Jobs and Unemployment in Macroeconomic Theory: A Turbulence Laboratory

Lars Ljungqvist Thomas J. Sargent*

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Abstract

We use three general equilibrium frameworks with jobs and unemployed workers to study the effects of government mandated unemployment insurance (UI) and employment protection (EP). To illuminate the forces in these models, we study how UI and EP affect outcomes when there is higher 'turbulence' in the sense of worse skill transition probabilities for workers who suffer involuntary layoffs. Matching and search-island models have labor market frictions and incomplete markets. The representative family model with employment lotteries has no labor market frictions and complete markets. The adverse welfare state dynamics coming from high UI indexed to past earnings that were isolated by Ljungqvist and Sargent (1998) are so strong that they determine outcomes in all three frameworks. Another force stressed by Ljungqvist and Sargent (2005), through which higher layoff taxes suppress frictional unemployment in less turbulent times, prevails in the models with labor market frictions, but not in the frictionless representative family model. In addition, the high aggregate labor supply elasticity that emerges from employment lotteries and complete insurance markets in the representative family model makes it impossible to include generous government-supplied unemployment insurance in that model without getting the unrealistic result that economic activity virtually shuts down.

Key words: Job, search, matching, lotteries, skills, turbulence, unemployment, unemployment insurance, employment protection, discouraged worker.

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*Ljungqvist: Stockholm School of Economics and CEPR (email: lars.ljungqvist@hhs.se); Sargent: New York University and Hoover Institution (email: thomas.sargent@nyu.edu). We are thankful to Riccardo Colacito, Constantino Hevia, Kevin Kallock, Sagiri Kitao, and Alejandro Rodriguez for excellent research assistance. For useful comments on earlier drafts, we thank David Backus, Gadi Barlevy, Marco Bassetto, Jeffrey Campbell, Dirk Krueger, Mariacristina De Nardi, Wouter DenHaan, David Domeij, Jesus Fernandez-Villaverde, Timothy Kehoe, Narayana Kocherlakota, Lisa Lynch, Edward C. Prescott, Richard Rogerson, François Velde, and seminar participants at Georgetown University, New York University, the Federal Reserve Banks of Chicago and New York, ITAM, the Stockholm School of Economics, the University of Helsinki, the University of Oregon, and the European University Institute. Ljungqvist's research was supported by a grant from the Jan Wallander and Tom Hedelius Foundation. Sargent's research was supported by a grant to the National Bureau of Economic Research from the National Science Foundation.

1 Introduction

Lucas (1987, p. 50) noted that real business cycle and other competitive equilibrium models have "... no sense in which anyone ... can be said to 'have a job' or lose, seek or find a job."[1] For analyzing positive and normative questions about unemployment compensation, Lucas wanted a theory that includes reasons 'why people allocate time to a particular activity – like unemployment ...' (p. 54), and also why there are 'jobs' in the sense of long term employee-employer relationships between particular workers and particular capitalists.

Macroeconomists disagree about the best models of jobs and the workers who do and don't have them. But we have several promising models. This paper compares three stochastic general equilibrium models containing jobs and uses them to study the effects of government supplied unemployment insurance (UI in the language of Mortensen and Pissarides (1999)) and employment protection provisions (EP in the language of Mortensen and Pissarides (1999)). To illuminate the economic forces stressed within each model, we study how increases in UI and EP interact with an increase in the probability of skill deterioration after involuntary layoffs, where following Ljungqvist and Sargent (1998), we refer to an enlarged skill deterioration probability as an increase in *turbulence.*

The models are (1) a suite of matching models inspired by Mortensen and Pissarides (1999) and DenHaan et al. (2001); (2) a variant of a search-island model of Alvarez and Veracierto (2001); and (3) an employment lotteries model with comprehensive economy-wide insurance arrangements in the spirit of Hansen (1985), Rogerson (1988), and Prescott (2002). The models share the same stochastic skill accumulation and deterioration technologies, but differ with respect to labor market frictions and arrangements for allocating risks. Each model has a theory of a job and of why unemployed workers choose to spend time in an activity that can be called unemployment.[2] In the matching models, workers without jobs wait in a matching function. In the search-island model, depending on their financial assets, human capital, and entitlement to benefits, some people spend more time unemployed than others because they exert less effort searching. In the representative family lotteries model, unemployed workers have won a lottery telling them to specialize in leisure.

1. The matching models have risk neutral workers, no asset accumulation decisions, an attenuated allocative role for wages, and a significant allocative role for the ratio of vacancies to unemployed workers.[3] They feature adverse congestion effects that unemployed workers impose on each other and that firms with vacancies impose on each

[1] Labor economists often use models in which there are no jobs. A useful example is the Rosen schooling model that is estimated by Ryoo and Rosen (2004).

[2] Because it has no theory of jobs, Lucas (1987, p. 53, footnote 4) indicated that he regarded the Hansen-Rogerson model to be an inappropriate tool for analyzing policy questions concerning UI. We extend the Hansen-Rogerson model to contain jobs. We include it among our suite of models partly because Prescott (2002), Rogerson (2005a), and Rogerson (2005b, text for SED plenary lecture) have forcefully advocated it as a vehicle for explaining the most pertinent European labor market outcomes, and also because we want to determine whether the interaction between turbulence and UI that we featured in Ljungqvist and Sargent (1998) also comes through in this environment.

[3] Hosios (1990) describes the matching framework as follows. "Though wages in the matching-bargaining

other, a wage bargaining process, and waiting times as equilibrating signals that reconcile the decisions of firms and workers.

2. The search-island model has risk averse workers whose decisions about how intensively to search when unemployed depend on their skills and benefit entitlements as well as their accumulations of a risk-free asset, the only savings vehicle available to them. The model features incomplete risk sharing through self-insurance against both unemployment and uncertain life spans in retirement. The wage rate is determined in a competitive labor market but is constrained to remain fixed in the face of idiosyncratic productivity shocks: workers receive a fixed wage per unit of skill for the duration of a job. As a result, there are socially wasteful separations.

3. In the lotteries model, risk averse agents have access to complete markets in history-contingent consumption claims. Labor contracts are identical to those in the search-island model. An indivisibility in the choice set of each worker requires him either to work a fixed number of hours or not at all. That would confront the individual worker with a nonconvexity in his opportunity set if he were isolated from other workers. But he is not. He is a member of an economy-wide representative family that uses lotteries to convexify its production opportunity set by assigning fractions of its members to specialize in work and in leisure. The employment lottery yields a high *aggregate* labor supply elasticity that makes the fraction of the national family that works respond sensitively to both tax wedges and UI benefits.

These alternative labor market arrangements create different avenues through which UI and EP influence outcomes, not just for unemployment, but for individual workers' consumption and labor market experiences including the incidence of long-term unemployment.

Our frameworks share some common limitations. First, we ignore the intensive margin of the labor supply decision by assuming that workers are either unemployed or employed full-time and working the same number of hours. Second, if it were not for the labor market frictions in the matching and search-island models and the labor indivisibility in the representative family employment lotteries model, under laissez-faire everyone of working-age would be employed. Hence, we are ignoring such non-market activities as education, child rearing, and other types of homework that explain why some people of working-age do not participate in the labor market.[4]

models are completely flexible, these wages have nonetheless been denuded of any allocating or signaling function: this is because matching takes place before bargaining and so search effectively precedes wage-setting. ... In conventional market situations, by contrast, firms design their wage offers in competition with other firms to profitably attract employees; that is, wage-setting occurs prior to search so that firms' offers can influence workers' search behavior and, in this way, firms' offers can influence the allocation of resources in the market."

[4]This means that we calibrate the representative family employment lotteries model under laissez faire to explain an unemployment rate rather than the employment/population ratio that is more often taken as the target by researchers using that model. See section 1.3.1 for elaboration of this point.

1.1 Why should *macro*economists care about unemployment?

Recent contributions by Prescott (2002, 2004), and Rogerson (2005a) compel us to face a modeling issue that Lucas (1987, pp. 67-68) posed sharply when he asked "... whether modeling *aggregative* employment in a competitive way as in the Kydland and Prescott model (and, hence, lumping unemployment together with 'leisure' and all other non-work activities) is a serious strategic error in trying to account for business cycles. I see no reason to believe that it is. If the hours people work – *choose* to work – are fluctuating, it is because they are substituting into some other activity. For some purposes, – designing an unemployment compensation scheme, for example – it will clearly be essential to break non-work hours into finer categories, including as one 'activity' unemployment. But such a finer breakdown need not substantially alter the problem Kydland and Prescott have tried to face of finding a parameterization of preferences over goods and hours that is consistent with observed employment movements."

Prescott (2002, 2004), and Rogerson (2005a) have extrapolated Lucas's reasoning by using a model that contains neither jobs nor a separate activity called unemployment to explain observed differences in per capita hours in terms of differences in measures of the pertinent flat-rate tax wedge across time and countries.[5] Although UI is substantial in many of the countries in their sample, Prescott's and Rogerson's calculations set UI to zero. Was that a good idea? We think not because the big labor supply elasticities that in their models make tax wedges so important also imply that UI will have big effects, making calculations that ignore UI misleading.[6]

Prescott and Rogerson abstract from *frictional* unemployment. In their frictionless economies, the idleness of some workers emerges from the combined workings of employment lotteries and complete markets for contingent claims. The random process that assigns people to work or to leisure and the complete sharing of labor income risk achieve an economy-wide coordination of decisions absent from the other two labor market models. Perhaps assuming such complete social cohesion is a good way to model employment fluctuations over the business cycle because, for example, U.S. companies resort to temporary

[5] They also postulate that tax revenues are handed back to households either as transfers or as goods and services, i.e., they isolate the substitution effect of taxation by arresting the wealth effect that would follow if the revenues of those taxes were not to be rebated lump sum.

[6] Hansen (1985) shows that the allocation in the employment lottery framework can be interpreted as the outcome of a competitive equilibrium in which households can purchase arbitrary amounts of UI. Hansen's argument pertains to *private* insurance, where the representative household properly internalizes the costs and benefits of the insurance arrangement. In contrast, when the household collects government supplied UI, it does not internalize the costs, and this makes a big difference in outcomes. In particular, since the representative household takes tax rates as given and independent of its own use of government supplied UI, the household will "abuse" the UI system by choosing a socially inefficiently high probability of not working: because the government subsidizes unemployment (leisure), households spend too much time unemployed. As we show below, the effect is large because the labor supply elasticity is so high in the representative family employment lotteries model. Hence, it is a mistake to think that government supplied UI facilitates implementing an optimal allocation either by completing markets or by substituting for private insurance in the employment lottery framework.

layoffs in downturns and because the experience rating in the U.S. UI system means that the private sector at least partially facilitates consumption sharing between those who are employed and unemployed.[7] But we doubt that the abstraction of complete social cohesion is also appropriate for understanding differences in employment observed on different sides of the Atlantic. It is difficult for us to believe that private contractual arrangements are what has required furloughing large numbers of European workers into extended periods of idleness, sometimes even into absorbing states of life-time idleness. The widespread sharing across all individuals in society that is achieved by the smooth functioning of employment lotteries and the elaborate private transfers between households in the representative family model contrast markedly with the friction-laded, individualistic worlds of the matching and search-island models. We will argue that European households today are more likely to empathize with the choices faced by workers in those latter two models, where households must fend for themselves and seek their own fortunes in labor markets (or in government welfare programs) while trading a limited array of financial assets that cannot replicate the outcomes of transactions in employment lotteries and consumption claims contingent on the outcomes of those household specific lotteries.[8]

To elaborate further about why macroeconomists should want to model unemployment, consider the following two issues. First, the nature of the activities that we call unemployment and the details of the process through which jobs and workers find each other can matter for aggregative labor market analysis. Analyzing the aggregate effects of layoff taxes provides one good example. Lucas and Prescott (1974) concluded that a layoff tax reduces unemployment in the search-island model, while Hopenhayn and Rogerson (1993) reached the opposite conclusion in the representative family model with employment lotteries. We shall investigate why the frictionless model in the latter study yields an opposite outcome from models that embody frictional unemployment. Second, as already mentioned, designating a worker as unemployed can entitle him or her to government-provided UI that is quite substantial in some countries. Without explicitly modelling these entitlements and the associated labor market dynamics, we cannot properly address some puzzling observations. For example, OECD (1994, chap. 8) reports that there was a negative cross-country correlation between UI benefit levels and unemployment in the 1960s and early 1970s. In the early 1960s, the lower unemployment in Europe, notwithstanding UI benefits that were more generous than in the US, attracted the attention of the Kennedy administration. The President's Committee to Appraise Employment and Unemployment Statistics (1962) concluded that differences in statistical methods and definitions did not explain the observed differences and, hence, European unemployment rates were indeed lower than that of the U.S.[9] But as we know, the picture changed in the late 1970s. The deterioration of European

[7]See footnote 6.

[8]For another critical evaluation of the aggregation theory in the representative family lotteries model, see Ljungqvist and Sargent (2004b).

[9]The Committee's verification of lower unemployment in Europe prompted Deputy Commissioner Myers (1964, pp. 172–173) at the Bureau of Labor Statistics to write: "From 1958 to 1962, when joblessness in [France, former West Germany, Great Britain, Italy and Sweden] was hovering around 1, 2, or 3 per cent, [the U.S.] rate never fell below 5 per cent and averaged 6 per cent. ... The difference between [the U.S.]

labor market outcomes is probably worse than indicated by the unemployment statistics because many unemployed Europeans have left unemployment by entering other government programs such as disability insurance and early retirement.[10]

The Prescott and Rogerson versions of the representative family framework lacks frictional unemployment and government-provided UI benefits, two features that we think are crucial for understanding the European employment experience. Instead, Prescott and Rogerson stress employment lotteries and trading of a complete set of consumption claims contingent on the outcomes of those lotteries, features that we find virtually impossible to combine with realistically calibrated government-provided UI benefits in Europe: when faced with generous government supplied UI benefits, the representative family with its high labor supply elasticity would simply furlough far too many workers into leisure.[11]

1.2 An excuse for using turbulence as a laboratory

We use turbulence as our laboratory because we believe that it, along with government supplied UI and EP, truly is an important ingredient in understanding labor market outcomes in Europe and the U.S.[12] During the 1950s and 1960s, unemployment rates were lower in Europe than in the United States, but during the 1980s and 1990s, they became persistently higher. In Ljungqvist and Sargent (2005), we reproduced those outcomes within models of an artificial 'Europe' and an artificial 'U.S.' that contain identical peoples and technologies but different labor market institutions. In Europe, as an employment protection device, there is a tax on job destruction that is absent in the U.S., and in Europe unemployment insurance benefits are longer and more generous than in the U.S. Our model imputes *both* the lower

unemployment rate and the average for these European countries was only a little more than 3 percentage points. But, if we could wipe out that difference, it would mean 2 million more jobs, and perhaps $40 to $50 billion in Gross National Product. We can surely be excused for looking enviously at our European friends to see how they do it. We have profited much in the past from exchange of ideas with Europe. It would be short-sighted indeed to ignore Europe's recent success in holding down unemployment."

[10]For example, Edling (2005) reported that in 2004 early retirees comprised 10% of the working age population in Sweden and in 2003 2.4% had been sick for more than a year. Edling asked rhetorically whether "unemployment is hidden in accounts other than those originally intended for the unemployed?" The incendiary nature of his question became clear when Edling's employer refused to publish the report, prompting Edling to resign after having served 18 years as an investigator at the largest trade union in Sweden.

[11]The impossibility of incorporating European sized social insurance in the employment lottery framework causes us to question the appropriateness of that abstraction for understanding the European employment experience. Prescott (2004, p. 8) voices no such concerns but instead lauds an apparent modelling success: "I am surprised that virtually all the large differences between the U.S. labor supply and those of Germany and France are due to differences in tax systems. I expected institutional constraints on the operation of labor markets and the nature of the unemployment benefit system to be of major importance."

[12]For other models of labor market institutions and turbulence, see Bertola and Ichino (1995) and Marimon and Zilibotti (1999). Bertola and Ichino show that high EP and a rigid wage can explain why unemployment increases in response to more volatile local demand shocks. Marimon and Zilibotti explore how high UI can cause unemployment to rise when there is an increase in the value of forming the 'correct matches' between heterogeneous workers and firms.

European unemployment of the 50s and 60s *and* the higher European unemployment of the 80s and 90s to these labor market institutions and how workers chose to respond to more 'turbulent' labor market prospects in the later period. We modelled turbulence by increasing a parameter that determines an instantaneous loss of human capital by workers who suffer an 'involuntary' job dissolution.[13] That representation of more turbulence also generates the increased volatilities of permanent and transitory components of earnings that have been documented by Gottschalk and Moffitt (1994), among others.[14,15]

We designed our model to focus on adverse incentives and to ignore the benefits in terms of consumption smoothing provided by social insurance: our risk-neutral workers care only about present values of after-tax earnings and benefits.[16] In the spirit of McCall (1970), our model takes a fixed distribution of wages as exogenous: there are no firms, no bargaining over wages, and no theory of endogenous wage determination. We added dynamics for skills to McCall's model and took earnings to be a wage draw times a skill level. Skills accrue during episodes of employment and deteriorate during periods of unemployment.

Because it omits many conceivable general equilibrium effects, our extended McCall model is fair game for criticism. How would outcomes differ in general equilibrium models that incorporate some of the features missing from our model, e.g., a matching function that captures adverse congestion effects, firms that post vacancies, equilibrium wages that emerge from bargaining, and a Beveridge curve; or risk-averse workers who engage in precautionary saving because of incomplete markets, firms that invest in physical capital, and a

[13]Pavoni (2003) reviews a substantial body of evidence bearing witness to such human capital loss. Heckman (2003) provides a broad ranging portrayal of the European labor market institutions and outcomes that is consistent with the idea that what we call turbulence has increased in recent decades and that that has had adverse effects on outcomes emerging from European labor markets.

[14]For a survey of the empirical evidence on increased earnings volatility, see Katz and Autor (1999). For the United States, there is evidence that between the 1980s to the 1990s, job losses were more likely to be associated with permanent separations rather than temporary layoffs, to cause displaced workers to switch industries, and prompt workers to move to smaller firms which meant on average incurring permanent losses in their wage levels. All of these indicate losses of job-specific human capital. Also, the incidence of permanent job loss among high wage workers became larger. See Farber (1997, 2005).

[15]Friedman (2005) is full of anecdotes about how reductions in communication costs and lower political barriers, a process that can be called globalization, can lead to rapid depreciation of recently valued human capital. For example, on page 20, Friedman quotes David Schlesinger, head of Reuters America, as saying about New London, Connecticut: " ... Jobs went; jobs were created. Skills went out of use; new skills were required. The region changed; people changed." On page 294, Friedman remarks that workers are paid for "general skills and specific skills" and "when you switch jobs you quickly discover which is which." See Jovanovic and Nyarko (1996) for an information theoretic model of human capital in which experience-acquired knowledge about parameters that calibrate a 'task' forms an important component of human capital that can be destroyed when technology changes.

[16]We justified the linear utility specification by referring to the very high levels of government provided insurance in Europe. Benefits with high replacement rates and indefinite duration drastically reduce the consumption risk faced by individual workers. Hence, our decision to abstract from risk aversion did not seem to be too confining in the European context. The focus of our risk-neutral workers' on the present values of benefits can also be compared to a finding by Cole and Kocherlakota (2001) who describe a setting in which their access to a private storage technology makes risk-averse agents evaluate alternative social insurance contracts in terms of the present values of income that they yield.

competitively determined labor market? And what about the calculations by Prescott (2002, 2004) and Rogerson (2005a) that show that one can ignore the differences across countries in UI and EP and use a model in which willing workers immediately get work with firms to explain the important differences between labor market outcomes in Europe and the United States?

The quantitative models in this paper answer these questions by including these general equilibrium effects. Common features in their physical environments allow us to represent the interactions between the transition dynamics for skills and both the unemployment benefit levels and the job destruction taxes that Ljungqvist and Sargent (2005) focussed on, and to assess the sensitivity of outcomes to the models' distinct features, such as the presence or absence of congestion effects, wage bargaining, risk aversion, and complete or incomplete markets.

1.3 Calibrations

1.3.1 Unemployment versus an employment-population ratio

The issues about UI that preoccupy us prompt us to calibrate the labor-leisure tradeoff in the representative family employment lotteries model to match an unemployment rate rather than the employment to population ratio that is more often the target in the real business cycle literature. In the laissez-faire version of the model, these different calibration targets would have no substantial consequences other than to scale up or down the fraction of the population of working age that is employed. However, these differences do matter in the welfare-state versions of the model because we assume that those who are not employed are entitled to government supplied UI. The distortionary tax rate needed to finance the UI system is increasing in the number of people counted as unemployed. For that reason, our decision to take the unemployment rate as the calibration target seems to be appropriate for studying the issues about UI addressed in this paper.

1.3.2 Microfoundations

It is appropriate to issue a couple of warnings about the microeconomic foundations of the quantitative results presented in this paper. To set out our first warning, recall how Lucas (1987, pp. 46–47) praised Kydland and Prescott (1982) for specifying their model in terms of parameters of preferences and technology because that enabled them to use extraneous microeconomic evidence to calibrate its parameters. "This is the point of 'microeconomic foundations' of macroeconomic models: to discover parameterizations that have interpretations in terms of specific aspects of preferences or of technology, so that the broadest range of evidence can be brought to bear on their magnitudes and their stability under various possible conditions. ... here is a macroeconomic model that actually makes contact with microeconomic studies in labor economics!"[17]

[17] Browning et al. (1999) question calibrations of the real business cycle model for sometimes not faithfully importing micro estimates. They also describe aggregate implications of general equilibrium theory that raise

However, "making contact with microeconomic studies in labor economics", in the sense of endowing their representative agents with the labor supply behavior of a typical individual detected in the micro studies, was certainly *not* the purpose of Hansen (1985) and Rogerson (1988) when they proposed their employment lotteries model. Instead, they showed that non-convexities at the household level in combination with well-functioning private insurance markets that smooth out the impact of those non-convexities on the households' budget constraints give rise to a high *economy-wide* labor supply elasticity even when the labor supply elasticity of each of the households being aggregated is small. That makes the labor supply elasticities that micro labor economists have estimated irrelevant for aggregative labor market analysis.

Now for our second warning. Parts of our three models are so highly stylized that they cannot readily be connected to micro evidence. For example, we arbitrarily specify truncated normal distributions of productivity levels rather than calibrate them to data. However, we can and do calibrate other parameters to match some micro observations. For example, the earnings potential of a high skill worker is twice that of a low skill one, and it takes 10 years on average to work your way from low skill to high skill. Note that our parameterization for the time it takes to accumulate skills pertains both to new inexperienced workers and to workers who have suffered skill loss and want to regain their earnings potential (see footnote 29).

1.3.3 Common parameterizations

We reiterate parameter values from previous studies but also, as far as possible, retain common parameterizations across models. Our practice of keeping parameters fixed across different frameworks can be criticized because the same values of these parameters imply different outcomes in the different frameworks. This is easy to see for the discount factor, which implies different outcomes for the risk-free interest rate in the search-island model, where precautionary savings motives are in play, and in the representative family model, where there are complete insurance markets.

Our justification for keeping common parameters, including the discount factor and the variance of the productivity distribution, is that it best serves our goal of focusing attention on the economic forces at work in the alternative frameworks. And despite the single parameterization in our study, we will argue that those economic forces are robust within a framework either by appealing to evidence accumulated from earlier studies in the literature or by noting that the pertinent quantitative effects are so large that no reasonable change in parameter values can make a substantial difference. An example of the latter is our finding that the representative family employment lotteries model delivers quantitatively unrealistic responses to government supplied UI. This finding stems from the high aggregate labor supply elasticity that characterizes the employment lotteries model and is relatively insensitive to the choice of parameter values.

doubts about whether the cross section and other micro empirical studies are pertinent for the parameters of aggregative models, thereby updating arguments made by Kuh and Meyer (1957).

