Comment on 'Fiscal Consequences for Mexico of Adopting the Dollar' by Christopher A. Sims

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1. Introduction

A main argument in Sims's paper is that by dollarizing, Mexico would for practical purposes stop issuing state-contingent debt, which it now issues as nominal debt. The nominal debt bears a random return because inflation is random. Sims points out that not issuing state-contingent debt in *some* form would restrict and therefore damage the fiscal policy of a benevolent government that faces neither commitment nor information problems, and that designs its fiscal policy optimally.

Sims formulates a model of tax-smoothing with state-contingent debt. He interprets the state-contingency as coming from random inflation and deflation, but the mathematics and economics are identical if the model is cast in terms of state-contingent indexed debt. Sims's tax-smoothing model is really not a version of Barro's (1979) but rather a special case of Lucas and Stokey's (1983) model of optimal fiscal policy with state contingent debt. Sims endorses one of Lucas and Stokey's main conclusions: that state-contingent debt issues are important components of an optimal fiscal policy. Sims interprets dollarization as a proposal that will eliminate the Mexican government's ability to issue state contingent debt, because he assumes that Mexico will not be able to borrow or lend with explicitly state-contingent indexed debt. He does not describe a model of why it won't be able to, an issue to which I return in my concluding section.

So how much worse would things be without state contingent debt? To evaluate the *quantitative* importance of Sims's recommendation to retain state-contingent government debt, it is important to compare the Lucas-Stokey-Sims model with outcomes under a version of Barro's (1979) tax smoothing model. In that model, the government chooses an optimal fiscal policy but can buy or sell only risk-free one-period securities. I propose using a version of Barro's model as a benchmark for evaluating the quantitative benefits of issuing state-contingent debt. To understand what parameters will determine the quantitative magnitude, it is useful to turn to precautionary savings models.

Sims's tax-smoothing models is isomorphic to a consumption-smoothing model. So is our version of Barro's. I'll briefly describe savings problems with and without state-contingent debt and link them to corresponding tax-smoothing models. Every issue in the consumption smoothing models surfaces in a corresponding tax smoothing model. Table 1.1 shows correspondences among pairs of variables in the consumption and tax models. On the basis of the isomorphism between these models, I will make the following points:

1. With state contingent debt and 'risk-neutral' state-contingent prices, a consumer finds it optimal completely to smooth consumption over time and across states. Optimal consumption is constant.
2. With state-contingent debt and 'risk-neutral' state-contingent prices, the optimal fiscal policy sets tax revenues to be constant across states and time. This is Sims's result. It is the analogue of result 1.
3. *Without* state-contingent debt, the consumer engages in precautionary saving to smooth income fluctuations across states and time. Under circumstances described by Chamberlain and Wilson (2000), a special instance of which we study below, and without constraints on the household's asset holdings, occasionally binding *borrowing* constraints cause the consumer's assets and consumption either to converge to their bliss levels or to diverge to $+\infty$.
4. Without state-contingent debt, the optimal fiscal policy also involves precautionary saving. Under conditions corresponding to those just mentioned for the consumption model, the optimal fiscal policy has government assets converge to a level high enough that government expenditures can always be financed from interest on government-owned assets; tax rates and revenues converge to zero.
5. In the models without state contingent debt, the asset limits determine whether a submartingale convergence theorem operates to push assets into the region where the

problem of distorting taxation might vanish, as mentioned in 3 and 4. To get government assets to converge to a non-trivial invariant distribution requires putting an ad hoc limit on government holdings of assets, thereby arresting the submartingale convergence theorem. When an ad hoc limit on government assets is imposed, consumption and tax revenues behave like martingales off corners, in the fashion of Chamberlain and Wilson (2000) and Barro (1979).

6. The welfare loss in giving up state-contingent government debt involves issues like those studied by Krusell and Smith (1998). In precautionary savings models with small serial dependence of income shocks, self-insurance is cheap and effective: allocations tend to be almost as good as those under access to complete markets. However, when shocks are highly serially dependent, allocations with self-insurance deteriorate relative to those with complete insurance. Analogous points arise in the tax smoothing models. Thus, the quantitative significance of Sims's recommendation to retain state-contingent debt depends on the serial dependence of the government expenditure process.
7. The analogy between consumption-smoothing models and tax-smoothing models can be exploited more than it has yet been in the literature, and this can help make contact with issues that arose frequently in discussions of various papers at the conference, including Sims's. Typically, proponents of dollarization at the conference appealed to commitment and information problems that somehow rendered dollarization more credible and more likely to produce good outcomes. Those proponents presented no models, however.

