Understanding European Unemployment with Matching and Search-Island Models

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Abstract

To understand European and American unemployment during the last 60 years, we use a search-island model and four matching models with workers who have heterogeneous skills and entitlements to government benefits. When there is higher turbulence, in the sense of worse skill transition probabilities for workers who suffer involuntary layoffs, high government mandated unemployment insurance (UI) and employment protection (EP) in Europe increase unemployment rates and durations. But when there is lower turbulence, high European EP suppresses unemployment rates despite high European UI. Four matching models differ in how they assign unemployed workers to matching functions. That affects how strongly unemployment responds to increases in turbulence. Heterogeneity among unemployed workers highlights the central role of adverse labor market externalities in matching models and reveals that the cost of posting vacancies is the lynchpin of a matching model.

KEY WORDS: Job, search, matching, skills, turbulence, discouraged worker, labor market externality, bargaining, unemployment, unemployment insurance, employment protection.

1 Introduction

This paper applies a search-island model and a suite of matching models to understand some striking differences in European and American unemployment across two broad subperiods of the twentieth century that we summarize in section 1.1. The models use a human capital risk and how it has changed over time to explain the macroeconomic facts in section 1.1, thereby telling a story about the interaction of institutions and the distribution of microeconomic shocks that Ljungqvist and Sargent (2007a) told with a McCall search model. But those similarities in macroeconomic outcomes conceal important differences in the models' microeconomic forces and the personal experiences of the workers who live within them. For example, in the matching models, unemployed workers are victimized by the actions of other workers who impose adverse congestion externalities, via a matching function, and by their own bargaining power when firms seek to recoup sunk hiring costs. But in the search-island model, with average luck, unemployed workers are fully responsible for their own situations. Applying these different models as we do is a good way to reveal what makes them tick.

We view it as a point in their favor that these models succeed in explaining the section 1.1 facts that have long puzzled macroeconomists. In Ljungqvist and Sargent (2007b), we explain why a representative family employment lotteries model is less successful in accomplishing this particular task.

1.1 Facts

As targets, we take the following facts about unemployment outcomes in two continents:[1]

1. In the 1950s and 1960s, unemployment rates were persistently *lower* in Europe than in the U.S. The difference was accounted for by a *higher* inflow rate into unemployment in the U.S.

2. After the 1970s, unemployment became persistently *higher* in Europe.[2]

3. Inflow rates into unemployment were roughly constant across periods within both Europe and U.S.

4. In Europe, average durations of unemployment were low in the 1950s and 1960s, but became high after the 1970s. Average duration in the U.S. stayed low.

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[1]These facts are documented by Layard et al. (1991).

[2]A proper account of unemployment would include a wider group of people than those officially counted as unemployed. OECD (2003, chapter 4) reports comprehensive measures of benefit dependency: "Some countries have now reached a position where most of the working-age population that is neither employed nor participating in education has an income-replacement benefit. ...Benefit recipients are a very heterogeneous group. Some of them may want to work, or can be 'activated' ...The largest categories in 1999 were disability, unemployment and [early retirement] ...a near-universal rise in the aggregate benefit dependency rate among the population of working age between 1980 and 1990, with Japan and the United States being the only exceptions."

5. In Europe, since the 1970s, hazard rates of leaving unemployment fall with increases in the duration of unemployment.

Next, as features that differentiated Europe and the U.S. and that we shall take as exogenous inputs into the models, we note the following facts about UI and EP:

1. In both periods, government supplied unemployment insurance (UI in the language of Mortensen and Pissarides (1999)) were generous with long durations in Europe, but they were stingy with short durations in the U.S.

2. Government mandated employment protection (EP in the language of Mortensen and Pissarides (1999)) was stronger in Europe throughout both periods.

As in Ljungqvist and Sargent (2007a), we accept the judgement of Krugman (1987) that higher UI and EP were in place throughout both periods and so take model parameters that govern UI and EP to be constant across model subperiods. Of course, to explain different outcomes in different subperiods, the models require that *some* exogenous variable changed across subperiods. We take this to be something that we call microeconomic turbulence.

In Ljungqvist and Sargent (2007a), we appeal to the following sources of evidence to support the hypothesis that turbulence increased after the 1970s:

1. *Displaced workers studies* document substantial human capital destruction after involuntary job loss (Jacobson et al. (1993), Farber (1997, 2005)).

2. There is evidence of increased *volatility of earnings* (Gottschalk and Moffitt (1994), Katz and Autor (1999)).

3. There has been an increase in *occupational and industry mobility* (Kambourov and Manovskii (2005)).

4. A sophisticated observer recently summarized the evidence by noting: "A growing body of evidence points to the fact that the world economy is more variable and less predictable today than it was 30 years ago.... [there is] more variability and unpredictability in economic life ..." Heckman (2003, pp. 30–31).

In our models, we define an increase in *turbulence* as an increase in the probability that an involuntarily displaced worker loses human capital.

1.2 Models

Our models tell how high levels of UI and EP interact with an increase in the probability of skill deterioration after involuntary layoffs. The models share the same stochastic skill accumulation and deterioration technologies, but differ with respect to labor market frictions. In the search-island model, depending on their financial assets, human capital, and entitlement to benefits, some people spend more time unemployed than others because they exert less

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effort searching. In the matching models, workers without jobs wait in one or more matching functions. Each model has a theory of a job and of why unemployed workers choose to spend time in an activity that can be called unemployment.[3]

1.2.1 Search-island model

The search-island model descends from models of Lucas and Prescott (1974) and Alvarez and Veracierto (2001) and features a search friction and incomplete risk sharing through self-insurance against both unemployment and uncertain life spans after retirement. The model has risk averse workers whose decisions about how intensively to search when unemployed depend on their skills and benefit entitlements as well as on their accumulations of a risk-free asset, the only available savings vehicle. The wage rate is determined in a competitive labor market but is constrained to remain fixed in the face of idiosyncratic productivity shocks. Workers receive a fixed wage per unit of skill for the duration of a job.[4]

1.2.2 Matching models

These models feature a matching function that sets the probabilities at which firms and unemployed workers meet bilaterally as functions of the sizes of pools of workers and firms who are waiting for invitations to match. An invisible hand adjusts sizes of pools of unemployed workers and job vacancies to reconcile the choices of workers and firms. Individual workers and firms within each pool face a constant probability of encountering a vacancy and an unemployed worker, respectively.[5] To highlight the economic forces at work, we construct a suite of matching models that are differentiated by how they group unemployed workers, in particular, by whether they put workers with heterogeneous skills and benefit entitlements into the same pool or into different pools. Our matching models have risk neutral workers, an attenuated allocative role for wages, and a significant allocative role for the ratio of vacancies to unemployed workers.[6] They feature adverse congestion effects that unemployed workers impose on each other and that firms with vacancies impose on each other, a wage

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[3]Our models share some limitations. First, they ignore the intensive margin of the labor supply decision by assuming that workers are either unemployed or employed full-time and working the same number of hours. Second, if it were not for the labor market frictions in the matching and search-island models, everyone of working age would be employed under laissez-faire. Hence, the models ignore such non-market activities as education and child rearing.

[4]As noted by Alvarez and Veracierto (2001), this greatly simplifies the task of computing an equilibrium. But a consequence is that there are socially wasteful separations.

[5]Thus, it would be misleading to call these pools 'queues'.

[6]Hosios (1990) describes the matching framework as follows: "Though wages in the matching-bargaining models are completely flexible, these wages have nonetheless been denuded of any allocating or signaling function: this is because matching takes place before bargaining and so search effectively precedes wage-setting. ... In conventional market situations, by contrast, firms design their wage offers in competition with other firms to profitably attract employees; that is, wage-setting occurs prior to search so that firms' offers can influence workers' search behavior and, in this way, firms' offers can influence the allocation of resources in the market."

bargaining process, and waiting times as equilibrating signals that reconcile the decisions of firms and workers.

1.3   Organization

Section 2 describes features by all of our models. These include: (1) two transition matrices for workers' skill levels, one for workers whose jobs continue or end endogenously, another for workers whose jobs terminate exogenously; (2) a probability distribution from which to draw productivity levels of new workers and transition matrices for the productivity levels of workers whose jobs continue; and (3) parameters that define a replacement ratio for UI and a layoff tax for EP. Section 3 describes the additional features that complete the search-island model: a discounted risk-averse utility function that is separable across consumption, search effort, time, and states and that imparts a precautionary savings motive; and firm-owned technologies for creating jobs and for converting labor and capital into output. Beyond the common features reported in section 2, section 4 describes the additional features in the matching models: a risk-neutral utility function of consumption and four sets of matching functions that define alternative market structures for assigning workers and firms to pools where workers wait for jobs and firms post vacancies. Section 5 describes calibrations. Sections 6 and 7 describe outcomes in the search-island and matching models. Section 8 describes the perspectives that the models put on the experiences and motives of typical unemployed European workers. Section 9 contains concluding remarks that by comparing the key forces at work in our search-island and matching models will remind us of the following message from Lucas and Prescott (1974, p. 206): "The question of whether there exist important external effects in *actual* labor markets remains, of course, to be settled."

Two appendixes describe Bellman equations and equilibrium conditions. A sequel (Ljung-qvist and Sargent 2007b) studies how UI and EP interact with heightened turbulence in a representative family employment lotteries model that, while it has complete markets and no labor market frictions, shares with the search-island model the specifications of human skill accumulation technology, government supplied UI and EP, and firm activities.

2   Common features of our environments

Figures 1 and 2 show the within-period timing of our models. The top halves of these figures are identical. Each of a continuum of potential workers faces a constant probability ρ of exiting the labor force. In the matching models, a worker immediately exits the model upon leaving the labor force. In the search-island model, ρ is the probability that a worker will retire and not be allowed to work, and σ is the probability that a retired worker dies. To keep the total population and the shares of workers and retirees constant over time, people who die are replaced by newborn workers.

There are three other exogenous sources of uncertainty. First, an employed worker faces a probability π^o that his job terminates. Second, workers experience stochastic accumulation

or deterioration of skills, conditional on employment status and instances of exogenous job terminations. Third, idiosyncratic shocks impinge on employed workers' productivities.

2.1 Skill dynamics

There are two possible skill levels, indexed by $h \in \{0, H\}$. All newborn workers enter the labor force with the low skill index, $h = 0$. An employed worker with skill index h faces a probability $p^n(h, h')$ that his skill at the beginning of next period is h', conditional on no exogenous job termination. In the event of an exogenous job termination, a laid off worker with last period's skill h faces a probability $p^o(h, h')$ that his skill becomes h'. A worker's skill remains unchanged during an unemployment spell. The skill transition matrices are:

$$p^n = \begin{bmatrix} 1 - \pi^u & \pi^u \\ 0 & 1 \end{bmatrix}, \tag{1}$$

$$p^o = \begin{bmatrix} 1 & 0 \\ \pi^d & 1 - \pi^d \end{bmatrix}. \tag{2}$$

Our notion of turbulence is encoded in the parameter $\pi^d \in [0, 1]$, where increased turbulence is represented as an increase in the parameter value π^d.

2.2 Firm formation and productivity

The process of uniting firms and workers differs across the two frameworks but has several common features. Firms incur a cost μ when posting a vacancy in the matching model or when creating a job in the search-island model. We model a new job opportunity as a draw of productivity z from a distribution $Q_h^o(z)$. The productivity of an ongoing job is governed by a Markov process: $Q_h(z, z')$ is the probability that next period's productivity is z', given current productivity z. For any two productivity levels z and $\hat{z} < z$, the conditional probability distribution $Q_h(z, z')$ first-order stochastically dominates $Q_h(\hat{z}, z')$, meaning that

$$\sum_{z' \leq \bar{z}} Q_h(z, z') < \sum_{z' \leq \bar{z}} Q_h(\hat{z}, z'), \qquad \text{for all } \bar{z}. \tag{3}$$

The probability distributions, $Q_h^o(z)$ and $Q_h(z, z')$, depend on the worker's skill h in the matching model, but not in the search-island model.

An employed worker retains his last period productivity with probability $(1 - \pi)$ and draws a new productivity with probability π from the distribution $Q_h^o(z')$, so that new productivities on existing jobs are drawn from the same distribution as the productivities at the time of job creation; $Q_h^o(z')$ depends on the worker's current skill index h in the matching model, but not in the search-island model.

2.3 Government mandated UI and EP

The government levies layoff taxes on job destruction and provides benefits to the unemployed. It imposes a layoff tax Ω on every endogenous job separation and on every exogenous job termination except retirement. The government pays unemployment benefits equal to a replacement rate η times a measure of past income. To determine his benefit entitlement, it suffices to keep track of a worker's skill in his last employment. Newborn workers are entitled to the lowest benefit level in the economy. The government finances unemployment benefits with revenues from the layoff tax and other model-specific taxes.

3 Search-island model

Our search-island model with incomplete markets features risk-averse workers who engage in precautionary saving; a non trivial choice of search effort by unemployed workers; and a competitive labor market for workers whose job searches are successful. The only vehicle for savings is a single risk-free one-period security. We consign various technical details to appendix A.

We create our model by altering the model of Alvarez and Veracierto (2001).[7] We adopt their specification of a worker's preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + A \frac{(1-s_t)^\gamma - 1}{\gamma} \right], \qquad \text{with } A > 0, \ \gamma > -1, \qquad (4)$$

where the expectation operator E_0 is taken over future states of employment, unemployment and retirement. β is the subjective discount factor, c_t is the worker's consumption, and $s_t \in [0, 1)$ is the worker's choice of search intensity if he is unemployed and of working age. A search intensity s_t determines an unemployed worker's probability s_t^ξ of finding a centralized labor market in the next period, where $0 \leq \xi \leq 1$. Workers who find the labor market get a job paying a market-clearing wage rate. To accommodate the feature, not present in Alvarez and Veracierto (2001), that workers differ in their skills, we let w denote the wage rate per unit of skill, where the skill level of a low-skilled worker is normalized to one and the skill level of a high-skilled worker is $1 + H$. Hence, a low-skilled worker earns w and a high-skilled worker earns $(1 + H)w$.

