Lotteries for consumers versus lotteries for firms

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ABSTRACT

Prescott emphasizes similarities between lotteries that smooth nonconvexities for firms and for consumer-workers. We emphasize their differences. We also argue that models with employment lotteries that are used to generate unemployed individuals in a frictionless framework can have very different implications than models embodying frictional unemployment. As an illustration, models with employment lotteries predict effects from job destruction taxes that are opposite to those in search models.

1. Introduction

James Tobin said that good macroeconomic analysis ignores distribution effects. But in general equilibrium theory, distribution effects usually can't be ignored. Edward Prescott's paper is an elegant summary of a very successful research agenda that manages to apply general equilibrium theory to macroeconomics by carefully setting up redistribution arrangements that serve to smooth the nonconvexities that are confronted by both firms and households and that thereby deliver both a stand-in household and a stand-in firm. Prescott's work continues the Tobin tradition not by ignoring distribution effects but by designing them to facilitate aggregate analysis.

There is much to admire and to copy in Prescott's work in general and in this paper in particular. This is a perfect paper to assign to graduate students. A beautiful aspect of the paper is that because it adheres to the rules for describing competitive equilibria, everything is in the open. We take advantage of this openness to highlight and challenge an important aspect of Prescott's analysis. Prescott focuses on how non-convexities at the level of individual households and production units affect outcomes in quantitative general equilibrium models of business cycles. His message is that

> One notable success of theory was the recognition that an aggregation result underlies the stand-in household in the aggregate theory. This result is analogous to the aggregation result that justifies the concave, constant-returns-to-scale, aggregate production function.

Prescott (2002, p. 4) has pointed out that while the aggregation theory behind the aggregate production function is well-known, there is also "some not-so-well-known aggregation theory behind the stand-in household utility function." Prescott emphasizes the formal similarities associated with smoothing out nonconvexities by aggregating over firms, on the one hand, and aggregating over consumers, on the other. We shall argue that the different economic interpretations that attach to these two types of aggregation make the two aggregation theories very different. Perhaps this difference explains why this aggregation method has been applied more to firms than to consumers.[1]

An important distinction between firms and households in general equilibrium theory is that firms have no independent preferences. They serve only as vehicles for generating rental payments for employed factors and profits for their owners. When a firm becomes inactive, that can be bad news for its stakeholders, but the 'firm' itself does not care whether it continues or ceases to exist. In contrast, individual consumers do have preferences and care about alternative states of the world. Although the aggregation theory that Prescott likes can be applied both to firms and to consumers to smooth out lumpy behavior at the micro level, the aggregation theory behind the stand-in household has an additional aspect that is not present in the theory that aggregates over firms, namely, it says how consumption and leisure are smoothed across people.

On the household side, Prescott emphasizes the non-convexity that arises when it is imposed that an individual is allowed only one workweek length. A stand-in household emerges when all individuals participate in an employment lottery that is supplemented with the exchange of state-contingent claims over lottery outcomes, as proposed by Hansen (1985) and Rogerson (1988). Aggregating the work-week length non-convexity with lotteries divides *ex ante* identical people into employed and non-employed individuals and creates a setting in which, despite the absence of search and information frictions, real shocks can give rise to fluctuations in the number of employed individuals. This creates the possibility of emulating fluctuations in employment over the business cycle and is the basis for the notable success that Prescott praises.

This comment points out that despite these possibly appealing aggregate implications, Prescott's aggregation strategy also has unattractive implications. We shall use a particular policy experiment to highlight the consequences of following Prescott in modelling employment variations as being driven by a high intertemporal elasticity of labor supply that emerges because the economy is effectively pooling all labor income and designing enforceable gambles over who gets to work. In particular, it matters very much that the framework embodies no frictional unemployment in the sense of Friedman and Stigler.

