty}\int(\hat{\tau}_t-\bar{\tau}_t)\Big[\int \bar{n}(x_{t+1},x_t)dx_{t+1}\Big]\,\bar{q}^{(t)}(x_t,x_0)dx_t = D \\
&= \sum_{t=1}^{\infty}\int(\hat{\tau}_t-\bar{\tau}_t)\bar{q}^{(t)}(x_t,x_0)dx_t
\end{aligned} \tag{37}$$

where the latter equality is (32), and where (31) defines D.

Equation (37) implies a restriction on $\{\hat{\tau}_t - \bar{\tau}_t\}_{t=1}^{\infty}$ that must hold under our timing assumptions if the policy experiment (29)-(32) is to be consistent with (34) and (35). Evidently, this restriction cannot be satisfied generally. For instance, if the initial ($^-$) equilibrium displays a (positive) constant net nominal interest rate, then $\int \bar{n}(x_{t+1}, x_t) \equiv (1+i)^{-1} < 1$, and (37) will fail to hold.

In order to satisfy the government budget constraint (14) under policy alterations that satisfy (29)-(31), it is necessary that taxes and currency creation obey

$$\sum_{t=1}^{\infty} \int \Big[(\hat{\tau}_t - \bar{\tau}_t) \int n\,(x_{t+1}, x_t) dx_{t+1} + \left(\frac{\hat{M}_{t+1} - \hat{M}_t}{\hat{p}_t}\right) - \left(\frac{\bar{M}_{t+1} - \bar{M}_t}{\bar{p}_t}\right) \Big] \, \bar{q}^{(t)}(x_t, x_0) dx_t = D. \tag{38}$$

Equation (38) asserts that the present value of tax revenues adjusted to compensate for the inflation tax plus the present value of seigniorage must be adjusted for the initial change in government seigniorage income D. The failure of (37) to hold in general indicates that, relative to the ($^-$) equilibrium, some additional seigniorage revenue typically must be raised in periods later than $t = 0$ in order to satisfy the government budget constraint and to preserve the relations $\hat{p}_t/\hat{p}_{t+1} = \bar{p}_t/\bar{p}_{t+1}$. This adjustment in seigniorage revenue is required in order to compensate for the deficiency of revenues that would be associated with a tax policy described by (32), a deficiency associated with the inflation tax that is imposed on government revenue collection. ∎

Special equivalence class of policies

While restrictions (37) are not generally satisfied for the wide class of tax policies that can support the open market operation under Lucas's timing, they can be satisfied by very special tax policies. Here is an example. Consider an initial ($^-$) equilibrium associated with the special nonstochastic economy: $\xi_t = \xi\,, \bar{g}_t = g\,, \bar{\tau}_t = \tau$ for all $t \geq 0$; $\bar{M}_{t+1}/\bar{M}_t = \alpha_e > 1$ for t even, $\bar{M}_{t+1}/\bar{M}_t = \alpha_o > 1$ for t odd, $t \geq 0$. Let g and τ be chosen so that they satisfy

$$\frac{g - \tau}{1 - \beta} = \left(\frac{\xi\beta}{1 - \beta^2}\right) \left[\left(\frac{\alpha_0 - 1}{\alpha_0}\right) + \beta \left(\frac{\alpha_e - 1}{\alpha_e}\right) \right] + \xi - \frac{M_0}{\bar{p}_0}$$

for some $\bar{p}_0 > 0$. This equation guarantees that the government budget constraint is satisfied with $\bar{q}^{(t)}(x_t, x_0) = \beta^t$ and $\int n(x_{t+1}, x_t) dx_{t+1} = \beta/\alpha_o$ for t even, β/α_e for t odd. The equilibrium values of $\bar{M}_{t+1}/\bar{M}_t$ and $\bar{p}_t/\bar{p}_{t+1}$ satisfy

$$\frac{\bar{M}_{t+1}}{\bar{M}_t} = \begin{cases} \alpha_e & t \text{ even} \\ \alpha_o & t \text{ odd} \end{cases}$$

and

$$\frac{\bar{p}_t}{\bar{p}_{t+1}} = \begin{cases} \frac{1}{\alpha_o} & t \text{ even} \\ \frac{1}{\alpha_e} & t \text{ odd.} \end{cases}$$