3.1 Firms

We suppress Alvarez and Veracierto's firm size dynamics and, in the spirit of our matching model, let each firm employ only one worker. Each firm also rents physical capital. The

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[7]The Alvarez and Veracierto (2001) model is descended from the model of Lucas and Prescott (1974) in which risk-neutral workers engage in effortless but time-consuming search across a large number of spatially distinct islands with idiosyncratic productivity shocks. The only search cost in that model is the opportunity cost of labor income foregone when moving between islands. Alonso-Borrego et al. (2004) use a two-market version of the Alvarez and Veracierto (2001) model to study how legal regulations that exempt fixed term labor contracts from layoff taxes affect equilibrium outcomes.

firm's production function is

$$z_t k_t^\alpha (1 + h_t)^{1-\alpha}, \qquad \text{with } \alpha \in (0,1), \tag{5}$$

where z_t is the current productivity level, $h_t \in \{0, H\}$ is the skill index of the firm's worker, and k_t is physical capital that depreciates at the rate δ. Output can be devoted to consumption, investment in physical capital, and startup costs. The rest of Alvarez and Veracierto's model of firms enters our framework as follows. Incurring a startup cost μ at time t allows a firm to create a job opportunity at $t+1$ by drawing a productivity level z from the distribution $Q^o(z)$. After seeing z, a firm decides whether to hire a worker from the centralized labor market. We retain Alvarez and Veracierto's key assumption that firms and workers first meet under a veil of ignorance about their partner's state vector: the firm hires a worker drawn randomly from a single pool of unemployed workers with a mix of low-skilled and high-skilled workers. Once hired, a firm observes a worker's skill, hires the appropriate physical capital, and pays the worker the market wage of w per unit of skill. A firm must retain a worker for at least one period.

We retain Alvarez and Veracierto's assumption that old as well as new hires earn the same wage rate (per unit of skill). The assumption that the market-determined wage rate is common to all workers and does not respond to idiosyncratic productivity shocks is restrictive. To avoid layoffs, workers would be willing to accept wage cuts in response to some adverse productivity shocks.[8]

3.2 Other features

Besides markets for goods and for renting labor and capital, workers can acquire non-negative holdings of risk-free assets that earn a net interest rate i. Following Alvarez and Veracierto (2001), we postulate a competitive banking sector that accepts deposits that it invests in physical capital and claims on firms. The banking sector rents physical capital to firms at the competitive rental rate $i + \delta$. Banks hold a diversified portfolio of all firms and therefore bear no risk.

In the spirit of Alvarez and Veracierto, we assume that a worker who dies is replaced by a newborn unemployed worker, to whom he is indifferent, but who nevertheless inherits his assets. Newborn workers have the low skill index, $h = 0$.

The government pays unemployment compensation equal to a replacement rate η times an unemployed worker's last labor earnings. Newborn workers are entitled to the lowest benefit level in the economy. The government receives revenues from layoff taxes, and from a flat-rate tax τ on labor earnings and unemployment benefits. The government balances its budget.

Figure 1 shows the within-period timing of events in our search-island model.

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[8]A main purpose of Alvarez and Veracierto (2001) is to quantify the potential welfare gains of a tax on job destruction that reduces socially wasteful separations. They acknowledge that the rigidity they impose on labor contracts and their assumption of no disutility from work cause them to overestimate those welfare gains. While this remains true in our model, notice that the rigidity has no direct effect on the incidence of skill losses since those are triggered by exogenous job terminations but not by endogenous separations.
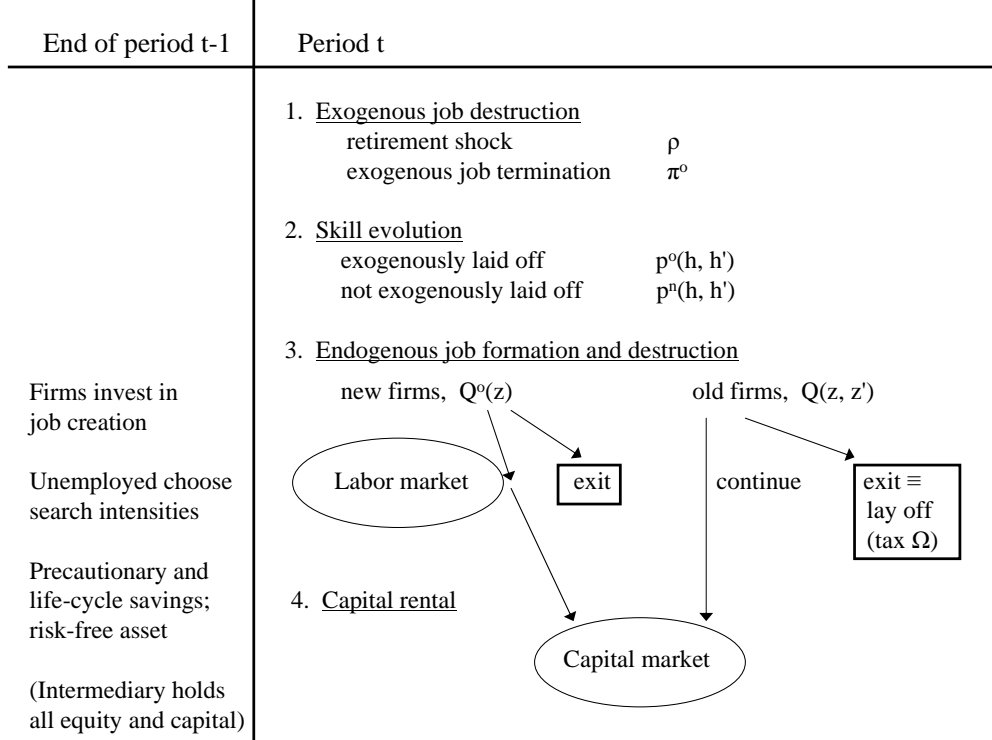
End of period t-1 | Period t

1. Exogenous job destruction
    retirement shock ρ
    exogenous job termination π^o

2. Skill evolution
    exogenously laid off $p^o(h, h')$
    not exogenously laid off $p^n(h, h')$

3. Endogenous job formation and destruction
    new firms, $Q^o(z)$ old firms, $Q(z, z')$

Firms invest in job creation

Unemployed choose search intensities

Labor market   exit   continue   exit $\equiv$ lay off (tax Ω)

Precautionary and life-cycle savings; risk-free asset

4. Capital rental

Capital market

(Intermediary holds all equity and capital)

Figure 1: Search-island model

4 Matching models

Like DenHaan et al. (2001), we include skill dynamics in a matching framework.[9] The probability distributions of the productivity levels for high-skilled ($h = H$) workers stochastically dominate corresponding probability distributions of low-skilled ($h = 0$) workers, i.e.,

$$\sum_{z' \leq \bar{z}} Q_H^o(z') < \sum_{z' \leq \bar{z}} Q_0^o(z') \qquad \text{and} \qquad \sum_{z' \leq \bar{z}} Q_H(z, z') < \sum_{z' \leq \bar{z}} Q_0(z, z'), \qquad (6)$$

for all $\bar{z}$, given that z is a permissible productivity level for both low-skilled and high-skilled workers. We follow DenHaan et al. (2001) and assume that benefits are determined by a replacement rate η on the average labor income in the worker's skill category when last employed.[10] Hence, we can index a worker's benefit entitlement by his skill in his

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[9]We thank Wouter DenHaan, Christian Haefke, and Garey Ramey for generously sharing their computer code, which we have modified. The matching framework originated in works of Diamond (1982), Mortensen (1982), and Pissarides (2000).

[10]We make two simplifications to DenHaan, Haefke, and Ramey's specification of the benefit system. First, newborn workers are entitled to the lowest benefit level without first having to work one period. Second, workers who experience an upgrade in skills are immediately entitled to the higher benefit level, even if the match breaks up immediately. These assumptions simplify solving the model. The second assumption enables us also to discard DenHaan, Haefke and Ramey's simplifying, but debatable, assumption that a skill upgrade is accompanied with a new productivity draw where the lower bound on possible draws is the reservation productivity of an ongoing match with a high-skilled worker. In our model, the distributions from which productivities are drawn do not change with endogenous reservation productivities.

last employment spell, $b \in \{0, H\}$, so that his benefit entitlement is some function $\bar{b}(b)$. Let $u(h, b)$ be the number of unemployed workers with current skill h and skill during his previous employment spell of b. The total number of unemployed workers is then

$$u = \sum_{h,b} u(h, b). \tag{7}$$

We drop the assumption of DenHaan et al. (2001) that there is an exogenous number of firms and instead impose a zero-profit condition that expresses the outcome of free entry. Let v be the endogenous number of vacancies and let $M(v, u)$ be an increasing, concave, and linearly homogeneous matching function:

$$M(v, u) = u \, M\left(\frac{v}{u}, 1\right) \equiv u \, m(\theta), \tag{8}$$

where the ratio $\theta \equiv v/u$ is the endogenously determined degree of "market tightness." The probability of finding a job, $M/u = m(\theta)$, is an increasing function of market tightness, and the probability of filling a vacancy, $M/v = m(\theta)/\theta$, is a decreasing function of market tightness. We first assume a single matching function for all vacancies and all unemployed workers, but later consider multiple matching functions.[11] For various technical details, see appendix B.

We form three models with separate matching functions (1) for unemployed workers with different skill levels, yielding equilibrium vacancies $v(h)$ for each $h \in \{0, H\}$; (2) for unemployed workers having different benefit entitlements, yielding equilibrium vacancies $v(b)$ for each $b \in \{0, H\}$; and (3) for unemployed workers indexed by both their current skill h and their skill b in their last employment (as a determinant of their benefit entitlement), yielding equilibrium vacancies $v(h, b)$ for each pair of values $(h, b) \in \{0, H\} \times \{0, H\}$. The last setup requires only three matching functions because, given our specification of the benefit system, there are no high-skilled unemployed workers with low benefits (see footnote 10).

We keep DenHaan, Haefke, and Ramey's specification that workers are risk neutral. Workers' preferences are ordered by

$$E_0 \sum_{t=0}^{\infty} \beta^t (1 - \rho)^t c_t, \tag{9}$$

where the worker discounts future utilities by the subjective discount factor $\beta \in (0, 1)$ and the survival probability $(1 - \rho)$. A matched worker and firm engage in Nash bargaining over the wage in every period. The government finances the unemployment compensation scheme with the revenues that it receives from the layoff tax and a flat-rate tax τ on firms' output.

Figure 2 shows the within-period timing of events in our matching model.

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[11]Davis (1995) is the first model with multiple matching functions of which we are aware. Among other things, Davis analyzes how, with heterogeneous workers, the number of matching functions impinges on the efficiency of outcomes. Mortensen and Pissarides (1999) study a matching model with heterogeneous workers endowed with different skill levels who match with firms in separate skill-specific matching functions. They are interested in the aggregate unemployment dynamics of a mean-preserving spread in the distribution of the labor force over skill types.
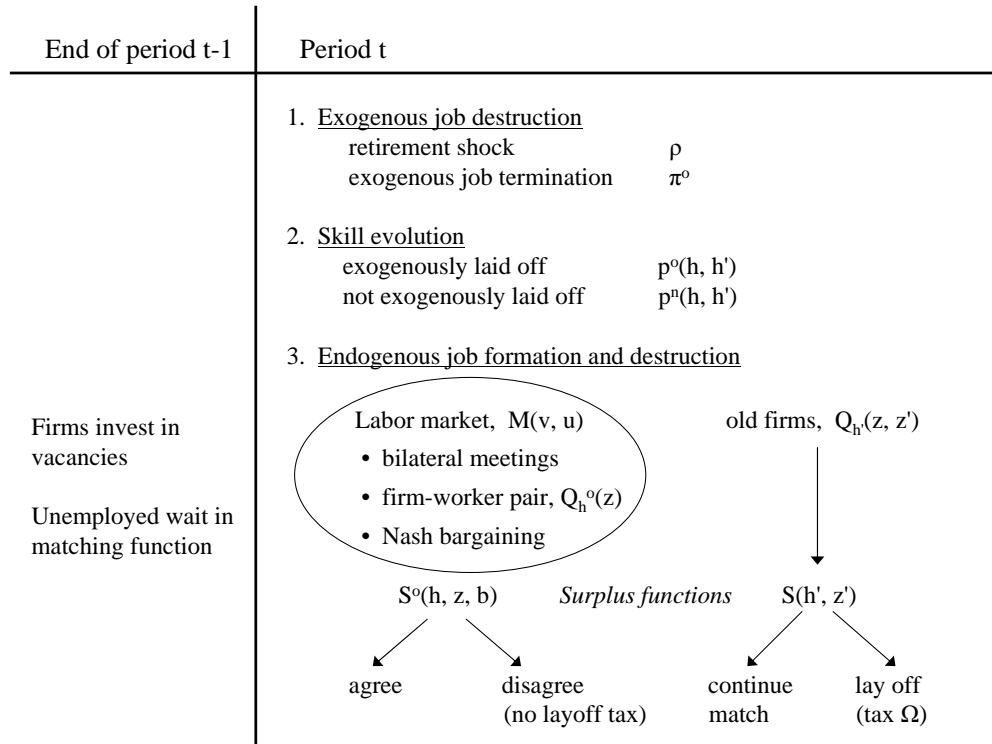
End of period t-1 | Period t

1. Exogenous job destruction
    retirement shock ρ
    exogenous job termination π^o

2. Skill evolution
    exogenously laid off $p^o(h, h')$
    not exogenously laid off $p^n(h, h')$

3. Endogenous job formation and destruction

Firms invest in vacancies

Unemployed wait in matching function

Labor market, $M(v, u)$
- bilateral meetings
- firm-worker pair, $Q_h^o(z)$
- Nash bargaining

old firms, $Q_{h'}(z, z')$

$S^o(h, z, b)$ *Surplus functions* $S(h', z')$

agree   disagree (no layoff tax)   continue match   lay off (tax Ω)

Figure 2: Matching model

5 Calibrations

5.1 Some caveats

Parts of our two models are too highly stylized to connect readily to micro evidence. For example, we arbitrarily specify truncated normal distributions of productivity levels rather than calibrate them to data. However, we can and do calibrate other parameters to match micro observations. For example, the earnings potential of a high skill worker is twice that of a low skill one, and it takes 10 years on average to work your way from low skill to high skill. Note that our parameterization for the time it takes to accumulate skills pertains both to inexperienced new workers and to workers who have suffered skill loss and want to regain their earnings potential (see footnote 12).