1.4 Conversations

Concerning the study of unemployment, Lucas (1987) reserved his highest praise for the McCall search model. "Indeed, the model's explicitness invites hard questioning." (p. 56). "Questioning a McCall worker is like having a conversation with an out-of-work friend: 'Maybe you are setting your sights too high', or 'Why did you quit your old job before you had a new one lined up?' This is real social science: an attempt to model, to *understand*, human behavior by visualizing the situation people find themselves in, the options they face and the pros and cons as they themselves see them." We too listen to the workers and firms in our models, especially in section 10, where we use these conversations as a way to make up our own minds about which models ring truest in terms of the choice problems faced by unemployed workers.[18]

1.5 Organization

Section 2 describes features that transcend the environments of all of our models. These include: (1) two transition matrices for workers' skill levels, one for workers whose jobs continue or end voluntarily, another for workers whose jobs terminate involuntarily; (2) a probability distribution for drawing productivity levels of new workers and transition matrices for the productivity levels of workers whose jobs continue; and (3) parameters that define a replacement ratio for UI and a layoff tax for EP. Section 3 describes the additional features that complete the matching models: a risk-neutral utility functional of consumption; and four sets of matching functions that define alternative market structures for sorting workers and firms into pools where workers wait for jobs and firms post vacancies. Beyond the common features reported in section 2, section 4 describes the additional features in the search-island model: a discounted risk-averse functional that is separable across consumption, search effort, time, and states and that imparts a precautionary savings motive; and firm-owned technologies for creating jobs and for converting labor and capital into output. Section 5 describes the special features of the representative family model. Firms face the same problem that they do in the search-island model. But there are complete markets to smooth consumption across states and time. An infinitely lived representative family consists of a continuum of lineages composed of workers who value leisure, consumption, and their offsprings' welfare. Section 6 describes calibrations of the three models. Sections 7, 8, and 9 describe calibrated outcomes in the matching, search-island, and representative family models, respectively. Section 10 critically evaluates the mechanisms at work in the three types of models, comments on the transcendence of the forces stressed by Ljungqvist and Sargent (1998, 2005), and critically evaluates the representative family model and its high labor aggregate supply elasticity as a vehicle for understanding cross-country differences in labor market outcomes. Three appendixes, one for each model, describe Bellman equations and equilibrium conditions.

[18] Ljungqvist and Sargent (2003) reported imaginary conversations with a European and American worker who had experienced identical sequences of labor market shocks, but who behaved differently at the ends of their lives.

2 Common features of our environments

Figures 1 and 2 show the within-period timing of our models. The top halves of these figures are identical. In all models, each of a continuum of potential workers faces a constant probability ρ of exiting the labor force. In the matching model, a worker immediately exits the model upon leaving the labor force while, in the other two frameworks, ρ is the probability that a worker will retire and become unable to work, and σ is the probability that a retired worker dies. To keep the total population and the shares of workers and retirees constant over time, people who exit a model are replaced by newborn workers.

There are three other exogenous sources of uncertainty. First, an employed worker faces a probability π^o that his job terminates. Second, workers experience stochastic accumulation or deterioration of skills, conditional on employment status and instances of exogenous job terminations. Third, idiosyncratic shocks impinge on employed workers' productivity.[19]

2.1 Skill dynamics

There are two possible skill levels, indexed by $h \in \{0, H\}$. All newborn workers enter the labor force with the low skill index, $h = 0$. An employed worker with skill index h faces a probability $p^n(h, h')$ that his skill at the beginning of next period is h', conditional on no exogenous job termination. In the event of an exogenous job termination, a laid off worker with last period's skill h faces a probability $p^o(h, h')$ that his skill becomes h'. A worker's skill remains unchanged during an unemployment spell. The skill transition matrices are:

$$p^n = \begin{bmatrix} 1-\pi^u & \pi^u \\ 0 & 1 \end{bmatrix}, \tag{1}$$

$$p^o = \begin{bmatrix} 1 & 0 \\ \pi^d & 1-\pi^d \end{bmatrix}. \tag{2}$$

2.2 Firm formation and productivity

The process of uniting firms and workers differs across the three frameworks but has several common features. Firms incur a cost μ when posting a vacancy in the matching model or when creating a job in the other two frameworks. We model a new job opportunity as a draw of productivity z from a distribution $Q_h^o(z)$. The productivity of an ongoing job is governed by a Markov process: $Q_h(z, z')$ is the probability that next period's productivity is z', given current productivity z. For any two productivity levels z and $\hat{z} < z$, the conditional probability distribution $Q_h(z, z')$ first-order stochastically dominates $Q_h(\hat{z}, z')$, meaning that

$$\sum_{z' \leq \bar{z}} Q_h(z, z') < \sum_{z' \leq \bar{z}} Q_h(\hat{z}, z'), \qquad \text{for all } \bar{z}. \tag{3}$$

[19]Endogenous job separations will impose additional risks on individual workers. Whether the workers can fully insure against such risks varies across our models.

The probability distributions, $Q_h^o(z)$ and $Q_h(z, z')$, depend on the worker's skill h in the matching model, but not in the other two frameworks.

2.3 Government mandated UI and EP

The government levies layoff taxes on job destruction and provides benefits to the unemployed. It imposes a layoff tax Ω on every endogenous job separation and on every exogenous job termination except retirement. The government pays unemployment benefits equal to a replacement rate η times a measure of past income. In all three models, it will suffice to keep track of a worker's skill in his last employment, to determine his benefit entitlement. Newborn workers are entitled to the lowest benefit level in the economy. The government finances unemployment benefits with revenues from the layoff tax and other model-specific taxes.

3 Matching models

Like DenHaan et al. (2001), we include skill dynamics in a matching framework.[20] The probability distributions of the productivity levels for high-skilled ($h = H$) workers stochastically dominate corresponding probability distributions of low-skilled ($h = 0$) workers , i.e.,

$$\sum_{z' \leq \bar{z}} Q_H^o(z') < \sum_{z' \leq \bar{z}} Q_0^o(z') \qquad \text{and} \qquad \sum_{z' \leq \bar{z}} Q_H(z, z') < \sum_{z' \leq \bar{z}} Q_0(z, z'), \tag{4}$$

for all $\bar{z}$, given that z is a permissible productivity level for both low-skilled and high-skilled workers. We follow DenHaan et al. (2001) and assume that benefits are determined by a replacement rate η on the average after-tax labor income in the worker's skill category when last employed.[21] Hence, we can index a worker's benefit entitlement by his skill in his last employment spell, $b \in \{0, H\}$, so that his benefit entitlement is some function $\tilde{b}(b)$. Let $u(h, b)$ be the number of unemployed workers with skill level h and skill during his previous employment spell of b. The total number of unemployed workers is then

$$\bar{u} = \sum_{h,b} u(h, b). \tag{5}$$

[20] We thank Wouter DenHaan, Christian Haefke, and Garey Ramey for generously donating their computer code, which we have adapted. The matching framework originated in works of Diamond (1982), Mortensen (1982), and Pissarides (2000).

[21] We make two simplifications to DenHaan, Haefke, and Ramey's specification of the benefit system. First, newborn workers are entitled to the lowest benefit level without first having to work one period. Second, workers who experience an upgrade in skills are immediately entitled to the higher benefit level, even if the match breaks up immediately. These assumptions simplify solving the model. The second assumption enables us also to discard DenHaan, Haefke and Ramey's simplifying, but questionable, assumption that a skill upgrade is accompanied with a new productivity draw where the lower bound on possible draws is the reservation productivity of an ongoing match with a high-skilled worker. In our model, exogenous distributions from which productivities are drawn do not change with endogenous reservation productivities.

We drop the assumption of DenHaan et al. (2001) that there is an exogenous number of firms and instead impose a zero-profit condition that expresses the outcome of free entry. Let v be the endogenous number of vacancies and let $M(v,\bar{u})$ be an increasing, concave, and linearly homogeneous matching function:

$$M(v,\bar{u}) = \bar{u}\, M\left(\frac{v}{\bar{u}},1\right) \equiv \bar{u}\, m(\theta), \tag{6}$$

where the ratio $\theta \equiv v/\bar{u}$ is the endogenously determined degree of "market tightness." The probability of finding a job, $M/\bar{u} = m(\theta)$, is an increasing function of market tightness, and the probability of filling a vacancy, $M/v = m(\theta)/\theta$, is a decreasing function of market tightness. We first assume a single matching function for all vacancies and all unemployed workers, but later consider multiple matching functions.[22] For various technical details, see appendix A.

We form three models with separate matching functions (1) for unemployed workers with different skill levels, yielding equilibrium vacancies $v(h)$ for each $h \in \{0,H\}$; (2) for unemployed workers having different benefit entitlements, yielding equilibrium vacancies $v(b)$ for each $b \in \{0,H\}$; and (3) for unemployed workers indexed by both their current skill h and their skill b in their last employment, yielding equilibrium vacancies $v(h,b)$ for each pair of values $(h,b) \in \{0,H\} \times \{0,H\}$. The last setup has only three matching functions because, given our specification of the benefit system, there are no high-skilled unemployed workers with low benefits.

We keep DenHaan, Haefke, and Ramey's specification that workers are risk neutral. Workers' preferences are ordered by

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-\rho)^t c_t, \tag{7}$$

where the worker discounts future utilities by the subjective discount factor $\beta \in (0,1)$ and the survival probability $(1-\rho)$. The government finances the unemployment compensation scheme with the revenues that it receives from the layoff tax and a flat-rate tax τ on firms' output.

Figure 1 shows the within-period timing of events in our matching model.

3.1 Conversation with a worker and a firm

In the spirit of the conversations with McCall workers in Lucas (1987) and Ljungqvist and Sargent (2003), it is informative to discuss the choices that confront our Diamond-Mortensen-Pissarides worker. He likes to consume, but is indifferent to when because his utility is linear in consumption and the inverse of the equilibrium gross interest rate equals his subjective

[22] Hornstein et al. (2003) use a matching model with one matching function, workers with one skill level, but physical capital of different vintages, as a tool for studying how different rates of embodied technical change impinge on equilibrium outcomes, including wage distributions.

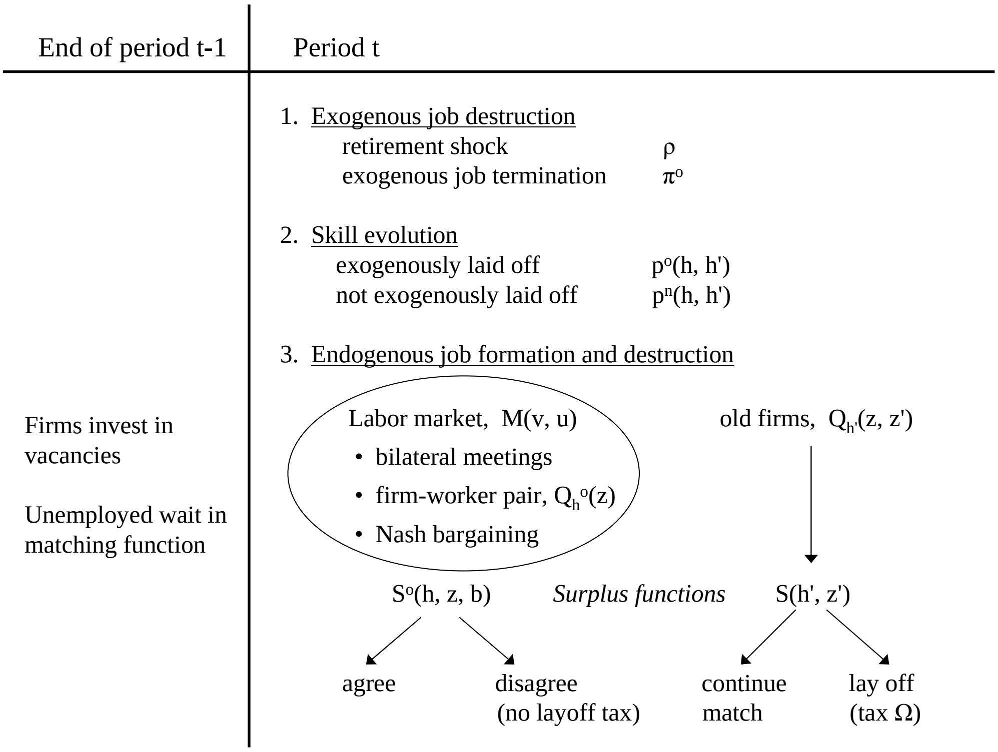


Figure 1: Matching model

discount factor times his constant survival probability. When employed, he works effortlessly. When unemployed, he waits and collects unemployment benefits without effort. When there are more unemployed workers, he waits longer, given the number of vacancies. After being newly matched, a firm and worker bargain about a wage. An acceptable initial bargain commences a worker-firm relationship that lasts until one of three events occur. First, a worker might retire. Second, nature might dissolve the match. Third, a worker and a firm might agree to dissolve a match in response to an idiosyncratic productivity shock. In each period that a worker and a firm remain matched, they bargain over how to split the match surplus. UI and EP influence the surpluses for both new and continuing matches, albeit in somewhat different ways because firms pay layoff taxes only if they had previously hired a worker. By increasing a worker's threat point, unemployment benefits also have a direct impact on bargaining.

Firms choose a number of vacancies to post at a constant per-vacancy resource cost. The matching function imposes congestion effects on both firms and workers. An increase in vacancies decreases the waiting time between matches for each unemployed worker and increases the waiting time between matches for each vacancy, given the number of unemployed workers. Free entry implies that firms that post vacancies can expect to earn zero profits. Therefore, the expected discounted cost of filling a vacancy equals the expected discounted value of a firm's share of future match surpluses.

4 Search-island model

Our search-island model with incomplete markets features risk-averse workers who engage in precautionary saving; a non trivial choice of search effort by unemployed workers; and a competitive labor market for workers whose job searches are successful. The only vehicle for savings is a single risk-free one-period security. Labor contracts cannot depend on the history of a firm's productivity. We consign various technical details to appendix B.

We create our model by altering the model of Alvarez and Veracierto (2001).[23] We adopt their specification of a worker's preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t (1-o_t)^t \left[\log(c_t) + A \frac{(1-s_t)^\gamma - 1}{\gamma} \right], \qquad \text{with } A > 0,\ \gamma > -1, \tag{8}$$

where $(1-o_t)$ is the survival probability with $o_t = \rho$ if the worker is of working age and $o_t = \sigma$ if the worker is retired; and $s_t \in [0,1)$ is the worker's choice of search intensity if he is unemployed and of working age. A search intensity s_t determines an unemployed worker's probability s_t^ξ of finding a centralized labor market in the next period, where $0 \leq \xi \leq 1$. Workers who find the labor market get a job paying a market-clearing wage rate. To accommodate the feature, not present in Alvarez and Veracierto (2001), that workers differ in their skills, we let w^* denote the wage rate per unit of skill, where the skill level of a low-skilled worker is normalized to one and the skill level of a high-skilled worker is $1+H$. Hence, a low-skilled worker earns w^* and a high-skilled worker earns $(1+H)w^*$.

4.1 Firms

We suppress Alvarez and Veracierto's firm size dynamics and, in the spirit of our matching model, let each firm employ only one worker. Each firm also rents physical capital. The firm's production function is

$$z_t k_t^\alpha (1+h_t)^{1-\alpha}, \qquad \text{with } \alpha \in (0,1), \tag{9}$$

where z_t is the current productivity level, $h_t \in \{0, H\}$ is the skill index of the firm's worker, and k_t is physical capital that depreciates at the rate δ. Output can be devoted to consumption, investment in physical capital, and startup costs. The rest of Alvarez and Veracierto's model of firms enters our framework as follows. Incurring a startup cost μ at time t allows a firm to create a job opportunity at $t+1$ by drawing a productivity level z from the distribution $Q^o(z)$. After seeing z, a firm decides whether to hire a worker from the centralized labor

[23] The antecedent of this search-island framework is a model of Lucas and Prescott (1974) in which risk-neutral workers engage in effortless but time-consuming search across a large number of spatially distinct islands with idiosyncratic productivity shocks. The only search cost in that model is the opportunity cost of labor income foregone when moving between islands. Alonso-Borrego et al. (2004) use a two-market version of the Alvarez and Veracierto (2001) model to study how legal regulations that exempt fixed term labor contracts from layoff taxes affect equilibrium outcomes.

market. We retain Alvarez and Veracierto's key assumption that firms and workers first meet under a veil of ignorance about their partner's state vector: the firm hires a worker drawn randomly from a single pool of unemployed workers with a mix of low-skilled and high-skilled workers. Once hired, a firm observes a worker's skill, hires the appropriate physical capital, and pays the worker the market wage of w^* per unit of skill. A firm must retain a worker for at least one period.

Notice that the wage cannot be indexed by the history of shocks z^t. Under a veil of ignorance, all unemployed workers who have been successful in their job search efforts are randomly matched with firms that have decided to create new jobs. The assumption that the market-determined wage rate per unit of skill is unchanged throughout an employment spell is restrictive. To avoid layoffs, workers would be willing to accept wage cuts in response to some adverse productivity shocks.[24]

4.2 Other features

Besides markets for goods and for renting labor and capital, workers can acquire non-negative holdings of risk-free assets that earn a net interest rate i. Following Alvarez and Veracierto (2001), we postulate a competitive banking sector that accepts deposits that it invests in physical capital and claims on firms. The banking sector rents physical capital to firms at the competitive rental rate $i + \delta$. Banks hold a diversified portfolio of all firms and so bear no risk.

In the spirit of Alvarez and Veracierto, we assume that a worker who dies is replaced by a newborn unemployed worker, to whom he is indifferent, but who nevertheless inherits his assets. Newborn workers have the low skill index, $h = 0$.

The government pays unemployment compensation equal to a replacement rate η times an unemployed worker's last labor earnings net of taxes. Newborn workers are entitled to the lowest benefit level in the economy. The government receives revenues from layoff taxes and other revenues from an income tax whose rate we represent in terms of the difference between before-tax and after-tax wage rates, $(w^* - w)$. The government balances its budget.

Figure 2 shows the within-period timing of events in our search-island model.

4.3 Conversations with a worker and a firm

Differentiated by their skill, benefit entitlements, assets, and employment-unemployment-retirement status, workers save or dissave. Employed workers supply labor effortlessly. Unemployed workers suffer more disutility when they search more intensively. After a successful search, a worker receives a job that pays a competitively determined wage. Although the wage rate per unit of skill is the same across all firms, a worker cares about the productivity

[24] A main purpose of Alvarez and Veracierto (2001) is to quantify the potential welfare gains of a tax on job destruction that reduces socially wasteful separations. They acknowledge that the rigidity they impose on labor contracts and their assumption of no disutility from work cause them to overestimate those welfare gains.

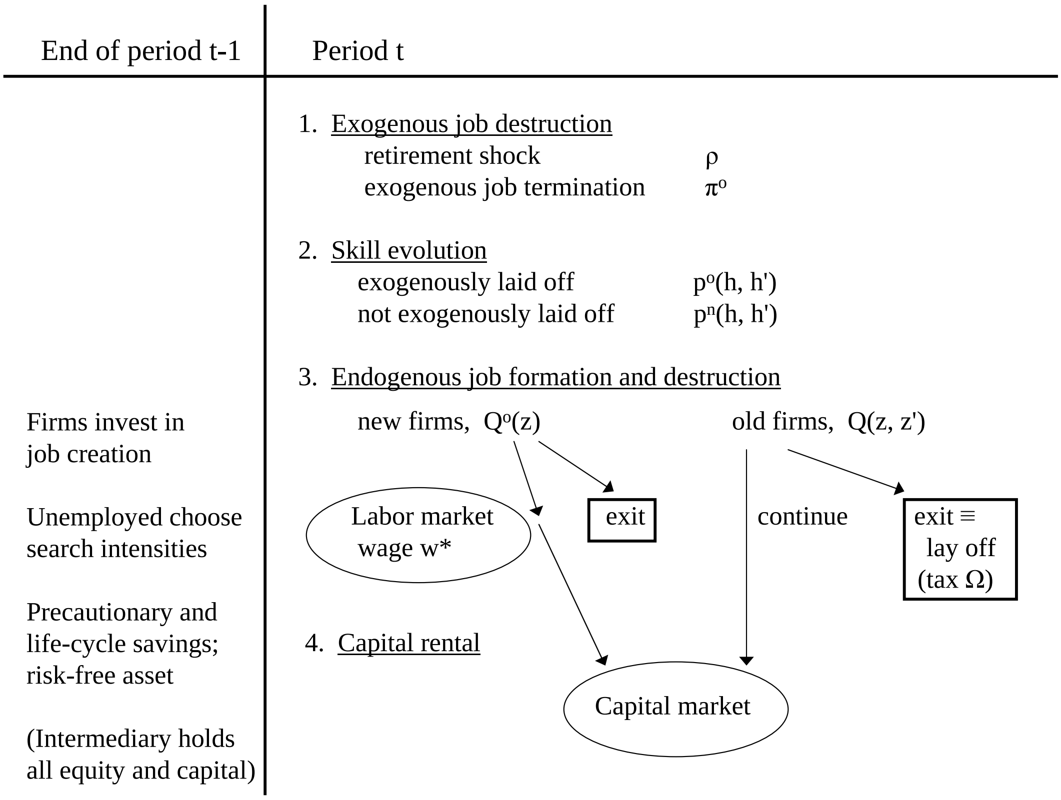


Figure 2: Search-island model

of the firm employing him. A firm's idiosyncratic productivity shock determines when a worker is cast into unemployment. An unemployed worker incurs the disutility from searching for a new job and consumes from his savings and whatever unemployment insurance he is entitled to.

At a constant per-job resource cost, firms create new jobs. Free entry ensures that all job creating firms can expect to earn zero profits. After observing the productivity of a new job, a firm decides whether to hire a worker at the market-clearing wage rate per unit of skill. There are no congestion effects. An unemployed worker inflicts no injury on other job seekers beyond what a seller of a good ordinarily imposes on his competitors.

5 Representative family model with lotteries

This section describes our representative family model with lotteries, with technical details consigned to appendix C. We make these changes to our search-island model: (1) agents care about their offspring, so that each lineage of agents has an infinite-horizon utility function where the survival probability is equal to one; (2) a disutility of working replaces a disutility of search; (3) no search, or other kinds of frictions impede workers from immediately finding the competitive labor market; and (4) agents have access to complete markets in all possible history-contingent consumption claims. We retain the assumption of the search-island and matching models that there is no 'intensive' margin for an agent's choice of work, i.e., each working-age agent faces a $\{0, 1\}$ choice of working or not working an exogenously amount of time in any given period. This indivisibility in agents' labor supply in combination with complete contingent claims markets can give rise to equilibrium outcomes with employment lotteries and following Hansen (1985), and Rogerson (1988), we model the economy as a representative family. Otherwise, from firms' viewpoint, the labor market is identical to that of the search-island model.

The representative family consists of a continuum of lineages indexed on the unit interval. Each lineage consists of an infinite sequence of workers, only one of whom is alive at any time. Each worker retires with probability ρ, then dies with probability σ. In each lineage, a newborn worker immediately replaces a deceased worker. The representative family has a utility function over consumption and the work of all of its lineage members:[25]

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t^j) - v(n_t^j)\Big] dj \; = \; \int_0^1 \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t^j) - n_t^j B\Big] dj \, , \tag{10}$$

where c_t^j is lineage j's consumption at time t and n_t^j equals one if the current member of lineage j is working and equals zero otherwise. We assume that $v(n)$ increases with increases in n. The indivisibility of the individual worker's labor supply makes $v(0)$ and $v(1)$ the only pertinent values. We let $v(0) = 0$ and $v(1) = B$, so the parameter $B > 0$ captures the disutility of working.

The technology and choices of a typical firm are identical with those in the search-island model described in section 4.1. So while there are no search or matching frictions for workers, a friction in the form of the job-creation cost μ still confronts firms. In each period, the arrival order of shocks is 1) exogenous job destruction due to retirement shock with probability ρ and exogenous job termination with probability π^o, and 2) skill evolution. Then firms and families take actions, and job seekers are matched with vacancies immediately and without friction. A firm hires a worker under a veil of ignorance about his skill and rents physical capital after seeing the worker's skill.

5.1 Conversations with a worker and a firm

Each family member does what he is told.[26] A collective entity called 'the family' determines fractions R_t, U_{0t}, $U_{\triangle t}$, U_{Ht}, N_{0t}, N_{Ht} of workers who are retired, unemployed with low skills and low benefits, unemployed with low skills and high benefits, unemployed with high skills and high benefits, employed with low skills, and employed with high skills, respectively; $R_t + U_{0t} + U_{\triangle t} + U_{Ht} + N_{0t} + N_{Ht} = 1$. The family adjusts these fractions in response to wage

[25] It is rewarding to compare the representative family model of Hansen (1985) and Rogerson (1988) with the household labor supply model of Chiappori (1992, 1997), and Browning and Chiappori (1998), and how differently they draw the line that separates families. In the representative family model, a family is an aggregate (national) economy. In applications of the household labor supply model, a family consists of what we usually think of as a nuclear family.

