Managing expectations and fiscal policy

Anastasios G. Karantounias

with Lars Peter Hansen and Thomas J. Sargent*

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Abstract

This paper studies an optimal fiscal policy problem of Lucas and Stokey (1983) but in a situation in which the representative agent's distrust of the probability model for government expenditures puts model uncertainty premia into history-contingent prices. This gives rise to a motive for expectation management that is absent within rational expectations and a novel incentive for the planner to smooth the shadow value of the agent's subjective beliefs in order to manipulate the equilibrium price of government debt. Unlike the Lucas and Stokey (1983) model, the optimal allocation, tax rate, and debt all become history dependent despite complete markets and Markov government expenditures.

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*First draft: November 2007. Karantounias (corresponding author): Federal Reserve Bank of Atlanta (e-mail: anastasios.karantounias@atl.frb.org); Hansen: University of Chicago (e-mail: l-hansen@uchicago.edu); Sargent: New York University and Hoover Institution (e-mail: thomas.sargent@nyu.edu). This paper circulated previously under the title "Ramsey taxation and fear of misspecification." We are thankful to David Backus, Marco Bassetto, Pierpaolo Benigno, Marco Cagetti, Steve Coate, Kris Gerardi, Ricardo Lagos, Andreas Lehnert, Guido Lorenzoni, Monika Piazzesi, Will Roberds, John Rust, Martin Schneider, John Shea, Karl Shell, Ennio Stacchetti, Tom Tallarini, Viktor Tsyrennikov, Tao Zha and to seminar participants at Birkbeck College, Cornell, EIEF, FRB, FRB Atlanta, FRB Chicago, Iowa, Maryland, NYU, Oxford, Penn State, Warwick and Wharton. Karantounias would like to thank, without implicating, the Research and Statistics Division of the FRB and the Monetary Policy Strategy Division of the ECB for their hospitality and support. Sargent's research was supported by a grant to the National Bureau of Economic Research from the National Science Foundation. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Atlanta or the Federal Reserve System.

1 Introduction

Optimal policy design problems heavily exploit the rational expectations assumption that attributes a unique and fully trusted probability model to all agents. That useful assumption precludes carrying out a coherent analysis that attributes fears of model misspecification to some or all agents.

It seems natural to ask the question: How should we approach policy design problems in macroeconomics when at least some agents distrust the model? This question is not just of academic interest but of particular practical relevance. Lack of confidence in models seems to have become pronounced in the recent financial crisis and has entered policy discussions. Caballero and Krishnamurthy (2008), for example, impute *sets* of probability models and a max-min criterion to private agents as a way to model Knightian uncertainty when a lender of last resort copes with flights to quality, whereas Uhlig (2009) appeals to uncertainty aversion to justify pessimism during bank runs. Our approach can be viewed as putting a particular structure on a decision maker's set of models and thereby on his pessimism. This additional structure opens up channels of influence for policy makers not present in the analyses of Caballero and Krishnamurthy (2008) and Uhlig (2009).[1]

This paper features a notion of *expectation management* that is absent from the standard rational expectations paradigm. We formulate an optimal policy problem in which private agents' fears of model misspecification cause them to adjust their expectations in ways that a Ramsey planner recognizes and exploits, bringing the household's endogenous beliefs into the forefront of an optimal policy design problem because they affect equilibrium prices.

We study a Ramsey fiscal policy problem in which a planner knows that a representative household distrusts a probability model for exogenous sequences of government expenditures, while the planner still trusts it. We start with the complete-markets economy without capital analyzed by Lucas and Stokey (1983), but modify the representative household's preferences to express his concerns about misspecification of the stochastic process for government expenditures. The planner can use a distortionary tax on labor income and issue state-contingent debt in order to finance the exogenous government expenditures. Our household expresses distrust of his model by ranking consumption plans according to the multiplier preferences of Hansen and Sargent (2001); when a multiplier parameter assumes a special value, the expected utility preferences of Lucas and Stokey (1983) emerge as a special case in which the decision maker completely trusts his probability model.[2]

[1]For further policy recommendations based on the insights of Caballero and Krishnamurthy (2008) see Caballero and Kurlat (2009).

[2]Multiplier preferences lead to tractable functional forms. See Maccheroni et al. (2006a,b) and Strzalecki

The Lucas and Stokey (1983) environment isolates essential dimensions of an optimal macroeconomic policy design problem in which a representative household's ambiguity about its statistical model creates an avenue that motivates the planner to manipulate the household's beliefs, because they affect equilibrium Arrow-Debreu prices.

More specifically, the Ramsey planner and the household share a common sequence of transition densities $\pi_{t+1}(g_{t+1}|g^t)$ for government spending g_{t+1} conditional on histories g^t of g_s for s from 0 to t. The household's concern about misspecification leads it to twist $\pi_{t+1}(g_{t+1}|g^t)$ pessimistically by multiplying it by a conditional likelihood ratio

$$m^*_{t+1}(g^{t+1}) = \frac{\exp\left(-\frac{V_{t+1}(g^{t+1})}{\theta}\right)}{\sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) \exp\left(-\frac{V_{t+1}(g^{t+1})}{\theta}\right)}, \tag{1}$$

where $V_{t+1}(g^{t+1})$ is a continuation value and $\theta \in [\underline{\theta}, +\infty]$ is a positive multiplier parameter expressing the household's distrust of $\pi_{t+1}(g_{t+1}|g^t)$. Multiplication by (1) raises probabilities of events that give rise to lower continuation values and lowers those giving rise to higher continuation values. The continuation values are themselves constructed recursively in a way that makes them depend on future distortions m^*_{t+j}. In a competitive equilibrium, the household's distorted probabilities $\tilde{\pi}_{t+1}(g_{t+1}|g^t) = m^*_{t+1}\pi_{t+1}(g_{t+1}|g^t)$ help determine the prices of date- and history-contingent securities. The Ramsey planner cares about these prices because he wants to manipulate the value of government debt passed from one period to the next. The fact that the distortions m^*_{t+1} depend on continuation values, which in turn depend on continuation allocations, means that the planner influences the household's beliefs and equilibrium prices by its choices of state contingent tax and borrowing strategies. The endogeneity of the household's pessimistic beliefs contributes additional restrictions on allocations that supplement implementability conditions already present in Lucas and Stokey's model. The planner's response to these additional implementability conditions injects a source of history dependence into taxes, government debt, and allocations that is not present in Lucas and Stokey (1983).

A salient feature of the Ramsey plan of Lucas and Stokey (1983) is the absence of history dependence in allocations, tax rates, and government debt. For example, with Markov government expenditures, the value of government debt at date t depends only on the date t value of the Markov state that drives government expenditures. Lucas and Stokey failed to rationalize the permanent-income like predictions of Barro (1979) that put extensive history

(2008) for axioms that rationalize multiplier preferences as expressions of model ambiguity.

dependence into tax rates and government debt. The impression that observed time series of government debt and taxes have apparently exhibited history dependence – observed series on government debt are much smoother series than the ones predicted by the Lucas-Stokey model and more like those in Barro's model – prompted Aiyagari et al. (2002) and Battaglini and Coate (2008) to put history dependence into a Ramsey plan, in the model of Aiyagari et al. (2002), or a political-economic bargaining equilibrium, in the model of Battaglini and Coate (2008), by dropping Lucas and Stokey's assumption of complete markets. In our setup, we retain the assumption of complete markets, but find that history dependence emerges as a consequence of optimal policy, reflecting the planner's management of the household's endogenous beliefs. For example, with quasi-linear preferences, dependence on the past can be quite striking: for small doubts about the model, we show that the tax rate becomes a random walk, whereas it would stay constant with full confidence in the model.

We use tools from the recursive contracts literature and utilize the Marcet and Marimon (1998) method to find state variables with which to cast a recursive representation of the Ramsey problem. Two martingales capture the history dependence of the optimal allocation.

What are the economic insights that emerge in our environment? Our state variables let us identify a novel intertemporal smoothing motive for the planner in the form of a desire to smooth the shadow value of the household's continuation value by making it a martingale. There is a simple intuition behind this finding that underscores the price manipulation that partly motivates the planner: the planner strives to make consumption claims cheaper when he buys them and more expensive when he sells them, by manipulating the household's endogenous beliefs.

To see that clearly, assume that the government expenditures take two values and that there is a sequence of high shocks. Complete markets allow the planner to hedge shocks by buying ex ante assets for the case of high shocks and selling ex ante assets (issuing government debt) for the case of low shocks. In the Lucas and Stokey (1983) model, the planner's optimal policy depends only on the realization of the shock every period, and therefore entails a constant tax rate, and a constant government deficit every period, corresponding to the sequence of high shocks. With doubts about the model though, the planner has an incentive to decrease the tax rate over time and increase the assets that he is buying ex ante. The reason is that by decreasing the tax rate the planner raises the household's utility and leads the household to assign a lower probability on these contingencies, as seen from (1), thereby decreasing the equilibrium price of consumption claims that the government buys. The opposite would happen in the case of a sequence of low shocks, leading to an increasing sequence of tax rates in order to increase the equilibrium price (decrease the interest rate) of debt that the government issues. Therefore the planner *front-loads* taxes in the case of a sequence of high government expenditure shocks and *back-loads* taxes in the case of a sequence of low government expenditure shocks in order to affect equilibrium prices through beliefs.

This illustrates the expectation management aspect of optimal policy. An important feature of our optimal policy problem that needs to be stressed is the fact that expectation management is not activated by a *difference* of beliefs between the planner and household. Clearly though, the heterogeneity in beliefs consists an additional force that shapes our results. A planner that does *not* doubt the model has an incentive to tax *more* when there are high fiscal shocks because he considers them less probable than the pessimistic household and *less* when there are low fiscal shocks since he considers more probable than the household, leading to a behavior that acts in the opposite direction than his price manipulation efforts.

1.1 Related literature

Other contributions that share our aim of attributing misspecification fears to at least some agents include Kocherlakota and Phelan (2008), who study a mechanism design problem using a max-min expected utility criterion and Hansen and Sargent (2007, ch. 16), who formulate a model in which a Stackelberg leader distrusts an approximating model while a competitive fringe of followers completely trusts it.[3] Hansen and Sargent's assumptions about the leader's and followers' specification concerns in effect reverse the ones made here. In several ways, Woodford (2008) is the most interesting previous paper for us because he also uses a general equilibrium model and because of how the timing is set up to conceal the private sector's beliefs from the government. In Woodford's model, while both the government and the private sectors fully trust their *own* models, the government distrusts its knowledge of the private sector's beliefs about prices. Arranging things so that this is possible is subtle because with enough markets, equilibrium prices and allocations reveal private sector beliefs. In contrast to Woodford, we set things up with complete markets whose prices fully reveal private sector beliefs to the Ramsey planner.

Any analysis with multiple subjective probability models requires a convenient way to express those models. Along with Woodford (2008), this paper uses the martingale representation of Hansen and Sargent (2005, 2006) and Hansen et al. (2006). From the point of view of the approximating model, these martingale perturbations look like multiplicative

[3]Our work is also linked in a general sense to that of Brunnermeier et al. (2007), who study a setting in which households choose their beliefs.

preference shocks. In the present context, the Ramsey planner manipulates those 'shocks'.