We use parameter values from previous studies but also, as far as possible, retain common parameterizations across models. Our practice of keeping parameters fixed across different frameworks can be criticized because the same values of these parameters imply different outcomes in the different frameworks. Our justification for keeping common parameters, including the discount factor and the variance of the productivity distribution, is that it well serves our goal of focusing attention on the economic forces at work in the alternative frameworks. We will argue that those economic forces are robust within a framework either by appealing to evidence accumulated from earlier studies in the literature or by noting that the pertinent quantitative effects are so large that reasonable changes in parameter values are unlikely to make a substantial difference.

We turn first to a set of parameter values shared by all models. Thereafter, we report calibrations of features that are unique to each framework. As far as possible, we reiterate parameter values from previous studies. For calibrating labor market frictions and disutility of searching, we target a laissez-faire unemployment rate in the range of 4 to 5%.

5.2   Common parameter values

Following Alvarez and Veracierto (2001), we set the model period equal to half a quarter, and specify a discount factor $\beta = 0.99425$ and a probability of retiring $\rho = 0.0031$ that are the same across models. People of working age have an annualized subjective discount rate of 4.7%. On average, they spend 40 years in the labor force.

Table 1 shows that the skill accumulation process is the same across models. We set transition probabilities to make the average durations of skill acquisition and skill deterioration agree with those in Ljungqvist and Sargent (1998, 2007a), who let it take a long time to acquire the highest skill level in order to match realistic shapes of wage-experience profiles.[12] We set a semiquarterly probability of upgrading skills $\pi^u = 0.0125$, so that it takes on average 10 years to move from low to high skill, conditional on no job loss. Exogenous layoffs occur with probability $\pi^o = 0.005$, i.e., on average once every 25 years. The probability of a productivity switch on the job equals $\pi = 0.05$, so that a worker expects to retain a given productivity level for 2.5 years.

Another common assumption is that productivities are drawn from a truncated normal distribution with mean 1.0 and standard deviation 1.0. Model-specific assumptions dictate how these productivity draws enter the production technology.

5.3   Search-island model

In addition to the discount factor and the probability of retiring, we take the following survival, technology, and preference parameters from Alvarez and Veracierto (2001): $\{\sigma, \delta, \xi, \gamma\}$ (see our Table 1). Since the model period equals half a quarter and the survival probability in retirement equals $\sigma = 0.0083$, the average duration of retirement is 15 years. The semiquarterly depreciation rate is $\delta = 0.011$. Our settings of exponents on the search technology ($\xi = 0.98$) and on the disutility of search ($\gamma = 0.98$), respectively, make these close to linear.

One-worker firms operate a constant-returns-to-scale Cobb-Douglas production technology with a capital share parameter $\alpha = 0.333$. Each firm has an idiosyncratic multiplicative productivity shock that is drawn from a distribution that is generated by truncating $\mathcal{N}(1, 1)$ to the interval [0,2] and then rescaling it to integrate to one. Low-skilled workers have one unit of human capital while high-skilled workers have twice that amount, $(1 + H) = 2$.

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[12]We thank Daniel Hamermesh for conversations about his data explorations of wage-experience profiles. Our assumption that work experience alone can double a worker's earnings seems to line up well with data for full-time male workers in the U.S. manufacturing industry. But the time required to attain such earnings gains are longer than what we assume. Note that the speed of skill accumulation in our model pertains to both inexperienced new workers and workers who have suffered skill loss and want to regain their earnings potential.

Parameters common to all models

Discount factor β	0.99425
Retirement probability ρ	0.0031
Probability of upgrading skills, π^u	0.0125
Probability of exogenous breakup, π^o	0.005
Probability of productivity change, π	0.05
Productivity distribution	truncated $\mathcal{N}(1,1)$

Additional parameters in search-island model

Probability of dying, σ	0.0083
Disutility of search, A	5.0
γ	0.98
Search technology, ξ	0.98
Capital share parameter, α	0.333
Depreciation rate, δ	0.011
Job creation cost, μ	5.0
Low skill level	1.0
High skill level, $(1+H)$	2.0

Additional parameters in matching models

Matching function, $M(v,u)$	$0.45\, v^{0.5} u^{0.5}$
Vacancy cost, μ	0.5
Worker's bargaining weight, ψ	0.5
Low-skilled workers' productivity:	truncated $\mathcal{N}(1,1)$
High-skilled workers' productivity:	truncated $\mathcal{N}(2,1)$

Table 1: Parameter values (one period is half a quarter)

The cost of starting a firm, i.e., of drawing anew from the distribution of productivities, equals 5. This can be measured against the laissez-faire outcome that only about 20% of all such draws exceed the optimally chosen reservation productivity when the firm hires a worker at a semiquarterly equilibrium wage rate equal to 6.4 for low-skilled workers. Hence, the average cost of recruiting a worker is approximately 6 months of the wage paid a low-skilled worker.

The disutility parameter A for job search equals 5, which generates a laissez-faire unemployment rate of 4.4%.

5.4 Matching models

Here we adopt most of the parameter values of Ljungqvist and Sargent (2004), who modify the matching framework of DenHaan et al. (2001). The calibration is reported in Table 1. The main substantial departures from Ljungqvist and Sargent (2004) are that (1) we replace the earlier uniform productivity distributions by truncated normal distributions; (2) instead of a fixed number of firms, we assume free entry; and (3) we introduce a Cobb-Douglas matching function and a vacancy cost μ.[13]

High-skilled workers' productivity distribution is a truncated $\mathcal{N}(2,1)$ and low-skilled workers' productivity distribution is a truncated $\mathcal{N}(1,1)$, both of which are rescaled to integrate to one. Both distributions are truncated to a range of 4 units where the midpoint of the range is the mean of the corresponding untruncated distributions. Thus, the range for high-skilled workers' productivities is [0,4] and the range for low-skilled workers' productivities is [-1,3]. The high-skilled workers' distribution is the low-skilled workers' distribution shifted to the right.

Table 1 shows that our parameterization of the matching technology and the Nash bargaining between workers and firms is fairly standard. A worker's bargaining weight equals $\psi = 0.5$, which is also the elasticity of the Cobb-Douglas matching function.

By computing the expected cost $\theta\mu/m(\theta)$ of filling a vacancy, we can interpret the semiquarterly vacancy cost $\mu = 0.5$. In the laissez-faire economy, this average recruitment cost equals 3.4, which can be compared to the average semiquarterly output of 2.3 goods per all workers. Our calibration of the matching model yields a laissez-faire unemployment rate of 5.0%.

6 Outcomes in the search-island model

Figures 3–7 show outcomes in our calibrated search-island model. For zero turbulence, the solid lines in figures 3 and 4, respectively show that equilibrium unemployment increases with increases in the UI replacement rate η and that it decreases with increases in the layoff tax Ω, *ceteris paribus*. The outcome that layoff taxes suppress unemployment also prevails

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[13]To ensure that the Cobb-Douglas matching technology generates permissible matching probabilities inside the unit interval, we assume that the number of matches equals $\min\{M(v,u), v, u\}$.
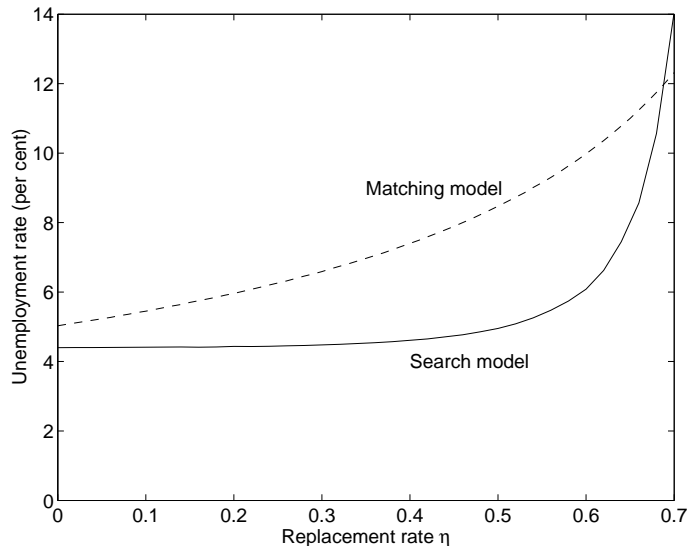
Figure 3: Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes. The solid line refers to the search-island model, and the dashed line refers to the matching model.

in the quantitative analysis of Alvarez and Veracierto (2001), who report that equilibrium unemployment falls by 1.8 percentage points in response to a layoff tax equal to 12 months of wages. In our search-island model, where workers's skill levels contribute an additional source of heterogeneity relative to Alvarez and Veracierto's model, figure 4 shows that unemployment falls by 0.9 percentage points in response to a layoff tax of 50 which corresponds to roughly 12 months of wages for a low-skilled worker. Alvarez and Veracierto do not study UI replacement rates but instead compute the effects of one-time payments from the government to laid off workers. Not surprisingly, since they are invariant to the length of unemployment spells, such severance payments have only a muted (positive) effect on equilibrium unemployment. However, it is interesting to note that Alvarez and Veracierto report that they assumed a 66% UI replacement rate when initially calibrating their model to U.S. policies and data, and they found the unemployment rate of that calibrated model to be not much higher than in the laissez-faire version of their model. This seems puzzling given our figure 3, where the unemployment rate increases dramatically at replacement rates in excess of 55–60%, but we conjecture that the explanation is that Alvarez and Veracierto assume that unemployed workers lose their eligibility for unemployment benefits with a constant probability in every period while our unemployed workers keep their benefits throughout their entire unemployment spells.

As a benchmark parameterization of the welfare state, we set the replacement rate equal to 0.55 and the layoff tax equal to the above mentioned 12 months of wages for a low-skilled worker, $(\eta, \Omega) = (.55, 50)$. That yields an equilibrium unemployment rate of 4.1%, which is lower than the laissez-faire unemployment rate of 4.4%. This is qualitatively the same
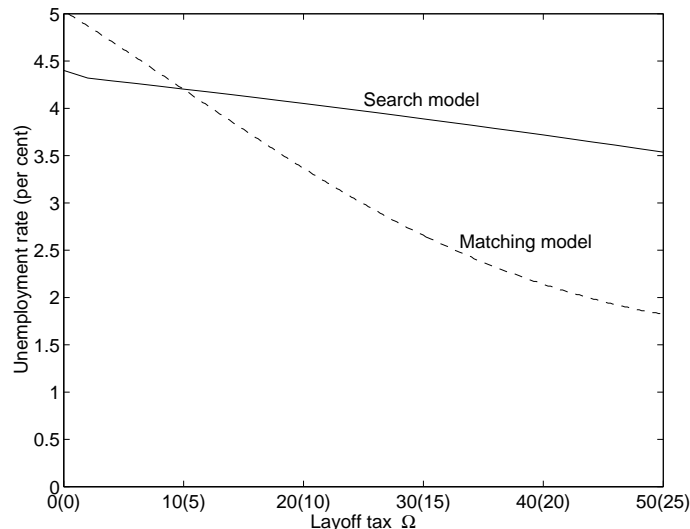
Figure 4: Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The solid (dashed) line refers to the search-island (matching) model for which the scale on the horizontal axis is expressed without (with) parentheses. In the search-island model, the magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6.4 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 50 corresponds to roughly one year of wage income for a low-skilled worker. In the matching model, the average semiquarterly output is 2.3 goods per worker in the laissez-faire economy, i.e., a layoff tax equal to 19 corresponds to approximately one year's of a worker's output.

outcome as in the analysis of the European unemployment experience in the 1950s and 1960s in Ljungqvist and Sargent (2007a). Next, we study how turbulence gives rise to qualitatively the same effects in the search-island model as in Ljungqvist and Sargent (2007a).

The two panels of figure 5 show the disparate effects of an increase in turbulence on unemployment rates in the welfare state $(\eta, \Omega) = (.55, 50)$ and laissez-faire $(\eta, \Omega) = (0, 0)$ economies. The figures show both total unemployment (solid lines) and the subgroup of unemployed workers who have suffered a skill loss after a lay off in their last job (dashed lines). The dashed lines reveal that the explosion of unemployment in the welfare state economy when turbulence π^d increases is attributable to greater unemployment of previously high-skilled workers who have suffered skill loss upon termination. In the laissez faire economy, unemployment involving that group increases only mildly with an increase in turbulence, an outcome that explains why the overall unemployment rate in the laissez-faire economy is not much affected by increases in turbulence.