[26] Alternatively, the family member responds to equilibrium prices. See Hansen (1985).

rates, unemployment benefits and taxes. Each worker participates in an employment lottery that determines whether he works as his personal history of skill and benefit entitlement unfolds. Winners of the lottery do not work. As in the models of Hansen (1985) and Rogerson (1988), the employment lotteries enlarge the *aggregate* labor supply elasticity relative to what would be chosen by an individual worker with preferences under the integral sign on the left side of (10) who did *not* face the $\{0,1\}$ restriction on his choice of n_t^j.[27]

Because (10) is additively separable, the family assigns the same consumption $\bar{c}_t$ to every family member. The family assigns an unretired person to work with a probability that depends on his or her skill and benefit entitlement.[28] The *unconditional* expected utility of a lineage in the representative family is

$$\sum_{t=0}^{\infty} \beta^t \Big[\log(\bar{c}_t) - \bar{n}_t B \Big] , \tag{11}$$

where $\bar{n}_t$ is the unconditional probability that a member of the lineage works in period t, which equals the fraction of all family members of both skill levels sent to work, $\bar{n}_t = N_{0t} + N_{Ht}$. Reformulation (11) of the family's utility function (10) embeds the outcomes of both the work-sharing lottery and trades in a comprehensive set of markets in history contingent claims to consumption.

The technologies and choices confronting firms are the same as in the search-island model. At a cost μ, a new one-job firm can be formed. After observing the productivity of new jobs, firms can choose to hire workers under a veil of ignorance and pay a market-clearing wage rate per unit of skill. Upon a non-retirement separation from a worker, a firm pays the layoff tax.

6 Calibrations

An employed worker keeps his productivity from last period with probability $(1-\pi)$ and draws a new productivity with probability π from the distribution $Q_h^o(z')$, so that new productivities on existing jobs are drawn from the same distribution as the productivities at the time of job creation; $Q_h^o(z')$ depends on the worker's current skill index h in the matching model, but not in the other two frameworks.

[27]The aggregate labor supply elasticity will be altered if we expand the number of points in the set of hours that an individual worker is allowed to work, thereby somewhat relaxing the severe indivisibility imposed in the original models of Hansen (1985) and Rogerson (1988). See the extension to handle straight-time and overtime work by Hansen and Sargent (1988).

[28]Relative to Hansen (1985) and Rogerson (1988), we extend the objects that the employment lottery randomizes across to include entire histories (actually, it is better to call them '*futures*') of skills and benefit entitlements that a worker might experience. For example, a worker who experiences a skill upgrade will never again be assigned to enjoy leisure, at least as long as he does not suffer an involuntary layoff with skill loss. In the spirit of Lucas (1987, p. 67, footnote 13) when he remarked that "in the Hansen and Rogerson papers, [unemployed] workers are happier than those who draw employment!," we can say that the real losers of our lotteries are the workers who have had successful careers with skill accumulation because they enjoy the same consumption as everyone else but none of the leisure. (For details, see appendix C.7.)

We turn first to a set of parameter values that are common to all models. Thereafter, we report calibrations of features that are unique to each framework. As far as possible, we reiterate parameter values from previous studies. For calibrating labor market frictions and disutilities of searching and working, we target a laissez-faire unemployment rate in the range of 4 to 5%.

6.1 Parameter values common to all models

Following Alvarez and Veracierto (2001), we set the model period equal to half a quarter, and specify a discount factor $\beta = 0.99425$ and a probability of retiring $\rho = 0.0031$ that are the same in all three frameworks. People of working age have an annualized subjective discount rate of 4.7%. On average, they spend 40 years in the labor force.

Table 1 shows that the skill accumulation process is the same across models. We set transition probabilities to make the average durations of skill acquisition and skill deterioration agree with those in Ljungqvist and Sargent (1998, 2005), who let it take a long time to acquire the highest skill level in order to match realistic shapes of wage-experience profiles.[29] We set a semiquarterly probability of upgrading skills $\pi^u = 0.0125$, so that it takes on average 10 years to move from low to high skill, conditional on no job loss. Exogenous layoffs occur with probability $\pi^o = 0.005$, i.e., on average once every 25 years. The probability of a productivity switch on the job equals $\pi = 0.05$, so that a worker expects to retain a given productivity level for 2.5 years.

Another common assumption is that productivities are drawn from a truncated normal distribution with mean 1.0 and standard deviation 1.0. Model-specific assumptions dictate how these productivity draws enter the production technology.

6.2 Matching models

Here we adopt most of the parameter values of Ljungqvist and Sargent (2004a), who modify the matching framework of DenHaan et al. (2001). The calibration is reported in Table 1. The main substantial departures from Ljungqvist and Sargent (2004a) are that (1) we replace the earlier uniform productivity distributions by truncated normal distributions; (2) instead of a fixed number of firms, we assume free entry; and (3) we introduce a Cobb-Douglas matching function and a vacancy cost μ.[30]

High-skilled workers' productivity distribution is a truncated $\mathcal{N}(2, 1)$ and low-skilled workers' productivity distribution is a truncated $\mathcal{N}(1, 1)$, both of which are rescaled to inte-

[29] We thank Dan Hamermesh for conversations about his data explorations of wage-experience profiles. Our assumption that work experience alone can double a worker's earnings seems to line up well with data for full-time male workers in the U.S. manufacturing industry. But the time required to attain such earnings gains are longer than what we assume. Note that the speed of skill accumulation in our model pertains to both new inexperienced workers and workers who have suffered skill loss and want to regain their earnings potential.

[30] To ensure that the Cobb-Douglas matching technology generates permissible matching probabilities inside the unit interval, we assume that the number of matches equals $\min\{M(v, \bar{u}),\, v,\, \bar{u}\}$.

Parameters common to all models	
Discount factor β	0.99425
Retirement probability ρ	0.0031
Probability of upgrading skills, π^u	0.0125
Probability of exogenous breakup, π^o	0.005
Probability of productivity change, π	0.05
Productivity distribution	truncated $\mathcal{N}(1,1)$
Additional parameters in matching models	
Matching function, $M(v,u)$	$0.45\, v^{0.5} u^{0.5}$
Vacancy cost, μ	0.5
Worker's bargaining weight, ψ	0.5
Low-skilled workers' productivity:	truncated $\mathcal{N}(1,1)$
High-skilled workers' productivity:	truncated $\mathcal{N}(2,1)$
Parameters common to search-island and representative family model	
Probability of dying, σ	0.0083
Capital share parameter, α	0.333
Depreciation rate, δ	0.011
Job creation cost, μ	5.0
Low skill level	1.0
High skill level, $(1+H)$	2.0
Additional parameters in search-island model	
Disutility of search, A	5.0
γ	0.98
Search technology, ξ	0.98
Additional parameter in representative family model	
Disutility of working, B	1.01

Table 1: Parameter values (one period is half a quarter)

grate to one. Both distributions are truncated to a range of 4 units where the midpoint of the range is the mean of the corresponding untruncated distributions. Thus, the range for high-skilled workers' productivities is [0,4] and the range for low-skilled workers' productivities is [-1,3]. The high-skilled workers' distribution is the low-skilled workers' distribution shifted to the right.

Table 1 shows that our parameterization of the matching technology and the Nash bargaining between workers and firms is fairly standard. Workers' bargaining weight equals $\psi = 0.5$, which also equals the matching elasticity of the Cobb-Douglas matching function.

By computing the expected cost $\theta\mu/m(\theta)$ of filling a vacancy, we can interpret the semiquarterly vacancy cost $\mu = 0.5$. In the laissez-faire economy, this average recruitment cost equals 3.4, which can be compared to the average semiquarterly output of 2.3 goods per all workers. Our calibration of the matching model yields a laissez-faire unemployment rate of 5.0%.

6.3 Search-island model

In addition to the discount factor and the probability of retiring, we take several other parameter values from Alvarez and Veracierto (2001): see our Table 1. We take the following survival, technology, and preference parameters from them: $\{\sigma, \delta, \xi, \gamma\}$. Since the model period equals half a quarter and the survival probability in retirement equals $\sigma = 0.0083$, the average duration of retirement is 15 years. The semiquarterly depreciation rate is $\delta = 0.011$. Our settings of exponents on the search technology ($\xi = 0.98$) and on the disutility of search ($\gamma = 0.98$), respectively, make these close to linear.

One-worker firms operate a constant-returns-to-scale Cobb-Douglas production technology with a capital share parameter $\alpha = 0.333$. Each firm has an idiosyncratic multiplicative productivity shock that is drawn from a distribution that is generated by truncating $\mathcal{N}(1,1)$ to the interval [0,2] and then rescaling it to integrate to one. Low-skilled workers have one unit of human capital while high-skilled workers have twice that amount, $(1 + H) = 2$.

The cost of starting a firm, i.e., of drawing anew from the distribution of productivities, equals 5. This can be measured against the laissez-faire outcome that only about 20% of all such draws exceed the optimally chosen reservation productivity when the firm hires a worker at a semiquarterly equilibrium wage rate equal to 6.4 for low-skilled workers. Hence, the average cost of recruiting a worker is approximately 6 months of the wage paid a low-skilled worker.

The disutility parameter A for job search equals 5, which generates a laissez-faire unemployment rate of 4.4%.

6.4 Representative family model with lotteries

Table 1 shows that the representative family model and the search-island model are calibrated alike, except for parameters pertaining to job search. There is neither a search technology nor a disutility of *searching*. Instead, the new parameter B represents disutility of *working*. Setting $B = 1.01$ makes the laissez-faire unemployment rate be 4.7%.

7 Quantitative findings in matching models

This section reports the effects of increases in turbulence on equilibrium outcomes in four matching models that differ in how they sort workers before assigning them to a matching function. How outcomes in these four models respond to increased turbulence illuminates the economic forces that equilibrate labor markets within matching models. Matching models feature adverse congestion effects that job-seeking workers impose on each other and that worker-seeking firms impose on each other. Unmatched workers and firms are concerned about both matching probabilities that are affected by the total stocks of unemployment and vacancies, and the bargaining situation that they will face in future matches. Within a labor pool defined by a matching function, market tightness, $v/u \equiv \theta$, is an important equilibrating variable that the invisible hand uses to reconcile the decisions of firms and workers.

7.1 Single matching function

In the model with a single matching function, figure 3 shows that the unemployment rate is positively related to the replacement rate in the unemployment insurance system. This result emerges in almost any model of unemployment, but it is useful to recall the forces that produce this outcome in the matching model. Unemployment benefits raise the value of a workers' outside option in the wage bargaining with employers. If nothing else changed, a higher threat point for workers would cause wages and the reservation productivity to rise. That would deteriorate firms' bargaining positions, leaving them unable to recover the expected cost of filling vacancies if their probability of encountering unemployed workers were to remain unchanged. Therefore, the invisible hand restores the profitability of firms by lowering the number of vacancies relative to the number of unemployed workers, i.e., the equilibrium measure of market tightness falls, which in turns implies a longer average duration of unemployment spells. Hence, unemployment rises because the duration and incidence of unemployment both increase.

Although UI benefits necessarily increase unemployment in the matching model, layoff taxes have countervailing effects on unemployment. Mortensen and Pissarides (1999) pointed out that layoff taxes reduce incentives both to create jobs and to destroy them. They show that the net effect of these forces on market tightness, and consequently on unemployment duration, is ambiguous, but that the reservation productivity for existing jobs decreases and therefore so does the incidence of unemployment.[31] Figure 4 shows that in our calibrated

[31] The ambiguous effect of layoff taxes upon unemployment is compounded further in our framework because we model a new job opportunity as a draw from a productivity distribution, so that there is an endogenous reservation productivity in job creation. In contrast, Mortensen and Pissarides (1999) assume that all new jobs begin with the same productivity level.

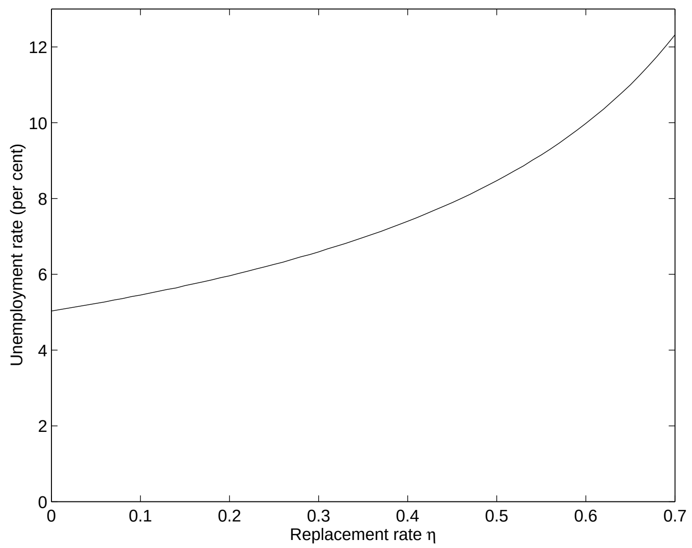


Figure 3: (Matching model) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

matching model with one matching function there is a strong negative relationship between layoff taxes and unemployment. This is also true for the calibration of Mortensen and Pissarides (1999). Unemployment falls because layoff taxes reduce labor reallocation and lock workers into their jobs, so that frictional unemployment falls. Although this negative relationship between layoff taxes and unemployment is the predominant outcome in the matching literature, there are exceptions, most notably Millard and Mortensen (1997). As explained by Ljungqvist (2002), such contradictory quantitative findings in the matching literature come not from differences in parameter values but from different assumptions about bargaining strengths. Millard and Mortensen (1997) assume that firms must also pay layoff taxes after encounters with job seekers who are not hired. That dramatically increases workers' bargaining strengths, making equilibrium market tightness plummet in order to level the playing field for firms. Under the more typical assumption that firms pay layoff taxes only for the workers they had chosen to hire and then had subsequently laid off, Ljungqvist (2002) concludes on the basis of a wide range of simulations that there is a presumption that layoff taxes reduce unemployment in the matching model.[32]

7.1.1 High layoff taxes and high benefits

As noted by Mortensen and Pissarides (1999), the countervailing forces of unemployment benefits and layoff taxes in the matching model can explain why the unemployment rate in a welfare state need not be high. For low values of the turbulence parameter π^d, a more

[32]Unlike Millard and Mortensen (1997), our models obey the dictum, "If a firm does not *hire*, it does not have to *fire*."

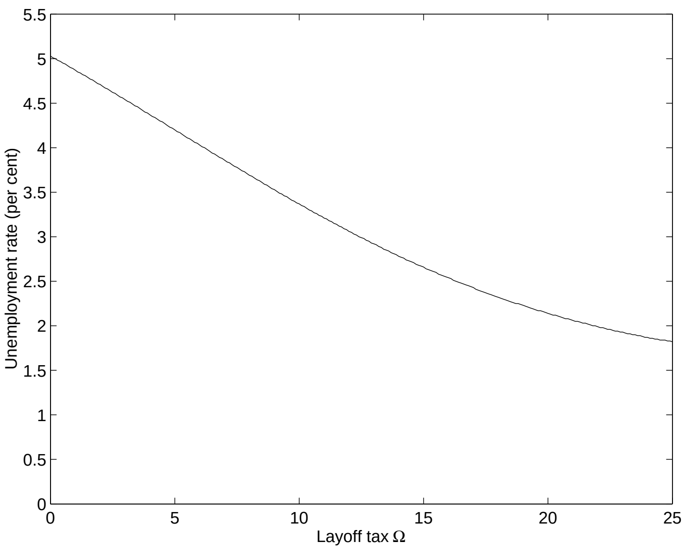


Figure 4: (Matching model) Unemployment rates for different layoff taxes Ω, given tranquil economic times and no benefits. The magnitude of the layoff tax can be compared to an average semiquarterly output of 2.3 goods per worker in the laissez-faire economy, i.e., a layoff tax equal to 19 corresponds to approximately one year's of a worker's output.

generous unemployment insurance system can accompany higher layoff taxes, leaving the equilibrium unemployment rate unchanged or even lower than the laissez-faire outcome. To illustrate this outcome, we pick a replacement rate of 70%, which on its own would have raised the unemployment rate from the laissez-faire level of 5.0% to 12.3% in figure 3, and we choose a layoff tax equal to 24, which corresponds to approximately 5 quarters of a worker's average output in laissez faire, which on its own would have lowered the unemployment rate from 5.0% to 1.9% in figure 4. The combination of these two policies yields an unemployment rate of 4.4%, which falls below the laissez-faire unemployment rate of 5.0%, as depicted in figure 5 at zero turbulence, $\pi^d = 0$. This is qualitatively the same outcome as in the analysis of the European unemployment experience in the 1950s and 1960s in Ljungqvist and Sargent (2005).

7.1.2 Turbulence and unemployment

In addition, the matching model confirms the finding of Ljungqvist and Sargent (1998, 2005) that increased turbulence causes unemployment to increase in the welfare state while it remains virtually unchanged in the laissez-faire economy, as shown in figure 5. The figure attributes the unemployment increase in the welfare state to the generous UI system, because government mandated EP on its own would not have caused unemployment to rise. Furthermore, the positive relationship between turbulence and unemployment is explained by the choices made by laid off workers who have suffered skill loss, as detailed in the left

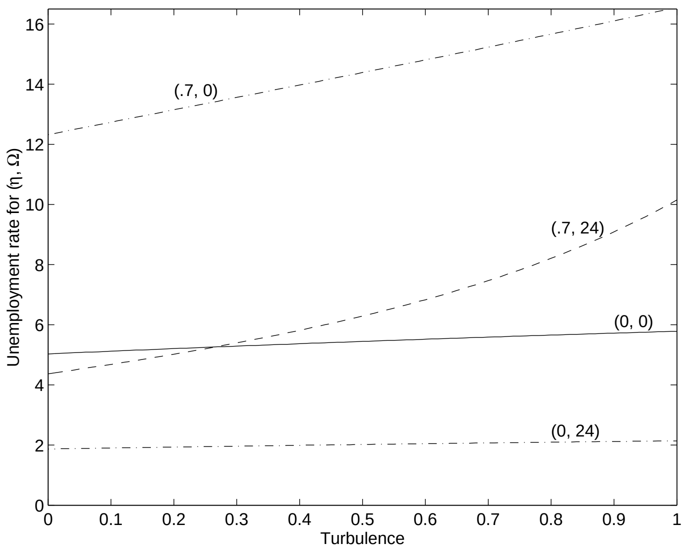
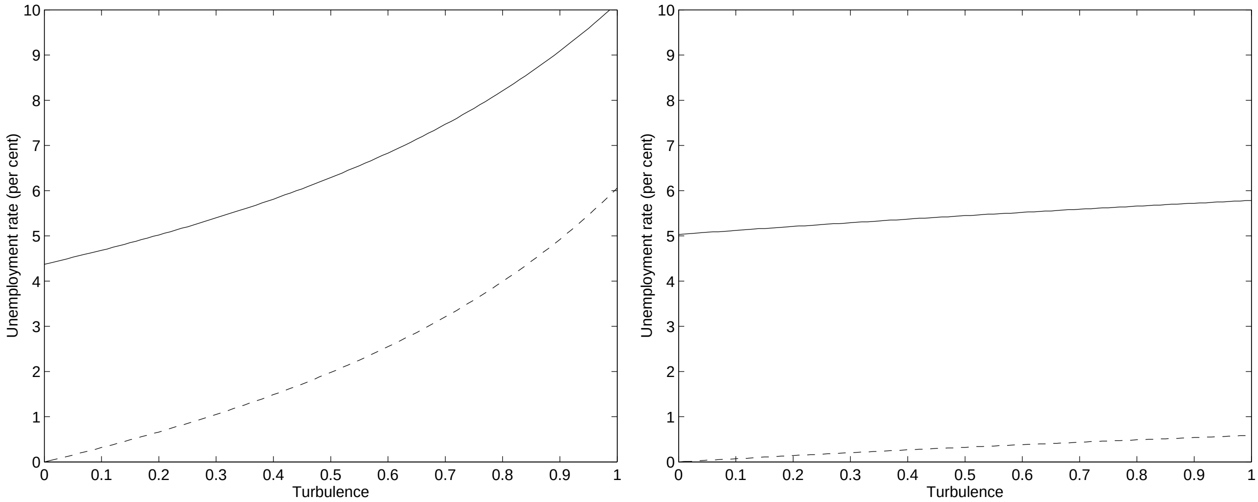


Figure 5: (Matching model) Unemployment rates for different government policies as indexed by (η, Ω) where η is the replacement rate and Ω is the layoff tax. The solid line indexed by $(0, 0)$ refers to the laissez-faire economy.

panel of figure 6.

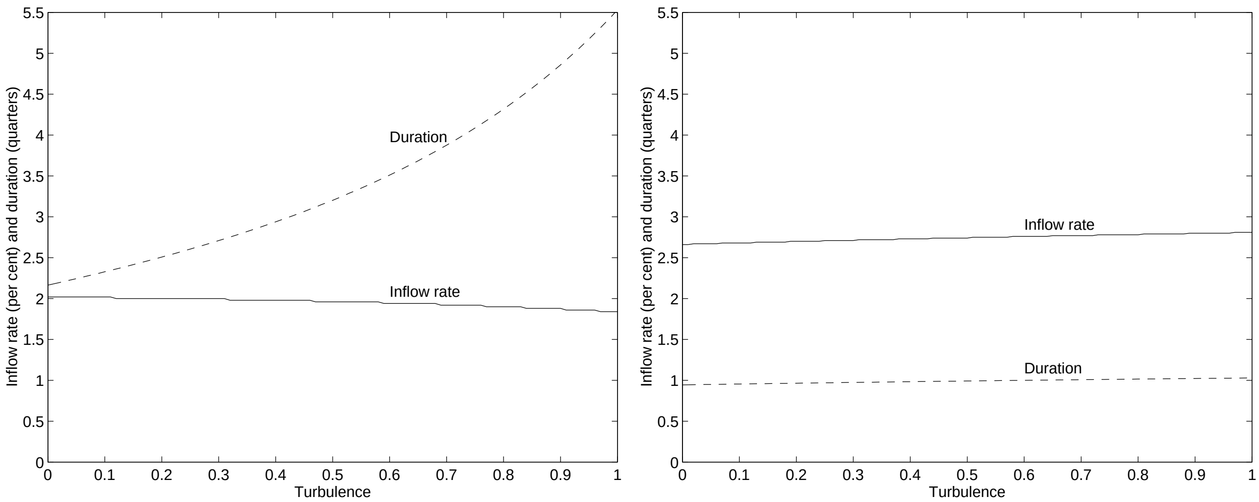
The left panel of figure 7 shows that in the welfare state an increase in turbulence increases the average duration of unemployment spells but leaves the inflow rate almost unchanged. The higher average duration of unemployment is not shared equally among unemployed workers. Although all unemployed workers face the same probability of encountering a vacancy because they enter a common matching function, job acceptance rates differ among workers who are heterogenous with respect to their skill levels and benefit entitlements. Thus, consider unemployed workers who have been laid off and suffered skill loss. Because unemployment benefits are indexed to past earnings, such workers receive benefits that are high compared to their current earnings potential. To give up their generous benefits, these workers must encounter vacancies with idiosyncratic productivities that are high enough to induce firms to offer more generous wages. Hence, low-skilled unemployed workers with high benefits encounter fewer acceptable matches than do low-skilled unemployed workers with low benefits. The unchanging inflow rate into unemployment is explained by the nearly constant reservation productivities that determine job destruction.Turning to the laissez-faire economy in the right panel of figure 7, both the inflow rate and the average duration of unemployment are virtually unaffected by turbulence. In the laissez-faire economy, firms and workers respond to turbulence in ways that leave both the optimal rate of job destruction and the optimal length of time to search for a job unchanged.

Since turbulence sharply increases the average duration of unemployment spells in the welfare state, after allowing for the equilibrium response in the reservation productivity for new jobs, one would expect a precipitous fall in market tightness $\theta = \frac{v}{u}$. Thus, the dotted



(a) Welfare state (b) Laissez-faire economy

Figure 6: (Matching model) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is $(\eta, \Omega) = (0.7, 24)$.



(a) Welfare state (b) Laissez-faire economy

Figure 7: (Matching model) Inflow rate and average duration of unemployment in the welfare state (panel a) and the laissez-faire economy (panel b). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is given by $(\eta, \Omega) = (0.7, 24)$.