This paper resides at the intersection of three strands of literature. Optimal policy analysis by Bassetto (1999), Chari et al. (1994), Zhu (1992), Angeletos (2002) and Buera and Nicolini (2004) in complete markets, or in incomplete markets by Aiyagari et al. (2002), Shin (2006) and Marcet and Scott (2009), and recursive representations as in Chang (1998) and Sleet and Yeltekin (2006) are all relevant antecedents of work. The multiplier preferences we are using are closely related to risk-sensitive preferences and to Epstein and Zin (1989) and Weil (1990) preferences and therefore our work is also related to Anderson (2005) and Tallarini (2000), who study the impact of risk-sensitivity on risk-sharing and on business cycles respectively, as well as to Hansen et al. (1999), who study the effect of doubts about the model on permanent income theory and asset prices. Another related line of work is Farhi and Werning (2008), who analyze the implications of recursive preferences in private information setups.

2 The economy

To create an avenue for the planner to manipulate beliefs, we modify the preferences of the representative consumer but not the planner in the model of Lucas and Stokey (1983). Time $t \geq 0$ is discrete and the horizon infinite. Labor is the only input into a linear technology that produces one perishable good that can be allocated to private consumption c_t or government consumption g_t. Markets are complete and competitive. The only source of uncertainty is an exogenous sequence of government expenditures g_t that potentially takes on a finite or countable number of values. Let $g^t = (g_0, ..., g_t)$ denote the history of government expenditures. Equilibrium plans for work and consumption have date t components that are measurable functions of g^t. A representative agent is endowed with one unit of time, works $h_t(g^t)$, enjoys leisure $l_t(g^t) = 1 - h_t(g^t)$ and consumes $c_t(g^t)$ at history g^t for each $t \geq 0$. One unit of labor can be transformed into one unit of the good. Feasible allocations satisfy

$$c_t(g^t) + g_t = h_t(g^t). \tag{2}$$

Competition makes the real wage $w_t(g^t) = 1$ for all $t \geq 0$ and any history g^t. The government finances its time t expenditures either by using a linear tax $\tau_t(g^t)$ on labor income or by issuing a vector of state-contingent debt $b_{t+1}(g_{t+1}, g^t)$ that is sold at price $p_t(g_{t+1}, g^t)$ at history g^t and promises to pay one unit of the consumption good if government expenditures are g_{t+1}

next period and zero otherwise. The one-period government budget constraint at t is

$$b_t(g^t) + g_t = \tau_t(g^t)h_t(g^t) + \sum_{g_{t+1}} p_t(g_{t+1}|g^t)b_{t+1}(g_{t+1}, g^t). \tag{3}$$

Equivalently, we can work with an Arrow-Debreu formulation in which all trades occur at date 0 at Arrow-Debreu history- and date-contingent prices $q_t(g^t)$. In this setting, the government faces the single intertemporal budget constraint

$$b_0 + \sum_{t=0}^{\infty}\sum_{g^t} q_t(g^t)g_t \leq \sum_{t=0}^{\infty}\sum_{g^t} q_t(g^t)\tau_t(g^t)h_t(g^t).$$

2.1 Fear of model misspecification

The representative agent and the government share an approximating model in the form of a sequence of joint densities $\pi_t(g^t)$ over histories g^t $\forall t \leq \infty$. Following Hansen and Sargent (2005), we characterize model misspecifications with multiplicative perturbations that are martingales with respect to the approximating model. The representative agent, but not the government, fears that the approximating model is misspecified in the sense that the history of government expenditures will actually be drawn from a joint density that differs from the approximating model but is absolutely continuous with respect to the approximating model over finite time intervals. Thus, by the Radon-Nikodym theorem there exists a non-negative random variable M_t with $E(M_t) = 1$ that is a measurable function of the history g^t and that has the interpretation of a change of measure. The random variable M_t, which we take to be a likelihood ratio $M_t(g^t) = \frac{\tilde{\pi}_t(g^t)}{\pi_t(g^t)}$ of a distorted density $\tilde{\pi}_t$ to the approximating density π_t is a martingale, i.e., $E_t M_{t+1} = M_t$ where E denotes expectation with respect to the approximating model. Here the tilde refers to a distorted model. Evidently, we can compute the mathematical expectation of a random variable $X_t(g^t)$ under a distorted measure as

$$\tilde{E}(X_t) = E(M_t X_t).$$

To attain a convenient decomposition of M_t, define

$$m_{t+1} \equiv \frac{M_{t+1}}{M_t} \text{ for } M_t > 0$$

and let $m_{t+1} \equiv 1$ when $M_t = 0$, (i.e., when the distorted model assigns zero probability to a particular history). Then

$$M_{t+1} = m_{t+1} M_t \qquad (4)$$
$$= M_0 \prod_{j=1}^{t+1} m_j.$$

The non-negative random variable m_{t+1} distorts the conditional probability of g_{t+1} given history g^t, so that it is a conditional likelihood ratio $m_{t+1} = \frac{\tilde{\pi}_{t+1}(g_{t+1}|g^t)}{\pi_{t+1}(g_{t+1}|g^t)}$. It has to satisfy the restriction that $E_t m_{t+1} = 1$ in order qualify as a distortion to the conditional measure. We measure discrepancies between conditional distributions by relative entropy, which is defined as

$$\varepsilon_t(m_{t+1}) = E(m_{t+1} \log m_{t+1} | g^t).$$

Note that relative entropy is zero if the approximating and perturbed models coincide and positive otherwise. Relative entropy is the expected log-likelihood ratio under the perturbed model.

2.2 Preferences

To represent fear of model misspecification, we use the multiplier preferences of Hansen and Sargent (2001) and Hansen et al. (2006) to describe how the representative consumer ranks consumption, leisure plans whose time t components are measurable functions of g^t: [4]

$$\min_{\{m_{t+1}, M_t\}_{t=0}^{\infty} \geq 0} \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi_t(g^t) M_t(g^t) U(c_t(g^t), 1 - h_t(g^t)) + \beta\theta \sum_{t=0}^{\infty} \sum_{g^t} \beta^t \pi_t(g^t) M_t(g^t) \varepsilon_t(m_{t+1}) \qquad (5)$$

[4] In effect, we constrain the set of perturbations by the following constraint on a measure of discounted entropy

$$\beta E\Big[\sum_{t=0}^{\infty} \beta^t M_t E(m_{t+1} \log m_{t+1} | g^t) \Big| g_0\Big] \leq \eta$$

where η measures the size of an entropy ball of models surrounding the approximating model. This constraint could be used to formulate the constraint preferences of Hansen and Sargent (2001). They discuss the relation between constraint preferences and the multiplier preferences featured in this paper and show how to construct η *ex post* as a function of the multiplier θ in (5) and other parameters.

where $U(c_t, 1-h_t)$ is the same period utility function assumed by Lucas and Stokey (1983) and the multiplier $\theta > 0$ is a penalty parameter that measures fear of model misspecification.[5] Higher values of the multiplier parameter θ represent more confidence in the approximating model π_t. Full confidence is captured by $\theta = \infty$, which reduces the above preferences to the expected utility preferences of the Lucas and Stokey household. Along with Lucas and Stokey, we assume that $U(c, 1-h)$ is strictly increasing, strictly concave, and thrice continuously differentiable.[6]

2.3 The representative household's problem

For any sequence of random variables $\{a_t\}$, let $a \equiv \{a_t(g^t)\}_{t,g^t}$. The problem of the consumer is[7]

$$\max_{c,h} \min_{M\geq 0, m\geq 0} \quad \sum_{t=0}^{\infty} \beta^t \sum_{g^t} \pi_t(g^t) M_t(g^t) \Big[U(c_t(g^t), 1-h_t(g^t)) \\ + \theta\beta \sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) m_{t+1}(g^{t+1}) \ln m_{t+1}(g^{t+1}) \Big]$$

subject to

$$\sum_{t=0}^{\infty} \sum_{g^t} q_t(g^t) c_t(g^t) \leq \sum_{t=0}^{\infty} \sum_{g^t} q_t(g^t)(1-\tau_t(g^t)) h_t(g^t) + b_0 \tag{6}$$

$$c_t(g^t) \geq 0, h_t(g^t) \in [0,1] \forall t, g^t \tag{7}$$

$$M_{t+1}(g^{t+1}) = m_{t+1}(g^{t+1}) M_t(g^t), M_0 = 1 \forall t, g^t \tag{8}$$

$$\sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) m_{t+1}(g^t) = 1, \forall t, g^t \tag{9}$$

Inequality (6) is the intertemporal budget constraint of the household. The right side is the discounted present value of after tax labor income plus an initial asset position b_0 that can assume positive (denoting government debt) or negative (denoting government assets) values.

[5]The multiplier preferences can be written recursively as

$$V_t = U(c_t, 1-h_t) + \beta \min_{m_{t+1}} \{E_t m_{t+1} V_{t+1} + \theta \varepsilon_t(m_{t+1})\}.$$

[6]Strict concavity is not satisfied for the quasi-linear example to be studied in subsection 6.1.

[7]We assume that uncertainty at $t = 0$ has been realized, so $\pi_0(g_0) = 1$. Thus, the distortion of the probability of the initial period is normalized to be unity, so that $M_0 \equiv 1$.

2.4 The inner problem: choosing beliefs

The inner problem chooses (M, m) to minimize the utility of the representative household subject to the law of motion of the martingale M and the restriction that the conditional distortion m integrates to unity. The optimal conditional distortion takes the exponentially twisting form:

$$m^*_{t+1}(g^{t+1}) = \frac{\exp\left(-\frac{V_{t+1}(g^{t+1})}{\theta}\right)}{\sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t)\exp\left(-\frac{V_{t+1}(g^{t+1})}{\theta}\right)}, \text{all } t \geq 0, g^t \tag{10}$$

where the asterisks denote optimal values and V_t is the utility of the household under the distorted measure, which follows the recursion

$$V_t = U(c_t, 1-h_t) + \beta[E_t m^*_{t+1} V_{t+1} + \theta E_t m^*_{t+1} \ln m^*_{t+1}]. \tag{11}$$

Equations (10) and (11) are the first-order conditions for the minimization problem with respect to m_{t+1} and M_t. Substituting (10) into (11) gives

$$V_t = U(c_t, 1-h_t) + \frac{\beta}{\sigma} \ln E_t(\exp(\sigma V_{t+1})) \tag{12}$$

where $\sigma \equiv -1/\theta$. Thus, the martingale distortion evolves according to

$$M^*_{t+1} = \frac{\exp\left(\sigma V_{t+1}(g^{t+1})\right)}{\sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t)\exp\left(\sigma V_{t+1}(g^{t+1})\right)} M^*_t, \quad M_0 \equiv 1. \tag{13}$$

Equation (13) asserts that the martingale distortion attaches higher probabilities to histories with low continuation utilities and lower probabilities to histories with high continuation utilities. Such exponential tilting of probabilities summarizes how the representative household's distrust of the approximating model produces conservative probability assessments that give rise to an indirect utility function that solves the recursion (12), an example of the discounted risk-sensitive preferences of Hansen and Sargent (1995).[8] For $\theta = \infty$ (or equivalently $\sigma = 0$) the conditional and unconditional distortion become unity $M^*_t = m^*_t = 1$, expressing the lack of doubts about the approximating model.

[8]The risk-sensitive recursion is closely related to the preferences of Epstein and Zin (1989) and Weil (1990).