Now consider a $(\hat{\ })$ tax policy given by

$$\hat{\tau}_t - \bar{\tau}_t = \begin{cases} \gamma_e & t \text{ even} \\ \gamma_o & t \text{ odd} \end{cases},$$

$\gamma_e \neq \gamma_o$. This tax policy along with the initial currency creation process will leave the $(^-)$ allocation an equilibrium so long as (γ_e, γ_o) are chosen to satisfy

$$\begin{pmatrix} \frac{\beta^2}{\alpha_e} & \frac{\beta^3}{\alpha_o} \\ \beta & \beta^2 \end{pmatrix} \begin{pmatrix} \gamma_o \\ \gamma_e \end{pmatrix} = (1 - \beta^2) \begin{pmatrix} \bar{D} \\ \bar{D} \end{pmatrix}$$

where $\bar{D}$ is the value of D in (37) associated with the $(^-)$ equilibrium. The above equation guarantees that (37) is satisfied. The $(\hat{\ })$ price system satisfies

$$\hat{p}_t = \bar{p}_t \cdot \frac{\hat{p}_0}{\bar{p}_0} \quad \text{for all } \ t \geq 0.$$

$$\int \hat{n}(x_{t+1}, x_t) dx_{t+1} = \int \bar{n}(x_{t+1}, x_t) dx_{t+1}.$$

This example illustrates that, when it can be satisfied at all, a very particular sequence of taxes is required to satisfy (37). It is not sufficient simply to set the present value of the sequence of taxes equal to the value required under Lucas's timing.

A Modigliani-Miller Theorem for Government Finance

A Modigliani-Miller theorem for government finance describes an equivalence class of government borrowing, tax, and currency creation policies that support the same allocation and the same price system as an equilibrium (see Wallace (1981) and Sargent and Smith (1987)). Under Lucas's timing, the fact that the quantity theory equation $p_t = M_{t+1}/\xi_t$

obtains precludes the existence of any such nontrivial equivalence class of financial policies that support the same price level process. But the modified quantity theory equation $p_t = M_{t+1}/(\xi_t + \tau_t)$ that obtains altered timing studied here makes room for a nontrivial equivalence class. We now construct this equivalence class of policies that support the same equilibrium.

Proposition 4: *Let the stochastic process $x_t = (\xi_t, g_t)$ and the initial currency stock $M_0 = \bar{M}_0$ be identical across two economies. Assume that there exists an initial equilibrium, denoted the $(^-)$ equilibrium. An alternative government financing strategy, denoted the $(\hat{\ })$ strategy, supports the $(^-)$ allocation and price system as an equilibrium if and only if*[7]

$$\frac{1}{\bar{p}_0}\int\Big[\hat{\ell}_1(x_1)-\bar{\ell}_1(x_1)\Big]\bar{n}(x_1,x_0)dx_1 = -\Big(\frac{\hat{M}_1-\bar{M}_1}{\bar{p}_0}\Big) \tag{39.a}$$

$$\frac{\hat{M}_{t+1}-\bar{M}_{t+1}}{\bar{p}_t} = \hat{\tau}_t - \bar{\tau}_t \quad \forall \;\; t \geq 0 \tag{40}$$

$$\hat{\tau}_0 - \bar{\tau}_0 + \sum_{t=1}^{\infty}\int(\hat{\tau}_t-\bar{\tau}_t)\bar{q}^{(t)}(x_t,x_0)dx_t = 0. \tag{41}$$

Proof: Consider two alternative government financing strategies, the $(\hat{\ })$ strategy and the $(^-)$ strategy that support an equilibrium. Assume that an equilibrium exists for the $(^-)$ government financing strategy. Let $\bar{q}^{(t)}(x_t, x_0)$ be the associated equilibrium pricing kernels for the $(^-)$ equilibrium. If the $(^-)$ price system is to constitute an equilibrium for both the $(\hat{\ })$ and $(^-)$ strategies, the government budget constraints (8) and (9) imply that

$$\frac{1}{\bar{p}_0}\int\Big[\hat{\ell}_1(x_1)-\bar{\ell}_1(x_1)\Big]\bar{n}(x_1,x_0)dx_1 = -\Big(\frac{\hat{M}_1-\bar{M}_1}{\bar{p}_0}\Big) \tag{39.a}$$