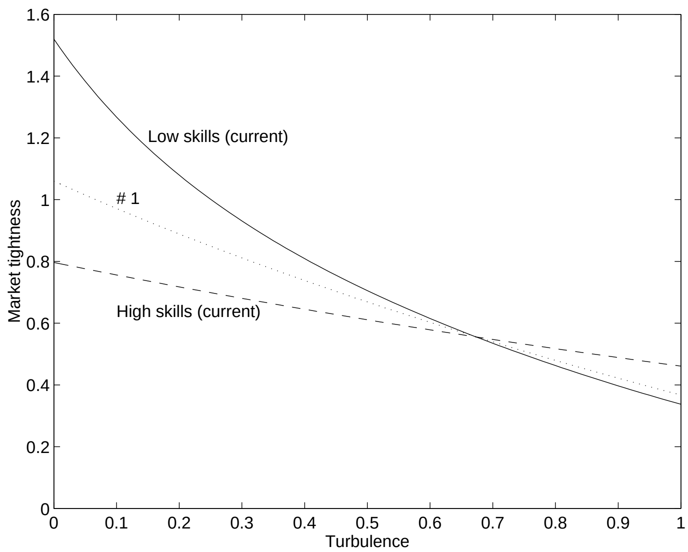


Figure 8: (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon their current skills; low skills (solid line) and high skills (dashed line). As a benchmark, the dotted line labelled #1 depicts market tightness in the economy with a single matching function. The government's policy is given by $(\eta, \Omega) = (0.7, 24)$.

line in figure 8 depicts how market tightness plummets in response to higher turbulence. To ensure that firms break even when posting vacancies in a more turbulent environment, the invisible hand increases the probability that a vacancy encounters an unemployed worker, and thereby weakens the effective bargaining strength of workers: the lower probability that an unemployed worker encounters a vacancy causes workers' outside value to fall. Decreased waiting times between matches for vacancies and the associated fall in a worker's outside value are how the invisible hand improves firms' prospects in response to two adverse forces on firms' profits. First and foremost, increased turbulence ignites adverse welfare-state dynamics because with a given replacement rate UI becomes more valuable compared to what can be earned working. This is most apparent in the case of laid off workers who have suffered skill loss and become low-skilled unemployed workers who are entitled to benefits that are generous relative to their reduced earnings potential. The invisible hand must compensate firms for meeting such workers, because these encounters are less likely to result in agreeable matches; and when such matches are formed, wage payments to low-skilled workers who are entitled to high benefits are higher than those to low-skilled workers who are entitled to low benefits. Second, our representation of turbulence implies a worse technology for skill accumulation and, therefore, higher turbulence has detrimental effects on match surpluses in both the welfare state and the laissez-faire economies. The invisible hand must improve firms' situations because they have to break even while financing the average cost for filling a vacancy out of a fixed fraction of the diminished match surpluses.

Of these two forces that make market tightness fall in response to increased turbulence, that driven by adverse welfare-state dynamics is more important. This assertion emerges from the outcome that laissez-faire unemployment in the right panel of figure 6 increases only slightly in response to increased turbulence: under laissez-faire, the second adverse force from increased turbulence operates, but not the first.

7.2 Multiple matching functions

7.2.1 Separate matching functions for different skills

Figure 8 also reports outcomes of an economy with two separate matching functions for unemployed workers sorted only according to their current skills. It is instructive to compare the market tightness across the two labor markets in such an economy. When $\pi^d = 0$, the zero-profit condition for job creation calls for more vacancies per unemployed worker assigned to the low-skill market. Workers with low skills enjoy a higher probability of encountering a vacancy because there is a larger match surplus to be shared in the case of a match and therefore more incentive for firms to post vacancies. The larger match surplus associated with a low-skilled worker arises from the possibility that employment might result in a skill upgrade that leads to a future capital gain for the match. However, the relative advantage for low-skilled unemployed in terms of market tightness erodes quickly as turbulence increases. When there is turbulence, the low-skill market includes not only low-skilled workers with low benefits but also laid off workers who have suffered skill loss and are now low-skilled but entitled to high benefits. As discussed above, firms think that low-skilled workers who are entitled to high benefits are poor job candidates and, therefore, the invisible hand must compensate firms that post vacancies in the low-skill market by assigning shorter times to encounter an unemployed worker. Thus, there has to be lower market tightness and an associated weakening of workers' effective bargaining strength. In terms of aggregate unemployment, figure 11 shows that the outcome in this economy with two matching functions is not much different from the model with a single matching function.

7.2.2 Separate matching functions for different unemployment benefits

In figure 9, we turn to an economy with separate matching functions for unemployed workers sorted only according to their benefits. The least desirable job candidates, the low-skilled workers entitled to high benefits, are now pooled with the high-skilled unemployed. When turbulence π^d increases, the high-benefit market experiences a precipitous fall in market tightness, while the decline in the low-benefit market is smaller, at least until turbulence reaches a critical level. When turbulence reaches 0.60, marked by a star in Figure 9, the market tightness $\theta = \frac{v}{u}$ in the high-benefit market has fallen so much that the probability that a vacancy meets an unemployed worker is equal to one. Higher levels of turbulence further depress market tightness and reduce the probability that an unemployed worker meets a vacancy, but the probability that a vacancy meets an unemployed worker remains one. The short end of the market, the number of vacancies, determines the total number of encounters. When turbulence reaches another critical level at 0.96 marked by a circle

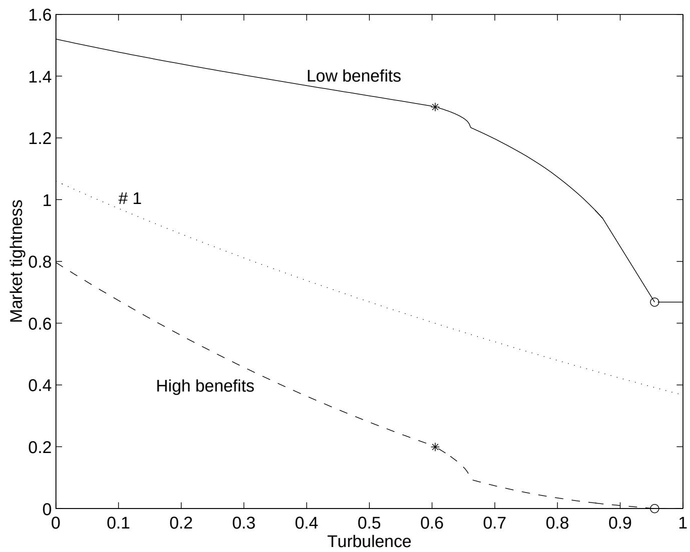


Figure 9: (Matching model) Market tightness θ when there are separate matching functions for unemployed workers sorted according to their benefits; low benefits (solid line) and high benefits (dashed line). As a benchmark, the dotted line labelled #1 depicts market tightness in the economy with a single matching function. The government's policy is $(\eta, \Omega) = (0.7, 24)$. For π^d above .6 denoted by the *, the probability that a vacancy meets a worker equals 1. For π^d above .96 (the circle), the high benefit market closes.

in figure 9, market tightness in the high-benefit market has fallen to zero and the market closes. At levels of turbulence above this critical point, a firm in the high-benefit market cannot expect to break even by posting a vacancy and meeting a worker with certainty in the next period, even though the worker's threat point is merely the outside value of remaining unemployed forever. Vacancies in the high-benefit market have become unprofitable because the odds of encountering a low-skilled worker rather than a high-skilled worker are too high. The point at which the high-benefit market breaks down obviously depends on the length of a model period. While a shortening of our semiquarterly period would delay and maybe eliminate the market breakdown, unemployment would still explode when higher turbulence drives market tightness closer to zero, as further discussed in section 7.2.5.

7.2.3 Comparisons across different pooling arrangements

Figure 11 depicts how aggregate unemployment increases more in response to turbulence in the economy with separate matching functions indexed by unemployed workers' benefits than it does in the economy with a single matching function or in the economy with separate matching functions for unemployed workers sorted according to their current skills. In all three economies, low-skilled unemployed workers entitled to high benefits harm the prospects of other workers with whom they are pooled within a matching function. However, the impact of these unwanted job candidates is diluted when there is a single matching function in the economy or, in the case of multiple matching functions, when these workers are pooled with a group of unemployed workers who can better withstand such a mixing. The low-skilled workers entitled to low benefits are the more resilient group of unemployed because their match surpluses include the prospects of capital gains associated with becoming high-skilled: if it is worthwhile to assign *any* workers to operate the economy's technology, it must show up in the match surplus of newborn workers. Thus, low-skilled unemployed workers entitled to low benefits are the most resilient group of unemployed in terms of bearing the burden of being pooled with low-skilled unemployed entitled to high benefits. If these unwanted job candidates are instead pooled with the less resilient group, namely, the high-skilled unemployed workers who are entitled to high benefits, as when the matching functions are indexed by benefits b but not skills h, aggregate unemployment increases more with turbulence and increases further at the level of turbulence where the probability of that a vacancy meets an unemployed worker in the high-benefit market has increased to its maximum of one, indicated by a star in figure 11. At higher levels of turbulence, the ever lower measures of market tightness in figure 9 cause aggregate unemployment virtually to explode in figure 11.[33] At the critical level of turbulence indicated by a circle, the high-benefit market shuts down and unemployment becomes an absorbing state for *all* skilled workers who suffer exogenous layoffs. It follows that the solutions to workers' and firms' optimization problems are no longer affected by the incidence of skill loss among the exogenously laid off. Hence, unemployment in figure 11 and measures of market tightness in figure 9 become constant for any turbulence above this critical level. A consequence of the adverse outcomes in the high-benefit market is that market tightness in the low-benefit market suffers a dramatic decline. The breakdown of the high-benefit market is tantamount to a drastic deterioration in the skill accumulation technology. The argument in section 7.1.2 explains why the invisible hand must lower market tightness in the low-benefit market to uphold the zero-profit condition for job creation in the face of what is analogous to a deterioration in the economy's technology.

7.2.4 Three matching functions: sorting according to *both* skills *and* benefits

Insights gleaned from the models with two matching functions help to understand the outcomes for the model in which three matching functions sort unemployed workers perfectly along all of their attributes. Under our calibration, the labor market for low-skilled workers who are entitled to high benefits operates only at very low levels of turbulence, and its market tightness in figure 10 is so low that the probability that a vacancy meets an unemployed worker always equals one. At the critical level of $\pi^d = 0.09$ indicated by a circle in figure 10, the market for low-skilled unemployed workers entitled to high benefits shuts down, just

[33] The unmarked point of inflection at turbulence 0.66 on aggregate unemployment in the economy with separate matching functions for unemployed workers based upon their benefits in figure 11 coincides with the endogenous job destruction involving high-skilled workers coming to an end. At higher levels of turbulence, high-skilled workers separate from their jobs only because of exogenous job destruction. This equilibrium outcome somewhat arrests the explosion in unemployment.

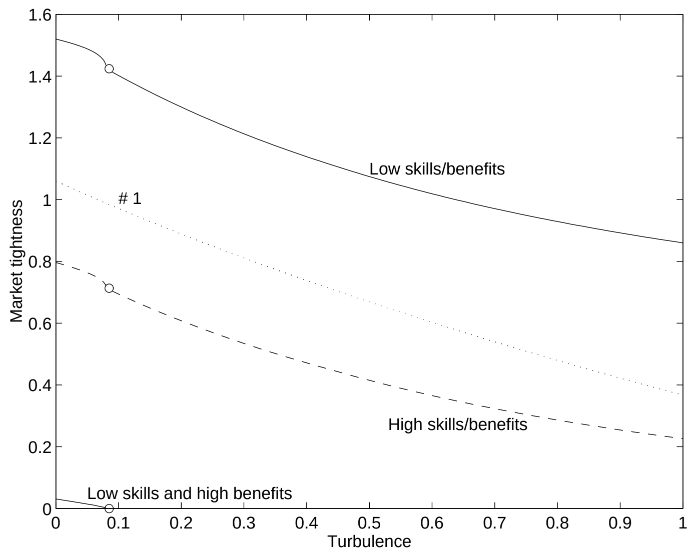


Figure 10: (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon both their current skills and benefits; low skills/benefits (upper solid line), high skills/benefits (dashed line) and low skills but high benefits (lower solid line). As a benchmark, the dotted line depicts market tightness in the economy with a single matching function. The government's policy is $(\eta, \Omega) = (0.7, 24)$.

as the high-benefit market did in the economy of subsection 7.2.2. The two active labor markets are characterized by smoothly falling measures of market tightness throughout the range of turbulence in figure 10. This particular outcome resembles the outcome of the single matching function model of section 7.1, but here the similarities end. The unemployment rate of the model with three matching functions explodes in response to increased turbulence in figure 11, both before and after the breakdown of the market for low-skilled unemployed entitled to high benefits. This is hardly surprising after the market breakdown because unemployment has then become an absorbing state for all workers experiencing skill loss. Hence, the ranks of the unemployed must then inevitably swell in response to higher incidence of skill loss.

7.2.5 A semimonthly model period

To illustrate how the breakdown of markets depends on the length of a model period, we temporarily adopt the assumption of a semimonthly rather than a semiquarterly model period.[34] The shorter model period makes the invisible hand less disposed to shut down

[34]We convert our semiquarterly calibration into a semimonthly one by rescaling the discount factor, all exogenous transition probabilities, and the multiplicative coefficient in the matching function. After doing so, the laissez-faire outcome is virtually unchanged, with the qualification that the similar numerical outcomes for output and wages now refer to a semimonthly frequency. Hence, to keep any layoff tax in the welfare

markets. The condition for a market breakdown now becomes that it is unprofitable to post a vacancy even if an unemployed worker is encountered with certainty after merely half a month rather than half a quarter as under our semiquarterly calibration.

Figure 12 depicts aggregate unemployment outcomes for both the semimonthly and the semiquarterly calibrations of the two models that exhibited market breakdowns under the semiquarterly calibrations. Starting with the economy with separate matching functions for unemployed workers sorted according to their benefits, we see how the shorter model period delays the market breakdown until a higher level of turbulence. Given the semimonthly calibration, the invisible hand finds it relatively easier to compensate firms that post vacancies in the market for unemployed workers entitled to high benefits by pulling the measure of market tightness ever closer to zero. This manifests itself by making unemployment rise at an ever increasing rate until the market finally shuts down. In the economy with three matching functions, the shorter model period results in an interior outcome for the probability that a vacancy meets an unemployed worker in the market for low-skilled workers who are entitled to high benefits until turbulence reaches the critical level of 0.23 that is indicated by a star in figure 12. At higher levels of turbulence, the invisible hand can improve firms' situation in that market only by lowering the probability that an unemployed worker meets a vacancy while keeping the probability that a vacancy meets an unemployed worker equal to one. Under those circumstances, we see that the market quickly shuts down at the point indicated by a circle in figure 12, without any perceptible range of turbulence between the star and the circle. (This contrasts with the outcome in the economy with two matching functions where the unemployed workers were sorted according to their benefits.) At this point, the aggregate unemployment rate immediately reaches the trajectory of our earlier semiquarterly calibration. Recall that the market for low-skilled workers entitled to high benefits in effect isolates the workers whom firms regard as poor job candidates. Not surprisingly, the invisible hand sets a low market tightness in this market, implying a high probability that a vacancy encounters an unemployed worker and a low probability that an unemployed worker encounters a vacancy. This market is truly characterized by long-term unemployment and its average duration of unemployment spells spirals ever higher as turbulence increases, until the market eventually breaks down.

7.2.6 Heterogeneity not duration dependence as source of falling hazard rate

For the semiquarterly calibration, figure 13 depicts the hazard rate of gaining employment in the most turbulent times ($\pi^d = 1.0$). The hazard rate is practically flat in the laissez-faire economy but declines sharply in the welfare state. The high incidence of long-term unemployment in the welfare state is conveyed graphically by a hazard rate that is low even at the start of unemployment spells. Compare this to the much higher and constant hazard rate in the laissez-faire economy. The hazard rate in the welfare state falls with the duration of spells entirely because of *heterogeneity*, not *duration dependence*. Our model

state comparable across calibrations, the tax in the semimonthly calibration has to be set thrice as large as the tax in the semiquarterly calibration.

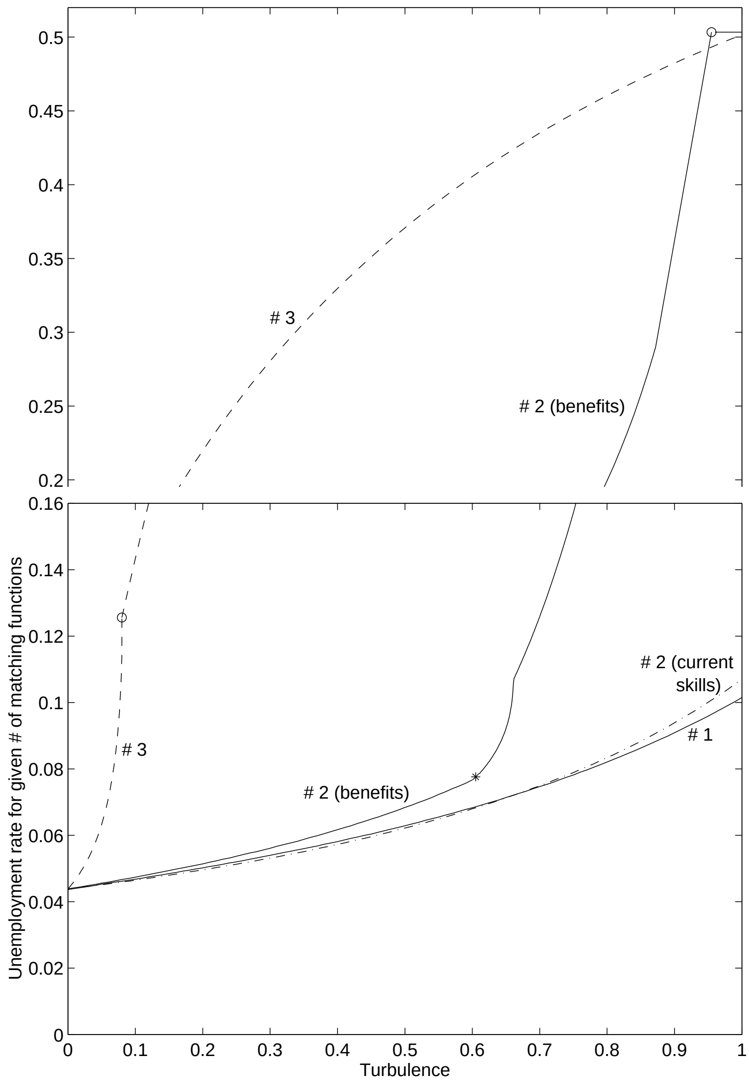


Figure 11: (Matching model) Aggregate unemployment rates for different number of matching functions. The lower solid line depicts the benchmark model with one matching function. The dash-dotted and the upper solid lines refer to the two models with two matching functions where the unemployed are sorted by their current skills and their benefits, respectively. The dashed line depicts the model with three matching functions, i.e., the unemployed are perfectly sorted along all of their attributes. The government's policy is $(\eta, \Omega) = (0.7, 24)$.

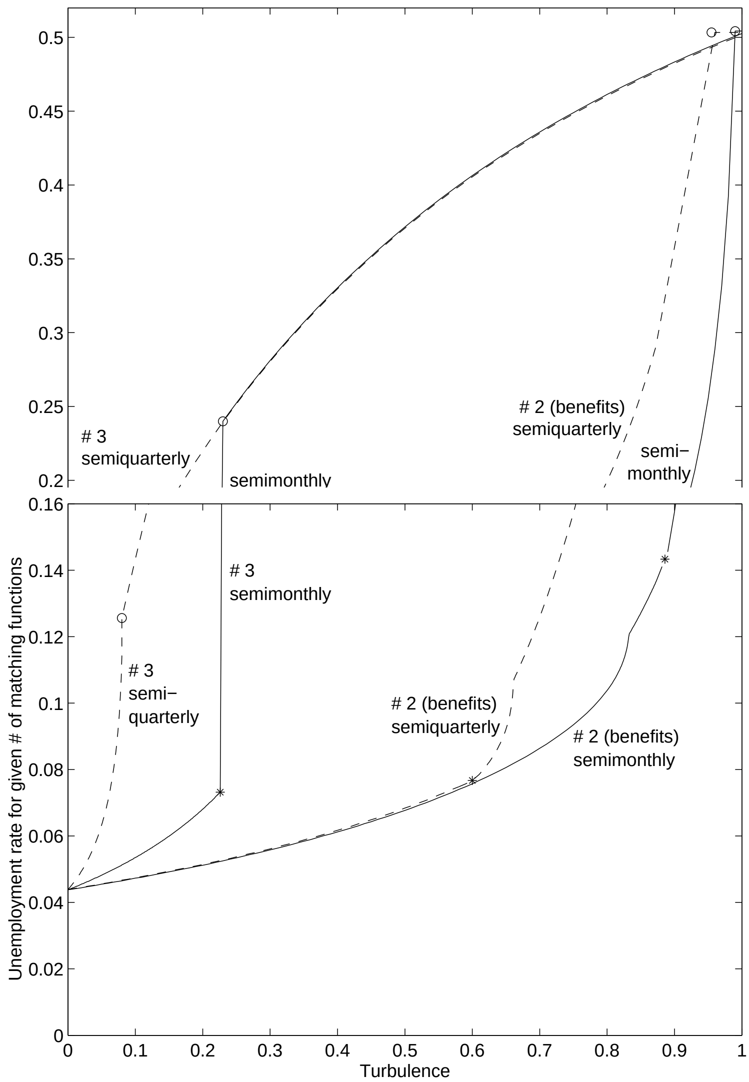


Figure 12: (Matching model) How aggregate unemployment rates are affected in the welfare state by the length of a period in the model with two matching functions where the unemployed are sorted by their benefits, and in the model with three matching functions. The dashed lines refer to the semiquarterly calibration of figure 11, and the solid lines depict the semimonthly version of that calibration.

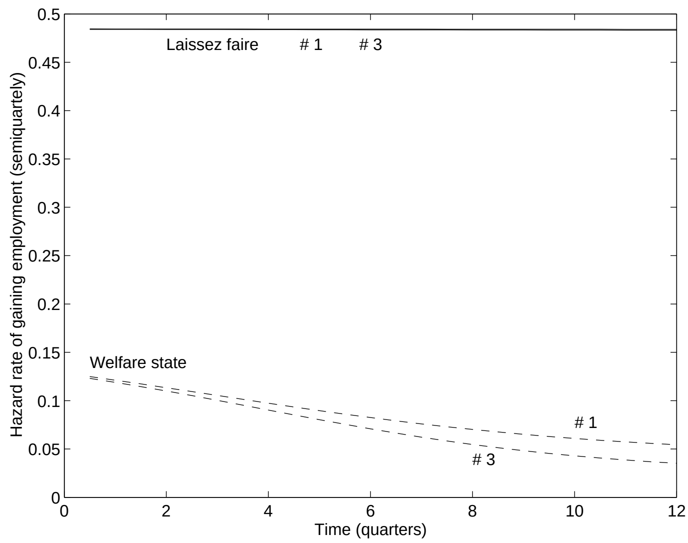


Figure 13: (Matching model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$. The two dashed lines indexed by #1 and #3 depict the welfare state with one and three matching functions, respectively. The solid line represents the laissez-faire economy with an almost perfect overlap of the outcomes associated with one and three matching functions, respectively. The policy of the welfare state is $(\eta, \Omega) = (0.7, 24)$.

allows for no duration dependence in a sense that would allow hazard rates to fall during an unemployment spell for a *given* unemployed worker. In contrast to Ljungqvist and Sargent (2005), who assume additional probabilistic skill losses while workers are unemployed and also probabilistic transitions between age classes, the state vector of an unemployed worker in our matching models is unchanged over the unemployment spell. That implies a constant hazard rate for a *given* unemployed worker. So the economy-wide hazard rate in figure 13 falls with the duration of spells in the welfare state because the least employable workers, those with low but constant hazard rates, constitute an ever larger share of the remaining unemployed at longer spells. These least employable workers are the low-skilled unemployed entitled to high benefits who have a low but positive hazard rate in the model with one matching function, and a zero hazard rate in the model with three matching functions.

7.3 Synthetic Beveridge curves

A time series scatter plot of $\bar{u}$ and v is called a Beveridge curve. Because we have analyzed only stationary equilibria of matching models with time-invariant fundamentals, we cannot deduce true Beveridge curves from our calculations. However, we can compute something akin to a Beveridge curve by varying the turbulence parameter π^d, while holding other parameters constant, thereby tracing out equilibrium $(\bar{u}, v)$ pairs . Figures 14 and 15 show the combinations of stationary equilibrium $(\bar{u}, v)$ pairs traced out as we vary π^d for models

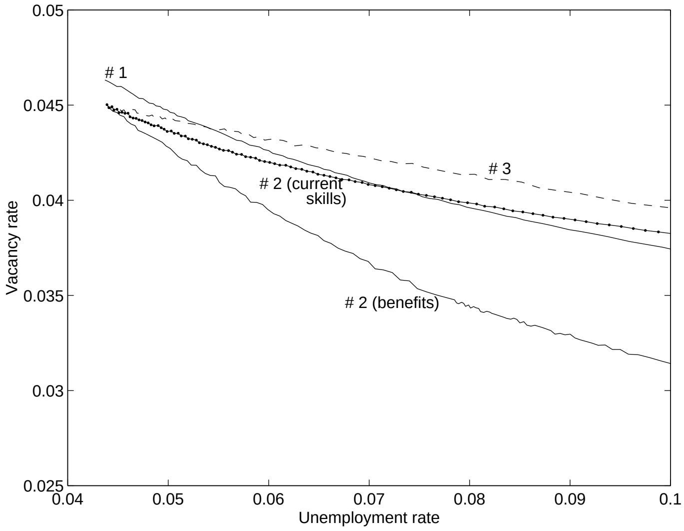


Figure 14: (Matching model) Synthetic Beveridge curves for steady states of welfare state $(\eta, \Omega) = (0.7, 24)$ economies with one, two, and three matching functions. Differences in turbulence π^d induce different steady state $(\bar{u}, v)$ pairs.