The two panels of figure 6 display how the inflow into unemployment and the average duration of unemployment respond to increases in turbulence π^d in the welfare state and laissez-faire economies. In the laissez-faire economy, the inflow rate and the duration are both impervious to increases in turbulence, while in the welfare state economy the average duration

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<table>
<tr><td>(a) Welfare state</td><td>(b) Laissez-faire economy</td></tr>
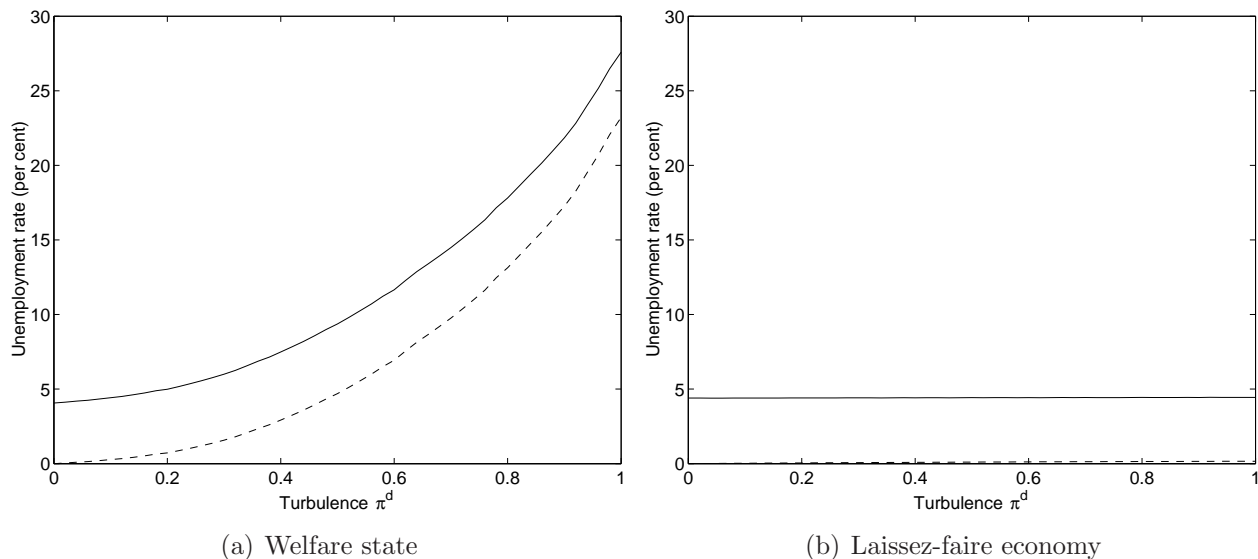</table>

Figure 5: (Search model) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.

grows markedly with increases in turbulence; especially at higher levels of turbulence, the inflow rate into unemployment actually falls modestly with increases in turbulence.

Figure 7 shows how in very turbulent times ($\pi^d = 1$), hazard rates of gaining employment behave very differently in the laissez faire and welfare state economies, being flat in the former and rapidly declining with the length of the unemployment spell in the latter economy.

These outcomes closely resemble those in the model of Ljungqvist and Sargent (2007a) that was based on the McCall search environment without many of the features of the search-island model such as risk aversion, precautionary saving, and competitive firms that hire capital and labor. The common feature of these different frameworks is the presence in turbulent times of unemployed workers who have suffered skill loss but are entitled to relatively generous unemployment benefits based on past earnings. In Ljungqvist and Sargent (2007a), these workers were more likely to become long-term unemployed because they chose relatively high reservation earnings as compared to their current earnings potential; and given the low likelihood of drawing such earnings from the wage offer distribution and the mere fact that the generous benefits made it less costly to stay unemployed, these workers also chose low search intensities. It is appropriate to call them "discouraged workers" because they have low probabilities of returning to gainful employment. In our search-island model, such workers' choice of low search intensities is the only avenue that operates because the abstraction of Alvarez and Veracierto (2001) has the equilibrium outcome that all workers are paid the same wage rate per unit of skill. Evidently, this channel by itself is sufficient to explain how unemployment explodes in the welfare state in response to turbulence while the laissez-faire unemployment rate remains virtually unchanged.

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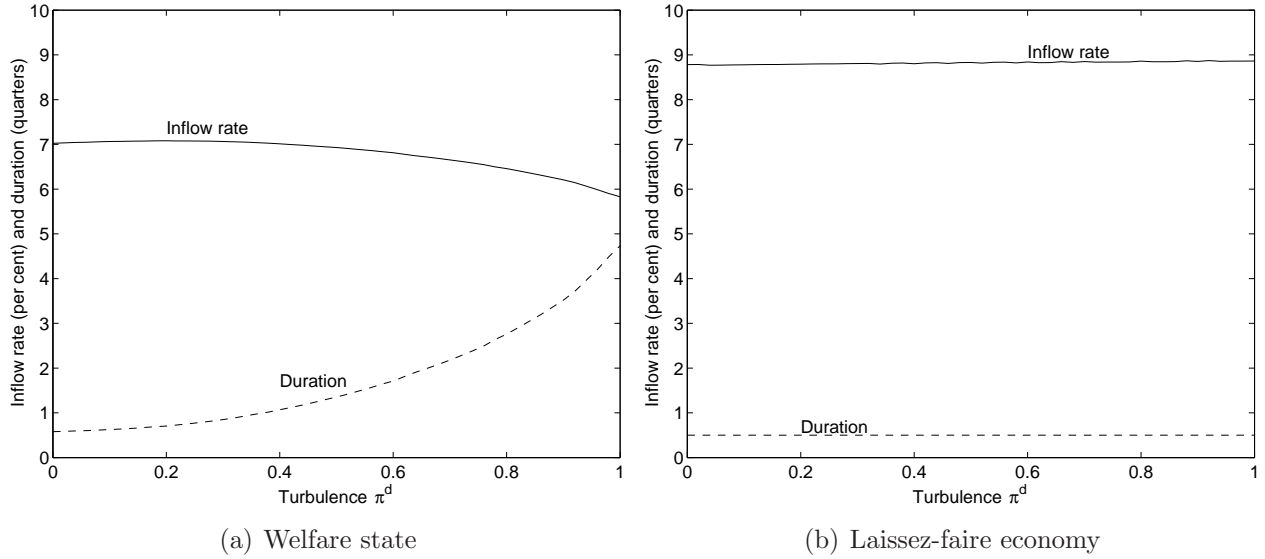(a) Welfare state   (b) Laissez-faire economy

Figure 6: (Search model) Inflow rate and average duration of unemployment in the welfare state (panel a) and the laissez-faire economy (panel b). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.
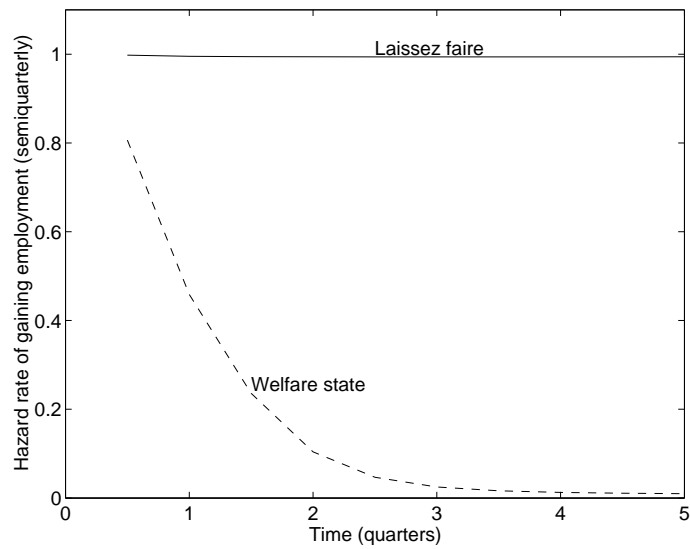

Figure 7: (Search model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$, in the welfare state (dashed line) and in the laissez-faire economy (solid line). The policy of the welfare state is $(\eta, \Omega) = (0.55, 50)$.

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Thus, forces present in the search-island model but neglected by the McCall model used in Ljungqvist and Sargent (1998, 2007a) (i.e., risk-aversion, precautionary savings, firms that hire capital, and wages determined in competitive markets) fail to blunt the main force captured by the McCall model: how the incentives for unemployed workers to search change with increasing turbulence. Workers' choices of search intensity now also depend on their financial assets and the curvature of their utility function. But these additional influences on search intensities don't alter the pattern of outcomes. By worsening the effective skill accumulation technology confronting workers, increased turbulence affects the relative returns to searching and collecting unemployment benefits.

7   Findings in matching models

This section reports the effects of increases in turbulence on equilibrium outcomes in four matching models that differ in how they assign workers to matching functions. How outcomes in these four models respond to increased turbulence illuminates the economic forces that equilibrate labor markets within matching models. Matching models feature adverse congestion effects that job-seeking workers impose on each other and that worker-seeking firms impose on each other. Unmatched workers and firms are concerned both about matching probabilities that are affected by the total stocks of unemployment and vacancies, and about the bargaining situation that they will face in future matches. Within a labor pool defined by a matching function, market tightness, $v/u \equiv \theta$, is an important equilibrating variable that the invisible hand uses to reconcile the decisions of firms and workers.

7.1   Single matching function

In the model with a single matching function, the dashed line in figure 3 shows that the unemployment rate is positively related to the replacement rate in the unemployment insurance system. This result emerges in many models of unemployment, but it is useful to recall the particular forces that produce this outcome in the matching model. Unemployment benefits raise the value of a workers' outside option in the wage bargaining with employers. If nothing else changed, a higher threat point for workers would cause wages and the reservation productivity to rise. That would deteriorate firms' bargaining positions, leaving them unable to recover the expected cost of filling vacancies if their probability of encountering unemployed workers were to remain unchanged. Therefore, the invisible hand restores the profitability of firms by lowering the number of vacancies relative to the number of unemployed workers, i.e., the equilibrium measure of market tightness falls, which in turns implies a longer average duration of unemployment spells. Hence, unemployment rises because the duration and incidence of unemployment both increase.

Although UI benefits necessarily increase unemployment in the matching model, layoff taxes have countervailing effects on unemployment. Mortensen and Pissarides (1999) pointed out that layoff taxes reduce incentives both to create jobs and to destroy them. They show that the net effect of these forces on market tightness, and consequently on unemployment

duration, is ambiguous, but that the reservation productivity for existing jobs decreases and therefore so does the incidence of unemployment.[14] The dashed line in figure 4 shows that in our calibrated matching model with one matching function there is a strong negative relationship between layoff taxes and unemployment. This is also true for the calibration of Mortensen and Pissarides (1999). Unemployment falls because layoff taxes reduce labor reallocation and lock workers into their jobs, so that frictional unemployment falls. Although this negative relationship between layoff taxes and unemployment is the most common outcome in the matching literature, there are exceptions, most notably Millard and Mortensen (1997). As explained by Ljungqvist (2002), such contradictory quantitative findings in the matching literature come not from differences in parameter values but from different setups. Millard and Mortensen assume that firms must also pay layoff taxes after encounters with job seekers who are not hired. That dramatically increases workers' bargaining strengths, making equilibrium market tightness plummet in order to level the playing field for firms. Under the more typical assumption that firms pay layoff taxes only for the workers they had chosen to hire and then had subsequently laid off, Ljungqvist (2002) concludes on the basis of a wide range of simulations that there is a presumption that layoff taxes reduce unemployment in the matching model.[15]

7.1.1 High layoff taxes and high benefits

As noted by Mortensen and Pissarides (1999), the countervailing forces of unemployment benefits and layoff taxes in the matching model can explain why the unemployment rate in a welfare state need not be high. For low values of the turbulence parameter π^d, a more generous unemployment insurance system can accompany higher layoff taxes, leaving the equilibrium unemployment rate unchanged or even lower than the laissez-faire outcome. To illustrate this outcome, we pick a replacement rate of 70%, which on its own would have raised the unemployment rate from the laissez-faire level of 5.0% to 12.3% in figure 3, and we choose a layoff tax equal to 24, which corresponds to approximately 5 quarters of a worker's average output in laissez faire, which on its own would have lowered the unemployment rate from 5.0% to 1.9% in figure 4. The combination of these two policies yields an unemployment rate of 4.4%, which falls below the laissez-faire unemployment rate of 5.0%, as depicted in figure 8 at zero turbulence, $\pi^d = 0$. Thus, both the matching model and the search-island model can rationalize why unemployment need not be high in the welfare state in tranquil times.

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[14]The ambiguous effect of layoff taxes upon unemployment is compounded in our framework because we model a new job opportunity as a draw from a productivity distribution, so that there is an endogenous reservation productivity in job creation. In contrast, Mortensen and Pissarides (1999) assume that all new jobs begin with the same productivity level.

[15]Unlike Millard and Mortensen (1997), our models obey the dictum, "If a firm does not *hire*, it does not have to *fire*."
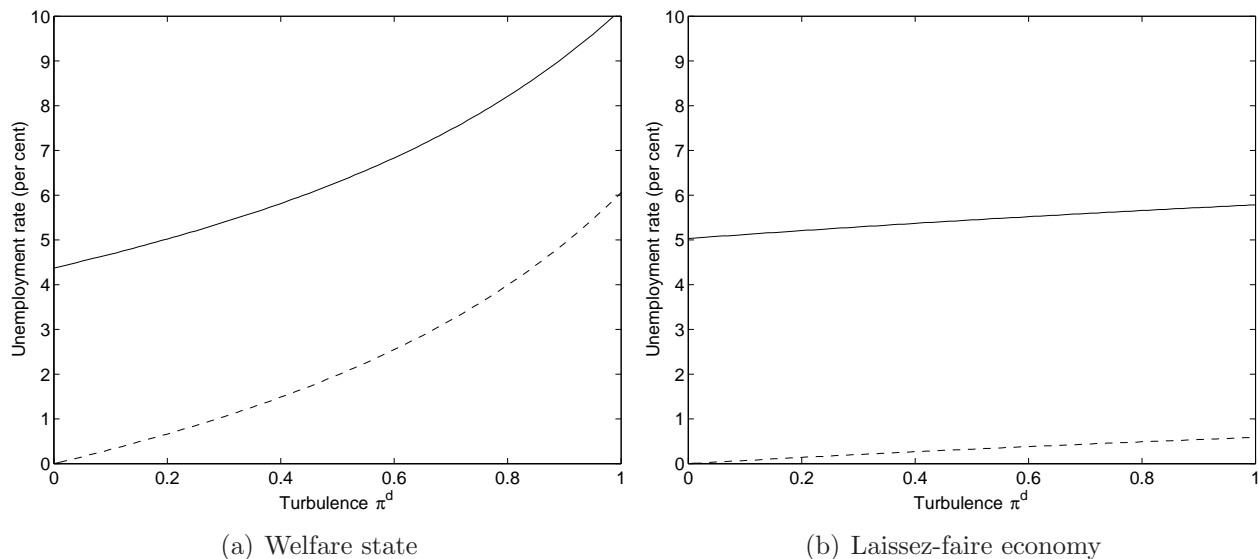
Figure 8: (Matching model) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. The dashed line shows the unemployed who have suffered skill loss. The policy of the welfare state is $(\eta, \Omega) = (0.7, 24)$.