As we shall see, the tax-smoothing model without state-contingent debt makes room for a policy that Hume described:

"It appears to have been the common practice of antiquity, to make provision, during peace, for the necessities of war, and to hoard up treasures before-hand, as the instruments either of conquest or defence; without trusting to extraordinary impositions, much less to borrowing, in times of disorder and confusion." David Hume, 'Of Public Credit,' 1777.

Table 1.1: Isomorphism between consumption and tax models

y_t	labor income	g_t	government purchases
c_t	consumption	T_t	tax revenues
$u(c_t)$	utility function	$W(T_t)$	$-$ deadweight loss
b_t	private assets	b_t^g	government assets

2. Consumption smoothing with state-contingent debt

A consumer has an exogenous endowment or 'labor income' process y_t that is governed by a discrete state Markov chain with transition matrix π and initial distribution π_0, with $y_t \in [y_{\min}, y_{\max}]$. We shall usually assume that the invariant distribution of π is unique and has full, or at least, nontrivial support. However, we'll also briefly consider the case in which g_t has an absorbing state. Conditional on an observed y_0, the Markov chain for y induces a sequence of distributions $\pi_t(y^t)$ over sequences of histories $y^t = [y_0, y_1, \dots, y_t]$ of y_s. Assume that one-period utility $u(c)$ is a function of the consumption of a single good and that $u(c)$ is increasing, strictly concave, with $\lim_{c \downarrow 0} u'(c) = +\infty$ and $\lim_{c \uparrow +\infty} u'(c) = 0$. The consumer has preference over stochastic processes for consumption that are ordered by

$$\sum_{t=0}^{\infty} \sum_{y^t} \beta^t u\left(c_t\left(y^t\right)\right) \pi_t\left(y^t\right), \tag{2.1}$$

where $\beta \in (0,1)$. At time 0, after y_0 has been observed, the consumer can purchase and sell history-y^t-contingent securities at prices $p_t(y^t)$, the prices of the Arrow-Debreu securities that trade at time 0 after y_0 has been realized. The consumer maximizes (2.1) subject to

$$\sum_t \sum_{y^t} p_t\left(y^t\right) c_t\left(y^t\right) \leq \sum_t \sum_{y^t} p\left(y^t\right) y_t. \tag{2.2}$$

Assume that state-contingent prices are exogenous and given by

$$p_t\left(y^t\right) = \beta^t \pi_t\left(y^t\right). \tag{2.3}$$

Under assumption (2.3), the solution of the household's optimization problem is

$$u'(c_t) = \lambda \quad \forall t \quad \forall y^t, \tag{2.4}$$

where λ is the Lagrange multiplier on the household's single Arrow-Debreu budget constraint (2.2). Equation (2.4) states consumption is constant. There is complete smoothing of consumption across all states and all dates. The multiplier λ and therefore the constant level of consumption c depend on π, π_0.

As usual, there is a version of this theory with Arrow securities. Here the household faces a sequence of budget constraints

$$c_t + b_{t+1}(y_{t+1})\, q_t(y_{t+1}|y_t) \leq y_t + b_t\left(y^t\right) \tag{2.5}$$

where $q_t(y_{t+1}|y_t)$ is the price kernel for one-period Arrow securities and $b_{t+1}(y_{t+1})$ is the amount of one-period Arrow securities purchased at t , and Arrow securities are constrained by

$$-b_{t+1}(y_{t+1}) \leq M(y_{t+1}|y_t),$$

where $M(y_{t+1}|y_t)$ is the 'natural borrowing' limit determined by the present value of the household's stream of labor income from time $t+1$, state y_{t+1}. If we assume that

$$q_t(y_{t+1}|y_t) = \beta\pi(y_{t+1}|y_t), \tag{2.6}$$

then the consumption theory with Arrow securities matches that with the Arrow-Debreu structure. Note that (2.6) implies that the risk-free gross one-period interest rate is β^{-1}

3. Consumption smoothing without state-contingent debt

To get a version of the 'savings problem' studied by Chamberlain and Wilson (2000), assume that the household can own or issue only one-period risk-free debt with gross rate of return between t and $t+1$ equal to R_t. The rest of the setup remains as above. Now the consumer maximizes (2.1) subject to the sequence of budget constraints

$$R_t^{-1} b_t + c_t \leq y_t + b_{t-1} \tag{3.1}$$

where b_t is denominated in time $t+1$ goods but measurable with respect to time $t-1$ information. This measurability restriction is what it means for the debt to be risk-free (see Marcet, Sargent, and Seppälä (2000)). We assume that

$$R_t = \beta^{-1}. \tag{3.2}$$

We must impose debt limits:

$$\underline{M} \leq b_t \leq \overline{M}. \tag{3.3}$$

There is a so-called 'natural borrowing limit' (Aiyagari (1994)):

$$\underline{M} = \frac{-y_{\min}}{\rho}$$

where $\beta = \frac{1}{1+\rho}$. This is the highest value of debt that the household can repay almost surely (even with consumption set to zero for all t). What about the upper limit $\overline{M}$? It is usually set at $+\infty$ in the theory of consumption, but this is a choice we will want to reconsider when we look at the tax-smoothing model that is isomorphic to the consumption model.