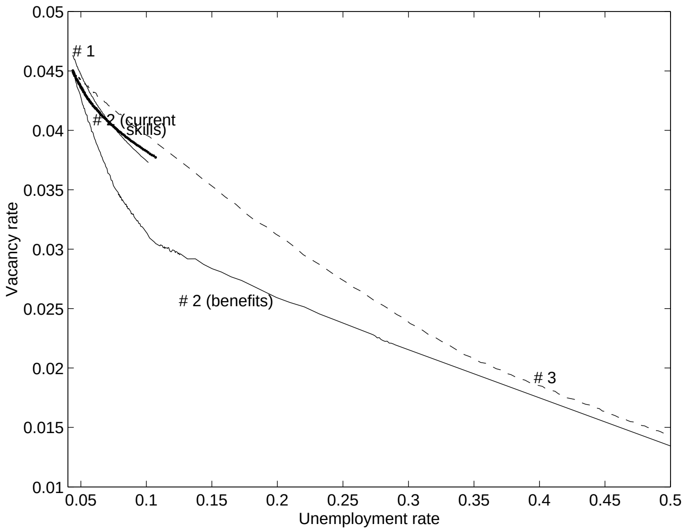


Figure 15: (Matching model) Synthetic Beveridge curves for welfare state economies for the entire range of $\pi^d \in [0, 1]$, and consequent bigger ranges of $\bar{u}$.

with one, two, and three matching functions. It is important to note that the same variation in π^d leads to different amounts of variation in $(\bar{u}, v)$, so that the variation in π^d associated with given movements along these curves differs across the models. For example, we move π^d over the entire range $[0, 1]$ to trace out the curve for the model with one matching model in figure 14; but variations of π^d over a smaller interval $[0, \bar{\pi}^d < 1]$ are used to trace out that for the multiple-matching function models. To make this point in a another way, in figure 15 we show the outcomes of varying π^d over the entire range $[0, 1]$ for all of the matching models.

Qualitatively, our synthetic Beveridge curves resemble real ones, sloping downward.

8 Quantitative findings in the search-island model

This section shows that laissez faire and welfare state versions of our search-island model exhibit the same responses of labor market outcomes as corresponding laissez faire and welfare state economies for the McCall search environment analyzed by Ljungqvist and Sargent (1998, 2005). Thus, forces neglected by the McCall model but present in the search-island model (i.e., risk-aversion, precautionary savings, firms that hire capital, and wages determined in competitive markets) fail to blunt the main force captured by the McCall model: how the incentives for unemployed workers to search change with increasing turbulence. By worsening the effective skill accumulation technology confronting workers, increased turbulence affects the relative returns to searching and collecting unemployment benefits. Though workers' choices of search intensity now also depend on their financial assets and the curvature of their utility function, considerations that are absent from the search model, these considerations fail to alter the pattern of outcomes.

Figures 16–20 show outcomes in our calibrated search-island model. For zero turbulence, figures 16 and 17, respectively show that equilibrium unemployment increases with increases in the UI replacement rate η and that it decreases with increases in the layoff tax Ω, *ceteris paribus*. The outcome that layoff taxes suppress unemployment also prevails in the quantitative analysis of Alvarez and Veracierto (2001), who report that equilibrium unemployment falls by 1.8 percentage points in response to a layoff tax equal to 12 months of wages. In our search-island model, where workers's skill levels contribute an additional source of heterogeneity relative to Alvarez and Veracierto's model, figure 17 shows that unemployment falls by 0.9 percentage points in response to a layoff tax of 50 which corresponds to roughly 12 months of wages for a low-skilled worker. Alvarez and Veracierto do not study UI replacement rates but instead compute the effects of one-time payments from the government to laid off workers. Not surprisingly, since they are invariant to the length of unemployment spells, such severance payments have only a muted (positive) effect on equilibrium unemployment. However, it is interesting to note that Alvarez and Veracierto report that they assumed a 66% UI replacement rate when initially calibrating their model to U.S. policies and data, and they found the unemployment rate of that calibrated model to be not much higher than in the laissez-faire version of their model. This seems puzzling given our figure 16 where the unemployment rate increases dramatically at replacement rates in excess of

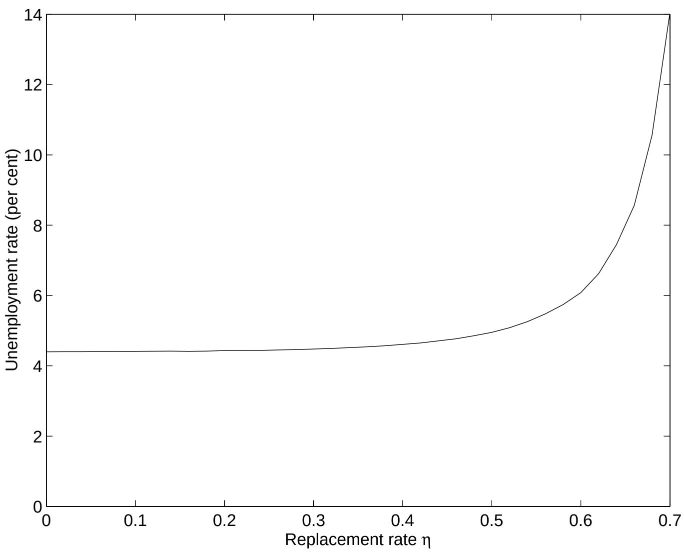


Figure 16: (Search model) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

55–60%, but we conjecture that the explanation is that Alvarez and Veracierto assume that unemployed workers lose their eligibility for unemployment benefits with a constant probability in every period while our unemployed workers keep their benefits throughout their entire unemployment spells.

As a benchmark parameterization of the welfare state, we set the replacement rate equal to 0.55 and the layoff tax equal to the above mentioned 12 months of wages for a low-skilled worker, $(\eta, \Omega) = (.55, 50)$. That yields an equilibrium unemployment rate of 4.1%, which is lower than the laissez-faire unemployment rate of 4.4%. Thus, both the search-island model and the matching model can rationalize why unemployment need not be high in the welfare state in tranquil times. (But as we will soon see, the representative family model with lotteries cannot rationalize this outcome.) Next, we study how turbulence gives rise to qualitatively the same effects in the search-island model as in the matching model.

The two panels of figure 18 show the disparate effects of an increase in turbulence on unemployment rates in the welfare state $(\eta, \Omega) = (.55, 50)$ and laissez-faire $(\eta, \Omega) = (0, 0)$ economies. The figures show both the total unemployment rate (solid lines) and the percentage of the workers who are unemployed and also had suffered a skill loss after a lay off in their last job (dashed lines). The dashed lines reveal that the explosion of unemployment in the welfare state economy when turbulence π^d increases is attributable to greater unemployment of previously high-skilled workers who have suffered skill loss upon termination. In the laissez faire economy, unemployment involving that group increases only mildly with an increase in turbulence, an outcome that explains why the overall unemployment rate in the laissez-faire economy is not much affected by increases in turbulence.

The two panels of figure 19 display how the inflow into unemployment and the average

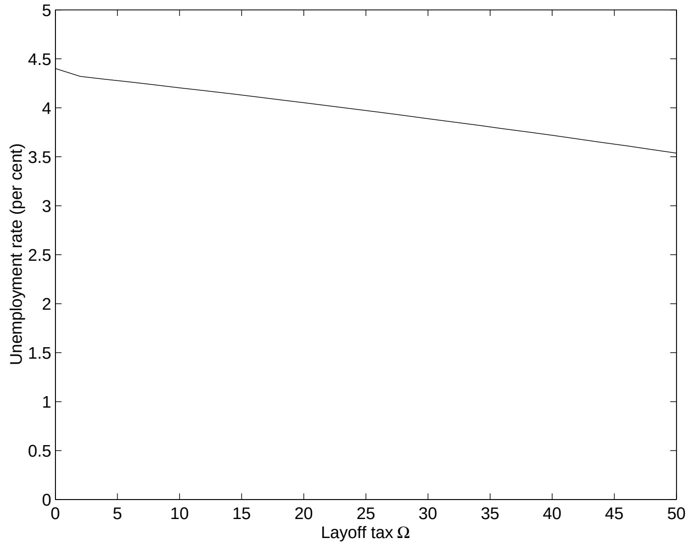
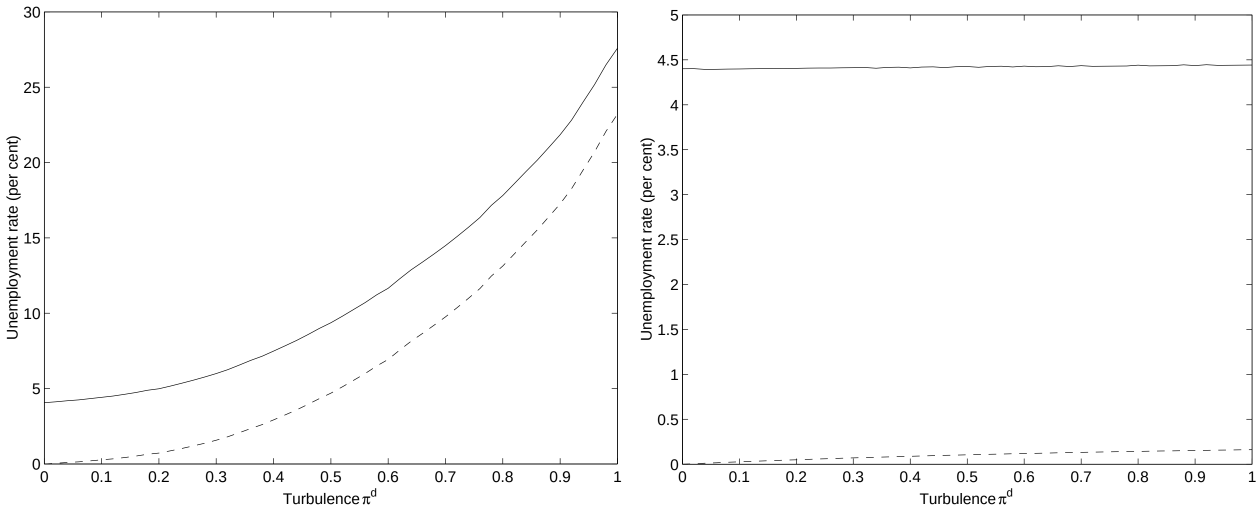


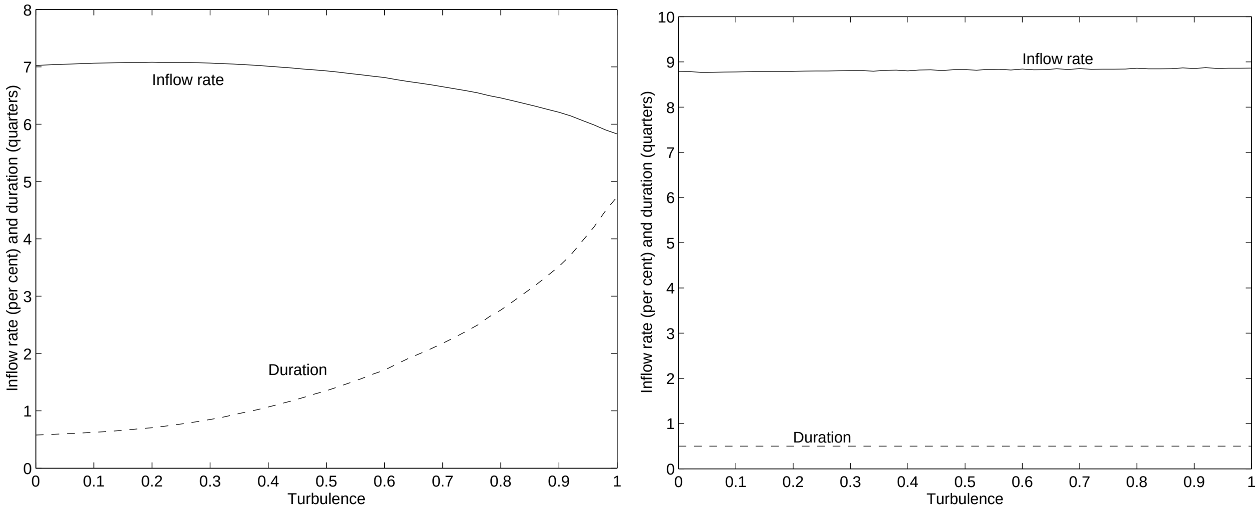
Figure 17: (Search model) Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6.4 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 50 corresponds to roughly one year of wage income for a low-skilled worker.



(a) Welfare state

(b) Laissez-faire economy

Figure 18: (Search model) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.



(a) Welfare state (b) Laissez-faire economy

Figure 19: (Search model) Inflow rate and average duration of unemployment in the welfare state (panel a) and the laissez-faire economy (panel b). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.

duration of unemployment respond to increases in turbulence π^d in the welfare state and laissez-faire economies. In the laissez-faire economy, the inflow rate and the duration are both impervious to increases in turbulence, while in the welfare state economy the average duration grows markedly with increases in turbulence; especially at higher levels of turbulence, the inflow rate into unemployment actually falls modestly with increases in turbulence.

Figure 20 shows how in very turbulent times ($\pi^d = 1$), hazard rates of gaining employment behave very differently in the laissez faire and welfare state economies, being flat in the former and rapidly declining with the length of the unemployment spell in the latter economy.

All of these findings are strikingly similar to the analysis by Ljungqvist and Sargent (2005), even though that earlier analysis was based on the McCall search environment without many of the features of the search-island model such as risk aversion, precautionary saving, and competitive firms that hire capital and labor. In turbulent times, the common feature in these seemingly very different frameworks is the presence of unemployed workers who have suffered skill loss but are entitled to relatively generous unemployment benefits based on past earnings. In Ljungqvist and Sargent (2005), these workers were more likely to become long-term unemployed because they chose relatively high reservation earnings as compared to their current earnings potential; and given the low likelihood of drawing such earnings from the wage offer distribution and the mere fact that the generous benefits made it less costly to stay unemployed, these workers also chose low search intensities. We can legitimately describe them as "discouraged workers" because they have low probabilities of

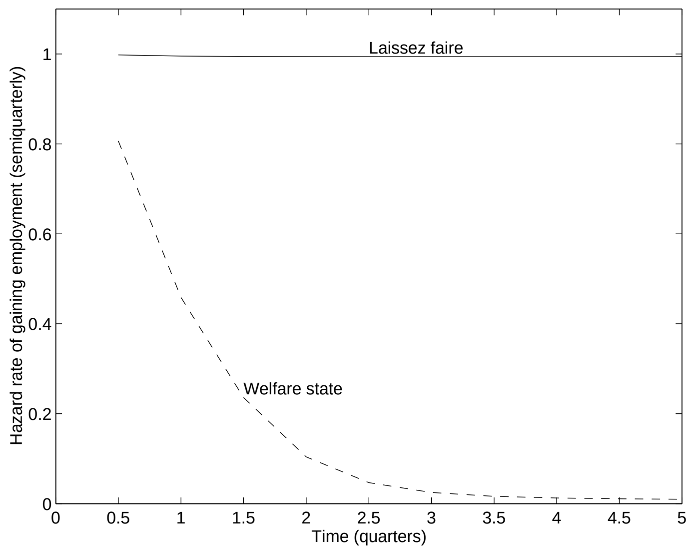


Figure 20: (Search model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$, in the welfare state (dashed line) and in the laissez-faire economy (solid line). The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.

returning to gainful employment. In our search-island model, such workers' choice of low search intensities is the only avenue that operates because the abstraction of Alvarez and Veracierto (2001) has the equilibrium outcome that all workers are paid the same wage rate per unit of skill. Evidently, this channel by itself is sufficient to explain how unemployment explodes in the welfare state in response to turbulence while the laissez-faire unemployment rate remains virtually unchanged.

8.1 Comparing the mechanics of the matching and search-island models

Even though the search-island and matching models have similar labor market outcomes, the mechanisms producing those outcomes differ between the two frameworks. As an illustration, consider figures 3 and 16 that show how the unemployment rate is positively related to the UI replacement rate. The curve for the matching model in figure 3 displays a more gradual increase, while the corresponding curve for the search-island model in figure 16 is nearly flat for a range of replacement rates below 40–50%. In the matching model, higher benefits affect the unemployment rate by increasing workers' threat points for wage bargaining. As described above, the invisible hand restores equilibrium after an increase in the replacement rate by lowering market tightness, thereby causing the probability that a worker encounters a vacancy to fall in order to compensate firms for the lower expected returns from posting vacancies that would occur if market tightness were not lowered. Hence, all unemployed workers suffer from increased congestion in the labor market. This mechanism by which higher benefits lead to higher unemployment is clearly continuous in the level of benefits. In contrast, the search-island model features a completely different mechanism through which higher benefits raise equilibrium unemployment because of the response in individual workers' search behavior. Higher benefits make it less costly to remain unemployed, and in response unemployed workers find it optimal to reduce their search intensities and so lessen the disutility of searching for new employment. But as seen in figure 16, this effect first becomes significant at relatively high replacement rates. The reason is that the unemployed workers must fend for themselves. Low replacement rates are of little comfort to the unemployed who must then finance their consumption with these low benefits and their savings, so their search intensities are not much affected and the unemployed concentrate instead on restoring their relatively higher labor market earnings. But at higher replacement rates, this mechanism for generating unemployment in the search-island model becomes very potent and workers in effect choose to furlough themselves into drawn out unemployment spells by setting low search intensities.

9 Quantitative findings in the representative family model with lotteries

This section shows that the representative family model with employment lotteries reproduces the responses to high UI and high turbulence found by Ljungqvist and Sargent (1998), but not the response to higher layoff taxes featured in Ljungqvist and Sargent (2005). Figure 21 shows that in tranquil times and with no layoff taxes, increasing the UI replacement rate η has a powerful effect of increasing the unemployment rate. This outcome agrees with those from the matching models and the search-island model, but the mechanism is different. Here there are neither the matching models' congestion effects nor the search-island model's search costs. Therefore, the change in unemployment is not an adjustment of the level of frictional unemployment that serves to alter firms' waiting times, thereby compensating them for their costs of posting vacancies, as in the matching models. Nor does it reflect a longer duration of unemployment coming from workers' diminished search activity, as in the search-island model. Instead, a higher UI replacement rate just makes leisure a more attractive use of the family's time.

Figure 22 reaffirms the findings of Hopenhayn and Rogerson (1993) and Ljungqvist (2002) that with zero turbulence in a representative family model with lotteries, an increase in the layoff tax causes the unemployment rate to *rise*, reversing the outcomes in Ljungqvist and Sargent (2005) and in the matching and search-island models of this paper. In the representative family lotteries model, anyone the family wants to put to work gets a job instantaneously, so the layoff tax does not have the effect of suppressing frictional unemployment that it does in our other models. Instead, an increase in the layoff tax decreases the equilibrium wage, prompting the household to substitute toward leisure.

The model's sensitivity to UI prompts us to choose a relatively low replacement rate, $\eta = 0.2$, in the benchmark parameterization of the welfare state. We do this to avoid

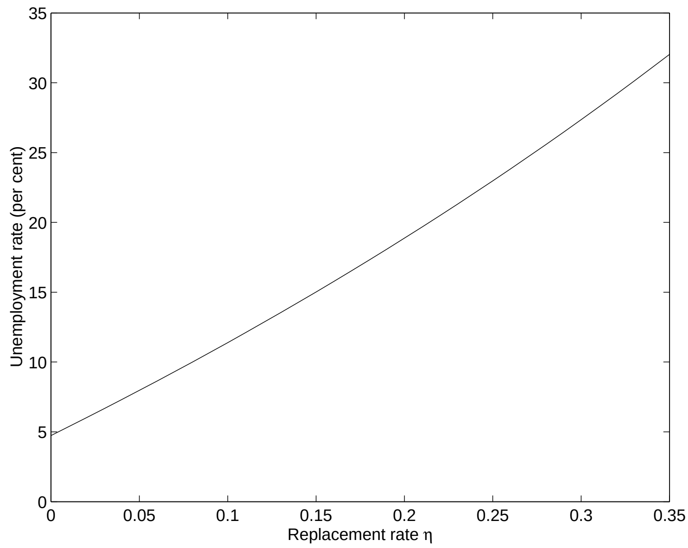


Figure 21: (Representative family) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

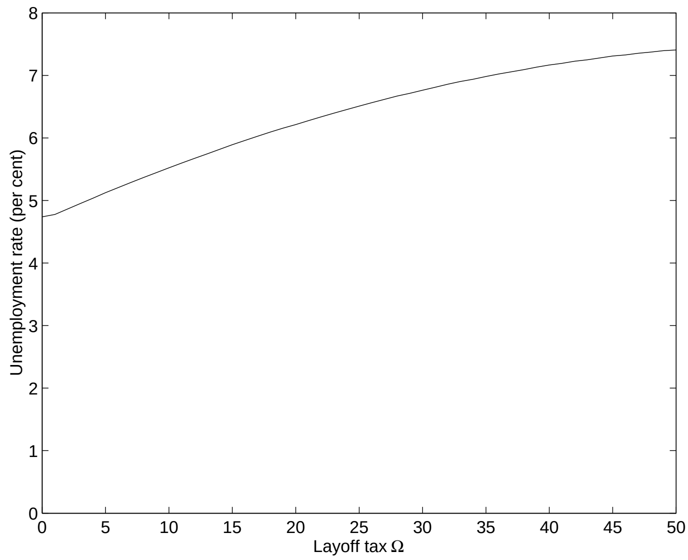
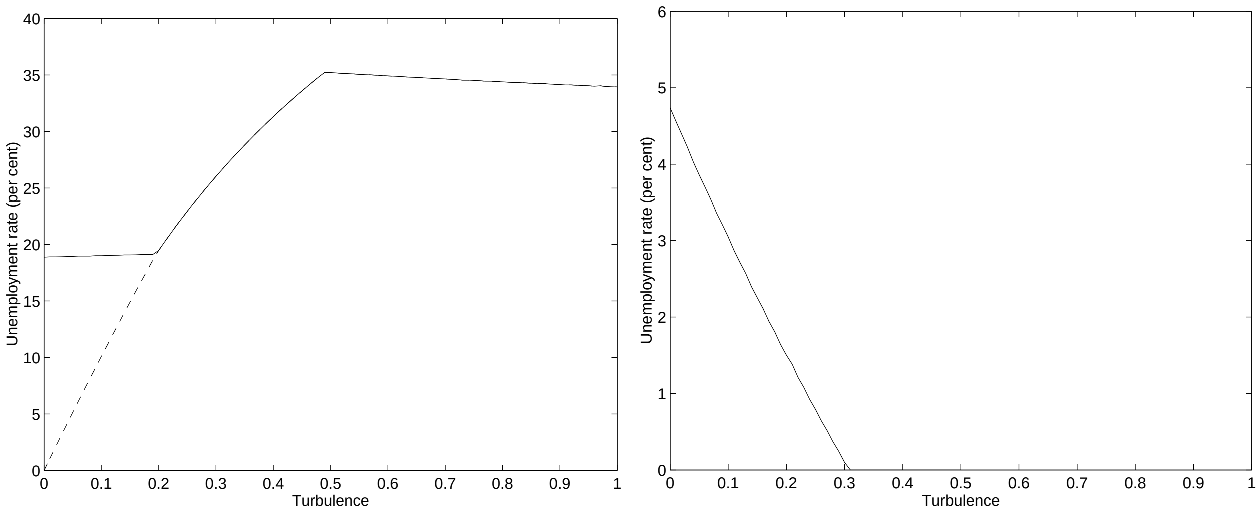


Figure 22: (Representative family) Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 48 corresponds to one year of wage income for a low-skilled worker.