7.1.2 Turbulence and unemployment

Along with the search-island model, the matching model confirms the finding of Ljungqvist and Sargent (1998, 2007a) that increased turbulence causes unemployment to increase in the welfare state while it remains virtually unchanged in the laissez-faire economy, as shown in figure 8. As in the search-island model, the positive relationship between turbulence and unemployment is explained by the choices made by laid off workers who have suffered skill loss (dashed line in panel a of figure 8).

Panel a of figure 9 shows that in the welfare state an increase in turbulence increases the average duration of unemployment spells but leaves the inflow rate almost unchanged. The higher average duration of unemployment is not shared equally among unemployed workers. Although all unemployed workers face the same probability of encountering a vacancy because they enter a common matching function, job acceptance rates differ among workers who are heterogeneous with respect to their skill levels and benefit entitlements. Thus, consider unemployed workers who have been laid off and suffered skill loss. Because unemployment benefits are indexed to past earnings, such workers receive benefits that are high compared to their current earnings potential. To give up their generous benefits, these workers must encounter vacancies with idiosyncratic productivities that are high enough to induce firms to offer more generous wages. Hence, low-skilled unemployed workers with high benefits encounter fewer acceptable matches than do low-skilled unemployed workers with low benefits. The unchanging inflow rate into unemployment is explained by almost
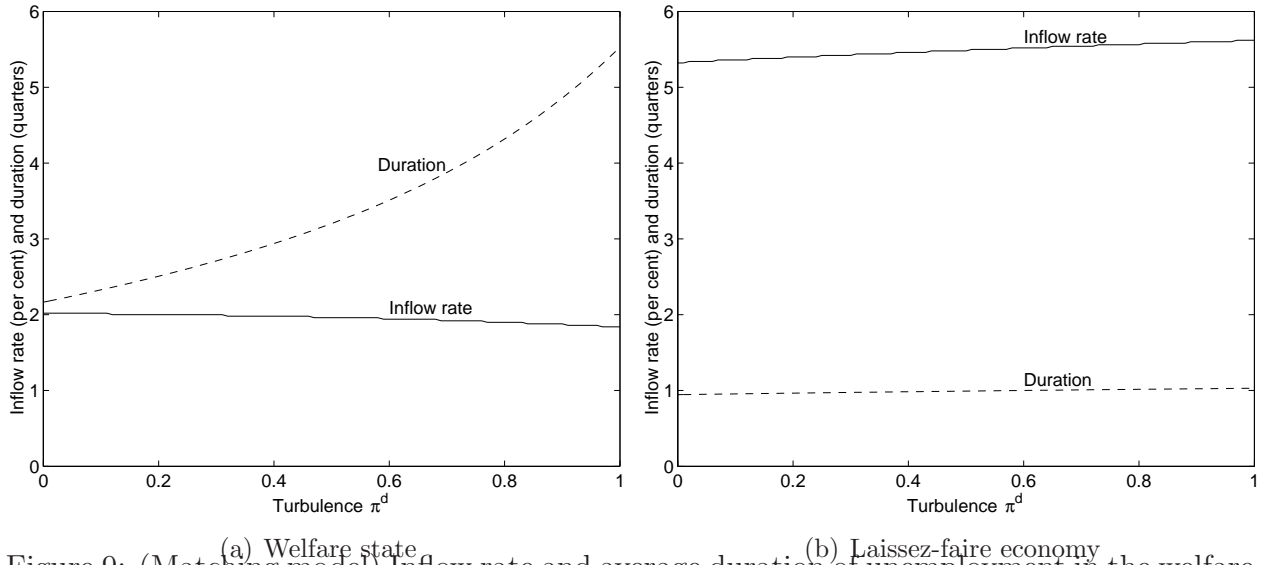
　　　　　　　　　　　　　　　　　(b) Laissez-faire economy

Figure 9: (Matching model) Inflow rate and average duration of unemployment in the welfare state (panel a) and the laissez-faire economy (panel b). The dashed line is the average duration of unemployment in quarters. The solid line depicts the quarterly inflow rate into unemployment as a per cent of the labor force. The policy of the welfare state is given by $(\eta, \Omega) = (0.7, 24)$.

unchanged reservation productivities that determine job destruction. Turning to the laissez-faire economy in panel b of figure 9, both the inflow rate and the average duration of unemployment are virtually unaffected by turbulence. In the laissez-faire economy, firms and workers respond to turbulence in ways that leave both the optimal rate of job destruction and the optimal length of time to search for a job unchanged.

Since turbulence sharply increases the average duration of unemployment spells in the welfare state, after allowing for the equilibrium response in the reservation productivity for new jobs, one would expect a precipitous fall in market tightness $\theta = v/u$. Thus, the dotted line in figure 10 depicts how market tightness plummets in response to higher turbulence. Alternatively, when varying the level of turbulence, we could have plotted combinations of stationary equilibrium (u, v) pairs in the space of unemployment and vacancy rates. Such a graph would show that our theory of increased turbulence is consistent with the observation that the Beveridge curve has shifted outward in Europe since the mid-1970s.[16]

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[16]In a survey of the matching function, Petrongolo and Pissarides (2001, p. 409) emphasize outward shifts in the Beveridge curve that "coincide with the secular rise in European unemployment, which started in the mid-1970s. The unemployment rate has increased despite the fact that the separation rate and the vacancy rate [both expressed relative to the level of employment] have not shown any trend. The implication for the matching function is that there are variables beside [unemployment and vacancies] that have played an important role in matching in the last two decades and these variables contributed to a deterioration of the matching rate." Our theory of increased turbulence is consistent with this account. The inflow rate into unemployment in panel a of figure 9, which is also the separation rate in our model, is almost unaffected by turbulence in the welfare state. All expressed relative to the level of employment, the separation rate falls

To ensure that firms break even when posting vacancies in a more turbulent environment, the invisible hand increases the probability that a vacancy encounters an unemployed worker, and thereby weakens the effective bargaining strength of workers: the lower probability that an unemployed worker encounters a vacancy causes workers' outside value to fall. Decreased waiting times between matches for vacancies and the associated fall in a worker's outside value are the invisible hand's way of improving firms' prospects in response to two adverse forces on firms' profits. First and foremost, increased turbulence ignites adverse welfare-state dynamics because, with a given replacement rate, UI becomes more valuable compared to what can be earned working. This is most apparent in the case of laid off workers who have suffered skill loss and become low-skilled unemployed workers who are entitled to benefits that are generous relative to their reduced earnings potential. The invisible hand must compensate firms for meeting such workers, because these encounters are less likely to result in agreeable matches; and when such matches are formed, wage payments to low-skilled workers who are entitled to high benefits are higher than those to low-skilled workers who are entitled to low benefits. Second, our representation of turbulence implies a worse technology for skill accumulation and, therefore, higher turbulence has detrimental effects on match surpluses in both the welfare state and the laissez-faire economies. The invisible hand must improve firms' situations because they have to break even while financing the average cost for filling a vacancy out of a fixed fraction of the diminished match surpluses.

Of these two forces that make market tightness fall in response to increased turbulence, that driven by adverse welfare-state dynamics is more important. This assertion emerges from the outcome that laissez-faire unemployment in panel b of figure 8 increases only slightly in response to increased turbulence: under laissez-faire, the second adverse force from increased turbulence operates, but not the first.

7.2 Multiple matching functions

7.2.1 Separate matching functions for different skills

Figure 10 also reports outcomes of an economy with two separate matching functions for unemployed workers sorted only according to their current skills. It is instructive to compare the market tightness across the two labor markets in such an economy. When $\pi^d = 0$, the zero-profit condition for job creation calls for more vacancies per unemployed worker assigned to the low-skill market. Workers with low skills enjoy a higher probability of encountering a vacancy because there is a larger match surplus to be shared in the case of a match and therefore more incentive for firms to post vacancies. The larger match surplus associated with a low-skilled worker arises from the possibility that employment might result in a skill upgrade that leads to a future capital gain for the match. However, the relative advantage for low-skilled unemployed in terms of market tightness erodes quickly as turbulence increases. When there is turbulence, the low-skill market includes not only

by merely 3%, while the unemployment rate depicted in panel a of figure 8 spans an increase of 128% with a corresponding vacancy rate (not shown) that declines by no more than 14 per cent.
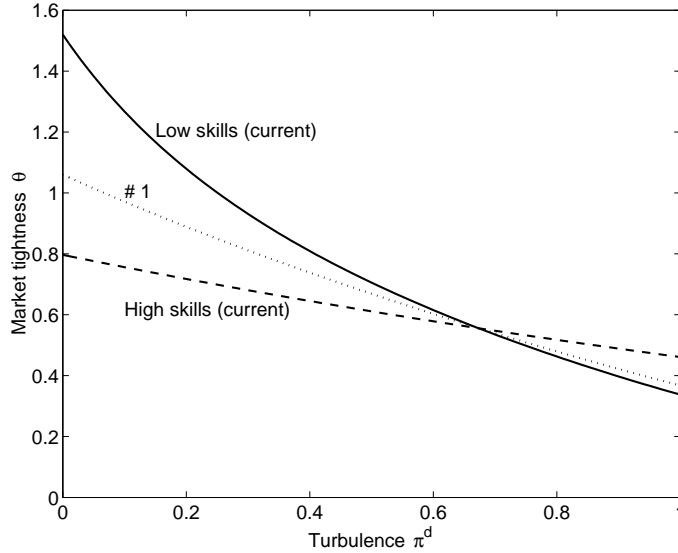
Figure 10: (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon their current skills; low skills (solid line) and high skills (dashed line). As a benchmark, the dotted line labeled #1 depicts market tightness in the economy with a single matching function. The government's policy is given by $(\eta, \Omega) = (0.7, 24)$.

low-skilled workers with low benefits but also laid off workers who have suffered skill loss and are now low-skilled but entitled to high benefits. As discussed above, firms think that low-skilled workers who are entitled to high benefits are poor job candidates and, therefore, the invisible hand must compensate firms that post vacancies in the low-skill market by assigning shorter times to encounter an unemployed worker. Thus, there has to be lower market tightness and an associated weakening of workers' effective bargaining strength. In terms of aggregate unemployment, figure 13 shows that the outcome in this economy with two matching functions is not much different from the model with a single matching function.

7.2.2 Separate matching functions for different unemployment benefits

In figure 11, we turn to an economy with separate matching functions for unemployed workers sorted only according to their benefits. The least desirable job candidates, the low-skilled workers who are entitled to high benefits, are now pooled with the high-skilled unemployed. When turbulence π^d increases, the high-benefit market experiences a precipitous fall in market tightness, while the decline in the low-benefit market is smaller, at least until turbulence reaches a critical level. When turbulence reaches 0.60, marked by a star in Figure 11, the market tightness $\theta = \frac{v}{u}$ in the high-benefit market has fallen so much that the probability that a vacancy meets an unemployed worker is equal to one. Higher levels of turbulence further depress market tightness and reduce the probability that an unemployed worker meets a vacancy, but the probability that a vacancy meets an unemployed worker remains one.
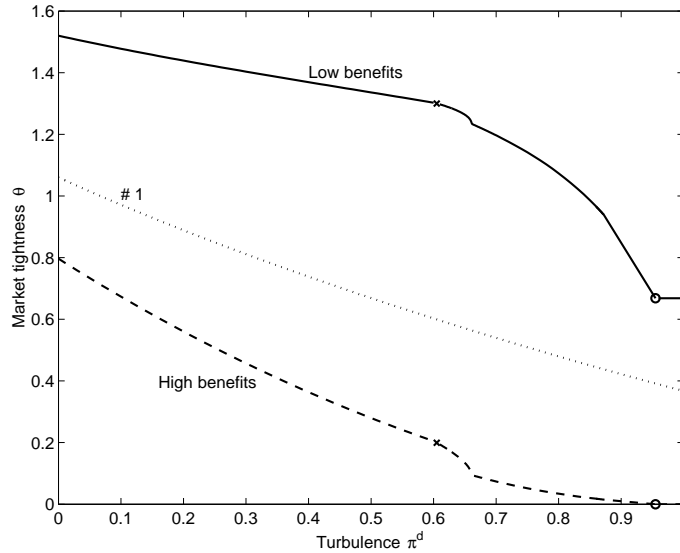
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Figure 11: (Matching model) Market tightness θ when there are separate matching functions for unemployed workers sorted according to their benefits; low benefits (solid line) and high benefits (dashed line). As a benchmark, the dotted line labeled #1 depicts market tightness in the economy with a single matching function. The government's policy is $(\eta, \Omega) = (0.7, 24)$. For π^d above .6 denoted by the *, the probability that a vacancy meets a worker equals 1. For π^d above .96 (the circle), the high benefit market closes.

The short end of the market, the number of vacancies, determines the total number of encounters. When turbulence reaches another critical level at 0.96 marked by a circle in figure 11, market tightness in the high-benefit market has fallen to zero and the market closes. At levels of turbulence above this critical point, a firm in the high-benefit market cannot expect to break even by posting a vacancy and meeting a worker with certainty in the next period, even though the worker's threat point is merely the outside value of remaining unemployed forever. Vacancies in the high-benefit market have become unprofitable because the odds of encountering a low-skilled worker rather than a high-skilled worker are too high. The point at which the high-benefit market breaks down obviously depends on the length of a model period. While a shortening of our semiquarterly period would delay and maybe eliminate the market breakdown, unemployment would still explode when higher turbulence drives market tightness closer to zero. (See footnote 18.)