Associated with the problem of maximizing (2.1) subject to (3.1), (3.2) are the following Euler inequalities:

$$E_t u'(c_{t+1}) = u'(c_t) \text{ if } \underline{M} < b_t < \overline{b} \tag{3.4a}$$

$$E_t u'(c_{t+1}) > u'(c_t) \text{ implies } b_t = \overline{M} \tag{3.4b}$$

$$E_t u'(c_{t+1}) < u'(c_t) \text{ implies } b_t = \underline{M}. \tag{3.4c}$$

We call $\overline{M} = +\infty$ the 'natural asset limit'. With $\overline{M} = +\infty$, inequalities (3.4) imply that

$$E_t u'(c_{t+1}) \leq u'(c_t), \tag{3.5}$$

which implies that marginal utility of consumption is a bounded supermartingale It is bounded below because $\lim_{c \uparrow +\infty} u'(c) = 0$. Then the supermartingale convergence theorem implies that $u'(c)$ converges a.s. There are two possibilities. (1) If π has an absorbing state for y_t, then c_t can converge to a constant. (2) With a nontrivial invariant distribution for y_t, $u'(c)$ converges to 0, and therefore c diverges to $+\infty$ or to the bliss point of consumption. This is a standard result in the literature on the savings problem. (See Chamberlain and Wilson (2000)).

If a finite ad hoc upper limit is put on the assets that a consumer can accumulate, the supermartingle for the marginal utility of consumption will be lost. Then it is possible for the distribution of the consumer's assets to converge to a nontrivial invariant distribution. The marginal utility of consumption will behave like a martingale only off corners, and consumption will slowly wander.

4. The tax smoothing model of Lucas and Stokey

Sims's version of Lucas and Stokey's tax-smoothing model comes from replacing y_t in the consumption model with g_t (exogenous government purchases), c_t with T_t (government revenues raised at t) and $u(c)$ with $W(T)$, defined to be *minus* the deadweight loss from resorting to distorting taxes. Thus, assume that g_t is Markov with transition matrix and initial distribution π, π_0 and that $g_t \in [g_{\min}, g_{\max}]$. As earlier, we'll usually assume that the invariant distribution has full support, but we'll also mention what happens when π has an absorbing state. Let $W(T)$ be defined on $T \in [0, T_{\max}] \equiv \mathcal{T}$. Assume that $W(T)$ is strictly *decreasing* on $\mathcal{T}$, twice continuously differentiable and strictly concave. Assume that $W(T)$ has a strict maximum of 0 at $T = 0$.

We now make what we shall call a 'small open economy' assumption:

$$p_t\left(g^t\right) = \beta^t \pi_t\left(g^t\right). \tag{4.1}$$

Later, we'll give a closed-economy, general equilibrium interpretation of this assumption.

The government's problem is to maximize

$$\sum_{t=0}^{\infty} \sum_{g^t} \beta^t W\left(T_t\right) \pi_t\left(g^t\right) \tag{4.2}$$

subject to

$$\sum_{t=0}^{\infty} \sum_{g^t} \left[T\left(g^t\right) - g_t\right] p_t\left(g^t\right) = 0. \tag{4.3}$$

The solution is

$$W'\left(T_t\right) = -\lambda_0, \tag{4.4}$$

where the single Lagrange multiplier λ depends on the π, π_0. Equation (4.4) implies constant taxes. As usual, there is a version with one-period Arrow-securities, whose construction mimics that for the consumption model above.

5. Tax smoothing in an almost Barro model

Following Marcet, Sargent, and Seppälä (2000), we now redo the tax smoothing problem assuming that the government can only issue or own one-period risk-free debt. We make what we now call a 'small open economy' assumption:

$$R_t^{-1} = \beta. \tag{5.1}$$

The government's problem is to choose $\{T_t, b_{t-1}^g\}_{t=0}^{\infty}$ to maximize (4.2) subject to the sequence of constraints

$$b_t^g \leq \beta^{-1}\left[g_t + b_{t-1}^g - T_t\right]$$
$$b_t^g \in \left[\underline{M}, \overline{M}\right]$$

For the limiting behavior of this taxes in this problem, what is most important is the limit on the government's asset holdings. There is a natural asset limit:

$$\overline{M} = -\frac{g_{\max}}{1-\beta}. \tag{5.2}$$

This is a value of government assets so large that the government could finance even the largest government purchases forever out of interest on its asset holdings.