(a) Welfare state (b) Laissez-faire economy

Figure 23: (Representative family) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. In the welfare state, the policy is $(\eta, \Omega) = (0.2, 0)$ and the dashed line shows the unemployed who have suffered skill loss (which is not a uniquely determined quantity in the laissez-faire economy and is therefore left out from panel b).

outrageously high unemployment in tranquil times. For the same reason, we set the layoff tax $\Omega = 0$, since it does not have the effect of suppressing unemployment that it does in the matching and search-island models. We thus admit defeat in our effort to use the representative family model in an explanation for how welfare states can sustain relatively low unemployment rates when $\pi^d = 0$, and proceed to ask whether the model predicts that unemployment erupts in the welfare state but not in the laissez-faire economy when turbulence π^d increases. Figure 23 provides an affirmative answer to this question: the two panels show how an increase in turbulence has very different effects in the welfare state $(\eta, \Omega) = (.2, 0)$ and the laissez faire $(\eta, \Omega) = (.0, 0)$ versions of this model. In the welfare state, unemployment increases with turbulence until turbulence reaches about .5, then unemployment falls slightly with further increases, while under laissez faire, increases in turbulence push unemployment down to zero.

To shed light on why the laissez-faire unemployment rate falls to zero as turbulence increases in the right panel of figure 23, it is useful to consider a simplified version of our representative family model that abstracts from retirement, startup costs for new firms, and shocks to firms' productivity. Let z denote a deterministic productivity level of all firms, which we assume takes a value that induces the family to choose an interior solution for employment. Given $\pi^u > 0$ and $\pi^d > 0$, the family chooses strictly positive steady-state fractions U, N_0, N_H of family members who are unemployed (with low skills), employed with low skills, and employed with high skills, respectively. The representative family's Euler equations and the equilibrium conditions that state that the family supplies the economy's labor and holds the aggregate capital stock imply the following steady-state values for k and n:

$$\bar{k} = \left[\frac{i+\delta}{\alpha z}\right]^{\frac{1}{\alpha-1}} [N_0 + (1+H)N_H] \,,$$

$$\bar{n} = N_0 + N_H = \frac{(1-\alpha)(i+\delta)(\pi^d+\pi^u)\left[1+\beta\pi^u(1+H)-\beta(1-\pi^d)\right]}{[i+\delta(1-\alpha)]\left[\pi^d+(1+H)\pi^u\right]\left[1+\beta\pi^u-\beta(1-\pi^d)\right]B} \,,$$

where $i = (\beta^{-1}-1)$ is the stationary net interest rate. Note that employment $\bar{n}$ does not depend on the productivity level z, as in real business cycle specifications with logarithmic preferences. But the employment effect of an increase in turbulence is strictly positive and given by

$$\frac{\partial \bar{n}}{\partial \pi^d} = \frac{(1-\alpha)(i+\delta)\pi^u(1-\beta)\left[1-\beta+2\beta(\pi^u+\pi^d)+\beta\pi^u H\right]}{[i+\delta(1-\alpha)]\left[\pi^d+(1+H)\pi^u\right]^2\left[1+\beta\pi^u-\beta(1-\pi^d)\right]^2 B} \, H \; > 0 \,.$$

Changes in the parameter π^d give rise to an equilibrium response in the representative family's behavior that can be understood in terms of substitution and wealth effects. An increase in π^d means that the return to working falls, which should reduce the family's labor supply because of the substitution effect, on one hand, and increase the family's labor supply due to the wealth effect from lower labor income, on the other hand. Evidently, the latter effect is stronger, since unemployment falls in response to increases in turbulence.

To understand the unemployment effects of increased turbulence in the welfare state, we divide the range of turbulence in the left panel of figure 23 into three regions; (a) the positive but relatively flat segment in region $\pi^d \in [0, .2]$, (b) the dramatic surge in region $\pi^d \in [.2, .5]$, and (c) the mildly downward-sloping segment in region $\pi^d \in [.5, 1]$. Recall that our specification of the skill deterioration technology (2) implies that at zero turbulence, there are no low-skilled workers entitled to high benefits. The only low-skilled workers are the newly born or workers who have not yet experienced a skill upgrade. At zero turbulence, the family decides to furlough enough of these workers into leisure that nearly 19% of the working-age population is unemployed.[35]

For low levels of turbulence, workers who suffer skill loss after layoffs constitute intramarginal workers in the family's leisure decision. It is trivially optimal to let these workers be the first to specialize in leisure because of their higher benefits while they share the same potential for future skill accumulation as the low-skilled workers who are entitled to low benefits; they remain intramarginal workers in that leisure decision so long as the family chooses unemployment for some of the latter workers. Hence, the family's marginal condition for assigning an additional worker to specialize in leisure at low levels of turbulence

[35] As discussed in appendix C.1, we have restricted parameters to guarantee that benefit policies are not so generous that they would induce families to accumulate skills simply to furlough high-skilled workers into unemployment and then forgo earning wages in order to collect UI. Hence, there is no steady state with unemployment among high-skilled workers.

pertains to a low-skilled worker who is entitled to *low* benefits: all low-skilled workers with *high* benefits have already been furloughed into unemployment. This marginal condition, which pertains to the entire region (a), yields an almost constant unemployment rate of approximately 19%. Thus, as turbulence π^d rises until it approaches .2, the family leaves the total amount leisure roughly constant and just changes the mix of types of workers who enjoy leisure. Low-skilled workers with low benefits are replaced by low-skilled workers with high benefits whom the family furloughs into permanent leisure for the rest of their lives. The slightly positive relationship between turbulence and unemployment in region (a) can be attributed to an increasing tax wedge on labor supply in response to the government's need to balance a budget with higher benefit expenditures (since the number of unemployed people who collect high benefits increases with turbulence). The higher tax rate on labor has only a substitution effect because the wealth effect is neutralized when the government pays out the tax proceeds as unemployment benefits to the representative family.

At a critical level of turbulence of approximately $\pi^d = 0.2$, the marginal condition for assigning a low-skilled worker who is entitled to low benefits to specialize in leisure holds with equality when actually no one of that category is ordered by the family to enjoy leisure. By then, turbulence has reached the critical level at which the last available unemployed low-skilled worker with low benefits has been replaced by a low-skilled worker entitled to high benefits – turbulence has entered region (b). Since the marginal condition holds with equality at that critical level of turbulence, it follows that the family's first-order condition for letting an additional low-skilled worker with high benefits specialize in leisure is a strict inequality when a further increase in turbulence causes that additional worker to 'materialize'. Thus, for levels of turbulence in region (b), the family strictly prefers to furlough into leisure *all* low-skilled workers who are entitled to high benefits but to furlough *none* of the low-skilled workers who are entitled to low benefits. As a result, the unemployment rate surges dramatically with increases in turbulence in region (b) because unemployment increases then one-for-one with the growing number of skill losers whom the family furloughs into leisure for the rest of their lives.

When turbulence reaches another critical level of approximately $\pi^d = 0.5$, the family's marginal condition for assigning a low-skilled worker who is entitled to high benefits to specialize in leisure starts to hold with equality – turbulence has entered region (c). The representative family sends now some of the skill losers back to work, and it forgoes their high benefits in exchange for their after-tax wages and their potential for renewed skill accumulation. In region (c), unemployment falls mildly with increases in turbulence because of the wealth effect from turbulence, an effect that was described in the case of the laissez-faire economy above. But one might then ask why the equilibrium response to the wealth effect is so meager in the welfare state as compared to the laissez-faire economy in the right panel of figure 23, where unemployment plummets in response to turbulence. The reason is that the representative family of the welfare state faces a very different opportunity set in which the labor income tax needed to finance the UI system is about 11.4% in region (c), and the unemployed workers collect an *effective* replacement rate of 40% on current earnings potential rather than the stipulated rate of 20%. (Recall that the stipulated replacement rate

$\eta = 0.2$ is applied to their past earnings, which are twice as high as their current earnings potential.) It follows that unemployment remains a relatively attractive option for the family in the welfare state as compared to the family in the laissez-faire economy in which neither taxes nor benefits distort its labor supply.

9.1 No frictional unemployment and high labor supply elasticity

The employment effects of turbulence in the laissez-faire economy in the right panel of figure 23 are reminders of the fact that the representative family model with employment lotteries does not embody frictional unemployment. While both the matching and search-island models generate unemployment because of labor market frictions, there is idleness in the frictionless representative family model only if the family finds it optimal to let some of its members specialize in leisure. Even though our preference specification is the same as that in many real business cycle analyses such as Hansen (1985), we have apparently lost the standard result that changes in productivity do not affect the steady-state employment to population ratio. The reason is that we have expanded the notion of productivity beyond the standard multiplicative productivity level in the aggregate production function to entail a skill technology in which work experience can enlarge individual workers' productivities. This technology introduces new intertemporal considerations into the representative family's labor supply decision that prevent substitution and wealth effects in response to steady-state changes in the productivity of that technology from cancelling each other. This makes possible the outcome that unemployment vanishes in our representative family model when turbulence increases – an outcome that is not possible in our matching and search-island models because these models embody frictional unemployment associated with the entry of new workers into the labor market and the churning of old workers between jobs.

The response of the representative family model to UI stems from the high labor supply elasticity of that framework, a feature that Hansen (1985) and Rogerson (1988) invoked to allow the model to explain business cycle fluctuations. That high labor supply elasticity that apparently works so well in the business cycle context yields unrealistically large equilibrium responses to the replacement rate in the UI system, a finding that perhaps explains why earlier researchers have refrained from incorporating European-style generous UI benefits into the representative family employment lotteries model.

10 Concluding remarks

Our three frameworks for labor market analysis differ: two incomplete markets models – the matching and search-island models – incorporate *different* labor market frictions, and the representative family employment lotteries model has *none*. Yet except for the positive response to more EP in the representative family model, the responses to more EP, increased UI, and increased turbulence are similar in the three frameworks. Though the detailed mechanisms through which UI and EP affect equilibrium unemployment differ, some common forces pull all three frameworks. In particular, adverse effects of high UI that are ignited by high turbulence predominate in all three. In subsection 10.1, we highlight the different mechanisms at work in the three models. In subsection 10.2, we comment on why the hypothesis advocated by Ljungqvist and Sargent (1998) is so robust that it comes through in three such different frameworks. In subsection 10.3, we summarize the ways the three frameworks teach us to think about unemployed European workers and offer our own views in subsection 10.4 about which description of the situation of those workers is most realistic.

10.1 Comparison of mechanisms

The three models use very different arrangements to match workers with jobs. This leads them to promote different ways of thinking about how unemployment is determined and what 'activity' it is that unemployed workers do.

1. In the matching models, postulated matching functions impose frictions and externalities on labor markets. By increasing the value of a newly matched worker's outside option, increases in UI diminish the surplus value of a match at a given initial draw of match productivity. To sustain firms' zero-profit condition for creating jobs, the invisible hand decreases 'market tightness', expressed as a ratio of vacancies to unemployment, thereby decreasing the time firms expect to wait until a vacancy is filled. That increases the time that an unemployed worker waits to land a job. The matching models thus focus on how the pools of unemployed workers and vacancies adjust to sustain firms' zero-profit condition for posting vacancies. Unemployed workers impose congestion costs on other unemployed workers, and firms that post vacancies impose congestion costs on other firms that post vacancies.

2. In the search-island model, the important friction is a costly technology that workers must use to find jobs.Unemployed workers personally bear search costs. They impose no congestion costs on other unemployed workers. Alterations in UI affect the unemployment rate by altering an unemployed worker's choice of search intensity, which determines the statistical distribution of time that a worker is unemployed and through it the equilibrium unemployment rate. Thus, the search-island model puts the spotlight on the individual worker's incentives to find a job.

3. In the representative family lotteries model, persons who are randomly selected to work face none of the matching or search frictions present in the matching and search-island models. Everyone selected to work is immediately hired. The representative family model focuses on how society chooses to allocate leisure among workers in different skill and benefit entitlement categories. The representative family allocates work and leisure in a way that is privately efficient by instructing those workers who are entitled to the best deals from the UI benefit agency to specialize in leisure. In the matching and search-island models, match and search frictions generate frictional unemployment. In the representative family model, there is no frictional unemployment, a difference that accounts for the models' differing responses to increases in EP.

4. In the matching and search-island models, market incompleteness separates individual workers and makes them bear the consequences of their own actions as well as of the good or bad luck that comes their way in the form of shocks to their skills and their employers' productivities. In the representative family lotteries model, a comprehensive family-wide consumption insurance arrangement protects individual workers from adverse consequences of those shocks. That protection, the employment lottery, and the associated high aggregate labor supply elasticity have important quantitative implications about the response to UI.

10.2 Robustness of the turbulence story

We have used our three models to reexamine the finding of Ljungqvist and Sargent (1998) that the persistent increase in European unemployment since the 1980s can be explained by how workers choose to respond to increased turbulence, modelled as a particular deterioration of a stochastic skill transition technology, when the government offers unemployment benefits. The economic force behind our finding is so strong that it comes through in three other types of general equilibrium model. In each of the matching models, the search-island model, and the representative family model, high unemployment erupts in a welfare state with generous benefits when laid off workers are exposed to more turbulence in their prospective earnings, while the unemployment rate in a laissez-faire economy remains unchanged (or actually decreases in the representative family model). The higher unemployment pool in a welfare state consists mainly of workers who have suffered sudden losses of immediate earnings prospects when laid off. The fact that welfare benefits are based on past earnings marginalizes these workers by leaving them unemployed for long periods of time. In the matching models, these unemployed workers with UI benefits that are high relative to their current earnings prospects encounter fewer acceptable matches because a vacancy's idiosyncratic productivity must be so much higher to yield a wage rate that can compete with the high UI benefits. If there are multiple matching functions, these workers have to contend with low market tightness and the associated relative scarcity of vacancies implies long waiting times before an unemployed worker succeeds in matching with a job. In the search-island model, workers with generous benefits but poor immediate labor market prospects choose low search intensities (they become 'discouraged workers'). In the representative family model, the family optimally allocates its labor resources by assigning laid off workers who have high benefits and low immediate earnings potential to specialize in leisure.

Ljungqvist and Sargent (2005) showed that within their McCall model with human capital, high layoff taxes can explain why, until the 1970s brought higher turbulence, unemployment rates were actually much *lower* in Europe than in the United States. Here, we have shown that their mechanism operates in two of our three alternative frameworks. In the matching model and the search-island model, layoff taxes delay the reallocation of labor, lock workers into their current employment, and lower frictional unemployment.[36] In

[36] Myers (1964, p. 180–181), whom we quoted in footnote 9, also conjectured that stringent EP might explain the low European unemployment rates in the 1950s and 1960s: "One of the differences [between the

contrast, in the representative family model, a layoff tax increases the equilibrium unemployment rate. Layoff taxes diminish labor turnover in the representative family model, but because there are no frictions in the labor market, there is no frictional unemployment to suppress. Unemployment *rises* with increases in layoff taxes because those increases reduce the equilibrium real wage rate. In response to a lower private return to work, the representative family substitutes away from consumption towards leisure by sending a smaller fraction of its members to work.[37]

In the matching and search-island models, government supplied UI and EP interact with turbulence in ways that can explain the European unemployment experience across the decades. Despite having levels of UI that were already generous in the 1950s and 1960s, Europe enjoyed low unemployment because high EP suppressed frictional unemployment by reducing turnover in the labor market. Inflow rates into unemployment were substantially lower in Europe than in the U.S., but durations of unemployment spells were approximately the same. The inflow rates into unemployment remained unchanged with the advent of turbulence in late 70s, but the length of unemployment spells exploded in Europe.

However, we have been unable to tell this story within the representative family lotteries model, first, because EP does not reduce unemployment in tranquil times, and, second, because generous UI generates too much unemployment in that framework at all levels of turbulence. We have also been frustrated in our attempt to use the lotteries to understand the European unemployment experience in terms of observations on the duration of unemployment spells. The reason is that there are multiple ways to design the employment lotteries that leave the aggregate allocation unaltered but that imply very different life histories for workers (see appendix C.7). Only for the middle region of the left panel of figure 23 is the duration of unemployment spells uniquely defined: in this region, the representative family furloughs all skill losers into unemployment for the rest of their lives, while no one else is unemployed. But absorbing states of unemployment could also be equilibrium outcomes in both the left and the right regions. In the left region, the unemployed people with low skills and low benefits could be newborn workers who have been furloughed into life-time unemployment. In the right region, all unemployed workers have low skills and high benefits, but since there also exist some workers with those same characteristic who *are* employed in that region, there exist multiple lottery designs that support the aggregate allocation, including

U.S. and Europe] lies in our attitude toward layoffs. The typical American employer is not indifferent to the welfare of his work force, but his relationship to his workers is often rather impersonal. The interests of his own employers, the stockholders, tend to make him extremely sensitive to profits and to costs. When business falls off, he soon begins to think of reduction in force ... In many other industrial countries, specific laws, collective agreements, or vigorous public opinion protect the workers against layoffs except under the most critical circumstances. Despite falling demand, the employer counts on retraining his permanent employees. He is obliged to find work for them to do. ... These arrangements are certainly effective in holding down unemployment. But they involve a very heavy cost. They partly explain the traditionally lower productivity and lower income levels in other countries. Here is something we can learn from our neighbors, therefore, but are we quite sure we want to learn it? Are there not better ways to reduce unemployment?"

[37]For a detailed comparison of the employment implications of layoff taxes in different frameworks, see Ljungqvist (2002).

one where a fraction of workers with these characteristics are furloughed into unemployment for the rest of their lives, while other skill losers are sent back to employment immediately after they are laid off. In the laissez-faire economy depicted in the right panel of figure 23, the only equilibrium restriction is that no high-skilled workers are ever unemployed, so there is a multiplicity of lottery designs for all levels of turbulence.

10.3 Different views of the European unemployment experience

It is time to think about whether our conversations with the unemployed European workers in our three models remind us, in Lucas's words, of 'talking to an unemployed [European] friend'.[38] Although an adverse interaction between high UI and high turbulence transcends all three models, the unemployed workers in these models have different alibis for why they aren't working.

10.3.1 Unemployed Europeans as winners of a lottery

In response to the question 'why aren't you working?', an unemployed worker in the representative family lotteries model would tell us that he has had the good luck to win a lottery prize that entitles him to specialize in enjoying leisure.[39] In response to 'but how will you eat?', the unemployed worker would tell us that the Europe he lives in functions like one big happy European-wide family that awards everyone the same consumption stream independently of his luck in the employment lottery. If he were well read, the unemployed European worker might detect an Anglo-Saxon drift to our questions and tell us that we are thinking incorrectly about unemployed European workers. He would remind us that in his Europe, individual workers are not isolated decision makers who are left to protect themselves as best they can against the random shocks that economic life throws at them.[40] Perhaps, to our surprise, he would credit this outcome to *private* financial markets – the complete contingent claims markets that enable sharing the labor income risk associated with the employment lotteries – not to the activities of his government. Seen from a private perspective, government-provided UI enriches the collective of workers engaging in the employment lotteries, but seen from a social perspective the associated adverse incentive effects impoverish the economy. Even modest amounts of government-provided UI threaten to send the economy down into an abyss.

The private insurance markets in the representative family model make government-provided unemployment insurance not only unnecessary but a recipe for disaster. Employment lotteries and contingent claims markets yield a high aggregative labor supply elasticity. Prescott (2004) vividly demonstrated the potency of that high labor supply elasticity in his

[38]We have added 'European' to Lucas's phrase.

[39]If we were to ask him when he had last worked, he might tell us about a job loss and skill transition event that, according to the lottery ticket he had drawn, declared him a leisure-specializing winner of the lottery contingent on the history of shocks corresponding to that event. Recall the discussion of figure 23.

[40]And he might tell us that Chiappori (1992, 1997), and Browning and Chiappori (1998) mistakenly drew the boundary of their family around a nuclear instead of a national family.

time series analysis of the American-European labor market divide. After observing similar numbers in the early 1970s, Prescott attributes the dramatic decline of 20 to 30 percent in hours worked per person of working age in Germany and France in the 1990s to tax increases of 7 and 10 percent, respectively.[41] Prescott incorporates neither government-provided UI nor any other welfare benefits (see footnote 11). In light of the large equilibrium responses to the tax increases studied by Prescott, think about what would happen if we were to introduce European-style social insurance into Prescott's framework: figure 21 shows how economic activity in the representative agent lotteries model shuts down with generous government supplied UI. Evidently, the high labor supply elasticity in the representative family lotteries model implies that generous government welfare programs would lead to the ultimate 'abuse' of such social insurance. Hence, a seemingly fatal shortcoming of that model for explaining the European employment experience is that the model predicts far too low a labor supply for realistically calibrated levels of government-provided UI. With realistic levels of government-provided UI, the puzzle for that model becomes, why do Europeans work at all?

10.3.2 Unemployed Europeans as victims of congestion and their own bargaining power

In the matching models, individual unemployed workers are isolated decision makers, not members of a national representative family. Because they are pooled with other workers who also wait in the same matching function, their employment prospects and anticipated durations of unemployment depend on the characteristics of those other workers and how costly it is for firms to create vacancies. The mixture of skills and UI benefit entitlements of the workers in a pool determine how profitable it is for firms to post vacancies there. In a matching function that includes low-skilled workers with different benefits, it is more costly for firms to match with workers with low skills who are entitled to high UI than to match with workers with low skills who are entitled to low UI. It requires a higher productivity draw to form successful matches with the former workers as compared to the latter workers and for a given productivity draw, bargaining with the former workers leads to higher wage payments as compared to bargaining with the latter workers. To induce firms to post vacancies in a pool with many low skill, high UI entitlement workers, the invisible hand must set market tightness $\theta = v/u$ to be low enough that firms can expect to match with workers frequently enough. But this means that workers can expect to match infrequently with firms, so that their expected durations of unemployment are high. Thus, if we were to ask an unemployed worker waiting in one of the matching models why he had been unemployed for so long, he could blame the other workers in his matching function whose bad characteristics are responsible for making market tightness low.

If pressed further about the reasons for his long unemployment spell, a low-skilled unem-

[41] Ljungqvist (2005) offers a qualification to this statement by showing that two thirds of the predicted employment effects in Prescott's analysis is due to the tax increases while one third is caused by movements in Prescott's estimate of a consumption to output ratio that determines the wealth effect of taxation.

ployed worker with high benefits would perhaps concede that his tough bargaining posture during 'job interviews' might contribute to his current predicament. Given his high benefits relative to his current skills, he will confront each prospective employer with a sizable threat point during the Nash bargaining process. If we were to ask the worker how he expects to get away with such outrageous wage demands when there are so many other unemployed workers also waiting in the same matching function, he could point to the fact that the firm would face him as the sole job candidate. If the firm would like to meet other candidates, it would have to expend additional resources on posting a vacancy and incur a delay of at least one more period before meeting someone else. Besides, any future encounters would also be on a bilateral basis. A firm could never expect to meet more than one job seeker at the negotiation table.

If the economy consigns low skill, high UI entitlement workers to their own matching function, it is true that unemployment rates for workers *not* in that 'skill-losers market' will be lower. But when UI benefits are high and turbulence is high, the existence of such an isolated market for losers of skills becomes a misfortune for the steady state of the economy, because high turbulence destines so many workers to pass through the skill-losers market in which firms choose optimally to post very few vacancies. Hence, the unemployment rate in such an economy literally explodes when turbulence increases because losers of skills become victims of long-term unemployment in their overly congested matching function. Having a single matching function in the economy dilutes the adverse consequences for unemployment of high turbulence, but even then, by creating more skill losers in the pool of prospective workers, high turbulence in conjunction with high UI causes equilibrium unemployment to grow in order to induce firms to post vacancies.[42]

10.3.3 Unemployed Europeans as discouraged workers

Because it emphasizes the factors that influence an unemployed worker's choice of search intensity, talking with an unemployed worker in the search-island model most closely resembles the conversation that Lucas imagined with an unemployed worker in the McCall model. An individual's search intensity is the only factor that determines his duration of unemployment. A worker's search intensity depends on his asset level, his skill level, his benefit entitlement, and, as a determinant of the skill accumulation technology, the level of the turbulence parameter π^d. For a given level of turbulence, workers with low skills, high accumulated financial assets, and high benefit entitlements choose the lowest search intensities. The weak incentives to search provided by their high UI entitlements and their

[42]Mortensen and Pissarides (1999) study a matching model with heterogeneous workers endowed with different skill levels who match with firms in separate skill-specific matching functions. They show that a mean-preserving spread in the distribution of the labor force over skill types can explain the divergent labor market performance in the U.S. and Europe, given differences in UI and EP. Pissarides (1992) analyzes skill accumulation in a two-period overlapping generations model where all workers match with firms in a single matching function. His focus is on how the externality in the matching process gives rise to a propagation mechanism where a temporary shock to employment can persist for a long time.

current low skills are what discourages these 'discouraged workers'.[43]

The search-island model and the representative family lotteries model have the common feature that there are no externalities in the labor market. A worker selling his labor services or a firm buying those services does not inflict injuries on others in the labor market beyond what a seller or a buyer of a good ordinarily imposes on competitors. Furthermore, since the wage rate is determined competitively in these two models, the holdup problem present in the bargaining setting in the matching model is absent. Thus, unemployment in the search-island model and the representative family lotteries model both directly reflect workers' decisions. After allowing for the time lag imposed by the time-to-create-jobs assumption, firms will create a number of new jobs that is adequate for all workers who are willing to work at the competitive wage, and who have 'found the labor market' in the search-island model.