For our laboratory, we follow the lead of Prescott's footnote 3, which refers to interesting quantitative general equilibrium analyses of labor market policies. In particular, we will contrast the ways that layoff

[1] Sherwin Rosen often used a lottery model for the household. Instead of analyzing why a particular individual chooses higher education, Rosen modelled a family with a continuum of members that allocates fractions of its members to distinct educational choices that involve different numbers of years of schooling. See Ryoo and Rosen (2003).

taxes affect employment in a no-frictional-unemployment lotteries model and in a frictional-unemployment island search model. The effects are very different. In the equilibrium of the lotteries model, unemployment *rises* in response to the introduction of a layoff tax because the private economy perceives higher layoff costs as equivalent to a less productive technology, prompting the stand-in household to substitute away from consumption towards leisure. The market outcome sets the employment lottery to give a lower probability of working. In the island-search model, introducing a layoff tax *reduces* unemployment through its effects on frictional unemployment, an avenue that is not present in the lotteries model.

We make the same assumptions that appear in most analyses of layoff taxes in the literature. The productivity of a job evolves according to a Markov process, and a sufficiently poor realization triggers a layoff. The government imposes a layoff tax τ on each layoff. The tax revenues are handed back as equal lump-sum transfers to all agents, denoted by T per capita. Here we assume the simplest possible Markov process for productivities. A new job has productivity p_0. In all future periods, with probability $\xi \in [0, 1)$, the worker keeps the productivity from last period, and with probability $1 - \xi$, the worker draws a new productivity from a distribution $G(p)$.

2. Layoff taxes in an employment lotteries model

This section shows analytically that introducing a layoff tax raises unemployment in an employment lotteries model.[2] A market-clearing wage w equates the demand and supply of labor. A constant returns to scale technology implies that an equilibrium wage is determined by the supply side as follows. At the beginning of a period, let the value to a firm of a worker with productivity p be $V(p)$, which satisfies the Bellman equation

$$V(p) = \max\Big\{p - w + \beta\Big[\xi V(p) + (1-\xi)\int V(p')\, dG(p')\Big],\; -\tau\Big\}. \tag{2.1}$$

Given a value of w, this Bellman equation determines a reservation productivity $\bar{p}$. If there exists an equilibrium with strictly positive employment, the equilibrium wage must be such that the firm breaks even on new hires:

$$V(p_0) = p_0 - w + \beta\Big[\xi V(p_0) + (1-\xi)\int V(p')\, dG(p')\Big] = 0$$
$$\Rightarrow w = p_0 + \beta(1-\xi)\tilde{V}, \tag{2.2}$$

where

$$\tilde{V} \equiv \int V(p')\, dG(p').$$

To compute $\tilde{V}$, we first look at the value of $V(p)$ when $p \geq \bar{p}$,

$$V(p)\Big|_{p\geq\bar{p}} = p - w + \beta\Big[\xi V(p) + (1-\xi)\tilde{V}\Big]$$
$$= \frac{p - w + \beta(1-\xi)\tilde{V}}{1-\beta\xi} = \frac{p - p_0}{1-\beta\xi}, \tag{2.3}$$

where we have successively substituted out for $V(p)$ and the last equality incorporates equation (2.2). We can then use equation (2.3) to find an expression for $\tilde{V}$,

$$\tilde{V} = \int_{-\infty}^{\bar{p}} -\tau\, dG(p) + \int_{\bar{p}}^{\infty} V(p)\, dG(p)$$
$$= -\tau\, G(\bar{p}) + \int_{\bar{p}}^{\infty} \frac{p - p_0}{1-\beta\xi}\, dG(p). \tag{2.4}$$

[2] Our result is the same as in Hopenhayn and Rogerson's (1993) numerical analysis of layoff taxes in a more elaborate employment lotteries framework with firm size dynamics.

From Bellman equation (2.1), the reservation productivity $\bar{p}$ satisfies

$$\bar{p} - w + \beta \left[\xi V(\bar{p}) + (1-\xi)\tilde{V} \right] = -\tau .$$

After imposing equation (2.2) and $V(\bar{p}) = -\tau$, we find

$$\bar{p} = p_0 - (1-\beta\xi)\tau \equiv \bar{p}(\tau) . \tag{2.5}$$

The equations (2.2), (2.4) and (2.5) can be used to solve for the equilibrium wage $w = w(\tau)$.