2.5 Outer problem: choosing $\{c_t, h_t\}$ plan

An interior solution to the maximization problem of the household satisfies the intratemporal labor supply condition

$$\frac{U_l(g^t)}{U_c(g^t)} = 1 - \tau_t(g^t) \tag{14}$$

that equates the MRS between consumption and leisure to the after tax wage rate and the intertemporal Euler equation

$$q_t(g^t) = \beta^t \pi_t(g^t) M_t^*(g^t) \frac{U_c(g^t)}{U_c(g_0)}. \tag{15}$$

Here we have normalized the price of an Arrow-Debreu security at $t = 0$ to unity, so $q_0(g_0) \equiv 1$. The implied price of one-period state-contingent debt (an Arrow security) is

$$p_t(g_{t+1}, g^t) = \beta \pi_{t+1}(g_{t+1}|g^t) \frac{\exp\left(\sigma V_{t+1}(g^{t+1})\right)}{\sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) \exp\left(\sigma V_{t+1}(g^{t+1})\right)} \frac{U_c(g^{t+1})}{U_c(g^t)}. \tag{16}$$

Remark 1. *Doubts about the model show up as a worst-case conditional density in the determination of the equilibrium price of an Arrow security. The stochastic discount factor under the approximating model has an additional multiplicative element which depends on continuation utility, an* endogenous *forward-looking object. The presence of continuation utilities creates a channel by which a Ramsey planner influences equilibrium prices, augmenting the marginal utilities channel that is already present in Lucas and Stokey.*

Definition. *A competitive equilibrium is a consumption-labor allocation (c, h), distortions to beliefs (m, M), a price system q, and a government policy (g, τ) such that (a) given $(q, \tau), (c, h)$ and (m, M) solve the household's problem, and (b) markets clear, so that $c_t(g^t) + g_t = h_t(g^t) \forall t, g^t$.*

3 Ramsey Problem

A Ramsey planner chooses at $t = 0$ state-contingent debt and distortionary taxes on labor income at every history. While the representative household distrusts the approximating model π, the Ramsey planner completely trusts it. Therefore, the Ramsey planner chooses a competitive equilibrium allocation that maximizes the expected utility of the representative household *under the approximating model*.[9]

We use the primal approach employed by Lucas and Stokey (1983). The Ramsey planner chooses allocations subject to the resource constraint (2) and implementability constraints imposed by competitive equilibrium.

Proposition 1. *The Ramsey planner faces the following implementability constraints*

$$\sum_{t=0}^{\infty}\beta^t\sum_{g^t}\pi_t(g^t)M_t^*(g^t)U_c(g^t)c_t(g^t)=\sum_{t=0}^{\infty}\beta^t\sum_{g^t}\pi_t(g^t)M_t^*(g^t)U_l(g^t)h_t(g^t)+U_c(g_0)b_0, \quad (17)$$

the law of motion for the martingale that represents distortions to beliefs (13), and the recursion for the representative household's value function (12).

Proof. Besides the resource constraint, a competitive equilibrium is characterized fully by the household's two Euler equations, the intertemporal budget constraint (6) that holds with equality at an optimum, and equations (13) and (12), which describe the evolution of the endogenous beliefs of the agent. Use (14) and (15) to substitute for prices and after tax wages in the intertemporal budget constraint to obtain (17). □

Definition. *The Ramsey problem is*

$$\max_{(c,h,M^*,V)}\sum_{t=0}^{\infty}\beta^t\sum_{g^t}\pi_t(g^t)U\big(c_t(g^t),1-h_t(g^t)\big)$$

subject to

$$\sum_{t=0}^{\infty}\beta^t\sum_{g^t}\pi_t(g^t)M_t^*(g^t)[U_c(g^t)c_t(g^t)-U_l(g^t)h_t(g^t)]=U_c(g_0)b_0 \quad (18)$$

$$c_t(g^t)+g_t=h_t(g^t),\forall t,g^t \quad (19)$$

$$M_{t+1}^*(g^{t+1})=\frac{\exp\left(\sigma V_{t+1}(g^{t+1})\right)}{\sum_{g_{t+1}}\pi_{t+1}(g_{t+1}|g^t)\exp\left(\sigma V_{t+1}(g^{t+1})\right)}M_t^*(g^t),M_0(g_0)=1,\forall t,g^t \quad (20)$$

$$V_t(g^t)=U(c_t(g^t),1-h_t(g^t))+\frac{\beta}{\sigma}\ln\sum_{g_{t+1}}\pi_{t+1}(g_{t+1}|g^t)\exp\big(\sigma V_{t+1}(g^{t+1})\big), \quad \forall t,g^t,t\geq 1 \quad (21)$$

[9] In Karantounias et al. (2007), we study alternative sets of assumptions that allow the Ramsey planner to doubt the approximating model and also possibly instruct the planner to evaluate expected utilities using the representative household's beliefs. The current setup isolates key forces that also operate in that alternative setting.

Remark 2. *The presence of the distorted beliefs in (17) contributes two additional implementability constraints to those already in Lucas and Stokey (1983). The Ramsey planner takes into account how the allocation* (c, h) *affects the utility of the agent* $V_t(g^t)$*, which determines in turn the* endogenous *likelihood ratio* $M_t^*(g^t)$ *and the consequent* worst-case *beliefs. Note that we could interpret the minimization problem of the household in the description of the preferences in (5) as the problem of a malevolent alter ego who, by choosing a worst-case probability distortion, motivates the household to value robust decision rules. Along the lines of this interpretation, the Ramsey problem becomes a Stackelberg game with one leader and* <u>*two*</u> *followers, namely, the representative household and the representative household's malevolent alter ego.*

3.1 First-best benchmark (i.e., lump-sum taxes available)

By first-best, we mean the allocation that maximizes the expected utility of the household under π subject to the resource constraint (2). Note that for *any* beliefs of the planner, the first-best is characterized by the condition $\frac{U_l(g^t)}{U_c(g^t)} = 1$ and the resource constraint (2), so the first-best allocation $(\hat{c}, \hat{h})$ is independent of probabilities π. Private sector beliefs affect asset prices through (15), but not the allocation. Because lump-sum taxes are not available in our model, the planner's and the household's beliefs both affect allocations.

3.2 Optimality conditions of the Ramsey problem

Define for convenience $\Omega(c_t(g^t), h_t(g^t)) \equiv U_c(g^t)c_t(g^t) - U_l(g^t)h_t(g^t)$ [10] and attach multipliers Φ, $\beta^t \pi_t(g^t)\lambda_t(g^t)$, $\beta^{t+1}\pi_{t+1}(g^{t+1})\mu_{t+1}(g^{t+1})$, and $\beta^t \pi_t(g^t)\xi_t(g^t)$ to constraints (17), (2), (13), and (12), respectively.

First-order necessary conditions[11] for an interior solution are

[10] Note that Ω_t represents the equilibrium government surplus or deficit in marginal utility terms, $\Omega_t = U_{ct}[\tau_t h_t - g_t]$.

[11] We set up the Lagrangian and derive the first-order condition with respect to $V_t(g^t)$ in a separate appendix available upon request.

$$c_t: \quad (1+\xi_t(g^t))U_c(g^t) + \Phi M_t^*(g^t)\Omega_c(g^t) = \lambda_t(g^t), t \geq 1 \tag{22}$$

$$h_t: \quad -(1+\xi_t(g^t))U_l(g^t) + \Phi M_t^*(g^t)\Omega_h(g^t) = -\lambda_t(g^t), t \geq 1 \tag{23}$$

$$M_t^*: \quad \mu_t(g^t) = \Phi\Omega(g^t) + \beta \sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t) m_{t+1}^*(g^{t+1})\mu_{t+1}(g^{t+1}), t \geq 1 \tag{24}$$

$$\begin{aligned} V_t: \quad & \xi_t(g^t) = \sigma m_t^*(g^t) M_{t-1}^*(g^{t-1}) \Big[\mu_t(g^t) - \sum_{g_t} \pi_t(g_t|g^{t-1}) m_t^*(g^t)\mu_t(g^t)\Big] \\ & + m_t^*(g^t)\xi_{t-1}(g^{t-1}), t \geq 1 \end{aligned} \tag{25}$$

$$c_0: \quad (1+\xi_0)U_c(g_0) + \Phi M_0 \Omega_c(g_0) = \lambda_0(g_0) + \Phi U_{cc}(g_0) b_0 \tag{26}$$

$$h_0: \quad -(1+\xi_0)U_l(g_0) + \Phi M_0 \Omega_h(g_0) = -\lambda_0(g_0) - \Phi U_{cl}(g_0) b_0. \tag{27}$$

In (24) and (25), we used expression (10) for the optimal conditional likelihood ratio m_{t+1}^* to save notation.

Remark 3. *In formulating the Ramsey problem, the last constraint (12) applies only from period one on since the value of the agent at* $t = 0$ V_0 *is not relevant to the problem due to the normalization* $M_0 \equiv 1$. *We can set the initial value of the multiplier equal to zero* $\xi_0 = 0$ *to accommodate this. Equivalently, we could maximize with respect to* V_0 *to get an additional first-order condition* $\xi_0 = 0$.

Remark 4. *Since* $\xi_0 = 0, M_0 = 1$, *the first-order conditions (26, 27) for the initial consumption-labor allocation* (c_0, h_0) *are the same as the respective first-order conditions for the special Lucas and Stokey (1983) case where the representative consumer fears no misspecification.*

The first-order conditions (22-27) together with the constraints (18-21) determine the Ramsey plan.

4 Characterizing the Ramsey plan

In the first-order condition (22)

$$U_c(g^t) + \xi_t(g^t)U_c(g^t) + \Phi M_t^*(g^t)\Omega_c(g^t) = \lambda_t(g^t)$$

the first term on the left is the marginal utility the Ramsey planner gets by increasing consumption by one unit. The second term on the left measures how increasing consumption affects the representative household's value function and consequently its worst-case model perturbation M_{t+1}^*. As we shall see later, the multiplier ξ_t serves as a state variable in a recursive statement of the Ramsey problem. Note that if the Ramsey planner were not to take into account that the worst-case beliefs of the representative household are endogenous, this term would be zero. The third term on the left represents the typical constraints that a competitive equilibrium allocation imposes on the Ramsey planner through the value of the government surplus and describes how the government surplus (in marginal utility terms) is affected when consumption is increased. Note though that there is a twist due to the presence of the likelihood ratio M_t^*, stemming from the fact that equilibrium prices reflect the representative household's worst-case beliefs $\tilde{\pi}_t$, which differ from the planner's beliefs π_t. Had the planner and the household shared the same beliefs, M_t^* would not show up in (22). The right side is the shadow value of output. Analogous interpretations apply to the first-order condition (23) for h_t.

4.1 Optimal wedge

Substituting the derivatives of Ω with respect to c and h into first-order conditions (22) and (23) and combining the resulting expressions to eliminate the shadow value of output λ_t delivers

$$\begin{aligned} U_l(g^t) - U_c(g^t) &= \frac{\Phi M_t^*(g^t)}{1+\xi_t(g^t)+\Phi M_t^*(g^t)}\big[U_{cc}(g^t)c_t(g^t) - U_{cl}(g^t)(c_t(g^t)+h_t(g^t)) \\ &+ U_{ll}(g^t)h_t(g^t)\big]. \end{aligned} \tag{28}$$

This condition describes the optimal wedge $(U_l - U_c)$ that determines the optimal tax rate for $t \geq 1$. [12]

Proposition 2. *The Ramsey allocation and taxes from period one onward are history dependent.*

Proof. Use the resource constraint (2) to substitute for h_t in (28) and solve for optimal consumption terms in terms of (g_t, M_t^*, ξ_t) and the multiplier Φ to get $c_t = c(g_t, M_t^*, \xi_t; \Phi)$. Analogously, we get $h_t = h(g_t, M_t^*, \xi_t; \Phi)$ and consequently from (14) the optimal tax rate $\tau_t = \tau(g_t, M_t^*, \xi_t; \Phi)$. Therefore, the allocation and taxes at t depends on the history of shocks as intermediated through M_t^* and ξ_t. □

Relative to the outcome in Lucas and Stokey (1983), the representative household's fear of misspecification makes the Ramsey allocation depend on two additional variables, namely, the likelihood ratio M_t^* and the multiplier ξ_t on the forward-looking constraint (12) that describes the evolution of the household's value function V_t. The multiplier ξ_t measures the shadow value to the planner of the representative household's value. It shows up in the optimal wedge (28) because increasing c_t, h_t affects V_t and therefore the household's worst-case distorted measure. The likelihood ratio M_t^* influences the Arrow-Debreu prices that become embedded in the implementability constraint (17) and shows up in the optimal wedge due to the fact that the planner does not doubt the model. These two variables are absent from Lucas and Stokey (1983), since for $\sigma = 0$ and from (10),(13) and (25), we see that $\xi_t(g^t) = 0, M_t^*(g^t) = 1, \forall t, g^t$. In this special case, equation (28) that determines the optimal wedge instructs us that only the current realization of the government shock g_t determines the optimal allocation and taxes $c_t^{\mathrm{LS}}(g^t) = c(g_t; \Phi)$. The only intertemporal link in this case occurs implicitly through the value of the multiplier Φ on the implementability constraint, and this by itself imparts no history dependence. Therefore, we have the well-known result that the Lucas and Stokey (1983) plan inherits the stochastic properties of government expenditures.