However, the search-island model shares two crucial features with the matching models: labor market frictions and incomplete markets. Those features enable these frameworks to explain why unemployment rates were lower in Europe than in the U.S. until the late 1970s, in spite of higher UI and more stringent EP in Europe – outcomes that remain a mystery for the representative family lotteries model.

10.4 Our opinion

We have let the models, and the unemployed workers living inside them, speak for themselves. It is wiser for the reader to listen to those workers than to us. But for what they are worth, we state our reactions to what these imaginary workers have told us. While all three models bear important insights, our conversation with the discouraged worker in the search-island model of section 10.3.3 rings truest to us. That is why we continue to advocate the extended McCall framework of Ljungqvist and Sargent (1998, 2005) as a good way to think about the choices confronting European workers and policy makers. The very low employment rates reported for the European economy with employment lotteries in section 9 do not capture the fact that the welfare states of Europe actually provided low unemployment rates before the 1970s. Viewing outcomes in Europe in the 1950s and 1960s from the point of view of the quantities that emerge from the search-island and matching models, on the one hand, and the representative family lotteries model, on the other, would make us think that those good European outcomes occurred until the 1970s *because* there were incomplete markets and no employment lotteries, forcing each household to seek its own fortune either in labor markets or in government welfare programs. We might say, "thank goodness that Europe has actually *not* had the nation-wide private consumption insurance and employment lotteries that in the presence of realistically calibrated UI benefits would have brought economic activity to a standstill!"

[43] If one could account for the discouraged workers who have taken advantage of early retirement and disability programs available in Europe, we suspect that, building on the ideas of Edling (2005), one would find even more discouraged workers than are recorded in the unemployment statistics. See the remarks in footnote 10.

Appendices

Three appendices describe the models in enough detail to prepare computer programs to compute their equilibria.

A Matching models

A.1 Single matching function

When there is a single matching function, the probability that a firm meets a worker with skill h and benefit entitlement b is

$$\lambda^f(h,b) = \frac{M(v,\bar{u})}{v}\,\frac{u(h,b)}{\bar{u}} = m(\theta)\,\frac{u(h,b)}{v} \tag{12}$$

and the probability that a worker with skill h and benefit entitlement b is matched with a vacancy is

$$\lambda^w(h,b) = \frac{M(v,\bar{u})}{\bar{u}} = m(\theta), \tag{13}$$

which is independent of (h,b). When we introduce multiple matching functions in subsection A.5, the probability that a worker with skill h and benefit entitlement b is matched with a vacancy will depend on (h,b).

A.2 Match surplus

When an unemployed worker with skill h and benefit entitlement b meets a firm with a vacancy, the firm-worker pair draws productivity z from a distribution $Q^o_h(z)$. The firm and the worker will stay together and produce if the match surplus $S^o(h,z,b)$ is positive:

$$\begin{aligned} S^o(h,z,b) \quad = \quad & \max_{\{\text{stay, depart}\}} \Big\{ (1-\tau)z - [1-\beta(1-\rho)]\,W(h,b) \\ & +\beta(1-\rho)\Big[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')S(h',z')\Big]\,,\ 0\Big\}, \end{aligned} \tag{14}$$

where $W(h,b)$ is the worker's outside value, and $S(h,z)$ is the surplus associated with a continuing match. A worker with skill h and benefit entitlement b has an outside option with value

$$W(h,b) \quad = \quad \tilde{b}(b) + \beta(1-\rho)\Big[W(h,b) + \lambda^w(h,b)\sum_z \psi S^o(h,z,b)Q^o_h(z)\Big]. \tag{15}$$

Free entry makes the firm's outside value zero. The firm and worker split the match surplus $S^o(h,z,b)$ through Nash bargaining, with outside values as threat points. Let $\psi \in (0,1)$ denote the worker's share of the match surplus. Because both parties want a positive match surplus, there is mutual agreement on whether to form a match. The reservation productivity $\bar{z}^o(h,b)$ satisfies

$$S^o(h, \bar{z}^o(h,b), b) = 0. \tag{16}$$

The surplus of a continuing match is

$$\begin{aligned} S(h,z) &= \max_{\{\text{continue, break up}\}} \Big\{ (1-\tau)z - [1-\beta(1-\rho)]\, W(h,h) \\ &\quad + \beta(1-\rho) \Big[-\pi^o\Omega + (1-\pi^o) \sum_{h',z'} p^n(h,h') Q_{h'}(z,z') S(h',z') \Big] ,\ -\Omega \Big\}. \end{aligned} \tag{17}$$

The government's policy of imposing a layoff tax Ω on matches that break makes (17) differ from expression (14).[44] A reservation productivity $\bar{z}(h)$ satisfying

$$S\left(h, \bar{z}(h)\right) = -\Omega \tag{18}$$

characterizes whether a match is dissolved.

A.3 Equilibrium condition

In equilibrium, firms expect to break even when posting a vacancy:

$$\mu \;=\; \beta(1-\psi) \sum_{h,z,b} \lambda^f(h,b) S^o(h,z,b) Q^o_h(z). \tag{19}$$

This condition will pin down the equilibrium value of market tightness θ.

A.4 Wage determination

Alternative wage structures support the same equilibrium allocation. We follow Mortensen and Pissarides (1999) and assume a two-tier wage system.[45] In particular, when a firm with a vacancy meets an unemployed worker with skill h and benefit entitlement b, they bargain. The worker's outside value is $W(h,b)$ and the firm's outside value is zero. Because they do not incur the layoff tax if they do not reach an agreement, the layoff tax does not directly affect the bargaining between a newly matched worker and firm. But if they succeed in forming a match, the firm must pay the layoff tax after any future breakup. We capture this by setting the firm's threat point equal to $-\Omega$ in future Nash bargaining.

[44]Another difference between expressions (14) and (17) is that an employed worker's benefit entitlement is encoded in his skill level h, so there is one less state variable in surplus expression (17).

[45]The risk neutral firm and worker would be indifferent between adhering to this two-tier wage system or one in which workers receive a fraction ψ of the match surplus $S(h,z)$ in every period (which would have the worker paying a share ψ of any future layoff tax). As emphasized by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Optimal reservation productivities remain the same. Under the two-tier wage system, a newly hired worker in effect posts a bond that equals his share of the future layoff tax.

These assumptions give rise to a two-tier wage system. There is one wage function $\tilde{w}^o(h,z,b)$ for the initial round of negotiations between a newly matched firm and worker, and another wage function $\tilde{w}(h,z)$ associated with renegotiations in an ongoing match. These wage functions satisfy

$$\begin{aligned} \tilde{w}^o(h,z,b) &= W(h,b) + \psi S^o(h,z,b) - \beta(1-\rho)\Big\{\pi^o \sum_{h'} p^o(h,h')W(h',h) \\ &\quad +(1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')\Big(\psi\big[S(h',z')+\Omega\big] + W(h',h')\Big)\Big\}, \quad (20) \\ \tilde{w}(h,z) &= W(h,h) + \psi\big[S(h,z)+\Omega\big] - \beta(1-\rho)\Big\{\pi^o \sum_{h'} p^o(h,h')W(h',h) \\ &\quad +(1-\pi^o)\sum_{h',z'} p^n(h,h')Q_{h'}(z,z')\Big(\psi\big[S(h',z')+\Omega\big] + W(h',h')\Big)\Big\}. \quad (21) \end{aligned}$$

A.5 Multiple matching functions

We entertain some alternative specifications that proliferate matching functions in the spirit of Mortensen and Pissarides (1999), who postulated that workers with different skill levels get matched with vacancies in separate but identical matching functions with market-specific inputs of unemployment and vacancies. We must modify their specification because they assumed that workers are permanently endowed with a particular skill level, and we don't. We consider three alternative specifications:

1. Separate matching functions for unemployed workers with different skill levels, yielding different equilibrium vacancies $v(h)$ for $h \in \{0, H\}$.
2. Separate matching functions for unemployed workers having different benefit entitlements, yielding different equilibrium vacancies $v(b)$ for each $b \in \{0, H\}$.
3. Separate matching functions for unemployed workers indexed by both their current skill h and their skill b in their last employment, yielding equilibrium vacancies $v(h,b)$ for each pair of values $(h,b) \in \{0,H\} \times \{0,H\}$.

Case 1: When workers are sorted according to their current skills h, tightness in market h is

$$\theta(h) = \frac{v(h)}{\sum_b u(h,b)}. \quad (22)$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h,b) = \frac{M\Big(v(h),\, \sum_b u(h,b)\Big)}{\sum_b u(h,b)} = m(\theta(h))\,, \tag{23}$$

$$\lambda^f(h,b) = \frac{M\Big(v(h),\, \sum_b u(h,b)\Big)}{v(h)} \, \frac{u(h,b)}{\sum_b u(h,b)} = m(\theta(h))\, \frac{u(h,b)}{v(h)}. \tag{24}$$

The zero-profit condition for posting a vacancy in the market for unemployed workers with skill h is

$$\mu = \beta(1-\psi) \sum_{z,b} \lambda^f(h,b) S^o(h,z,b) Q^o_h(z), \tag{25}$$

where μ is the cost of posting a vacancy.

Case 2: When workers are sorted according to their skills b when last employed, the tightness in market b is

$$\theta(b) = \frac{v(b)}{\sum_h u(h,b)}. \tag{26}$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h,b) = \frac{M\Big(v(b),\, \sum_h u(h,b)\Big)}{\sum_h u(h,b)} = m(\theta(b))\,, \tag{27}$$

$$\lambda^f(h,b) = \frac{M\Big(v(b),\, \sum_h u(h,b)\Big)}{v(b)} \, \frac{u(h,b)}{\sum_h u(h,b)} = m(\theta(b))\, \frac{u(h,b)}{v(b)}. \tag{28}$$

The zero-profit condition for posting a vacancy in the market for unemployed workers whose skills were b in their last employment becomes

$$\mu = \beta(1-\psi) \sum_{h,z} \lambda^f(h,b) S^o(h,z,b) Q^o_h(z). \tag{29}$$

Case 3: When workers are sorted both according to their present skills h and their skills b when last employed, the tightness in each separate market, indexed by (h,b), is given by

$$\theta(h,b) = \frac{v(h,b)}{u(h,b)}. \tag{30}$$

The probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h,b) \;=\; \frac{M\Big(v(h,b),\, u(h,b)\Big)}{u(h,b)} = m(\theta(h,b))\,, \tag{31}$$

$$\lambda^f(h,b) \;=\; \frac{M\Big(v(h,b),\, u(h,b)\Big)}{v(h,b)} = m(\theta(h,b))\;\frac{1}{\theta(h,b)}. \tag{32}$$

The zero-profit condition for posting a vacancy for unemployed workers with current skill h and skill b when last employed is

$$\mu \;=\; \beta(1-\psi)\lambda^f(h,b)\sum_z S^o(h,z,b)Q^o_h(z). \tag{33}$$

B Search-island model

B.1 Firm's problem

The Bellman equations of an existing firm are

$$V^f(h,z) \;=\; \max\Big\{\tilde{V}^f(h,z)\,, -\Omega\Big\}\,, \tag{34}$$

$$\tilde{V}^f(h,z) \;=\; \max_k \Big\{zk^\alpha\,(1+h)^{1-\alpha} - w^*\,(1+h) - (i+\delta)\,k\Big\} + \frac{1-\rho}{1+i}\Big[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'} p^n(h,h')\,V^f(h',z')\,Q\,(z,z')\Big]. \tag{35}$$

The first-order condition for capital in problem (35) is

$$z\alpha k^{\alpha-1}\,(1+h)^{1-\alpha} = (i+\delta)\,, \tag{36}$$

which can be solved for k to obtain the firm's policy function for choosing capital,

$$k\,(h,z) = \left[\frac{z\,\alpha}{i+\delta}\right]^{\frac{1}{1-\alpha}}\,(1+h)\,. \tag{37}$$

Associated with the solution to an existing firm's optimization problem is a reservation productivity $\bar{z}(h)$ that satisfies

$$\tilde{V}^f(h,\bar{z}(h)) = -\Omega. \tag{38}$$

Define the following indicator function

$$\Lambda\,(h,z) = \begin{cases} 1, & \text{if } z \geq \bar{z}(h); \\ 0, & \text{otherwise.} \end{cases} \tag{39}$$

The break-even condition for starting a new firm is

$$\mu = \frac{1}{1+i} \sum_{z} \max \left\{ (1-\phi)\tilde{V}^f(0,z) + \phi \tilde{V}^f(H,z) \,,\, 0 \right\} Q^o(z) \,, \tag{40}$$

where μ is the start-up cost and ϕ is the fraction of high-skilled workers among all new hires. The maximization in (40) implies a reservation productivity $\bar{z}^o$ that determines whether a new firm hires a worker after it observes its productivity level. The reservation productivity satisfies

$$(1-\phi)\tilde{V}^f(0,\bar{z}^o) + \phi \tilde{V}^f(H,\bar{z}^o) = 0. \tag{41}$$

Define the following indicator function

$$\Lambda^o(z) = \begin{cases} 1, & \text{if } z \geq \bar{z}^o; \\ 0, & \text{otherwise.} \end{cases} \tag{42}$$

The productivity distribution of new firms that hire workers is

$$\Gamma(z) = \frac{\Lambda^o(z)\, Q^o(z)}{\sum_{z'} \Lambda^o(z')\, Q^o(z')}. \tag{43}$$

B.2 Household's problem

We define three value functions $V^n(a,h,z)$, $V^u(a,h,b)$, and $V^r(a)$ for an employed worker, an unemployed worker, and a retired worker, respectively. The state variables are last period's assets (a), skill index (h), the firm's current productivity level if employed (z), and the worker's benefit entitlement if unemployed (b). The benefit entitlement is determined by the worker's last earnings, which we index by $b \in \{0, H\}$, his skill index when he last worked. Both newborn unemployed workers and laid off unskilled workers have a benefit entitlement indicated by index $b = 0$.

The Bellman equation of an employed worker is

$$\begin{aligned} V^n(a,h,z) \;=\; \max_{c,a'} \Bigg[& \log c + \beta\rho V^r(a') + \beta(1-\rho) \Bigg(\pi^o \sum_{h'} p^o(h,h')\, V^u(a',h',h) \\ & + (1-\pi^o) \sum_{h',z'} p^n(h,h') \Big\{ V^n(a',h',z')\, \Lambda(h',z') \\ & + V^u(a',h',h)\, [1-\Lambda(h',z')] \Big\} Q(z,z') \Bigg) \Bigg] \end{aligned} \tag{44}$$

subject to

$$\begin{aligned} c + a' &\leq (1+i)\, a + (1+h)\, w \,, \\ c,\, a' &\geq 0 \,. \end{aligned}$$

Policy functions $\bar{c}^n(a,h,z)$ and $\bar{a}^n(a,h,z)$ give the employed worker's optimal levels of consumption and savings, respectively.

The Bellman equation of an unemployed worker is

$$\begin{aligned} V^u(a,h,b) &= \max_{c,a',s}\Bigg[\log c + A\frac{(1-s)^\gamma - 1}{\gamma} + \beta\rho V^r(a') + \beta(1-\rho) \\ &\quad \cdot\left(\left(1-s^\xi\right)V^u(a',h,b) + s^\xi\sum_{z'}V^n(a',h,z')\,\Gamma(z')\right)\Bigg] \end{aligned} \tag{45}$$

subject to

$$\begin{aligned} c + a' &\leq (1+i)\,a + \eta\,(1+b)\,w\,, \\ c,\, a' &\geq 0\,, \quad s \in [0,1). \end{aligned}$$

Policy functions $\bar{c}^u(a,h,b)$, $\bar{a}^u(a,h,b)$, and $\bar{s}\,(a,h,b)$ give the unemployed worker's optimal levels of consumption, savings, and search effort, respectively.

The Bellman equation of a retired worker is

$$V^r(a) = \max_{c,a'}\Big[\log c + \beta(1-\sigma)V^r(a')\Big] \tag{46}$$

subject to

$$\begin{aligned} c + a' &\leq (1+i)\,a\,, \\ c,\, a' &\geq 0\,. \end{aligned}$$

Policy functions, $\bar{c}^r(a)$ and $\bar{a}^r(a)$, give optimal consumption and savings, respectively.

B.3 Steady state

In a steady state, a time-invariant measure $N\,(h,z)$ describes the number of firms operating with workers of skill index $h \in \{0, H\}$ and productivity level z. This measure must be consistent with the stochastic process for idiosyncratic shocks and the employment decisions of firms. If v is the number of newly created firms, then $N\,(\cdot\,,\cdot)$ must satisfy

$$\begin{aligned} N\,(0,z') &= vQ^o(z')\Lambda^o(z')(1-\phi) + (1-\rho)(1-\pi^o)\Lambda(0,z') \\ &\quad \cdot\sum_{h,z}p^n(h,0)N(h,z)Q(z,z'), \end{aligned} \tag{47}$$

$$\begin{aligned} N\,(H,z') &= vQ^o(z')\Lambda^o(z')\phi + (1-\rho)(1-\pi^o)\Lambda(H,z') \\ &\quad \cdot\sum_{h,z}p^n(h,H)N(h,z)Q(z,z'). \end{aligned} \tag{48}$$

Time-invariant measures $y^n(a,h,z)$, $y^u(a,h,b)$, and $y^r(a)$, respectively, describe the numbers of employed, unemployed, and retired households with various individual characteristics.

These measures are implied by the optimal decision rules by firms and households:

$$
\begin{aligned}
y^n(a',h',z') &= (1-\rho)\Bigg[(1-\pi^o)\Lambda\left(h',z'\right)\sum_{a,h,z:\bar{a}^n(a,h,z)=a'} p^n(h,h')\,y^n(a,h,z)\,Q\left(z,z'\right) \\
&\qquad +\Gamma\left(z'\right)\sum_{a,b:\bar{a}^u(a,h',b)=a'} \bar{s}(a,h',b)^{\xi}\,y^u(a,h',b)\Bigg]\,; \qquad (49)
\end{aligned}
$$

$$
\begin{aligned}
y^u(a',h,b) &= (1-\rho)\Bigg\{\pi^o\sum_{a,z:\bar{a}^n(a,b,z)=a'} p^o(b,h)\,y^n(a,b,z) \\
&\qquad +(1-\pi^o)\sum_{a,z,z':\bar{a}^n(a,b,z)=a'} p^n(b,h)\,y^n(a,b,z)\left[1-\Lambda\left(h,z'\right)\right]Q\left(z,z'\right) \\
&\qquad +\sum_{a:\bar{a}^u(a,h,b)=a'} y^u(a,h,b)\left[1-\bar{s}\left(a,h,b\right)^{\xi}\right]\Bigg\} \\
&\qquad +I(h,b)\,\sigma\sum_{a:\bar{a}^r(a)=a'} y^r(a)\;, \qquad (50)
\end{aligned}
$$

$$
\begin{aligned}
y^r(a') &= (1-\sigma)\sum_{a:\bar{a}^r(a)=a'} y^r(a) \\
&\qquad +\rho\Bigg[\sum_{a,h,z:\bar{a}^n(a,h,z)=a'} y^n(a,h,z)+\sum_{a,h,b:\bar{a}^u(a,h,b)=a'} y^u(a,h,b)\Bigg]\,, \qquad (51)
\end{aligned}
$$

where $I(h,b)$ is an indicator function that equals one if $h=b=0$ and zero otherwise.

Following Alvarez and Veracierto (2001),we consider steady-state equilibria without public debt. The government balances its budget every period, implying

$$
0 = (w^*-w)\sum_{h,z}(1+h)N\left(h,z\right)+\Omega D-\eta w\sum_{a,h,b}(1+b)y^u(a,h,b)\,, \qquad (52)
$$

where the amount of job destruction D is

$$
D=(1-\rho)\Bigg\{\pi^o\sum_{h,z}N\left(h,z\right)+(1-\pi^o)\sum_{h,h',z,z'}p^n(h,h')\left[1-\Lambda\left(h',z'\right)\right]N\left(h,z\right)Q\left(z,z'\right)\Bigg\}. \qquad (53)
$$

The market-clearing condition in the goods market is

$$
\bar{c}+\delta\bar{k}+\mu v=\sum_{h,z}N(h,z)\,z\,k(h,z)^{\alpha}(1+h)^{1-\alpha}, \qquad (54)
$$

where aggregate consumption and the aggregate capital stock, respectively, are

$$
\begin{aligned}
\bar{c} &= \sum_{a,h,z}\bar{c}^n(a,h,z)\,y^n(a,h,z)+\sum_{a,h,b}\bar{c}^u(a,h,b)\,y^u(a,h,b)+\sum_{a}\bar{c}^r(a)\,y^r(a)\,, \qquad (55) \\
\bar{k} &= \sum_{h,z}N(h,z)\,k(h,z). \qquad (56)
\end{aligned}
$$

There are two equilibrium conditions in the labor market. First, the measure of new firms that hire workers, $v \sum_z \Lambda^o(z) Q^o(z)$, must equal the measure of unemployed workers who accept employment. Second, the skill ratio ϕ among new hires that the firm takes as exogenous must equal the equilibrium skill ratio among new hires. We can use the time-invariant population measures to express these equilibrium conditions as:

$$v = \frac{(1-\rho) \displaystyle\sum_{a,h,b} \bar{s}(a,h,b)^{\xi}\, y^u(a,h,b)}{\displaystyle\sum_z \Lambda^o(z)\, Q^o(z)}, \tag{57}$$

$$\phi = \frac{\displaystyle\sum_{a,b} \bar{s}(a,H,b)^{\xi}\, y^u(a,H,b)}{\displaystyle\sum_{a,h,b} \bar{s}(a,h,b)^{\xi}\, y^u(a,h,b)}. \tag{58}$$

Households' aggregate demand for assets

$$\bar{a} = \sum_{a,h,z} a\, y^n(a,h,z) + \sum_{a,h,b} a\, y^u(a,h,b) + \sum_a a\, y^r(a)\ , \tag{59}$$

should equal the supply of assets, which consists of the aggregate capital stock $\bar{k}$ and the value of claims to the economy's firms:

$$\bar{a} = \bar{k} + \frac{\sum_{h,z}\Big[z\, k(h,z)^{\alpha}(1+h)^{1-\alpha} - w^*(1+h) - (i+\delta)k(h,z)\Big]N(h,z) - \mu v - \Omega D}{i}. \tag{60}$$

C Representative family model with lotteries

C.1 Permissible benefit policies

We assume benefit policies that are not so generous that they would induce families to accumulate skills simply to furlough high-skilled workers into unemployment and then forgo earning wages in order to acquire benefits from the public sector. This assumption implicitly generates a restriction on benefit policies that can be derived by taking a steady state in which the family initially enjoys leisure by keeping some of its low-skilled workers unemployed, then asking how the family's wealth would change were it to send an unemployed low-skilled family member to work with the intention of furloughing him into unemployment after he has attained the higher skill level. We impose that during the skill accumulation phase for that worker, the family keeps its leisure unchanged by temporarily furloughing an already high-skilled worker into unemployment. This strategy gives rise to stochastic streams of costs during the worker's skill accumulation phase and payoffs after the worker has attained the higher skill level. These can be exchanged for their expected present values evaluated at a stationary interest rate equal to $(1+i) = \beta^{-1}$.

During the accumulation phase, when the low-skilled worker replaces the high-skilled worker in the labor market, the family gains an amount $(1-\eta)w$ per period from sending the low-skilled worker to work but loses an amount $(1-\eta)(1+H)w$ from furloughing the high-skilled worker into unemployment. Thus, the impact on the family's disposable income during the accumulation phase is $-(1-\eta)Hw$ per period. This loss continues for another period with probability $(1-\pi^u)$, i.e., so long as the low-skilled worker does not experience an upgrade in skills.[46] But with probability π^u, the low-skilled worker attains the higher skill level. When that happens, the family sends the originally high-skilled worker back to work and furloughs the originally low-skilled but newly high-skilled worker into unemployment. That originally low-skilled worker is now entitled to benefits that exceed his earlier benefit level by ηHw. The family keeps this stream of a higher disposable income until the worker with the newly upgraded skill level retires.