$$\begin{aligned}&(\hat{\tau}_{t-1}-\bar{\tau}_{t-1})\frac{\bar{p}_{t-1}}{\bar{p}_t} + \frac{\hat{M}_{t+1}-\bar{M}_{t+1}}{\bar{p}_t} - \Big(\frac{\hat{M}_t-\bar{M}_t}{\bar{p}_{t-1}}\Big)\Big(\frac{\bar{p}_{t-1}}{\bar{p}_t}\Big) - \Big(\frac{\hat{\ell}_t(x_t)-\bar{\ell}_t(x_t)}{\bar{p}_t}\Big)\\ &+\frac{1}{\bar{p}_t}\int\Big[\hat{\ell}_{t+1}(x_{t+1})-\bar{\ell}_{t+1}(x_{t+1})\Big]\bar{n}(x_{t+1},x_t)dx_{t+1} = 0; \quad t\geq 1.\end{aligned} \tag{39.b}$$

must be satisfied by the $(\hat{\ })$ and $(^-)$ policies. Also, in order to maintain the price level process $\{\bar{p}_t\}$, from (15) it is necessary that

$$\frac{\hat{M}_{t+1}-\bar{M}_{t+1}}{\bar{p}_t} = \hat{\tau}_t - \bar{\tau}_t. \tag{40}$$

In addition, in order for the budget constraint of private agents to be satisfied by $\{\bar{c}_t\}$ at the given pricing kernels $\bar{q}^{(t)}(x_t, x_0)$, it is the case that $\{\hat{\tau}_t\}$ and $\{\bar{\tau}_t\}$ must satisfy

$$\hat{\tau}_0 - \bar{\tau}_0 + \sum_{t=1}^{\infty} \int (\hat{\tau}_t - \bar{\tau}_t)\bar{q}^{(t)}(x_t, x_0)dx_t = 0 \tag{41}$$

By virtue of (40), (41) is equivalent to

$$\frac{\hat{M}_1 - \bar{M}_1}{\bar{p}_0} + \sum_{t=1}^{\infty} \int \Big(\frac{\hat{M}_{t+1} - \bar{M}_{t+1}}{\bar{p}_t}\Big)\bar{q}^{(t)}(x_t, x_0)dx_t = 0. \tag{42}$$

Finally, satisfaction of the market clearing condition (3) requires that

$$\hat{\ell}^p_{t+1}(x_{t+1}) = \hat{\ell}_{t+1}(x_{t+1}) \quad \forall \;\; x_{t+1}, \quad \forall \;\; t.$$

Equation (41) states that the present value of tax levies must be held constant across the ($\hat{}$) and the ($\bar{}$) equilibria. Equation (42) has a less obvious interpretation, but if the government could collect its tax revenue in the "shopping session," as under Lucas's timing, (42) would hold the present value of seigniorage revenue constant across the ($\hat{}$) and ($\bar{}$) equilibria as well.

Consider any government strategies ($\hat{}$) that satisfy (39.a), (40), and (41). We claim that any such strategies, along with the ($\bar{}$) allocation and price system, constitute an equilibrium. First, substituting (41) into the expression (13) for the budget constraint of private agents, it is clear that policies satisfying (41) leave the private agent's budget set unaltered at the given pricing kernels $\bar{q}^{(t)}(x_t, x_0)$. Then $\{\bar{c}_t\}$ continues to solve the utility maximization problem of private agents. Moreover, private agents will be content to set $\hat{\ell}^p_{t+1}(x_{t+1}) = \hat{\ell}_{t+1}(x_{t+1})$ and $\hat{s}_t = 1$ at these prices (see Sargent (1987), proposition 5.1). This fact, along with (40), implies satisfaction of the market clearing conditions (1)-(3).

It remains to verify that, under the ($\bar{}$) pricing system, the ($\hat{}$) policies satisfy the government budget constraints, or in other words, that they satisfy (39). (39.a) is, of course, satisfied trivally. Satisfaction of (39.b) is implied by (40) and by equations (6) and (7). In particular, subtracting (6) for the ($\bar{}$) equilibrium from (6) for the ($\hat{}$) equilibrium yields

$$\begin{aligned} \hat{\theta}_t(x_t) - \bar{\theta}_t(x_t) = \frac{\hat{M}_{t+1} - \bar{M}_{t+1}}{\bar{p}_t} + \\ \frac{1}{\bar{p}_t} \int \Big[\hat{\ell}_{t+1}(x_{t+1}) - \bar{\ell}_{t+1}(x_{t+1})\Big] \bar{n}(x_{t+1}, x_t)dx_{t+1} \end{aligned} \tag{43}$$