Discussion In the Lucas and Stokey case, the time-additive expected utility assumption and the completeness of the markets allows the government to ignore the past and pay attention only to the current shock in its efforts to smooth tax distortions. With doubts

[12] The optimal wedge at the initial period is

$$U_l(g_0) - U_c(g_0) = \frac{\Phi}{1+\Phi}\left[U_{cc}(g_0)(c_0 - b_0) - U_{cl}(g_0)(c_0 - b_0 + h_0) + U_{ll}(g_0)h_0\right].$$

In the absence of initial debt $b_0 = 0$, the optimal wedge at $t = 0$ would be determined by (28) for $(M_0, \xi_0) = (1, 0)$. Initial consumption is a function of (g_0,b_0) and Φ, $c_0 = c(g_0, b_0; \Phi)$.

about the model, even though there is a *unique* intertemporal budget constraint due to the complete markets assumption, history dependence emerges because intertemporal marginal rates of substitution (and therefore equilibrium prices) are interconnected across histories through continuation utilities.

4.2 Tax rate

We can express the optimal tax in terms of the allocation and (M^*, ξ) as follows. Dividing (28) by $-U_c(g^t)$ and using $\tau_t = 1 - \frac{U_l}{U_c}$, we get for $t \geq 1$

$$\tau_t(g^t) = \frac{\Phi M_t^*(g^t)}{1 + \xi_t(g^t) + \Phi M_t^*(g^t)} \left[\gamma_{RA}(g^t) + \frac{U_{cl}(g^t)}{U_c(g^t)}(c_t(g^t) + h_t(g^t)) - \frac{U_{ll}(g^t)}{U_c(g^t)} h_t(g^t) \right], \quad (29)$$

where $\gamma_{RA}(g^t) \equiv -U_{cc}c/U_c$, the coefficient of relative risk aversion.[13]

Remark 5. *Formula (29) shows that the planner chooses smaller tax wedges at histories that the representative household thinks are less probable than does the Ramsey planner, i.e., when $M_t^*(g^t)$ is small. The planner chooses low tax rates for high values of the multiplier ξ_t.*

4.3 State variables

The problem under commitment has history dependence that adds state variables to the exogenous state variable g_t. Proposition 2 suggests that the endogenous variables M_t^* and ξ_t are natural candidates for state variables in a recursive formulation of the Ramsey problem, which we pursue along the lines of Marcet and Marimon (1998). [14]

Proposition 3. *Let the approximating model of government expenditures be Markov. Then the Ramsey problem from period one onward can be represented recursively by keeping as a state variable the vector (g_t, M_t^*, ξ_t). The likelihood ratio M_t^* and the multiplier ξ_t follow*

[13] It is easy to show that the tax rate is positive when $U_{cl} \geq 0$. According to (29), this amounts to showing that expression $1+\xi_t+\Phi M_t^*$ is positive, despite the fact that ξ_t can take negative values. Calculating Ω_c in the first-order condition (22) and rearranging delivers expression $U_{ct}(1+\xi_t+\Phi M_t^*) = \lambda_t - \Phi M_t^*(U_{cc,t}c_t - U_{cl,t}h_t) > 0$, since the shadow value of output λ_t is positive and $U_{cl} \geq 0$.

[14] The Bellman equation that describes the planner's problem is constructed in a separate appendix available upon request.

laws of motion [15]

$$M_t^* = M^*(g_t, g_{t-1}, M_{t-1}^*, \xi_{t-1}; \Phi)$$
$$\xi_t = \xi(g_t, g_{t-1}, M_{t-1}^*, \xi_{t-1}; \Phi),$$

with initial values $(M_0, \xi_0) = (1, 0)$. *The policy functions for consumption, household utility and debt for* $t \geq 1$ *are* [16]

$$c_t = c(g_t, M_t^*, \xi_t; \Phi)$$
$$V_t = V(g_t, M_t^*, \xi_t; \Phi)$$
$$b_t = b(g_t, M_t^*, \xi_t; \Phi).$$

Discussion The logic of the Marcet and Marimon (1998) method (and in fact of any method that tries to represent commitment problems recursively) is to augment the state space appropriately in order to capture the restrictions that the planner has to respect every period. The multiplier ξ_t (the *co-state* variable) on the forward-looking implementability constraint (21) becomes a state variable, with initial value zero, which reflects the fact that the planner at period one is not constrained to commit to the shadow value of his utility promises to the household. The nature of our problem also requires the inclusion in the state of the likelihood ratio M_t^* with law of motion (13). This augmented state allows us to express the policy problem as a functional saddle point problem.

4.4 Interpretation of μ_t

Fear of misspecification alters the essentially static nature of the Lucas and Stokey problem by injecting the probability distortion and the multiplier (M_t^*, ξ_t). In order to understand how the planner tries to affect equilibrium prices though the channel of beliefs, it helps to interpret the first-order conditions with respect to (M_t^*, V_t).

Consider μ_t, the multiplier on the evolution equation for the likelihood ratio (13), which represents the shadow value to the planner of distorting the probability of history g^t. Using

[15] In the case of an i.i.d. approximating model we would have $M_t^* = M^*(g_t, M_{t-1}^*, \xi_{t-1}; \Phi)$ and $\xi_t = \xi(g_t, M_{t-1}^*, \xi_{t-1}; \Phi)$.

[16] Note that, as our notation suggests, the policy functions are contingent on a value of the multiplier Φ. After finding the policy functions for $c_t = c(g_t, M_t^*, \xi_t; \Phi), t \geq 1$ and $c_0(g_0, b_0; \Phi)$, the multiplier Φ is adjusted so that the intertemporal budget constraint is satisfied.

the definition of $\Omega(g^t)$ in equation (24) gives

$$\mu_t(g^t) = \Phi[U_c(g^t)c_t(g^t) - U_l(g^t)h_t(g^t)] + \beta \sum_{g_{t+1}} \pi_{t+1}(g_{t+1}|g^t)m^*_{t+1}(g^{t+1})\mu_{t+1}(g^{t+1}).$$

Increasing $M^*_t(g^t)$ to make history g^t more probable has two effects: first, it affects prices at history g^t which is measured by the first term on the right, which is the marginal utility of a government surplus Ω_t multiplied by the shadow value of distortionary taxation Φ. Second, it alters the unconditional probability of next period's history, which is reflected by the second term on the right.

Let q^t_{t+i} denote the equilibrium price of an Arrow-Debreu security in terms of consumption at history g^t,

$$q^t_{t+i}(g^{t+i}) \equiv \frac{q_{t+i}(g^{t+i})}{q_t(g^t)} = \beta^i \pi_{t+i}(g^{t+i}|g^t) \prod_{j=1}^{i} m^*_{t+j}(g^{t+j}) \frac{U_c(g^{t+i})}{U_c(g^t)}.$$

Solving forward (24) allows us to express μ_t in terms of future government surpluses,

$$\mu_t(g^t) \quad = \quad \Phi U_c(g^t) \sum_{i=0}^{\infty} \sum_{g^{t+i}|g^t} q^t_{t+i}(g^{t+i})[\tau_{t+i}(g^{t+i})h_{t+i}(g^{t+i}) - g_{t+i}]. \tag{30}$$

Using the intertemporal budget constraint at time t, μ_t can be rewritten as

$$\mu_t(g^t) = \Phi U_c(g^t) b_t(g^t). \tag{31}$$

The government's budget constraint implies that the asset position of the government $b_t(g^t)$ equals the present discounted value of all future government surpluses, which renders $b_t(g^t)$ the relevant variable for capturing all *intertemporal* effects of changing the equilibrium price of an Arrow-Debreu security $q_t(g^t)$ by means of the likelihood ratio M^*_t.

4.5 Dynamics of ξ_t

Consider now the first-order condition with respect to $V_t(g^t)$, which delivers the law of motion of the state variable ξ_t

$$\xi_t = \sigma m^*_t M^*_{t-1}[\mu_t - E_{t-1} m^*_t \mu_t] + m^*_t \xi_{t-1}, t \geq 1, \xi_0 = 0 \tag{32}$$

where E denotes mathematical expectation under the approximating model. The planner faces intricate dynamic tradeoffs. By increasing $V_t(g^t)$, he affects the household's expectation at $t-1$ for the current period t. However, the planner is constrained to confirm the value $V_t(g^t)$ that he had earlier *promised* the household according to the promise-keeping constraint (21). The shadow value to the planner of the promised utility to the household is captured by the multiplier ξ_{t-1}, the *second* term on the right side of (32). The planner is able to steer the household's worst-case beliefs via equation (20) by committing to respect the recursion of the household's utility (21).

Furthermore, increasing $V_t(g^t)$ affects household beliefs for the current period by decreasing the probability of the history g^t and all subsequent future histories emanating from g^t, an effect measured by the multiplier μ_t in the first term of the right side. But decreasing by means of V_t the probability of this particular node of the event tree implies increasing probabilities attached to the other nodes, so that probabilities add to unity, as can be seen clearly from equation (10). The total effect is measured by $\eta_t \equiv \mu_t - E_{t-1} m_t^* \mu_t$, i.e., the innovation in μ_t under the distorted measure $\tilde{\pi}$, or in other words, the one-step ahead forecast error of μ_t with conditional mean under the distorted measure $E_{t-1} m_t^* \eta_t$ equal to zero by construction. This is the *first* term of the right-hand side. Optimality requires that the sum of the two effects must equal the shadow value of the promised utility of next period ξ_t, the left side of equation (32). Besides that, ξ_t has the following property:

Lemma 1. *The multiplier ξ_t is a martingale under the approximating model.*

Proof. Taking conditional expectation with respect to the approximating model π given history g^{t-1} in the law of motion (32) for ξ and remembering that variables dated at t are measurable functions of the history g^t, we get

$$\begin{aligned} E_{t-1}\xi_t &= \sigma M_{t-1}^* E_{t-1} m_t^* [\mu_t - E_{t-1} m_t^* \mu_t] + \xi_{t-1} E_{t-1} m_t^* \\ &= \sigma M_{t-1}^* [E_{t-1} m_t^* \mu_t - E_{t-1} m_t^* \cdot E_{t-1} m_t^* \mu_t] + \xi_{t-1} E_{t-1} m_t^* \\ &= \xi_{t-1}, \end{aligned}$$

since $E_{t-1} m_t^* = 1$. □

Remark 6. *Since the two state variables (M_t^*, ξ_t) are martingales with respect to the approximating model, any transitory shock in government expenditures will have a permanent effect on the state variables that are driving the Ramsey plan.*

An immediate corollary of lemma 1 is that the mean value of the ξ_t is zero since $E(\xi_t) =$

$E(E_0\xi_t) = E(\xi_0) = 0$. Note also that ξ_t can take both positive and negative values because constraint (12) can bind in both directions.