Let κ_0^H be the capitalized value of this whole strategy on its inception, and let κ_H^H be the capitalized value of the higher benefit stream at the time when the low-skilled worker gains the higher skill level and is furloughed into unemployment. These capitalized values satisfy

$$\kappa_0^H = -(1-\eta)Hw + \beta\Big[\pi^u\kappa_H^H + (1-\pi^u)\kappa_0^H\Big], \tag{61}$$

$$\kappa_H^H = \eta Hw + \beta(1-\rho)\kappa_H^H. \tag{62}$$

After solving for κ_H^H from equation (62) and substituting into equation (61), we can solve for the capitalized value associated with this strategy,

$$\kappa_0^H = \frac{-(1-\eta) + \dfrac{\beta\pi^u\eta}{1-\beta(1-\rho)}}{1-\beta(1-\pi^u)}\, Hw. \tag{63}$$

We require that a permissible benefit policy make this strategy unprofitable, so that $\kappa_0^H \leq 0$. This implies that

$$\beta\pi^u\eta \leq [1-\beta(1-\rho)](1-\eta). \tag{64}$$

This condition implies an upper bound on the generosity of the replacement rate η. Alternatively, for a given replacement rate η, expression (64) states that the probability π^u of experiencing an upgrade and the subjective discount factor β together must be sufficiently low that it is not worthwhile to accumulate skills just in order to collect benefits at the higher skill level. Thus, we set benefit levels so that it is in the representative family's interest to reap the returns from any skill accumulation that come from working.

C.2 Steady-state employment and population dynamics

We study an economy in a stochastic steady state. A representative family runs the household sector. In a steady state, the family's optimal policy is characterized by two flow rates into

[46] The retirement probability ρ does not enter these calculations, because if either the low-skilled or the high-skilled worker retires while the strategy is being executed, the family will just replace that worker with another person from his skill category.

unemployment: a fraction e_0 of newborns that enter life-time unemployment, and a fraction $e_\triangle$ of all laid off workers with skill losses who enter unemployment for the rest of their lives.[47] When the benefit policy satisfies restriction (64), there is no unemployment among high-skilled workers.

At time t, let R_t be the fraction of a family's members who are retired. The remaining working-age members are divided into four categories. Let U_{0t}, $U_{\triangle t}$, N_{0t}, and N_{Ht} be the fractions of a family's members who are unemployed from birth, unemployed after suffering skill loss, employed with low skills, and employed with high skills, respectively. These fractions satisfy

$$R_t + U_{0t} + U_{\triangle t} + N_{0t} + N_{Ht} = 1. \tag{65}$$

For given flow rates $(e_0, e_\triangle)$, the laws of motion are

$$\begin{aligned}
R_t &= (1-\sigma)R_{t-1} + \rho\Big[U_{0t-1} + U_{\triangle t-1} + N_{0t-1} + N_{Ht-1}\Big], && (66)\\
U_{0t} &= (1-\rho)U_{0t-1} + e_0\sigma R_{t-1}, && (67)\\
U_{\triangle t} &= (1-\rho)\Big[U_{\triangle t-1} + \pi^o\pi^d e_\triangle N_{Ht-1}\Big], && (68)\\
N_{0t} &= (1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]N_{0t-1} + \pi^o\pi^d(1-e_\triangle)N_{Ht-1}\Big\} + (1-e_0)\sigma R_{t-1}, && (69)\\
N_{Ht} &= (1-\rho)\Big\{\Big[1-\pi^o\pi^d\Big]N_{Ht-1} + (1-\pi^o)\pi^u N_{0t-1}\Big\}. && (70)
\end{aligned}$$

We can use equations (65) and (66) to solve for the stationary fraction of retired members

$$R = \frac{\rho}{\sigma+\rho}, \tag{71}$$

which can be substituted into equation (67) to obtain the stationary fraction of family members who have been unemployed since birth

$$U_0 = \frac{e_0\,\sigma}{\sigma+\rho}. \tag{72}$$

To compute the stationary labor allocation, we start with equation (70) and express N_H in terms of N_0,

$$N_H = \frac{(1-\rho)(1-\pi^o)\pi^u}{1-(1-\rho)(1-\pi^o\pi^d)}N_0, \tag{73}$$

which can be substituted together with equation (71) into equation (69) and then solved for

$$N_0 = \frac{\Big[1-(1-\rho)(1-\pi^o\pi^d)\Big](1-e_0)\sigma\rho}{\chi^e(\sigma+\rho)}, \tag{74}$$

[47] These flow rates into unemployment correspond to one particular design of the employment lottery, but there are many other designs that implement the same steady-state aggregate labor allocation and yield the same expected utility to workers. See section C.7.

where

$$\chi^e \equiv 1 - (1-\rho)\Big\{1 + \rho\Big[1 - \pi^o\pi^d - (1-\pi^o)\pi^u\Big] - (1-\rho)\pi^o\pi^d e_\triangle (1-\pi^o)\pi^u\Big\} > 0; \quad (75)$$

χ^e is strictly positive since

$$\chi^e \geq 1 - (1-\rho)\Big\{1 + \rho\Big[1 - \pi^o\pi^d - (1-\pi^o)\pi^u\Big]\Big\} \geq 1 - (1-\rho)(1+\rho) = \rho^2 > 0.$$

By using equations (73) and (74), we can solve for $U_\triangle$ from equation (68),

$$U_\triangle = \frac{(1-\rho)^2\pi^o\pi^d e_\triangle(1-\pi^o)\pi^u(1-e_0)\sigma}{\chi^e(\sigma+\rho)}. \quad (76)$$

It is interesting to note that the skill composition of employed workers is a function only of exogenous parameters and does not depend on the choice of flow rates $(e_0, e_\triangle)$. Use equation (73) to compute

$$\phi_N = \frac{N_H}{N_0 + N_H} = \frac{(1-\rho)(1-\pi^o)\pi^u}{\rho + (1-\rho)\pi^o\pi^d + (1-\rho)(1-\pi^o)\pi^u} \in (0,1). \quad (77)$$

C.3 A perturbation of employment

Before turning to equilibrium labor dynamics in a steady state, we examine two perturbations from a steady-state labor allocation. We will use these perturbations to compute a steady state.

Suppose that the steady state is such that the representative family has a positive measure of unemployed workers who have suffered a skill loss. We can then ask: how would the family's wealth change if the set of unemployed workers who have suffered skill loss is permanently reduced by one worker? That is, the family considers sending one such worker to the labor market and, when he retires, replacing him with another unemployed worker who has suffered skill loss. Such a succession of workers will give rise to a stochastic stream of labor income that the family can immediately exchange for the expected present value of the stream discounted at the stationary interest rate $(1+i) = \beta^{-1}$.

Let $\kappa_0^\triangle$ be the capitalized value of the labor income associated with this strategy of reducing unemployment among workers who have suffered skill loss. Let $\kappa_h^\triangle$ be the capitalized value of the stream of labor income at a future time when this worker (or one of his successors) has attained high skills. These capitalized values satisfy

$$\kappa_0^\triangle = w + \beta(1-\rho)\Big\{\Big[1 - (1-\pi^o)\pi^u\Big]\kappa_0^\triangle + (1-\pi^o)\pi^u\kappa_H^\triangle\Big\} + \beta\rho\kappa_0^\triangle, \quad (78)$$

$$\kappa_H^\triangle = (1+H)w + \beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^\triangle + \pi^o\pi^d\kappa_0^\triangle\Big\} + \beta\rho\kappa_0^\triangle, \quad (79)$$

where w is the market-clearing after-tax wage rate.

We can use equation (79) to solve for $\kappa_H^{\triangle}$,

$$\kappa_H^{\triangle} = \frac{(1+H)w + \beta\Big[(1-\rho)\pi^o\pi^d + \rho\Big]\kappa_0^{\triangle}}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{80}$$

which can be substituted into equation (78),

$$\kappa_0^{\triangle} = \frac{1-\beta(1-\rho)(1-\pi^o\pi^d) + \beta(1-\rho)(1-\pi^o)\pi^u(1+H)}{\chi^0}\, w > 0, \tag{81}$$

where

$$\begin{aligned}\chi^0 \;&\equiv\; \Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u\Big]-\beta\rho\Big\} \\ &\qquad -\beta(1-\rho)(1-\pi^o)\pi^u\beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big] \\ &=\; (1-\beta)\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u-\pi^o\pi^d\Big]\Big\} > 0. \end{aligned} \tag{82}$$

C.4 A second perturbation of employment

Suppose that in the steady state that the representative family has a positive measure of unemployed workers who have never been employed. We ask: how would the family's wealth change if the set of unemployed workers who have never worked is permanently reduced by one worker? That is, the family considers sending one such worker to the labor market and, when he retires, replacing him with an unemployed worker who has never worked. This gives rise to a stochastic stream of labor income that the family can immediately exchange for the present value of the stream's expected value discounted at the stationary interest rate $(1+i)=\beta^{-1}$.

We add a twist to this strategy. Whenever the worker (or one of his successors) has become high-skilled and then loses those skills after an exogenous layoff, the strategy furloughs the worker into unemployment indefinitely and replaces him in the work force with another unemployed family member who has never worked. This switch of workers yields a gain to the family because a stream of low unemployment benefits becomes a stream of high unemployment benefits. The uncertainty associated with retirement makes the gain of benefits stochastic, but the associated stochastic stream of gains can be sold immediately for its expected present value, as given by κ_H^H in expression (62), so that $\kappa_H^H = \frac{\eta H w}{1-\beta(1-\rho)}$.[48]

Let κ_0^0 be the capitalized value of the labor income associated with this strategy of reducing unemployment among the workers who have never been employed. Moreover, let κ_H^0 be the capitalized value of the stream of labor income at a future point in time when this worker (or one of his successors) has attained the high skill level. These capitalized values

[48]Recall that newborn workers are also entitled to the lower benefit level ηw.

satisfy

$$\kappa_0^0 \;=\; w + \beta(1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]\kappa_0^0 + (1-\pi^o)\pi^u\kappa_H^0\Big\} + \beta\rho\kappa_0^0, \tag{83}$$

$$\kappa_H^0 \;=\; (1+H)w + \beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^0 + \pi^o\pi^d(\kappa_0^0+\kappa_H^H)\Big\} + \beta\rho\kappa_0^0. \tag{84}$$

Equation (84) implies that

$$\kappa_H^0 = \frac{(1+H)w + \beta(1-\rho)\pi^o\pi^d\kappa_H^H + \beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big]\kappa_0^0}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{85}$$

which can be substituted into equation (83),

$$\begin{aligned}\kappa_0^0 &= \frac{\Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]w + \beta(1-\rho)(1-\pi^o)\pi^u\Big[(1+H)w+\beta(1-\rho)\pi^o\pi^d\kappa_H^H\Big]}{\chi^0}\\ &= \kappa_0^{\triangle} + \frac{\beta^2(1-\rho)^2(1-\pi^o)\pi^o\pi^u\pi^d\eta Hw}{\Big[1-\beta(1-\rho)\Big]\chi^0},\end{aligned} \tag{86}$$

where $\kappa_0^{\triangle}$ and χ^0 are given by equation (81) and (82), respectively.

C.5 Steady-state consumption

The representative family takes wages and interest rates as given. Since the utility function is additively separable in consumption and leisure, it is optimal for the family to assign equal consumption to each of its members. In a steady state with constant consumption, the stationary interest rate must equal $1+i=\beta^{-1}$ and the family must be content to hold a constant level of wealth in the form of physical capital and claims to firms.

Given the family's optimal labor decisions encoded in flow rates into unemployment $(e_0, e_{\triangle})$, the representative family has fractions N_0 and N_H of its members employed with low skills and high skills, respectively, as determined by equations (73) and (74). The stationary production of consumption goods per worker c implies per-capita consumption

$$\bar{c} \;=\; \bar{n}c \tag{87}$$

where $\bar{n}$ is the fraction of employed workers among all members of the family,

$$\bar{n} \;=\; N_0 + N_H = \frac{\Big\{1-(1-\rho)\Big[(1-\pi^o\pi^d)-(1-\pi^o)\pi^u\Big]\Big\}(1-e_0)\rho\sigma}{\chi^e(\sigma+\rho)}. \tag{88}$$

The representative family's utility in a steady state is

$$\int_0^1 \sum_{t=0}^{\infty}\beta^t u(c_t^j,\, n_t^j)dj \;=\; \frac{\log(\bar{c})-\bar{n}B}{1-\beta}. \tag{89}$$

C.6 Steady-state labor dynamics

It remains to describe optimal labor decisions in a steady state. When the benefit policy satisfies restriction (64), all high-skilled workers will be employed in a steady state. Unemployment will be confined to workers who currently have low skills. There are two possibilities concerning steady-state outcomes:

1. $e_0 = 0$ and $e_\triangle \in [0, 1]$;
2. $e_0 \in (0, 1]$ and $e_\triangle = 1$.

If there is any unemployment among low-skilled workers with low benefits, all high-skilled workers who suffer skill losses must flow into unemployment, i.e., $e_\triangle = 1$. If that were not true, the family would be better off working a low-skilled worker with low benefits instead of a laid off high-skilled worker who has just suffered a skill loss. Both workers are equally productive, but the latter is entitled to higher unemployment compensation. Hence, the steady-state labor dynamics must fall into either class 1 or 2.

What is the optimal setting of the two flow rates into unemployment, $(e_0, e_\triangle)$? To check whether a candidate pair of flow rates constitutes a steady state, we consider the welfare effects of the perturbations to employment that we described above. If the candidate $(e_0, e_\triangle)$ falls into class 1, we examine the first type of perturbation in which the set of unemployed workers who have suffered skill loss is permanently reduced by one worker. That increases the family's labor income by a capitalized value equal to $\kappa_0^\triangle$. In a steady state with equilibrium gross interest rate β^{-1}, it would be optimal for the family to convert this capitalized value into an annuity flow of $(1-\beta)\kappa_0^\triangle$ and permanently to increase consumption by that amount. The utility derived from this extra flow of consumption should be compared to the loss of benefits $\eta(1 + H)w$ and the loss of leisure. The condition for an interior optimum is

$$u_c(\bar{c}, \bar{n})\Big[(1-\beta)\kappa_0^\triangle - \eta(1+H)w\Big] + u_n(\bar{c}, \bar{n}) = 0, \tag{90}$$

where the marginal utilities of consumption and leisure are evaluated at the candidate steady-state allocation. Given our particular utility function, which is additively separable in the logarithm of consumption and a linear disutility term for labor, the condition for an interior optimum in class 1 becomes

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^\triangle - \eta(1+H)w\Big] = B. \tag{91}$$

If the candidate $(e_0, e_\triangle)$ falls into class 2, we consider the second type of perturbation in which the set of unemployed workers who have never worked is permanently reduced by one worker. Analyzing this perturbation leads to the following condition for an interior optimum:

$$u_c(\bar{c}, \bar{n})\Big[(1-\beta)\kappa_0^0 - \eta w\Big] + u_n(\bar{c}, \bar{n}) = 0, \tag{92}$$

which with our preference specification implies

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^0 - \eta w\Big] = B. \tag{93}$$

C.7 Employment lotteries

Although the aggregate allocation of labor is unique in an employment lotteries model, many possible lottery designs that randomly assign different tasks to individual workers can implement that allocation and yield the same expected utility to workers. In real business cycle models like the one of Hansen (1985), the identical workers could be randomizing over an arbitrary number of periods of working and leisure, possibly contingent on the phase of the business cycle. Alternatively, at the beginning of each period, there could just be an employment lottery for that period's labor supply. In our model with *ex post* heterogeneous workers, there are two kinds of multiplicity in the design of lotteries. First, the optimal lottery design is not unique with respect to the identity of low-skilled unemployed workers who are entitled to low benefits. For example, the workers would be indifferent between the device proposed above of randomly furloughing newborn workers into life-time unemployment and other devices that repeatedly randomize employment status among low-skilled workers who are entitled to low benefits. So long as the devices result in identical aggregate employment outcomes, workers would derive the same *ex ante* expected life-time utility. Second, there is nonuniqueness with respect to the identity of unemployed workers with skill losses whenever the optimal allocation requires some of these people to work. For example, workers would be indifferent between the above device of randomly furloughing a fraction of laid off workers with skill losses into unemployment for the rest of their lives and alternative devices with higher inflow rates but correspondingly shorter unemployment spells among laid off workers who experience skill losses.

The equilibrium conditions do restrict the multiplicity of lottery designs in one important respect. Since the representative family faces no frictions in the labor market and there is a single wage rate per unit of skill, the *family* is indifferent between, on the one hand, lotteries that include only the newly born and laid off old workers and, on the other hand, lotteries that include all working-age members and that entail furloughing some lottery winners who are employed into leisure. But *firms* are not indifferent to these alternative lottery designs because the latter would result in extra layoff taxes and the loss of firms' prior investments in job creation, at least if we assume that productivity processes are lost whenever there are worker separations, as we have assumed in the case of retirements. It follows that steady-state lottery designs cannot include employed workers because otherwise firms would have the incentive to offer 'back-loaded' wage payments – making the representative family strictly prefer to assign leisure to the new born and laid off old workers, rather than to furlough employed workers into leisure.

References

Alonso-Borrego, César, Jesús Fernández-Villaverde, and José E. Galdón-Sánchez. 2004. Evaluating Labor Market Reforms: A General Equilibrium Approach. Unpublished.

Alvarez, Fernando and Marcelo Veracierto. 2001. Severance Payments in an Economy with Frictions. *Journal of Monetary Economics* 47:477–498.

Bertola, Giuseppe and Andrea Ichino. 1995. Wage Inequality and Unemployment: United States vs. Europe. In *NBER Macroeconomics Annual*, edited by Ben S. Bernanke and Julio J. Rotemberg. Cambridge Mass.: MIT Press.

Browning, Martin and Pierre-Andre Chiappori. 1998. Efficient Intra-Household Allocation : A General Characterization and Empirical Tests. *Econometrica* 66 (6):1241–78.

Browning, Martin, Lars Peter Hansen, and James Heckman. 1999. Micro Data and General Equilibrium Models. In *Handbook of Macroeconomics, Vol. 1A*, edited by John Taylor and Michael Woodford. Amsterdam: North Holland.

Chiappori, Pierre-Andre. 1992. Collective Labour Supply and Welfare. *Journal of Political Economy* 100:437–67.

———. 1997. Introducing Household Production in Collective Models of Labor Supply. *Journal of Political Economy* 105:191–209.

Cole, Harold L. and Narayana Kocherlakota. 2001. Efficient Allocations with Hidden Income and Hidden Storage. *Review of Economic-Studies* 68 (3):523–42.

DenHaan, Wouter, RChristian Haefke, and Garey Ramey. 2001. Shocks and Institutions in a Job Matching Model. NBER Working Paper 8463.

Diamond, Peter A. 1982. Wage Determination and Efficiency in Search Equilibrium. *Review of Economic Studies* 49:217–227.

Edling, Jan. 2005. Alla behovs: Blott arbetsmarknadspolitik skapar inga nya jobb [Everybody is needed: Labor market policies on their own do not create any new jobs]. Unpublished.

Farber, Henry. 1997. The Changing Face of Job Loss in the United States, 1981-1995. *Brookings Papers on Economic Activity: Microeconomics* 55–128.

———. 2005. What Do We Know about Job Loss in the United States? Evidence from the Displaced Worker Survey, 1984-2004. Princeton University working paper 498.

Friedman, Thomas. 2005. *The World is Flat: a Brief History of the Twenty-First Century.* New York: Farrar, Straus, and Giroux.

Gottschalk, Peter and Robert Moffitt. 1994. The Growth of Earnings Instability in the U.S. Labor Market. *Brookings Papers on Economic Activity* (2):217–272.

Hansen, Gary D. 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics* 16:309–327.

Hansen, Gary D. and Thomas J. Sargent. 1988. Straight Time and Overtime in Equilibrium. *Journal of Monetary Economics* 21 (2/3):281–308.

Heckman, James J. 2003. Flexibility, Job Creation and Economic Performance. University of Chicago and American Bar Association.

Hopenhayn, Hugo and Richard Rogerson. 1993. Job Turnover and Policy Evaluation: A General Equilibrium Analysis. *Journal of Political Economy* 101:915–938.

Hornstein, Andreas, Per Krusell, and Giovanni L. Violante. 2003. Vintage Capital as an Origin of Inequalities. Unpublished.

Hosios, Arthur J. 1990. On the Efficiency of Matching and Related Models of Search and Unemployment. *Review of Economic Studies* 57:279–298.

Jovanovic, Boyan and Yaw Nyarko. 1996. Learning by Doing and the Choice of Technology. *Econometrica* 64 (6):1299–1310.

Katz, Lawrence F. and David H. Autor. 1999. Changes in the Wage Structure and Earnings Inequality. In *Handbook of Labor Economics, Vol. 3A*, edited by Orley Ashenfelter and David Card. Amsterdam: Elsevier.

Kuh, Edwin and John. R. Meyer. 1957. How Extraneous are Extraneous Estimates? *The Review of Economics and Statistics* 39 (4):380–393.

Kydland, Finn E. and Edward C. Prescott. 1982. Time to Build and Aggregate Fluctuations. *Econometrica* 50 (6):1345–1371.

Ljungqvist, Lars. 2002. How Do Layoff Costs Affect Employment? *Economic Journal* 112:829–853.

———. 2005. Comment. In *NBER Macroeconomics Annual*, edited by Mark Gertler and Kenneth Rogoff. Cambridge Mass.: MIT Press.

Ljungqvist, Lars and Thomas J. Sargent. 1998. The European Unemployment Dilemma. *Journal of Political Economy* 106:514.

———. 2003. European Unemployment: From a Worker's Perspective. In *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor or Edmund S. Phelps*, edited by Joseph Stiglitz Philippe Aghion, Roman Frydman and Michael Woodford. Princeton: Princeton University Press.

———. 2004a. European Unemployment and Turbulence Revisited in a Matching Model. *Journal of the European Economic Association* 2:456–468.

———. 2004b. Lotteries for Consumers Versus Lotteries for Firms. In *Frontiers in Applied General Equilibrium Modeling*, edited by Timothy J. Kehoe, T.N. Srinivasan, and John Whalley. Cambridge, UK: Cambridge University Press.

———. 2005. The European Unemployment Experience: Uncertainty and Heterogeneity. Departments of Economics, Stockholm School of Economics and New York University.

Lucas, Robert E., Jr. 1987. *Models of Business Cycles.* Oxford and New York: Basil Blackwell.

Lucas, Robert E., Jr. and Edward C. Prescott. 1974. Equilibrium Search and Unemployment. *Journal of Economic Theory* 7:188–209.

Marimon, Ramon and Fabrizio Zilibotti. 1999. Unemployment vs. Mismatch of Talents: Reconsidering Unemployment Benefits. *Economic Journal* 109:266–291.

McCall, John J. 1970. Economics of Information and Job Search. *Quarterly Journal of Economics* 113–126.

Millard, Stephen P. and Dale T. Mortensen. 1997. The Unemployment and Welfare Effects of Labor Market Policy: A Comparison of the USA and the UK. In *Unemployment Policy: Government Options for the Labor Market*, edited by Dennis Snower and Guillermo de la Dehesa. Cambridge, UK: Cambridge University Press.

Mortensen, Dale T. 1982. Property Rights and Efficiency in Mating, Racing, and Related Games. *American Economic Review* 72:968–979.

Mortensen, Dale T. and Christopher A. Pissarides. 1999. Unemployment Responses to 'Skill-Biased' Technology Shocks: The Role of Labor Market Policy. *Economic Journal* 109:242–265.

Myers, Robert J. 1964. What Can We Learn from European Experience? In *Unemployment and the American Economy*, edited by Arthur M. Ross. New York: John Wiley & Sons, Inc.

OECD. 1994. *The OECD Jobs Study: Evidence and Explanations*. Paris.

Pavoni, Nicola. 2003. Optimal Unemployment Insurance, Human Capital Accumulation and Duration Dependence. University College, London.

Pissarides, Christopher A. 1992. Loss of Skill During Unemployment and the Persistence of Employment Shocks. *Quarterly Journal of Economics* 107:1371–1391.

———. 2000. *Equilibrium Unemployment Theory*. Cambridge, Massachusetts: MIT Press, second ed.

Prescott, Edward C. 2002. Prosperity and Depression. *American Economic Review* 92:1–15.

———. 2004. Why Do Americans Work So Much More Than Europeans? *Federal Reserve Bank of Minneapolis Quarterly Review* 28:2–13.

Rogerson, Richard. 1988. Indivisible Labor, Lotteries, and Equilibrium. *Journal of Monetary Economics* 21:3–16.

———. 2005a. Structural Transformation and the Deterioration of European Labor Market Outcomes. Unpublished.

———. 2005b. Understanding Differences in Hours Worked. Unpublished text of plenary talk at 2005 SED conference.

Ryoo, Jaewoo and Sherwin Rosen. 2004. The Engineering Labor Market. *Journal of Political Economy* 112 (1):S110–S140.