This model is isomorphic to the consumption model with only risk-free borrowing and lending. Under a natural asset limit, the marginal deadweight loss of revenues follows a submartingale that is bounded above by zero. Therefore, the marginal deadweight loss and revenues both converge almost surely. Again, we consider two cases: (1) If π has an absorbing state, R_t can converge to a positive constant; (2) if π has a nontrivial invariant distribution, R converges almost surely to zero as the government eventually acquires a stock of assets sufficient to finance all expenditures from interest earnings. Evidently, the tail of the tax-policy converges to the first-best value of zero revenues with zero distortions.

As in the consumption smoothing model, putting an upper limit on government assets will arrest convergence of government revenues to zero, allow government assets to converge to a nontrivial invariant distribution, and allow tax collections to behave as a martingale off corners, as Barro (1979) suggested.

6. A general equilibrium formulation

As promised, we now give a general equilibrium rationalization of the 'small country' assumptions in the tax-smoothing models. Marcet, Sargent, and Seppälä (2000) described restrictions on Lucas and Stokey's (1983) pure exchange economy that lead to the above ad-hoc tax smoothing model. Thus, Lucas and Stokey assumed a representative household that orders stochastic processes of consumption c_t and leisure x_t according to

$$E_0 \sum_{t=0}^{\infty} \beta^t v\left(c_t, x_t\right),$$

where $v(c, x)$ is strictly increasing and concave in c and x, where the technology is

$$c_t + g_t = 1 - x_t.$$

The government raises revenues $T_t = \tau_t(1 - x_t)$ by imposing a flat-rate tax on labor. Assume that g_t is Markov as above. To rationalize the above 'small country' asset pricing outcomes, assume that

$$u\left(c, x\right) = c + H\left(x\right) \tag{6.1}$$

where $H' > 0, H'' < 0, \lim_{x \downarrow 0} H'(x) = +\infty, H'(1) < 1$. Then under complete markets the household's first-order conditions imply

$$p_t\left(g^t\right) = \beta^t \pi_t\left(g^t\right)$$

$$1 - H'\left(x_t\right) = \tau_t$$

Notice that

$$T = \left(1 - H'\left(x\right)\right)\left(1 - x\right).$$

Let x_1 be first best $(1 = H'(x))$ and x_2 be the point of maximum revenues. Then $T(x_1) = 0, T'(x_1) > 0, T'' < 0, T'(x_2) = 0$. We can invert $T(x)$ on $[x_1, x_2]$ to get

$$x = x\left(T\right)$$

on $T \in [0, T(x_2)]$. Here x is a convex increasing function on $[0, T(x_2)]$.

It follows that

$$u(c, x) = 1 - g - x(T) + H(x(T))$$

which we can write as

$$u(c, x) = 1 - g + W(T),$$

where $W(T)$ is *minus* the deadweight loss function described above. This verifies that Lucas and Stokey's Ramsey problem reduces to the one solved above.

7. Conclusions

The analogy between consumption-smoothing models and tax-smoothing models is instructive. It can be exploited still more than it has been in the literature. There exist natural extensions that make contact with key issues that arose frequently in discussions of various papers at the conference, including Sims's. In their papers and verbal discussions, proponents of dollarization often appealed to commitment and information problems that somehow render dollarization more credible and more likely to produce good outcomes. Those proponents presented no models of how dollarization was connected with credibility. We need some models. Consumption smoothing models with information and enforcement problems contain mechanisms that can be extended to the tax-smoothing literature. Three examples are (1) Thomas and Worrall's (1990) model of consumption smoothing in the face of unobserved income, (2) Kocherlakota's (1996) model of consumption smoothing with perfect information but an enforcement problem, and (3) Atkeson's (1988, 1991) model of consumption-smoothing in the face of both information and enforcement problems. By using the correspondence in Table 1.1, we might start to build models of optimal fiscal policy in situations where information and enforcement problems hinder governments from issuing state contingent debt. Such models could help extend Sims's discussion of why the government of a country like Mexico might not be able to issue state-contingent indexed debt to smooth fiscal shocks (because of information problems like those in Atkeson's model?) although it now offers its nominal debt holders a state contingent return via random fluctuations in inflation.

8. References

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