where the market clearing conditions and $\hat{g}_t = \bar{g}_t$ have been used in (43). Similarly, equation (7) lagged one period implies that

$$\hat{\theta}_t(x_t) - \bar{\theta}_t(x_t) = \frac{\hat{\ell}_t(x_t) - \bar{\ell}_t(x_t)}{\bar{p}_t} + \frac{\hat{M}_t - \bar{M}_t}{\bar{p}_t} - \left(\frac{\bar{p}_{t-1}}{\bar{p}_t}\right)(\hat{\tau}_{t-1} - \bar{\tau}_{t-1}) \tag{44}$$

But (43) and (44) imply (39.b), completing the argument.

It is also the case that any alterations of government policies that fail to satisfy (39.a), (40), and (41) cannot preserve the ($^-$) allocation and price system as an equilibrium. First, if (39.a) fails to hold, the time $t = 0$ government budget constraint will be violated. If (41) fails to hold, the ($^-$) allocation and price system will violate the budget constraint of private agents (equation (13)). Failure of (40) implies that the ($^-$) allocation and price system cannot be unaltered. ▮

Disappearance of A Ricardian Equivalence Proposition

Suppose as above that there is an initial ($^-$) equilibrium under a government policy ($^-$). Then consider an alternative government policy strategy, denoted the ($\hat{}$) strategy, obtained as follows. Let

$$\hat{\tau}_0 - \bar{\tau}_0 = -\int \left[\hat{b}_1(x_1) - \bar{b}_1(x_1)\right] \bar{q}(x_1, x_0) dx_1 \tag{45}$$

$$\begin{aligned} &\int \left[\hat{b}_{t+1}(x_{t+1}) - \bar{b}_{t+1}(x_{t+1})\right] \bar{q}(x_{t+1}, x_t) dx_{t+1} \\ &= \sum_{j=1}^{\infty} \int (\hat{\tau}_{t+j} - \bar{\tau}_{t+j}) \, \bar{q}^{(j)}(x_{t+j}, x_t) dx_{t+j}; \quad t \geq 0 \end{aligned} \tag{46}$$

$$\hat{M}_t = \bar{M}_t \qquad \forall \quad t \geq 0 \tag{47}$$

hold. Relative to the ($^-$) policy, the ($\hat{}$) policy represents just rearranges the timing of taxation.

Under Lucas's timing, the ($\hat{}$) policy, along with the ($^-$) allocation and price system, continues to constitute an equilibrium.[8] Thus with Lucas's timing, there prevails a Ricardian equivalence proposition that asserts the irrelevance of the timing of taxation.

Corollary to Proposition 4: *Under the symmetric timing assumptions considered here, the* ($\hat{}$) *policy is not consistent with the retention of the* ($^-$) *allocation and price system as an equilibrium.*

Proof: Since equation (47) holds the stochastic process for the money supply fixed, equation (15) implies that the process for the price level must differ across the ($\hat{}$) and the ($^-$) equilibria. Thus the timing of taxation must affect the nature of an equilibrium. ∎

While in our pure exchange economy only the price level is affected by the timing of taxation, it is easy to extend the argument just given to a production economy with cash goods and credit goods, as in Lucas (1984, 1987) and Lucas and Stokey (1987). In such an economy, allocations would in general be affected by the timing of taxation, as an examination of equation (12) of Lucas (1987) indicates.

Concluding Remarks

The monetary theory associated with a cash-in-advance model is all about restrictions on the timing of trading activities. Among the most important of these restrictions are ones that force some agents to hold the low rate of return asset currency between periods. In equilibria with positive nominal interest rates, there are incentives to evade holding currency that the model builder must somehow thwart. It matters which classes of agents the model builder forces to hold currency between periods.

In this paper, we have studied how things depend on whether the government is forced to hold its tax receipts in the form of currency between periods. Some but not all of the classical propositions delivered by a simple cash-in-advance model remain intact when the government gets stuck holding currency between periods. In particular, the timing of tax collections must generally "matter" when the government, as well as the private sector, is obligated to hold currency between periods. Both Ricardian equivalence theorems and classical theorems that open market operations are equivalent with pure currency unit changes require excusing the government from the requirement to hold cash between periods.

8. For a proof see Sargent (1987), proposition 5.3.

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