In a stationary equilibrium, let μ be the mass of new jobs created in every period. The mass of jobs with productivity $p_0 = 1$ that have not yet experienced a new productivity draw can then be expressed as

$$\mu \sum_{i=0}^{\infty} \xi^i = \frac{\mu}{1-\xi} , \tag{2.6}$$

and the mass of jobs that have experienced a new productivity draw and are still operating is given by

$$\begin{aligned} &\sum_{i=0}^{\infty} \xi^i \mu (1-\xi) [1 - G(\bar{p})] \sum_{j=0}^{\infty} \{\xi + (1-\xi)[1-G(\bar{p})]\}^j \\ &\qquad = \frac{\mu}{1-\xi} \frac{1-G(\bar{p})}{G(\bar{p})} . \end{aligned} \tag{2.7}$$

After equating the sum of these two kinds of jobs to N (which we use to denote the total mass of all jobs), we get the following steady-state relationship,

$$\mu = N G(\bar{p}) (1-\xi) . \tag{2.8}$$

By letting the continuum of agents be indexed on the unit interval, the total mass of jobs $N \in [0,1]$ is equal to the fraction of all employed agents, which also equals the probability that an individual agent works. This probability is a utility-maximizing choice of the representative agent. We adopt Prescott's log-linear preference specification, $\sum_{t=0}^{\infty} \beta^t (\log(C_t) - \gamma N_t)$. In a stationary equilibrium with wage w and a gross interest rate $1/\beta$, the representative agent's optimization problem reduces to a static problem of the form,

$$\begin{aligned} &\max_{C,N} \ \log C - \gamma N , \\ &\text{subject to} \quad C \leq Nw + \Pi + T , \quad C \geq 0 , \quad N \in [0,1] , \end{aligned} \tag{2.9}$$

where the profits from firms, Π, and the lump-sum transfer of layoff-tax revenues from the government, T, are taken as given by the agents. The optimal choice of the probability of working is then

$$N = \frac{1}{\gamma} - \frac{T + \Pi}{w} . \tag{2.10}$$

The sum of aggregate profits and lump-sum transfers can be computed by using the masses of jobs in expressions (2.6) and (2.7),

$$\begin{aligned} \Pi + T &= \frac{\mu}{1-\xi}(p_0 - w) + \frac{\mu}{1-\xi}\frac{1-G(\bar{p})}{G(\bar{p})} \int_{\bar{p}}^{\infty} \frac{p-w}{1-G(\bar{p})}\, dG(p) \\ &= N \left[G(\bar{p})(p_0 - w) + \int_{\bar{p}}^{\infty} (p-w)\, dG(p) \right] , \end{aligned} \tag{2.11}$$

where the last inequality invokes relationship (2.8).

We now adopt the special assumption that $G(p)$ is a uniform distribution on the unit interval $[0, 1]$, and the initial productivity of a new job is $p_0 = 1$. Expressions (2.4) and (2.11) can then be evaluated as follows,

$$\tilde{V} \;=\; -\tau\,\bar{p} + \left[\frac{1+\bar{p}}{2} - 1\right]\frac{1-\bar{p}}{1-\beta\xi}\,, \tag{2.12}$$

and

$$\Pi + T \;=\; N\left[\bar{p} + (1-\bar{p})\,\frac{1+\bar{p}}{2} - w\right]. \tag{2.13}$$

From equations (2.2) and (2.12),

$$w \;=\; 1 + \beta\,(1-\xi)\left[-\tau\bar{p} - \frac{(1-\bar{p})^2}{2\,(1-\beta\xi)}\right],$$

and after substituting for $\bar{p}$ from (2.5)

$$w \;=\; 1 - \beta\,(1-\xi)\,\tau\left[1 - \frac{(1-\beta\xi)\,\tau}{2}\right] \equiv w\,(\tau)\;. \tag{2.14}$$

By substituting (2.13) into (2.10) and using expressions (2.5) and (2.14), we arrive at an equilibrium expression for N,

$$N\,(\tau) = \frac{2w\,(\tau)}{\gamma\left[2\bar{p}\,(\tau) + 1 - \bar{p}\,(\tau)^2\right]}$$

with its derivative

$$\frac{d\,N\,(\tau)}{d\,\tau} = \frac{-2\beta\,(1-\xi)\,\bar{p}\,(\tau)\left[2\bar{p}\,(\tau) + 1 - \bar{p}\,(\tau)^2\right] + 4\,(1-\beta\xi)\,[1-\bar{p}\,(\tau)]\,w\,(\tau)}{\gamma\left[2\bar{p}\,(\tau) + 1 - \bar{p}\,(\tau)^2\right]^2}\,.$$