7.2.3 Comparisons across different pooling arrangements

Figure 13 depicts how aggregate unemployment increases more in response to turbulence in the economy with separate matching functions indexed by unemployed workers' benefits than it does in the economy with a single matching function or in the economy with separate matching functions for unemployed workers sorted according to their current skills. In all three economies, low-skilled unemployed workers entitled to high benefits harm the prospects

of other workers with whom they are pooled within a matching function. However, the impact of these unwanted job candidates is diluted when there is a single matching function in the economy or, in the case of multiple matching functions, when these workers are pooled with a group of unemployed workers who can better withstand such a mixing. In our model, the low-skilled workers who are entitled to low benefits are the more resilient group of unemployed because their match surpluses include the prospects of capital gains associated with becoming high-skilled. Hence, low-skilled unemployed workers who are entitled to low benefits can better bear the burden of being pooled with low-skilled unemployed who are entitled to high benefits. If these unwanted job candidates are instead pooled with the least resilient group, namely, high-skilled unemployed workers who are entitled to high benefits, as when the matching functions are indexed by benefits b but not skills h, aggregate unemployment increases more with turbulence and increases further at the level of turbulence where the probability of that a vacancy meets an unemployed worker in the high-benefit market has increased to its maximum of one, indicated by a star in figure 13. At higher levels of turbulence, the ever lower measures of market tightness in figure 11 cause aggregate unemployment virtually to explode in figure 13.[17] At the critical level of turbulence indicated by a circle, the high-benefit market shuts down and unemployment becomes an absorbing state for *all* skilled workers who suffer exogenous layoffs. It follows that the solutions to workers' and firms' optimization problems are no longer affected by the incidence of skill loss among the exogenously laid off. Hence, unemployment in figure 13 and measures of market tightness in figure 11 become constant for any turbulence above this critical level. A consequence of the adverse outcomes in the high-benefit market is that market tightness in the low-benefit market suffers a dramatic decline. The breakdown of the high-benefit market is tantamount to a drastic deterioration in the skill accumulation technology. The argument in section 7.1.2 explains why the invisible hand must lower market tightness in the low-benefit market to uphold the zero-profit condition for job creation in the face of what is like a deterioration in the economy's technology.

7.2.4   Three matching functions: sorting according to *both* skills *and* benefits

Insights gleaned from the models with two matching functions help to understand the outcomes for the model in which three matching functions sort unemployed workers perfectly along all of their attributes. Under our calibration, the labor market for low-skilled workers who are entitled to high benefits operates only at very low levels of turbulence, and its market tightness in figure 12 is so low that the probability that a vacancy meets an unemployed worker always equals one. At the critical level of $\pi^d = 0.09$ indicated by a circle in figure 12, the market for low-skilled unemployed workers entitled to high benefits shuts down, just as the high-benefit market did in the economy of subsection 7.2.2. The two active labor

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[17]The unmarked point of inflection at turbulence 0.66 on aggregate unemployment in the economy with separate matching functions for unemployed workers based upon their benefits in figure 13 coincides with the endogenous job destruction involving high-skilled workers coming to an end. At higher levels of turbulence, high-skilled workers separate from their jobs only because of exogenous job destruction. This equilibrium outcome somewhat arrests the explosion in unemployment.
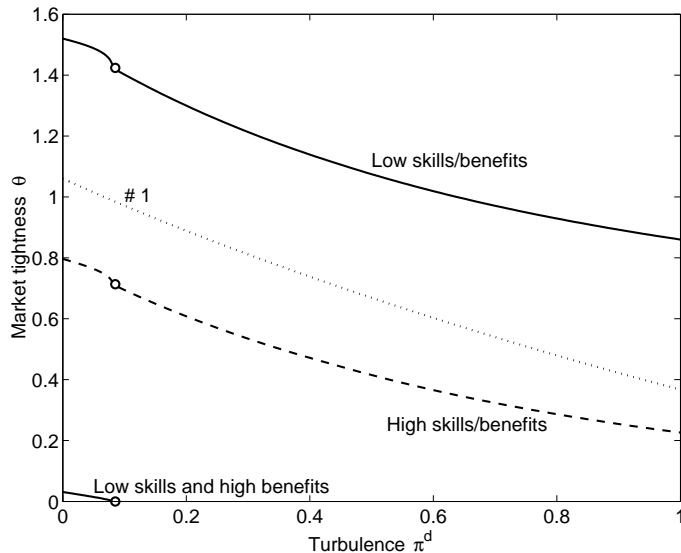
Figure 12: (Matching model) Market tightness θ when there are separate matching functions for unemployed based upon both their current skills and benefits; low skills/benefits (upper solid line), high skills/benefits (dashed line) and low skills but high benefits (lower solid line). As a benchmark, the dotted line depicts market tightness in the economy with a single matching function. The government's policy is $(\eta, \Omega) = (0.7, 24)$.

markets are characterized by smoothly falling measures of market tightness throughout the range of turbulence in figure 12. This particular outcome resembles the outcome of the single matching function model of section 7.1, but here the similarities end. The unemployment rate of the model with three matching functions explodes in response to increased turbulence in figure 13, both before and after the breakdown of the market for low-skilled unemployed entitled to high benefits. This is hardly surprising after the market breakdown because unemployment has then become an absorbing state for all workers experiencing skill loss. Hence, the ranks of the unemployed must then inevitably swell in response to higher incidence of skill loss.[18]

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[18]To examine the effects of a shorter model period, we converted our semiquarterly calibration into a semi-monthly one by rescaling the discount factor, all exogenous transition probabilities, and the multiplicative coefficient in the matching function so that the laissez-faire outcomes were unchanged. Under the semi-monthly calibration, the corresponding version of figure 13 depicts two lower curves that do not change much while the two upper curves are pushed to the right when the shorter model period delays market breakdowns until higher levels of turbulence. Specifically, in the economy with three matching functions, there is an interior solution for the probability that a vacancy meets an unemployed worker in the market for low-skilled workers with high benefits until turbulence reaches the critical level of 0.23, before which the aggregate unemployment rate lies somewhere in between the curve labeled #3 and the other curves in figure 13. But at higher levels of turbulence, the market quickly shuts down with the result that aggregate unemployment reaches the trajectory of the semiquarterly calibration (curve labeled #3 in figure 13).
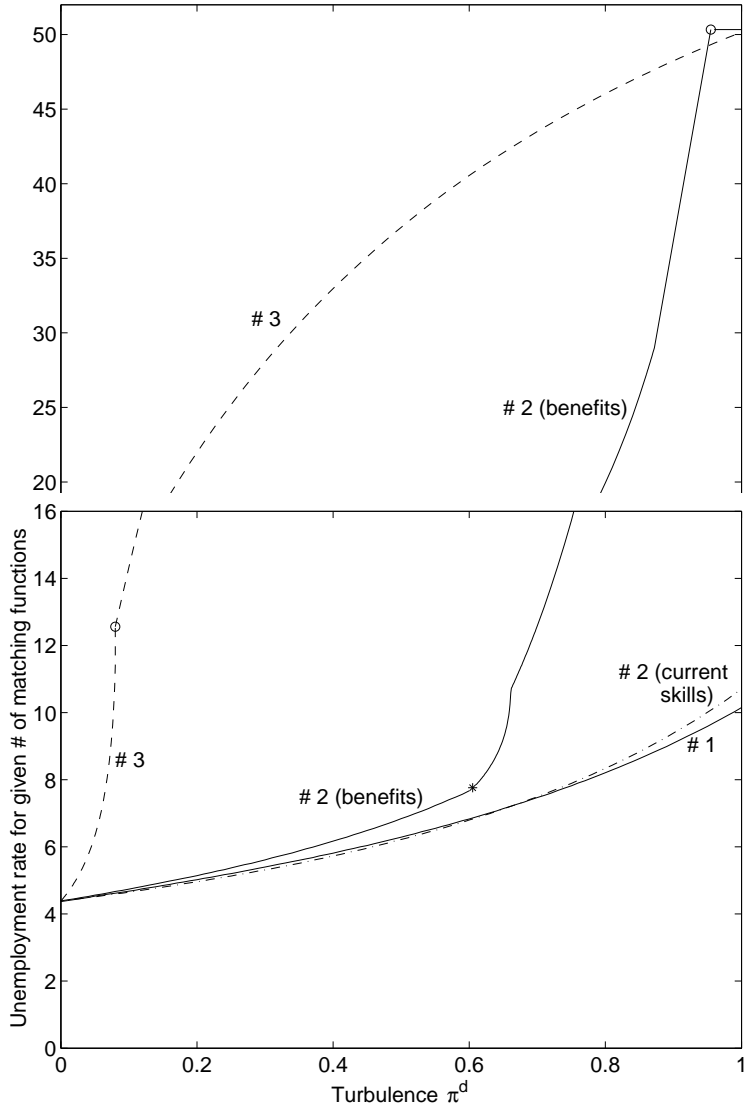
Figure 13: (Matching model) Aggregate unemployment rates for different number of matching functions. The lower solid line depicts the benchmark model with one matching function. The dash-dotted and the upper solid lines refer to the two models with two matching functions where the unemployed are sorted by their current skills and their benefits, respectively. The dashed line depicts the model with three matching functions, i.e., the unemployed are perfectly sorted along all of their attributes. The government's policy is $(\eta, \Omega) = (0.7, 24)$.
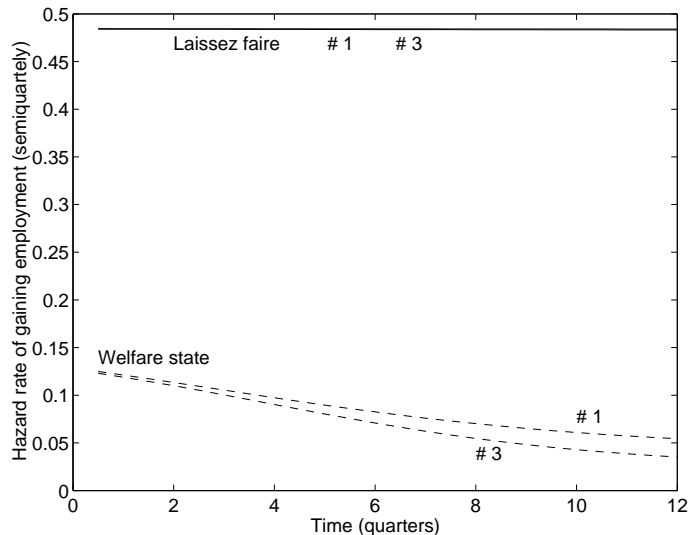
Figure 14: (Matching model) Semiquarterly hazard rates of gaining employment in turbulent economic times, $\pi^d = 1.0$. The two dashed lines indexed by #1 and #3 depict the welfare state with one and three matching functions, respectively. The solid line represents the laissez-faire economy with an almost perfect overlap of the outcomes associated with one and three matching functions, respectively. The policy of the welfare state is $(\eta, \Omega) = (0.7, 24)$.

7.2.5 Heterogeneity not duration dependence as source of falling hazard rate

Figure 14 depicts the hazard rate of gaining employment in the most turbulent times ($\pi^d = 1.0$). The hazard rate is practically flat in the laissez-faire economy but declines sharply in the welfare state. The high incidence of long-term unemployment in the welfare state is conveyed graphically by a hazard rate that is low even at the start of unemployment spells. Compare this to the much higher and constant hazard rate in the laissez-faire economy. The hazard rate in the welfare state falls with the duration of spells, not because of *duration dependence*, but because of *heterogeneity* with respect to skills and benefit entitlements. Our model allows for no duration dependence in a sense that would allow hazard rates to fall during an unemployment spell for a *given* unemployed worker. In contrast to Ljungqvist and Sargent (2007a), who assume additional probabilistic skill losses while workers are unemployed and also probabilistic transitions between age classes, the state vector of an unemployed worker in our matching models is unchanged over the unemployment spell. That implies a constant hazard rate for a *given* unemployed worker. So the economy-wide hazard rate in figure 14 falls with the duration of spells in the welfare state because the least employable workers, those with low but constant hazard rates, constitute an ever larger share of the remaining unemployed at longer spells. These least employable workers are the low-skilled unemployed entitled to high benefits. They have a low but positive hazard rate in the model with one matching function, and a zero hazard rate in the model with three matching functions.

8 Alternative views of unemployed European workers

Although they have broadly similar macroeconomic outcomes, our two classes of models tell different stories about the motivations and experiences of individual European workers. Lucas (1987, p. 56) praised the McCall because ". . . the model's explicitness invites hard questioning." "Questioning a McCall worker is like having a conversation with an out-of-work friend: 'Maybe you are setting your sights too high', or 'Why did you quit your old job before you had a new one lined up?' This is real social science: an attempt to model, to *understand*, human behavior by visualizing the situation people find themselves in, the options they face and the pros and cons as they themselves see them." A useful way to summarize differences among the models is to ask how well they remind us, in Lucas's words, of 'talking to an unemployed [European] friend'.[19] Although an adverse interaction between high UI and high turbulence transcends the models, the unemployed workers in these models have different alibis for why they aren't working.

8.1 Unemployed Europeans as discouraged workers

Because it emphasizes the factors that influence an unemployed worker's choice of search intensity, talking with an unemployed worker in the search-island model most closely resembles the conversation that Lucas carried on with an unemployed worker in the McCall model. An individual's search intensity is the only factor that determines his duration of unemployment. A worker's search intensity depends on his asset level, his skill level, his benefit entitlement, and, as a determinant of the skill accumulation technology, the level of a parameter π^d that governs skill obsolescence at the time of an exogenous job termination and that we use to measure turbulence. For a given level of turbulence, workers with low skills, high accumulated financial assets, and high benefit entitlements choose the lowest search intensities. The weak incentives to search provided by their high UI entitlements and their current low skills are what discourages these 'discouraged workers'.[20]

The search-island model has no externalities in the labor market. A worker selling his labor services or a firm buying those services does not inflict injuries on others in the labor market beyond what a seller or a buyer of a good ordinarily imposes on competitors. Furthermore, since the wage rate is determined competitively in these two models, the holdup problem that is present in the bargaining setting of the matching model is absent in the search-island model. Thus, unemployment in the search-island model directly reflects workers' decisions. After allowing for the time lag imposed by the time-to-create-jobs assumption, firms will create a number of new jobs that is adequate for all workers who are willing to work at the competitive wage and who have 'found the labor market' in the search-island model.

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[19]We have added 'European' to Lucas's phrase.

[20]We suspect that properly accounting for the discouraged workers who have taken advantage of the early retirement and disability programs available in Europe at government expense would turn up even more discouraged workers than are recorded in the unemployment statistics. See footnote 2.