5 Manipulation of expectations and prices

In this section we show that the state variables depict two distinct forces that are shaping the Ramsey plan: price manipulation of government debt through the management of the household's expectations (ξ_t) and the belief heterogeneity between planner and household (M_t^*).

5.1 Incentives to manipulate continuation utilities and beliefs

The household's doubts about the model make continuation utilities among the determinants of equilibrium prices. This contributes a multiplier ξ_t that influences the Ramsey plan.[17] We will use as a guide the evolution of ξ_t (32) in order to interpret how the planner manipulates equilibrium prices via continuation utilities and to describe implications for the tax rate. There are two dimensions along which the household's doubts about the model alter the Ramsey plan, an intratemporal one (among realizations of government expenditures shocks for a given t) and an intertemporal one. We explore the intratemporal dimension first by focusing on μ_t.

The multiplier μ_t is the shadow value for the planner of the likelihood ratio M_t^* and captures the marginal benefit or cost of increasing the equilibrium price via the worst-case beliefs of the household. It is easy to see how the likelihood ratio M_t^* is associated with the government bond holdings from the equilibrium intertemporal budget constraint

$$U_{c0}b_0 = \Omega_0 + \beta E_0 m_1^* \Omega_1 + ... + \beta^{t-1} E_0 M_{t-1}^* \Omega_{t-1} + \underbrace{\beta^t E_0 M_t^* U_{ct} b_t}_{U_{c0} \sum_{g^t} q_t(g^t) b_t(g^t)},$$

which makes transparent the fact that μ_t is associated with the debt obligations of the government $\mu_t = \Phi U_{ct} b_t$, as we have shown before. Note that the multiplier μ_t would be *zero* if the marginal cost of distortionary taxation Φ were zero. This reflects the fact that in a first-best world where lump-sum taxes are available, the equilibrium price of state-contingent debt is not relevant for the planner's problem. [18] The multiplier μ_t takes positive or negative

[17] Remember that with full confidence in the model we have $\xi_t = 0$, which reflects the absence of a role for continuation utilities in equilibrium prices.

[18] Φ could also be zero if there were sufficiently large initial assets $b_0 \ll 0$, a case that would render the

values depending on the asset position of the government. More specifically, the planner wants to *increase* the likelihood ratio and therefore the equilibrium price of consumption claims at contingencies where the government has outstanding debt obligations ($b_t > 0$) and wants to *decrease* it when the public is a net debtor to the government ($b_t < 0$).

The way to interpret the planner's incentives is as follows. Positive government debt at a particular contingency means that the government is ex-ante *selling* consumption claims. Thus, the planner strives to make claims to consumption more expensive through the channel of worst-case beliefs when he is engaged in selling them. In the opposite situation of government assets, the planner is ex-ante *buying* consumption claims, with the resulting incentive of *decreasing* the price of claims by decreasing M_t^*.

Of course, the planner is affecting the likelihood ratio M_t^* by means of continuation utility, which by (10) will affect also the rest of the conditional likelihood ratios at time t, so that they integrate to unity. Therefore, the relevant object which properly takes into account the *total* effect of changing continuation utility is actually the *innovation* in μ_t under the household's worst-case beliefs, which we defined in the previous section as $\eta_t \equiv \mu_t - E_{t-1} m_t^* \mu_t$. So the innovation η_t captures the intratemporal dimension of the price manipulation effect by determining the increment to the martingale ξ_t according to the law of motion (32) and is associated by (31) with the government asset position as $\eta_t = \Phi[U_{ct} b_t - E_{t-1} m_t^* U_{ct} b_t]$. A convenient way to think about η_t and the total effect is by defining $\omega_{t-1} \equiv E_{t-1} m_t^* U_{ct} b_t$, an object that can be thought of as the market value at $t-1$ of the government *portfolio* of state-contingent debt at time t, and by noting that $\eta_t = \Phi(U_{ct} b_t - \omega_{t-1})$. [19] So the planner increases the price by lowering continuation utilities at those contingencies upon which he sells consumption claims *above* the market value of the government portfolio and decreases the price in the opposite case. [20]

Tax rate implications Furthermore, price manipulation through continuation utilities as encoded in ξ_t has a direct impact on the Ramsey plan's consumption and leisure allocation and therefore on the tax rate through (29). A positive innovation $\eta_t > 0$ leads to lower ξ_t and therefore - keeping everything else equal - higher tax rates. Thus, when the planner wants to sell claims, he tries to increase equilibrium prices by lowering continuation utilities

optimal taxation problem trivial.

[19] More precisely, we have $\omega_{t-1} = \frac{U_{ct}}{\beta} \sum_{g_t} p_{t-1}(g_t, g^{t-1}) b_t(g^t)$.

[20] The market value of the government portfolio ω_{t-1} and the innovation η_t are not just convenient devices but very closely related to each other: η_t has a direct interpretation as the marginal change in the value of the government portfolio induced by a change in continuation utility since $\frac{\partial \omega_{t-1}}{\partial V_t} = \sigma \pi_{t|t-1} m_t^* (U_{ct} b_t - \omega_{t-1}) = \sigma \pi_{t|t-1} m_t^* \Phi^{-1} \eta_t$.

and increasing the tax rate, whereas when he wants to buy claims ($\eta_t < 0$), he increases continuation utilities and lowers tax rates.

Our discussion above is in terms of marginal incentives, since we focus on first-order conditions and multipliers. Along with the marginal benefits of price manipulation through continuation utilities, the planner has obviously to take into account the effects of changing the marginal utility of consumption and leisure U_c and U_l, together with his financing needs as captured by the multiplier Φ.

The preceding description of the incentives confronting the planner is evidently static. On the *intertemporal* dimension, note that continuation utilities are forwarding-looking objects and that any change in $V_t(g^t)$ will affect all past continuation utilities $\{V_1(g^1), ..., V_{t-1}(g^{t-1})\}$ along the history g^t through recursion (12), and as a result all the past worst-case beliefs of the household and associated equilibrium prices. Therefore a change in V_t besides altering the market value of the government portfolio at the end of period $t-1$, ω_{t-1}, affects also all *past* market values of the government portfolios $\omega_i, i < t-1$. Consequently, the past innovations in government debt (in marginal utility terms) $\eta_i, i \leq t$ will matter since they measure the shadow value for the planner of altering the respective equilibrium prices along history g^t through the channel of utility V_t. This can be easily seen by iterating backward the law of motion of the multiplier ξ_t (32)

$$\begin{aligned} \xi_t &= \sigma M_t^* (\eta_t + \cdots + \eta_1) \\ &= \sigma M_t^* H_t, \end{aligned} \tag{33}$$

where H_t is the cumulative forecast error $H_t \equiv \sum_{i=1}^t \eta_i, H_0 \equiv 0$. The cumulative forecast error captures the essence of commitment: the planner must take into account how a g^t-contingent action chosen at $t = 0$ affects the choices of the household along the history g^t. Thus the *history* of the shocks as summarized by the state variable ξ_t matters because it tracks dates in the past that the government was lending or borrowing on average, indicating a corresponding incentive to reduce or increase the equilibrium price of consumption claims and a respective utility promise.

Smoothing That the planner makes the shadow value of continuation utilities ξ_t a martingale with respect to his beliefs reflects a deeper *smoothing* motive that makes the planner want to keep the shadow value of continuation utilities constant over time. There is a useful analogy: With full confidence in the probability model, the standard prescription in the optimal taxation literature is to "smooth" *marginal* utilities of consumption and leisure. On the other hand, when the household has doubts about the model, there is an intertemporal dimension coming from continuation utilities and their effects on equilibrium prices and the optimal prescription is to smooth the shadow value of the additional channel of *continuation* utilities. Note that the additional channel of continuation utilities is bringing back into the picture the debt dynamics through the martingale ξ_t, a feature absent in the full-confidence complete-markets economy of Lucas and Stokey (1983).

5.2 Heterogeneity in ambiguity attitudes and impact on tax rate

The likelihood ratio M_t^* of the worst-case beliefs of the household over the beliefs of the planner also influences the optimal wedge (28) and therefore the optimal tax rate in (29) and functions as a state variable which reflects the *heterogeneity* in ambiguity attitudes between the household and the planner. From (29), the planner has an incentive to tax more heavily histories with a high likelihood ratio M_t^*, i.e., histories that the household considers more probable than the planner, keeping everything else equal. The source of this incentive is that the representative household and the planner disagree about the evaluation of welfare losses stemming from a tax $\tau_t(g^t)$ at history g^t because they evaluate the likelihood of this contingency in a different way. [21] The welfare loss of a given tax at a contingency that the household considers more probable than the planner is higher for the household than for the planner. In this case, the planner has an incentive to *increase* the tax rate, since the resulting loss coming from this action is not regarded as being so high by the planner. In opposite cases, i.e. in contingencies that the planner considers more probable than the household (low M_t^*), the planner tries to tax less, since high taxes carry high loss in the planner's evaluation of utility. For example, in the extreme case where M_t^* approaches zero, the tax rate approaches zero according to (29).

6 The effects of small doubts about the model

We can illustrate sharply the impacts of ambiguity aversion on the Ramsey plan by exploiting the fact that Lucas and Stokey 's plan can be easily calculated due to the history independence property. This feature allows us to use a perturbation method that expands in $\sigma \equiv -\theta^{-1}$ around $\sigma = 0$, i.e. around the entire stochastic path associated with the Ramsey

[21] This is exactly the reason why the likelihood ratio shows up in the first-order conditions (22),(23) and therefore in the optimal wedge (28).

plan of Lucas and Stokey, and tells us how introducing a small fear of misspecification perturbs the particular plan.[22] To make things concrete, the first-order approximation around $\sigma = 0$ for a generic endogenous random variable z_t takes the form

$$z_t(g^t, \sigma) \simeq z_t(g^t, 0) + \sigma z_\sigma(g^t, 0) \tag{34}$$

where $z_t(g^t, 0) = z(g_t, 0)$ is the history-independent plan of Lucas and Stokey (1983). For the rest we will use the notational convention $z_t(\sigma) \equiv z_t(g^t, \sigma)$ and $z_t'(\sigma) \equiv z_\sigma(g^t, \sigma)$.[23]

As discussed in detail in previous sections the optimal plan is influenced by the two state variables (M_t^*, ξ_t). In the approximated solution this influence occurs through the respective partial derivatives $(M_t^{*\prime}(0), \xi_t'(0))$. Observe that for any fear of misspecification ($\sigma \leq 0$), the martingale property of $M_t^*(\sigma)$ and $\xi_t(\sigma)$ is bequeathed to the martingale derivatives, $E_{t-1}M_t^{*\prime}(\sigma) = M_{t-1}^{*\prime}(\sigma)$ and $E_{t-1}\xi_t'(\sigma) = \xi_{t-1}'(\sigma)$. In particular, the laws of motion of the martingale derivatives for $\sigma = 0$ are

Result 1. *(Laws of motion of martingale derivatives)*

$$M_t^{*\prime}(0) = m_t^{*\prime}(0) + M_{t-1}^{*\prime}(0), M_0^{*\prime} \equiv 0 \tag{35}$$

$$\xi_t'(0) = \eta_t(0) + \xi_{t-1}'(0), \xi_0' \equiv 0 \tag{36}$$

with respective increments

$$\begin{aligned} m_t^{*\prime}(0) &= V_t(0) - E_{t-1}V_t(0) \\ \eta_t(0) &= \Phi(0)\big[U_{ct}(0)b_t(0) - E_{t-1}U_{ct}(0)b_t(0)\big]. \end{aligned}$$

Two remarks about the increments to the martingales are in order. First, note that exponential tilting in (10) is reflected in the *innovation* in the household's utility in the Lucas and Stokey economy. Doubts about the model make the household assign a higher conditional probability to the realization of an expenditure shock next period that is associated with a negative innovation in the household's utility, since $m_t^*(\sigma) \simeq 1 + \sigma\big(V_t(0) - E_{t-1}V_t(0)\big)$. Second, we are going to focus on the bond portfolio of the *Lucas and Stokey* economy, because it determines the increment to the martingale derivative $\xi_t'(0)$.