Evaluating the derivative at $\tau = 0$, where $\bar{p}(0) = p_0 = 1$, we have

$$\left.\frac{d\,N\,(\tau)}{d\,\tau}\right|_{\tau=0} = \frac{-\beta\,(1-\xi)}{\gamma} < 0.$$

This states that in general equilibrium, employment falls in response to the introduction of a layoff tax. This happens because agents respond to a higher layoff costs in the same way that they would to a less productive technology. Thus, the stand-in household substitutes away from consumption towards leisure and so chooses a lower probability of working in the lottery over employment.[3]

[3] The substitution effect prevails over the income effect because to a first-order approximation the latter effect is neutralized, since layoff costs are assumed to be a layoff tax where the tax revenues are handed back lump-sum to the agents.

3. Layoff taxes in an island model

The employment effects of introducing a layoff tax in an island framework are opposite to those for the lotteries model. Thus, Lucas and Prescott (1974, p. 205) analyzed such effects in an island model and found that

> "The result is a decrease in unemployment and a decrease in the equilibrium present value of wages. (This example shows that lower average unemployment is not, in general, associated with higher welfare for workers.) It may well be, though one could hardly demonstrate it at this level of abstraction, that differences of this sort in the actual or perceived costs of changing jobs can help to account for the observed differences in average unemployment across occupations and among countries."

Why does the island model yield the opposite outcome from the employment lottery model? Both models have reservation productivity falling and job tenures lengthening in response to an increase in the layoff tax. The difference is that in the island model there is no aggregate mechanism that allows individuals to substitute away from working – individual workers are fending for themselves: those who want to consume must also work. Layoff taxes in an island model reduce unemployment because there are fewer transitions between jobs/islands and therefore less frictional unemployment.

4. Concluding remarks

Rogerson and Hansen's lottery-based model of a stand-in household is elegant and analytically tractable. The lotteries smooth out nonconvexities arising from work-week restrictions and make the stand-in household 'one big happy family' that is very willing to reallocate its labor supply over time. Nevertheless, it gives us pause for thought that the theoretical consequences of an important public policy like employment protection differs so completely between a model with employment lotteries and an island model.[4] The negative employment effects of layoff taxes in an employment lottery model stem directly from the property of that framework that Prescott characterizes as being so important.

> Kydland and Prescott (1982) found that the growth model displays business cycle fluctuations if and only if the aggregate intertemporal elasticity of labor supply is high, a fact that was not then accepted by most labor economists. The labor economists ignored the consequences of aggregation in the face of non-convexities in coming to their incorrect conclusion that the aggregate elasticity of labor supply is small. Non-convexities at the household level imply high intertemporal elasticity of labor supply even if the intertemporal elasticity of labor supply of the households being aggregated is small.

For the sake of argument, let us set aside the question of frictional unemployment and focus on the substitution effect that is the driving force in the employment lottery model. If labor economists were asked about the substitution effects associated with layoff taxes, they would probably direct their attention to the joint employment decision of spouses within a family. For an environment that offers families the limited options of sending one or two persons to the labor market either full or part time, labor economists would estimate a low substitution effect in response to layoff taxes. Prescott would presumably argue that those estimates are mistaken because they fail to recognize that it would be possible for a large group of families to join together to randomize over who should be sent to work and who should stay home, while also trading state-contingent claims that would provide consumption for the people who not work.

This market arrangement and randomization device stand at the center of the employment lottery model. To us, it seems that they make the aggregation theory behind the stand-in household fundamentally different than the well-known aggregation theory for the firm side. Prescott's example of a non-convex production technology in section 6 illustrates this point very well. The plants that do not find any workers stay idle; that is just as well for those idle plants because the plants in operation earn zero rents. In short, whether individual production units operate or exit (or remain idle) is the end of the story in the aggregation theory

[4] For a detailed discussion of the employment implications of layoff taxes in different frameworks including the matching model, see Ljungqvist (2001).

behind the aggregate production function. But in the aggregation theory behind the stand-in household's utility function, it is really just the beginning.

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