8.2 Unemployed Europeans as victims of congestion and their own bargaining power

In the matching models, unemployed workers join others who wait in the same matching function, so their employment prospects and their anticipated durations of unemployment depend on the characteristics of those other workers. The mixture of skills and UI benefit entitlements of the workers in a pool determine how profitable it is for firms to post vacancies there. In a matching function that includes low-skilled workers with different benefits, it is more costly for firms to match with workers with low skills who are entitled to high UI than to match with workers with low skills who are entitled to low UI. It requires a higher productivity draw to form successful matches with the former workers as compared to the latter workers and for a given productivity draw, bargaining with the former workers leads to higher wage payments as compared to bargaining with the latter workers. To induce firms to post vacancies in a pool with many low skill, high UI entitlement workers, the invisible hand must set market tightness $\theta = v/u$, the ratio of the pools of job vacancies v and unemployed workers u, to be low enough that firms can expect to match with workers frequently enough. But this means that workers can expect to match infrequently with firms, so that their expected durations of unemployment are high. Thus, if we were to ask an unemployed worker waiting in one of the matching models why he had been unemployed for so long, he could blame the other workers in his matching function whose bad characteristics are responsible for making market tightness low.

If pressed further about the reasons for his long unemployment spell, a low-skilled unemployed worker with high benefits might concede that his tough bargaining posture during 'job interviews' might contribute to his current predicament. His high benefits relative to his current skills give him a high threat point during the Nash bargaining process with any future prospective employer. If we were to ask the worker how he expects to get away with such outrageous wage demands when there are so many other unemployed workers also waiting in the same matching function, he could point to the fact that a firm will have him as the sole job candidate. If the firm would like to meet other candidates, it would have to expend additional resources on posting a vacancy and incur a delay of at least one more period before meeting someone else. Besides, any future encounters would also be on a bilateral basis. A firm could never expect to meet more than one job seeker at the negotiation table.

If the economy consigns low skill, high UI entitlement workers to their own matching function, it is true that unemployment rates for workers *not* in that 'skill-losers market' will be lower. But when UI benefits are high and turbulence is high, the existence of such an isolated market for losers of skills becomes a misfortune for the steady state of the economy because high turbulence means that it is the destiny of many workers to pass through the skill-losers market in which firms optimally post very few vacancies. Hence, the unemployment rate in such an economy virtually explodes when turbulence increases because losers of skills become victims of long-term unemployment in their overly congested matching function. Having a single matching function in the economy dilutes the adverse consequences of high turbulence for unemployment, but even then, by creating more skill

losers in the pool of prospective workers, high turbulence in conjunction with high UI causes equilibrium unemployment to grow in order to induce firms to post vacancies.

9 Concluding remarks

9.1 The lynchpin of matching models

Although Petrongolo and Pissarides (2001) call matching functions a "black box," they commend them as a tractable device for building labor market frictions into equilibrium macroeconomic models of wages, employment, and unemployment. But as we have seen, seemingly natural modifications of the black box, such as assigning groups of similar workers to their own matching functions, can have dramatic effects on equilibrium outcomes, effects that emerge from how the invisible hand adjusts its key signal, the vacancy-unemployment ratio, enough to compensate firms for the cost of posting a vacancy. Our experiments with multiple matching functions demonstrate how the cost of posting vacancies is the lynchpin, or to use a less kind metaphor, the tail that wags the dog, of matching models.

That the cost of posting vacancies is the lynchpin of matching models is also a main message of recent contributions of Shimer and Hall. Shimer (2005) argues that a standard calibration of a matching model with a fixed Nash bargaining weight ψ cannot explain the high volatility in the vacancy-unemployment ratio (market tightness θ) relative to the low volatility in wages that we observe. But Shimer (2005, p. 42) also shows how the two volatilities can be rationalized under an alternative assumption that specifies that workers' bargaining power ψ changes stochastically (and persistently) over time. Shimer's calculations highlight how the economics of recovering a firm's cost of posting a vacancy is a key part of matching models. Firms must recover their sunk hiring costs by earning returns over a worker's job tenure. Because a worker's cumulative output over an average job tenure is so much greater than the average recruitment cost, with free entry the equilibrium outcome awards firms only a small fraction of each period's output. This allows even modest wage fluctuations induced by a stochastic bargaining weight to have large impacts on firms' small claims to output. Therefore, the 'invisible hand' must make large adjustments to market tightness in order to sustain profitability in firms' recruitment efforts.[21]

While we retain the standard Nash bargaining assumption, firms' recovery of recruitment costs remains the central force in our models with displaced workers who have suffered skill loss and are entitled to generous benefits. Firms fear these workers because their generous

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[21]Hall (2005) seizes on the multitude of wage rules that can be postulated for dividing the match surplus and shows how tampering with assumptions about wage stickiness vastly alters the sensitivity of the matching model to technology shocks. Perhaps to restore discipline to modeling process, Hall (2006) returns to the standard Nash bargaining assumption but adds the assumption that workers' bargaining power ψ varies inversely with market tightness. It might seem odd that workers' bargaining power increases in times of difficulties to find jobs. But Hall and Milgrom (2007) explain this outcome by using a bargaining theory that imparts a limited role to outside options with the rationale that realistic threats are to extend, not to terminate, bargaining.

benefits mean that they have to draw high match-specific productivities to employ them, and, even then, only at higher wages than for workers with the same skills but lower benefits. In contrast to the empirical literature on matching and heterogeneity that focuses on how job finding rates depend on individual characteristics, as summarized by Petrongolo and Pissarides (2001), we highlight the theoretical consequences of the externalities that heterogeneous workers impose on each other and how those externalities depend on workers' groupings into matching functions. Given the evidence about important heterogeneities among the unemployed in Europe, as for example reflected in groups of long-term unemployed, we suggest that worker heterogeneity and the appropriate number and grouping of matching functions deserve more attention.[22]

9.2   No externalities in search-island models

Our search-island and matching models have similar labor market outcomes, but the mechanisms producing them differ. As an illustration, consider figure 3 that shows how the unemployment rate is positively related to the UI replacement rate in the two types of models. The curve for the search-island model is nearly flat for a range of replacement rates below 40–50%, while the corresponding curve for the matching model displays a more gradual increase. In the matching model, higher benefits affect the unemployment rate by increasing workers' threat points for wage bargaining. The invisible hand restores equilibrium after an increase in the replacement rate by lowering market tightness, thereby causing the probability that a worker encounters a vacancy to fall in order to compensate firms for the lower expected returns from posting vacancies that would occur if market tightness were not lowered. Hence, high benefits make all unemployed workers suffer from increased congestion in the labor market. This mechanism by which higher benefits lead to higher unemployment is clearly continuous in the level of benefits. In contrast, the search-island model features a different mechanism through which higher benefits raise equilibrium unemployment through the response of individual workers' search behavior. Higher benefits make it less costly to remain unemployed. In response, unemployed workers find it optimal to reduce their search intensities and so lessen the disutility of searching for a new job. But as seen in figure 3, this effect becomes quantitatively important only at relatively high replacement rates. The reason is that unemployed workers must fend for themselves. Low replacement rates are of little comfort to unemployed workers who must finance their consumption with these low benefits and their savings, so their search intensities are not much affected and unemployed workers concentrate instead on restoring their relatively higher labor market earnings. But at higher replacement rates, this mechanism for generating unemployment in the search-

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[22]See footnote 11. An early illustration of the tensions inherit in matching models with heterogeneity is Pissarides' (1992) analysis of skill accumulation in a two-period overlapping generations model with a single matching function. His focus is on how the externality in the matching process gives rise to a propagation mechanism in which a temporary shock to employment can persist for a long time. Hornstein et al. (2003) use a matching model with one matching function, workers with one skill level, but physical capital of different vintages, as a tool for studying how different rates of embodied technical change impinge on equilibrium outcomes, including wage distributions.

island model becomes very potent and workers choose to furlough themselves into drawn out unemployment spells by setting low search intensities.

9.3   Responsibility for being unemployed

In a matching model, an unemployed worker can legitimately shift much of the blame for his situation to the adverse congestion effects induced by the behavior of the other workers and the firms with whom he waits in a matching function. In a search model, the responsibility for being unemployed falls squarely on an unemployed individual. Luck plays a role in the search process, but an unemployed worker cannot blame congestion externalities for his predicament, so he can't blame the behavior of firms or other unemployed workers. In our search-island model, an unemployed worker masters his own destiny by choosing his or her search intensity[23] and in equilibrium the economy creates as many jobs as workers are willing to find. Blanchard (2006, p. 24) provides an empirical observation that is consistent with this benign view of job creation: "even in economies with high unemployment, exogenous movements in the labor force – due to demography or repatriation, such as the return of European nationals after the independence of former colonies – translate fairly quickly into movements in employment."

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[23]In the search model of Ljungqvist and Sargent (2007a), an unemployed worker also chooses a reservation wage.

Appendices

Two appendices describe the models in enough detail to allow a reader to prepare computer programs to compute equilibria.

A   Search-island model

A.1   Firm's problem

The Bellman equations of an existing firm are

$$V^f(h,z) = \max\left\{\tilde{V}^f(h,z), -\Omega\right\}, \tag{10}$$

$$\tilde{V}^f(h,z) = \max_k\left\{zk^\alpha(1+h)^{1-\alpha} - w(1+h) - (i+\delta)k\right\}$$
$$+ \frac{1-\rho}{1+i}\left[-\pi^o\Omega + (1-\pi^o)\sum_{h',z'}p^n(h,h')V^f(h',z')Q(z,z')\right]. \tag{11}$$

The first-order condition for capital in problem (11) is

$$z\alpha k^{\alpha-1}(1+h)^{1-\alpha} = (i+\delta), \tag{12}$$

which can be solved for k to obtain the firm's policy function for choosing capital,

$$k(h,z) = \left[\frac{z\,\alpha}{i+\delta}\right]^{\frac{1}{1-\alpha}}(1+h). \tag{13}$$

Associated with the solution to an existing firm's optimization problem is a reservation productivity $\bar{z}(h)$ that satisfies

$$\tilde{V}^f(h,\bar{z}(h)) = -\Omega. \tag{14}$$

Define the following indicator function

$$\Lambda(h,z) = \begin{cases} 1, & \text{if } z \geq \bar{z}(h); \\ 0, & \text{otherwise.} \end{cases} \tag{15}$$

The break-even condition for starting a new firm is

$$\mu = \frac{1}{1+i}\sum_z \max\left\{(1-\phi)\tilde{V}^f(0,z) + \phi\tilde{V}^f(H,z), 0\right\}Q^o(z), \tag{16}$$

where μ is the start-up cost and ϕ is the fraction of high-skilled workers among all new hires. The maximization in (16) implies a reservation productivity $\bar{z}^o$ that determines whether a new firm hires a worker after it observes its productivity level. The reservation productivity satisfies

$$(1-\phi)\tilde{V}^f(0,\bar{z}^o) + \phi\tilde{V}^f(H,\bar{z}^o) = 0. \tag{17}$$

Define the following indicator function

$$\Lambda^o(z) = \begin{cases} 1, & \text{if } z \geq \bar{z}^o; \\ 0, & \text{otherwise.} \end{cases} \tag{18}$$

The productivity distribution of new firms that hire workers is

$$\Gamma(z) = \frac{\Lambda^o(z)\, Q^o(z)}{\sum_{z'} \Lambda^o(z')\, Q^o(z')}. \tag{19}$$

A.2   Household's problem

We define three value functions $V^n(a, h, z)$, $V^u(a, h, b)$, and $V^r(a)$ for an employed worker, an unemployed worker, and a retired worker, respectively. The state variables are last period's assets (a), skill index (h), the firm's current productivity level if employed (z), and the worker's benefit entitlement if unemployed (b). The benefit entitlement is determined by the worker's last earnings, which we index by $b \in \{0, H\}$, his skill index when he last worked. Both newborn unemployed workers and laid off unskilled workers have a benefit entitlement indicated by index $b = 0$.

The Bellman equation of an employed worker is

$$
\begin{aligned}
V^n(a, h, z) = \max_{c, a'} \Bigg[& \log c + \beta \rho V^r(a') + \beta(1 - \rho)\bigg(\pi^o \sum_{h'} p^o(h, h')\, V^u(a', h', h) \\
& + (1 - \pi^o) \sum_{h', z'} p^n(h, h') \Big\{ V^n(a', h', z')\, \Lambda(h', z') \\
& \qquad\qquad + V^u(a', h', h)\left[1 - \Lambda(h', z')\right] \Big\} Q(z, z') \bigg) \Bigg]
\end{aligned}
\tag{20}
$$

subject to

$$
\begin{aligned}
c + a' &\leq (1 + i)\, a + (1 - \tau)(1 + h)\, w\,, \\
c,\, a' &\geq 0\,.
\end{aligned}
$$

Policy functions $\bar{c}^n(a, h, z)$ and $\bar{a}^n(a, h, z)$ give the employed worker's optimal levels of consumption and savings, respectively.

The Bellman equation of an unemployed worker is

$$
\begin{aligned}
V^u(a, h, b) = \max_{c, a', s} \Bigg[& \log c + A \frac{(1 - s)^\gamma - 1}{\gamma} + \beta \rho V^r(a') + \beta(1 - \rho) \\
& \cdot \bigg(\left(1 - s^\xi\right) V^u(a', h, b) + s^\xi \sum_{z'} V^n(a', h, z')\, \Gamma(z') \bigg) \Bigg]
\end{aligned}
\tag{21}
$$

subject to

$$
\begin{aligned}
c + a' &\leq (1 + i)\, a + (1 - \tau)\eta\,(1 + b)\, w\,, \\
c,\, a' &\geq 0\,, \quad s \in [0, 1)\,.
\end{aligned}
$$

Policy functions $\bar{c}^u(a, h, b)$, $\bar{a}^u(a, h, b)$, and $\bar{s}(a, h, b)$ give the unemployed worker's optimal levels of consumption, savings, and search effort, respectively.

The Bellman equation of a retired worker is

$$V^r(a) = \max_{c, a'}\left[\log c + \beta(1 - \sigma)V^r(a')\right] \tag{22}$$

subject to

$$c + a' \leq (1 + i)a,$$
$$c, a' \geq 0.$$

Policy functions, $\bar{c}^r(a)$ and $\bar{a}^r(a)$, give optimal consumption and savings, respectively.