[22]There are examples of similar in spirit expansions in asset pricing and portfolio choice theory by Hansen et al. (2007) and Kogan and Uppal (2002).

[23]The details of the derivations are in a separate appendix available upon request.

6.1 Quasi-linear utility

We proceed with a quasi-linear example. Linearity in consumption eliminates the effects of marginal utility on labor supply and on equilibrium prices. Let the period utility of the agent take the form

$$U(c,l) = c + v(1-l)$$

Quasi-linear utility leads to Arrow-Debreu prices $q_t = \beta^t \pi_t M_t^*$. Thus, with this preference specification, the planner cannot manipulate the marginal utility to allocate distortions over histories. But he can still manipulate the endogenous beliefs of the agent through the channel of continuation utilities. For the rest of this section, we assume that $v(1-l) = v(h) = -\frac{1}{2}h^2$, a specification which together with (28) delivers the following labor allocation and tax rate:

$$h_t = \frac{1+\xi_t+\Phi M_t^*}{1+\xi_t+2\Phi M_t^*} \text{ and } \tau_t = 1-h_t = \frac{\Phi M_t^*}{1+\xi_t+2\Phi M_t^*}.$$

Equations (18-21), together with (24) and (25) determine the dynamics of (ξ_t, M_t^*) and the size of Φ.

6.1.1 No fear of misspecification case ($\sigma = 0$)

The optimal plan prescribes *constant* taxes and labor over time and across states. The lack of dependence of taxes and labor on the realization of g_t is a special feature of quasi-linear utility that will make more transparent the effects of the representative household's fear of misspecification. In particular,

$$\begin{aligned} h_t(0) &\equiv h = \frac{1+\Phi(0)}{1+2\Phi(0)} \\ \tau &= 1-h \\ c_t(0) &= h - g_t \\ b_t(0) &= \frac{\tau h}{1-\beta} - E_t \sum_{i=0}^{\infty} \beta^i g_{t+i} \end{aligned}$$

where $\Phi(0)$ represents the value of the multiplier for the Lucas and Stokey economy, where agents fully trust their model. [24] Having obtained the Ramsey plan for $\sigma = 0$, we can

[24]Since $\Phi(0) > 0$, we have $h^{LS} \in (1/2, 1]$. Labor (and therefore $\Phi(0)$) can be pinned down from the intertemporal budget constraint, which leads to finding the root of equation $Q(h) \equiv h^2 - h + G$, where $G \equiv (1-\beta)[b_0 + E_0 \sum_{t=0}^{\infty} \beta^t g_t]$. Assume that G is smaller than $1/4$, so that a solution exists, and larger than 0 in order to rule out an initial surplus that can finance the present value of government expenditures,

proceed to the expansion.

6.1.2 Ramsey allocation for small robustness

The quasi-linearity allows us to derive simple formulas for our expansion. In particular, the increments to the martingale derivatives are

$$m_t^{*\prime}(0) = V_t(0) - E_{t-1}V_t(0) = -(E_t - E_{t-1})[\sum_{i=0}^{\infty} \beta^i g_{t+i}] \quad (37)$$

$$\eta_t(0) = \Phi(0)[b_t(0) - E_{t-1}b_t(0)] = -\Phi(0)(E_t - E_{t-1})[\sum_{i=0}^{\infty} \beta^i g_{t+i}] \quad (38)$$

Equations (37) and (38) show that the dynamics are determined by the innovation in the present value of government expenditures. [25]

To attain more concrete results, assume a Wold moving average representation for the approximating model of government expenditures

$$g_t = \mu_g + \gamma(L)\varepsilon_t \quad (39)$$

where $\varepsilon_t \sim \text{iid}(0, \sigma_\varepsilon^2)$ and $\gamma(\beta) > 0$. [26]

Then the innovation in the present value of government expenditures is

$$(E_t - E_{t-1})[\sum_{i=0}^{\infty} \beta^i g_{t+i}] = \gamma(\beta)\varepsilon_t \quad (40)$$

where $\gamma(\beta)$ is the present value of the coefficients in the infinite order moving average representation, which allows us to get convenient approximate formulas for the pessimistic conditional distribution of ε_t.

Result 2. *The distortion to the conditional distribution of ε_t is approximately equal to*

$$m_t^* = 1 + \frac{1}{\theta}\gamma(\beta)\varepsilon_t$$

and discard the root that is less than 1/2 to get $h = \frac{1+\sqrt{1-4G}}{2}$.

[25] The innovation to the present value of the income of a representative consumer who fears misspecification plays an important role in determining market prices of risk. For example, see Barillas et al. (2007).

[26] Note that here we drop the restriction that g lives on a countable space. We also assume that ε has a bounded support, so that government expenditures remain positive.

The worst-case mean and variance of ε_t are approximately equal to

$$\begin{aligned}\tilde{E}_t \varepsilon_{t+1} &\equiv E_t m^*_{t+1}\varepsilon_{t+1} = \frac{1}{\theta}\gamma(\beta)\sigma_\varepsilon^2 \\ \tilde{V}ar_t(\varepsilon_{t+1}) &\equiv E_t m^*_{t+1}(\varepsilon_{t+1} - \tilde{E}_t\varepsilon_{t+1})^2 = \sigma_\varepsilon^2 + \frac{\gamma(\beta)}{\theta}E_t\varepsilon_{t+1}^3\end{aligned}$$

Result (2) implies that the household assigns higher probability on the event of a positive innovation in the present value of government expenditures and lower probability on the event of a negative innovation. Note in this example that the perturbations around the approximating model take the form of an *increased* conditional mean for government expenditures innovations ε_t and an *unaltered* conditional variance, assuming a zero third moment according to the approximating model ($E_t\varepsilon_{t+1}^3 = 0$).

The labor allocation and the tax rate are quantities that are constant in the economy of Lucas and Stokey (1983). With small doubts about the model though, the history-dependence of the Ramsey plan takes a particularly sharp form:

Result 3. *For small household fears of model misspecification, we find that:*

1. *The labor allocation, tax rate, and tax revenues $T_t \equiv \tau_t h_t$ follow random walks with respect to π:*

$$\begin{aligned}h_t - h_{t-1} &= -\frac{1}{\theta}\frac{\Phi(0)(1-\Phi(0))}{(1+2\Phi(0))^2}\gamma(\beta)\varepsilon_t \\ \tau_t - \tau_{t-1} &= \frac{1}{\theta}\frac{\Phi(0)(1-\Phi(0))}{(1+2\Phi(0))^2}\gamma(\beta)\varepsilon_t \\ T_t - T_{t-1} &= \frac{1}{\theta}\frac{\Phi(0)(1-\Phi(0))}{(1+2\Phi(0))^3}\gamma(\beta)\varepsilon_t\end{aligned}$$

with initial conditions $(h_0, \tau_0, T_0) = (h + \frac{1}{\theta}\frac{\Phi'(0)}{(1+2\Phi(0))^2}, \tau - \frac{1}{\theta}\frac{\Phi'(0)}{(1+2\Phi(0))^2}, \tau h - \frac{1}{\theta}\frac{\Phi'(0)}{(1+2\Phi(0))^3})$.

2. *The optimal debt policy takes the form*

$$b_t = \frac{\tau h}{1-\beta} - E_t\sum_{i=0}^{\infty}\beta^i g_{t+i} + \frac{1}{\theta}\frac{(1-\beta)^{-1}\Phi(0)}{(1+2\Phi(0))^3}(\xi_t'(0) - M_t^{*\prime}(0)).$$

The household's fear of misspecification puts non-stationarity into the Ramsey plan.[27]

[27] The random walk result holds as long as the upper and lower bounds for labor (and correspondingly for the tax rate) are not violated, i.e. the non-stationarity is local.

Note the amount of history dependence in this example: even in the case of i.i.d. government expenditures ($\gamma_0 = 1, \gamma_i = 0, i \geq 1$), the tax rate remains a random walk, a result reminiscent of the Barro (1979) and Aiyagari et al. (2002) results in incomplete markets setups. Here though, the source of the random walk property is the effort of the planner to smooth continuation utilities and his disagreement with the agent and not the lack of insurance markets.

More specifically, the price manipulation and the heterogeneity forces are entering into the determination of the Ramsey plan though the *difference* in the two martingale derivatives $\xi' - M^{*\prime}$, which is a random walk with respect to π:

$$\xi_t'(0) - M_t^{*\prime}(0) = \xi_{t-1}'(0) - M_{t-1}^{*\prime}(0) + (1 - \Phi(0))\gamma(\beta)\varepsilon_t. \tag{41}$$

The increment to the random walk is directly associated with the price manipulation effect $\Phi(0)\gamma(\beta)\varepsilon_t$ and the heterogeneity effect $\gamma(\beta)\varepsilon_t$. The size of $\Phi(0)$ depicts the relative strength of the price manipulation effect. Consider for example the tax rate in result 3. A positive innovation in the present value of government expenditures leads on the one hand to a higher ξ_t and on the other hand to a positive innovation in the likelihood ratio M_t^*. The increase in ξ_t expresses the planner's incentive to increase continuation utility in order to decrease the equilibrium price which leads to a *reduction* in the tax rate, whereas the fact that the planner doesn't consider this event so probable leads to an *increase* in the tax rate.[28] Notice that the two forces operate in *opposite* directions. This is not a coincidence – high government expenditures reduce continuation utilities (which increases M_t^*) and reduce government debt issued ex ante for these contingencies (which increases ξ_t), due to the planner's desire to use the possibilities that complete markets offer and insure against high government expenditure shocks. If $\Phi(0) > 1$, i.e. if there is a high marginal cost of distortionary taxation in the Lucas and Stokey (1983) economy, – a fact which makes equilibrium prices relatively important– the price manipulation effect dominates the heterogeneity effect, whereas when $\Phi(0) < 1$, the opposite happens. The size of $\Phi(0)$ depends on the present value of government expenditures and on initial debt b_0. The two effects exactly cancel for the borderline case $\Phi(0) = 1$, which would correspond to a labor allocation in the Lucas and Stokey (1983) economy of $h = 2/3$.

[28]The optimal tax rate formula (29) for small fear of misspecification and quasi-linear preferences takes the form $\tau_t(\sigma) = \tau + \frac{\Phi(0)}{(1+2\Phi(0))^2}[(M_t^*(\sigma) - 1) - \xi_t(\sigma)] + \sigma\frac{\Phi'(0)}{(1+2\Phi(0))^2}$, which makes clearer how (M_t^*, ξ_t) enter the solution. Taking first differences delivers the random walk result in Result 3.

Similar comments are due for the optimal debt policy implied by the above allocation. Assume for convenience that the approximating model is i.i.d. Then with full confidence in the model the optimal debt is

$$b_t(0) = \frac{\tau h - \mu_g}{1-\beta} - \varepsilon_t,$$

an expression that indicates that the government uses state-contingent debt to smooth the distortions across histories by hedging fiscal shocks–insuring against high shocks with low indebtedness towards the private sector and against low shocks with high indebtedness. Note also how the optimal state-contingent debt inherits the i.i.d. nature of fiscal shocks. With doubts about the model, the expression in Result 3 simplifies to

$$b_t(\sigma) \;=\; \frac{\tau h - \mu_g}{1-\beta} - \varepsilon_t + \frac{1}{\theta}\frac{(1-\beta)^{-1}\Phi(0)}{(1+2\Phi(0))^3}(1-\Phi(0))\sum_{i=1}^{t}\varepsilon_i,$$

which shows that fear of misspecification adds a unit-root component to the initially i.i.d. optimal debt. When the heterogeneity effect is larger than the price manipulation effect ($\Phi(0) < 1$), positive past innovations in government expenditures ($\varepsilon_i, i \leq t-1$) lead to an increase in debt.[29] This is because past positive innovations lead to a high current tax rate, which by the logic of the budget constraint leads to higher amount of government debt that can be sustained at the optimum. The opposite conclusions hold for $\Phi(0) > 1$. As expected, in the case of $\Phi(0) = 1$ the two opposite effects cancel, implying - for small fear of misspecification - the *same* optimal debt as in the Lucas and Stokey economy.

It is useful to calculate the derivative $\Phi'(0)$ in the quasi-linear case

$$\begin{aligned}\Phi'(0) &= (1-\beta)(1+2\Phi(0))^3 E_0 \sum_{t=0}^{\infty}\beta^t M_t^{*\prime}(0) g_t \\ &= -(1-\beta)(1+2\Phi(0))^3 \gamma(\beta)\sigma_\varepsilon^2 \sum_{t=0}^{\infty}\beta^t \sum_{i=0}^{t-1}\gamma_i, \qquad (42)\end{aligned}$$

where the second line follows from our particular moving average representation (39). Expression (42) measures the change in the multiplier if we impute fear of misspecification to the representative household. If we assume that $\sum_{i=0}^{t-1}\gamma_i > 0$ ($\gamma_i > 0$ would be sufficient for

[29]The contemporaneous effect of an innovation ε_t on b_t is more likely to have a negative effect on debt for small σ due to the insurance motive of the government.

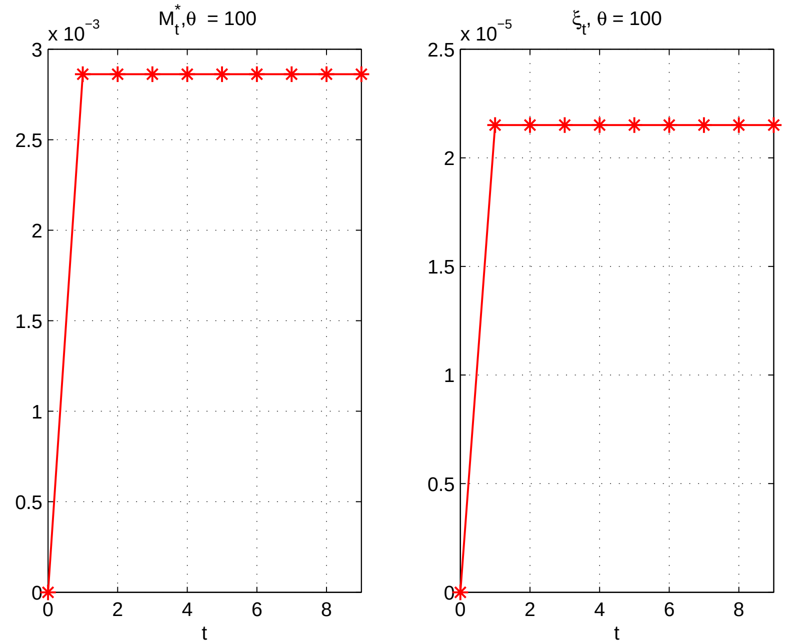

Figure 1: Impulse response functions of the unconditional likelihood ratio M^*_t (left panel) and of the multiplier ξ_t (right panel) to a positive fiscal shock at $t = 1$ for $\theta = 100$.

this to hold) then we have the result that $\Phi'(0) < 0$. Thus, the household's fear of misspecification *increases* the cost of distortionary taxation [30] (remember that $\sigma < 0$). For example, if we assume an AR(1) process with persistence $\rho \in [0, 1)$ for g_t, then $\gamma_i = \rho^i, i = 0, 1, ..$ and $\Phi'(0) = -\beta \frac{(1+2\Phi(0))^3}{(1-\beta\rho)^2} \sigma^2_\varepsilon$.

6.2 Aiyagari et al. (2002) utility function

We can utilize our expansion and analyze the effects of small doubts about the model also for utility functions where the marginal utility channel is present. Consider for example the preference specification of Aiyagari et al. (2002):

$$U(c, l) = \frac{c^{1-\gamma} - 1}{1 - \gamma} + a_l \frac{l^{1-\psi} - 1}{1 - \psi}$$

The parameter $-1/\psi$ controls the elasticity of leisure with respect to the after-tax wage. We follow for our illustrative purposes the calibration of Aiyagari et al. and set $(\beta, \gamma, \psi, a_l) =$

[30] The sign of the derivative $\Phi'(0)$ depends in principle on the specifics of the problem and is not necessarily negative.

$(0.95, 0.5, 2, 1)$. Furthermore, we scale up the amount of leisure available to the household to $\bar{l} = 100$ and set the initial debt equal to zero $b_0 = 0$.

Shock process of Aiyagari et al. We use as an approximating model for government expenditures the same process as Aiyagari et al. (2002), namely, an i.i.d. $N(30, 2.5^2)$ and we analyze the impulse responses of various variables of interest to a fiscal shock at $t = 1$ of a size of roughly one standard deviation above mean. [31]

A positive fiscal shock at $t = 1$ leads in figure 1 to an increase in the two state variables (M_t^*, ξ_t), due to a negative innovation in utility and bonds. The increase is *permanent*, since the two variables are martingales. As a result, the fiscal shock will have in part also a permanent effect on the rest of the variables of interest. Consider figure 2, which shows the impulse response functions for the Lucas and Stokey economy and for the case when the household doubts the model with $\theta = 100$, a penalty parameter that leads to a worst-case distribution that does not essentially differ from the approximating distribution, as figure 3 attests. Note that in the Lucas and Stokey case all variables return to zero after the shock at $t = 1$ due to the history-independence of the solution, whereas with doubts about the model, they do not. This is not discernible for consumption and barely discernible for labor. However, the effects of fear of misspecification are clearer for the tax rate and government debt, which stay permanently above zero after $t = 1$. Remember that the (M_t^*, ξ_t) affect the various variables in opposite directions. It turns out that the heterogeneity effect dominates the price manipulation effect in our example, which leads to a higher tax rate and higher debt (or less assets) at $t = 1$ in comparison to the Lucas and Stokey (1983) case.

War-peace example. Assume now that the approximating model for government expenditures is i.i.d. and that g can take two values $(g_L = 20, g_H = 40)$ with probabilities $(\pi_L, \pi_H) = (0.9, 0.1)$. Let the household doubt the model with $\theta = 100$, which leads to a worst-case scenario in figure 4 that is practically the same as the approximating model. However, the optimal tax rate and debt differ considerably with fear of misspecification. Figure 5 depicts the time path of the tax rate and state-contingent debt for a sequence of five high shocks (war) followed by five low shocks (peace) with and without full confidence in the model. The Lucas and Stokey plan prescribes a high tax rate for times of war and a low for peace. At times of war the government runs a deficit which is financed by issuing

[31] The distribution is approximated with gaussian quadrature with 11 nodes and the initial realization is set to be equal to the mean of government expenditures $g_0 = \bar{g} = 30$. We consider the level of each variable at history $g^t = (\bar{g}, \bar{g}, ..., \bar{g})$ and at history $\hat{g}^t = (\bar{g}, g', \bar{g}, ..., \bar{g})$, and plot the change among the two paths. We use a fiscal shock of size $g' = 32.32$, which corresponds to the 7th node in our approximation scheme.

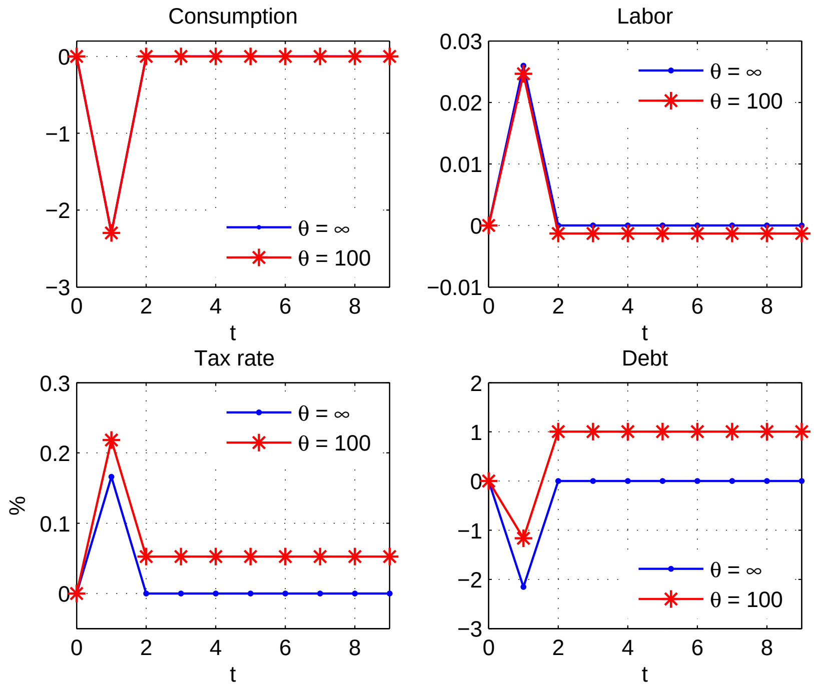

Figure 2: Impulse response functions to a positive fiscal shock for the case of full confidence in the model and for the case of fear of misspecification with $\theta = 100$.

government debt. At peace the government runs a surplus which is used to pay back the government debt. With doubts about the model, the tax rate is not staying constant at a high or low level, but is instead increasing for the first five periods and decreasing afterwards, with a respective increasing and decreasing debt. Note that in this example the heterogeneity effect again dominates the price manipulation effect.

As our two examples indicate, the basic insights we obtained about the Ramsey plan in the quasi-linear case, are valid also in the more general case when the marginal utility channel of the period utility index is present.