A.3 Steady state

In a steady state, a time-invariant measure $N(h, z)$ describes the number of firms operating with workers of skill index $h \in \{0, H\}$ and productivity level z. This measure must be consistent with the stochastic process for idiosyncratic shocks and the employment decisions of firms. If v is the number of newly created firms, then $N(\cdot, \cdot)$ must satisfy

$$N(0, z') = vQ^o(z')\Lambda^o(z')(1 - \phi) + (1 - \rho)(1 - \pi^o)\Lambda(0, z')$$
$$\cdot \sum_{h, z} p^n(h, 0)N(h, z)Q(z, z'), \tag{23}$$

$$N(H, z') = vQ^o(z')\Lambda^o(z')\phi + (1 - \rho)(1 - \pi^o)\Lambda(H, z')$$
$$\cdot \sum_{h, z} p^n(h, H)N(h, z)Q(z, z'). \tag{24}$$

Time-invariant measures $y^n(a, h, z)$, $y^u(a, h, b)$, and $y^r(a)$, respectively, describe the numbers of employed, unemployed, and retired households with various individual characteristics. These measures are implied by the optimal decision rules by firms and households:

$$y^n(a', h', z') = (1 - \rho)\left[(1 - \pi^o)\Lambda(h', z') \sum_{a, h, z: \bar{a}^n(a, h, z) = a'} p^n(h, h')\, y^n(a, h, z)\, Q(z, z')\right.$$
$$\left. + \Gamma(z') \sum_{a, b: \bar{a}^u(a, h', b) = a'} \bar{s}(a, h', b)^\xi\, y^u(a, h', b)\right]; \tag{25}$$

$$y^u(a', h, b) = (1 - \rho)\left\{\pi^o \sum_{a, z: \bar{a}^n(a, b, z) = a'} p^o(b, h)\, y^n(a, b, z)\right.$$
$$+ (1 - \pi^o) \sum_{a, z, z': \bar{a}^n(a, b, z) = a'} p^n(b, h)\, y^n(a, b, z)\left[1 - \Lambda(h, z')\right]Q(z, z')$$

$$+ \sum_{a:\bar{a}^u(a,h,b)=a'} y^u(a,h,b) \left[1 - \bar{s}(a,h,b)^\xi \right] \Bigg\}$$

$$+ I(h,b)\,\sigma \sum_{a:\bar{a}^r(a)=a'} y^r(a)\,, \tag{26}$$

$$y^r(a') = (1-\sigma) \sum_{a:\bar{a}^r(a)=a'} y^r(a)$$

$$+ \rho \left[\sum_{a,h,z:\bar{a}^n(a,h,z)=a'} y^n(a,h,z) + \sum_{a,h,b:\bar{a}^u(a,h,b)=a'} y^u(a,h,b) \right], \tag{27}$$

where $I(h,b)$ is an indicator function that equals one if $h = b = 0$ and zero otherwise.

Following Alvarez and Veracierto (2001),we consider steady-state equilibria without public debt. The government balances its budget every period, implying

$$0 = \tau w \sum_{h,z} (1+h) N(h,z) + \Omega D - (1-\tau)\eta\, w \sum_{a,h,b} (1+b) y^u(a,h,b)\,, \tag{28}$$

where the amount of job destruction D is

$$D = (1-\rho) \left\{ \pi^o \sum_{h,z} N(h,z) + (1-\pi^o) \sum_{h,h',z,z'} p^n(h,h') \left[1 - \Lambda(h',z') \right] N(h,z)\, Q(z,z') \right\}. \tag{29}$$

The market-clearing condition in the goods market is

$$\bar{c} + \delta\bar{k} + \mu v = \sum_{h,z} N(h,z)\, z\, k(h,z)^\alpha (1+h)^{1-\alpha}, \tag{30}$$

where aggregate consumption and the aggregate capital stock, respectively, are

$$\bar{c} = \sum_{a,h,z} \bar{c}^n(a,h,z)\, y^n(a,h,z) + \sum_{a,h,b} \bar{c}^u(a,h,b)\, y^u(a,h,b) + \sum_a \bar{c}^r(a)\, y^r(a)\,, \tag{31}$$

$$\bar{k} = \sum_{h,z} N(h,z)\, k(h,z). \tag{32}$$

There are two equilibrium conditions in the labor market. First, the measure of new firms that hire workers, $v \sum_z \Lambda^o(z) Q^o(z)$, must equal the measure of unemployed workers who accept employment. Second, the skill ratio ϕ among new hires that the firm takes as exogenous must equal the equilibrium skill ratio among new hires. We can use the time-

invariant population measures to express these equilibrium conditions as:

$$v = \frac{(1 - \rho) \sum\limits_{a,h,b} \bar{s}(a,h,b)^\xi \, y^u(a,h,b)}{\sum\limits_z \Lambda^o(z) \, Q^o(z)} \,, \tag{33}$$

$$\phi = \frac{\sum\limits_{a,b} \bar{s}(a,H,b)^\xi \, y^u(a,H,b)}{\sum\limits_{a,h,b} \bar{s}(a,h,b)^\xi \, y^u(a,h,b)} \,. \tag{34}$$

Households' aggregate demand for assets

$$\bar{a} = \sum_{a,h,z} a \, y^n(a,h,z) + \sum_{a,h,b} a \, y^u(a,h,b) + \sum_a a \, y^r(a) \,, \tag{35}$$

should equal the supply of assets, which consists of the aggregate capital stock $\bar{k}$ and the value of claims to the economy's firms:

$$\bar{a} = \bar{k} + \frac{\sum_{h,z} \left[z \, k(h,z)^\alpha (1+h)^{1-\alpha} - w(1+h) - (i+\delta)k(h,z) \right] N(h,z) - \mu v - \Omega D}{i} \,. \tag{36}$$

B Matching models

B.1 Single matching function

When there is a single matching function, the probability that a firm meets a worker with skill h and benefit entitlement b is

$$\lambda^f(h,b) = \frac{M(v,u)}{v} \frac{u(h,b)}{u} = m(\theta) \frac{u(h,b)}{v} \tag{37}$$

and the probability that a worker with skill h and benefit entitlement b is matched with a vacancy is

$$\lambda^w(h,b) = \frac{M(v,u)}{u} = m(\theta), \tag{38}$$

which is independent of (h,b). When we introduce multiple matching functions in subsection B.5, the probability that a worker with skill h and benefit entitlement b is matched with a vacancy will depend on (h,b).

B.2 Match surplus

When an unemployed worker with skill h and benefit entitlement b meets a firm with a vacancy, the firm-worker pair draws productivity z from a distribution $Q_h^o(z)$. The firm and the worker will stay together and produce if the match surplus $S^o(h, z, b)$ is positive:

$$S^o(h, z, b) = \max\Big\{ (1 - \tau)z - [1 - \beta(1 - \rho)] W(h, b)$$
$$+ \beta(1 - \rho)\Big[-\pi^o \Omega + (1 - \pi^o) \sum_{h', z'} p^n(h, h') Q_{h'}(z, z') S(h', z') \Big], 0 \Big\}, \quad (39)$$

where $W(h, b)$ is the worker's outside value, and $S(h, z)$ is the surplus associated with a continuing match. A worker with skill h and benefit entitlement b has an outside option with value

$$W(h, b) = \bar{b}(b) + \beta(1 - \rho)\Big[W(h, b) + \lambda^w(h, b) \sum_z \psi S^o(h, z, b) Q_h^o(z) \Big]. \quad (40)$$

Free entry makes the firm's outside value zero. The firm and worker split the match surplus $S^o(h, z, b)$ through Nash bargaining, with outside values as threat points. Let $\psi \in (0, 1)$ denote the worker's share of the match surplus. Because both parties want a positive match surplus, there is mutual agreement on whether to form a match. The reservation productivity $\bar{z}^o(h, b)$ satisfies

$$S^o(h, \bar{z}^o(h, b), b) = 0. \quad (41)$$

The surplus of a continuing match is

$$S(h, z) = \max\Big\{ (1 - \tau)z - [1 - \beta(1 - \rho)] W(h, h)$$
$$+ \beta(1 - \rho)\Big[-\pi^o \Omega + (1 - \pi^o) \sum_{h', z'} p^n(h, h') Q_{h'}(z, z') S(h', z') \Big], -\Omega \Big\}. \quad (42)$$

The government's policy of imposing a layoff tax Ω on matches that break makes (42) differ from expression (39).[24] A reservation productivity $\bar{z}(h)$ satisfying

$$S(h, \bar{z}(h)) = -\Omega \quad (43)$$

characterizes whether a match is dissolved.

B.3 Zero-profit condition

In equilibrium, firms expect to break even when posting a vacancy:

$$\mu = \beta(1 - \psi) \sum_{h, z, b} \lambda^f(h, b) S^o(h, z, b) Q_h^o(z). \quad (44)$$

This condition will pin down the equilibrium value of market tightness θ.

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[24] Another difference between expressions (39) and (42) is that an employed worker's benefit entitlement is encoded in his skill level h, so there is one less state variable in surplus expression (42).

B.4 Wage determination

Alternative wage structures support the same equilibrium allocation. We follow Mortensen and Pissarides (1999) and assume a two-tier wage system.[25] In particular, when a firm with a vacancy meets an unemployed worker with skill h and benefit entitlement b, they bargain. The worker's outside value is $W(h, b)$ and the firm's outside value is zero. Because they do not incur the layoff tax if they do not reach an agreement, the layoff tax does not directly affect the bargaining between a newly matched worker and firm. But if they succeed in forming a match, the firm must pay the layoff tax after any future breakup. We capture this by setting the firm's threat point equal to $-\Omega$ in future Nash bargaining.

These assumptions give rise to a two-tier wage system. There is one wage function $w^o(h, z, b)$ for the initial round of negotiations between a newly matched firm and worker, and another wage function $w(h, z)$ associated with renegotiations in an ongoing match. These wage functions satisfy

$$
\begin{aligned}
w^o(h, z, b) \;=\; & W(h, b) + \psi S^o(h, z, b) - \beta(1-\rho)\Bigg\{ \pi^o \sum_{h'} p^o(h, h') W(h', h) \\
& + (1-\pi^o) \sum_{h', z'} p^n(h, h') Q_{h'}(z, z') \Big(\psi\big[S(h', z') + \Omega \big] + W(h', h') \Big) \Bigg\}, \quad (45)
\end{aligned}
$$

$$
\begin{aligned}
w(h, z) \;=\; & W(h, h) + \psi\big[S(h, z) + \Omega \big] - \beta(1-\rho)\Bigg\{ \pi^o \sum_{h'} p^o(h, h') W(h', h) \\
& + (1-\pi^o) \sum_{h', z'} p^n(h, h') Q_{h'}(z, z') \Big(\psi\big[S(h', z') + \Omega \big] + W(h', h') \Big) \Bigg\}. \quad (46)
\end{aligned}
$$

B.5 Multiple matching functions

We entertain some alternative specifications that proliferate matching functions in the spirit of Mortensen and Pissarides (1999), who postulated that workers with different skill levels get matched with vacancies in separate but identical matching functions with market-specific inputs of unemployment and vacancies. We must modify their specification because they assumed that workers are permanently endowed with a particular skill level, and we don't. We consider three alternative specifications:

1. Separate matching functions for unemployed workers with different skill levels, yielding different equilibrium vacancies $v(h)$ for $h \in \{0, H\}$.

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[25]The risk neutral firm and worker would be indifferent between adhering to this two-tier wage system or one in which workers receive a fraction ψ of the match surplus $S(h, z)$ in every period (which would have the worker paying a share ψ of any future layoff tax). As emphasized by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Optimal reservation productivities remain the same. Under the two-tier wage system, a newly hired worker in effect posts a bond that equals his share of the future layoff tax.

2. Separate matching functions for unemployed workers having different benefit entitlements, yielding different equilibrium vacancies $v(b)$ for each $b \in \{0, H\}$.

3. Separate matching functions for unemployed workers indexed by both their current skill h and their skill b in their last employment, yielding equilibrium vacancies $v(h, b)$ for each pair of values $(h, b) \in \{0, H\} \times \{0, H\}$.

With a slight abuse of notation, for all three specifications, we use the function $v(j)$ to denote vacancies in market j, where $j = h$ in the first specification, $j = b$ in the second one, and $j = (h, b)$ in the third one. Similarly, for all three specifications, we let $\theta(j)$ denote market tightness in market j which is equal to

$$\theta(j) = \frac{v(j)}{\sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j)} u(\tilde{h}, \tilde{b})}, \tag{47}$$

where the summation set $\Upsilon(j)$ depends on the specification: $\Upsilon(h) = \{(h, 0), (h, H)\}$ in the first one, $\Upsilon(b) = \{(0, b), (H, b)\}$ in the second one, and $\Upsilon(h, b) = \{(h, b)\}$ in the third one. The corresponding probabilities that an unemployed worker finds a vacancy and that a firm with a vacancy finds a worker, respectively, equal

$$\lambda^w(h, b)\bigg|_{(h, b) \in \Upsilon(j)} = \frac{M\left(v(j), \sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j)} u(\tilde{h}, \tilde{b})\right)}{\sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j)} u(\tilde{h}, \tilde{b})} = m(\theta(j)), \tag{48}$$

$$\lambda^f(h, b)\bigg|_{(h, b) \in \Upsilon(j)} = \frac{M\left(v(j), \sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j)} u(\tilde{h}, \tilde{b})\right)}{v(j)} \frac{u(h, b)}{\sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j)} u(\tilde{h}, \tilde{b})} = m(\theta(j)) \frac{u(h, b)}{v(j)}. \tag{49}$$

The zero-profit condition for posting a vacancy in market j is

$$\mu = \beta(1 - \psi) \sum\limits_{(\tilde{h}, \tilde{b}) \in \Upsilon(j); z} \lambda^f(\tilde{h}, \tilde{b}) S^o(\tilde{h}, z, \tilde{b}) Q_{\tilde{h}}^o(z), \tag{50}$$

where μ is the cost of posting a vacancy, and match surpluses are determined as described in section B.2. Wage determination is also the same as in section B.4.

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