7 Concluding remarks

This paper answers the question of how to design optimal policy in an environment where agents have doubts about the probability model governing exogenous shocks. Those generate endogenous subjective beliefs that are reflected in equilibrium prices and motivate the

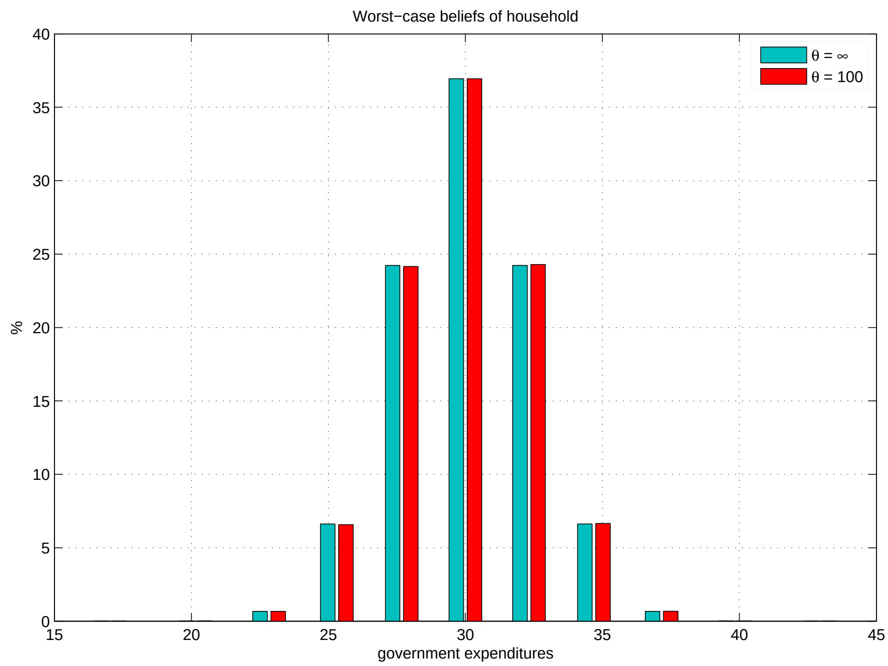

Figure 3: Shock distribution of Aiyagari et al. (2002) ($\theta = \infty$) and household's worst-case distribution for $\theta = 100$.

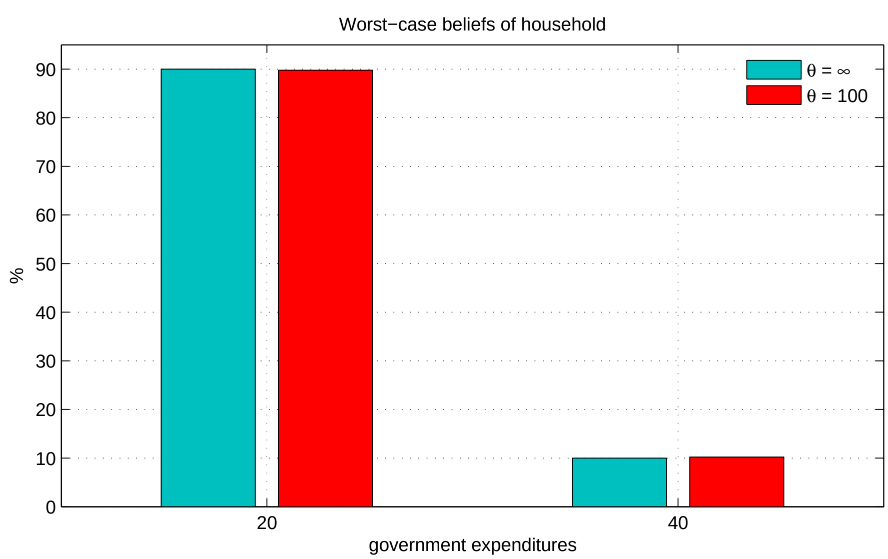

Figure 4: Approximating and worst-case model ($\theta = 100$) of government expenditures: $(\pi_L, \pi_H) = (0.9, 0.1)$ and $(\tilde{\pi}_L, \tilde{\pi}_H) = (0.8978, 0.1022)$.

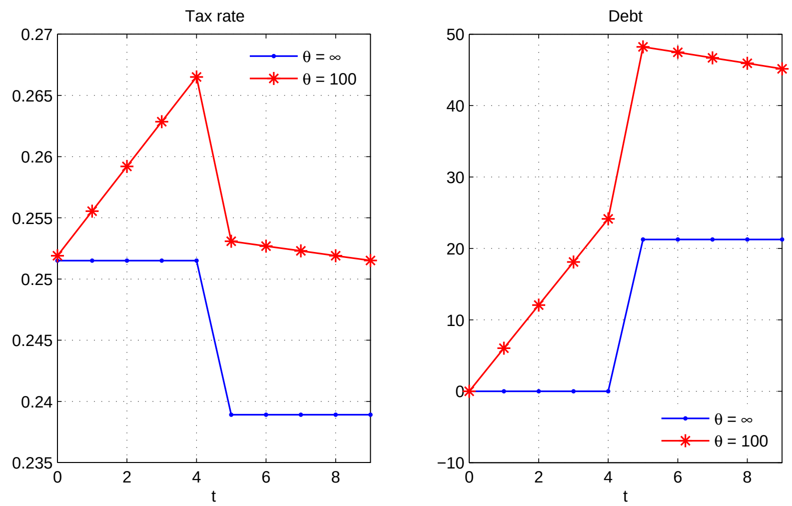

Figure 5: Time path of tax rate and debt with and without confidence in the model for a sequence of five high shocks ($g_H = 40$) followed by five low shocks ($g_L = 20$).

planner to manipulate them in a way that puts history dependence into the Ramsey plan.

The history independence of the Lucas and Stokey (1983) Ramsey plan is a statement that optimal policy treats the nodes of the event tree as atemporal objects. This is allowed because there is a full set of securities at each node *and* because of the absence of any other link between the nodes due to the time-additive expected utility assumption. However, with doubts about the model, the nodes are interconnected through continuation values, which show up in intertemporal marginal rates of substitution and therefore in equilibrium prices, an object that the planner cares about.

The intertemporal links introduced by continuation values would play a non-trivial role also in settings with capital accumulation as in the complete-markets economy of Chari et al. (1994) and Zhu (1992). For example, Chari et al. show that for a special class of utility functions (power utility of consumption and separability between consumption and labor) the Ramsey plan without fear of misspecification prescribes a zero tax on capital income after period zero. That will not be true with concerns about misspecification. We leave the analysis of this topic for future research.

References

Aiyagari, S. Rao, Albert Marcet, Thomas J. Sargent, and Juha Seppala. 2002. Optimal Taxation without State-Contingent Debt. *Journal of Political Economy* 110 (6):1220–1254.

Anderson, Evan W. 2005. The dynamics of risk-sensitive allocations. *Journal of Economic Theory* 125:93–150.

Angeletos, George-Marios. 2002. Fiscal Policy with Non-Contingent Debt and the Optimal Maturity Structure. *Quarterly Journal of Economics* 117:1105–1131.

Barillas, Francisco, Lars Peter Hansen, and Thomas J. Sargent. 2007. Volatility or Doubts? Unpublished paper, University of Chicago and New York University.

Barro, Robert J. 1979. On the Determination of the Public Debt. *Journal of Political Economy* 87 (5):940–71.

Bassetto, Marco. 1999. Optimal Fiscal Policy with Heterogeneous Agents. Mimeo, Federal Reserve Bank of Chicago.

Battaglini, Marco and Stephen Coate. 2008. A Dynamic Theory of Public Spending, Taxation and Debt. *American Economic Review* 98 (1):201–236.

Brunnermeier, Markus K, Christian Gollier, and Jonathan A Parker. 2007. Optimal Beliefs, Asset Prices and the Preference for Skewed Returns. *American Economic Review: Papers and Proceedings* 97 (2):159–165.

Buera, Francisco and Juan-Pablo Nicolini. 2004. Optimal maturity of government debt without state contingent bonds. *Journal of Monetary Economics* 51:531–554.

Caballero, Ricardo J. and Arvind Krishnamurthy. 2008. Collective Risk Management in a Flight to Quality Episode. *Journal of Finance* LXIII (5):2195–2230.

Caballero, Ricardo J. and Pablo Kurlat. 2009. The "Surprising" Origin and Natur of Financial Crises: A Macroeconomic Policy Proposal. Prepared for the Jackson Hole Symposium on Financial Stability and Macroeconomic Policy, August 2009.

Chang, Roberto. 1998. Credible Monetary Policy in an Infinite Horizon Model: Recursive Approaches. *Journal of Economic Theory* 81 (2):431–461.

Chari, V.V., Lawrence J. Christiano, and Patrick J. Kehoe. 1994. Optimal Fiscal Policy in a Business Cycle Model. *Journal of Political Economy* 102 (4):617–652.

Epstein, L. and S. Zin. 1989. Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework. *Econometrica* 57:937–969.

Farhi, Emmanuel and Ivan Werning. 2008. Optimal savings distortions with recursive preferences. *Journal of Monetary Economics* 55 (1):21–42.

Hansen, L. P. and T. J. Sargent. 2001. Robust Control and Model Uncertainty. *American Economic Review* 91:60–66.

Hansen, L. P., T. J. Sargent, G. A. Turmuhambetova, and N. Williams. 2006. Robust Control, Min-Max Expected Utility, and Model Misspecification. *Journal of Economic Theory* 128:45–90.

Hansen, Lars Peter and Thomas Sargent. 1995. Discounted Linear Exponential Quadratic Gaussian Control. *IEEE Transactions on Automatic Control* 40 (5):968–971.

Hansen, Lars Peter and Thomas J. Sargent. 2005. Robust Estimation and Control under Commitment. *Journal of Economic Theory* 124 (2):258–301.

———. 2006. Recursive Robust Estimation and Control without Commitment. Unpublished.

———. 2007. Robustness. Princeton University Press, forthcoming.

Hansen, Lars Peter, Thomas Sargent, and Thomas Tallarini. 1999. Robust Permanent Income and Pricing. *Review of Economic Studies* 66:873–907.

Hansen, Lars Peter, John Heaton, Junghoon Lee, and Nikolai Roussanov. 2007. Intertemporal Substitution and Risk Aversion. *Handbook of Econometrics* 6A.

Karantounias, Anastasios G., Lars Peter Hansen, and Thomas J. Sargent. 2007. Ramsey taxation and fear of misspecification: doubts of the planner. Unpublished paper, New York University and University of Chicago.

Kocherlakota, Narayana and Christopher Phelan. 2008. On the robustness of laissez-faire. *Journal of Economic Theory* Forthcoming.

Kogan, Leonid and Raman Uppal. 2002. Risk Aversion and Optimal Portfolio Policies in Partial and General Equilibrium Economies. Mimeo, Sloan School of Management and London Business School.

Lucas, Robert Jr. and Nancy L. Stokey. 1983. Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics* 12 (1):55–93.

Maccheroni, Fabio, Massimo Marinacci, and Aldo Rustichini. 2006a. Ambiguity Aversion, Robustness, and the Variational Representation of Preferences. *Econometrica* 74 (6):1447–1498.

———. 2006b. Dynamic Variational Preferences. *Journal of Economic Theory* 128:4–44.

Marcet, Albert and Ramon Marimon. 1998. Recursive Contracts. Mimeo, Universitat Pompeu Fabra and European University Institute.

Marcet, Albert and Andrew Scott. 2009. Debt and deficit fluctuations and the structure of bond markets. *Journal of Economic Theory* 144.

Shin, Yongseok. 2006. Ramsey meets Bewley: Optimal Government Financing with Incomplete Markets. Mimeo, University of Wisconsin.

Sleet, Christopher and Sevin Yeltekin. 2006. Optimal taxation with endogenously incomplete debt markets. *Journal of Economic Theory* 127.

Strzalecki, Tomasz. 2008. Axiomatic Foundations of Multiplier Preferences. Mimeo, Harvard University.

Tallarini, Thomas D. 2000. Risk-sensitive real business cycles. *Journal of Monetary Economics* 45 (3):507–532.

Uhlig, Harald. 2009. A model of a systemic bank run. NBER working paper 15072.

Weil, Philippe. 1990. Non-Expected Utility in Macroeconomics. *Quarterly Journal of Economics* CV (1):29–42.

Woodford, Michael. 2008. Robustly Optimal Monetary Policy Under Near-Rational Expectations. *American Economic Review* Forthcoming.

Zhu, Xiaodong. 1992. Optimal fiscal policy in a stochastic growth model. *Journal of Economic Theory* 58 (2):250–289.