A p Theory of Taxes and Debt Management*

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Abstract

Distortions induce a benevolent government that must finance an exogenous expenditure process to smooth taxes. An optimal fiscal plan determines the marginal cost $-p'$ of servicing government debt and makes government debt risk-free. A convenience yield tilts debts forward and taxes backward. The government's option to default determines debt capacity. Debt-GDP ratio dynamics are driven by 1) a primary deficit, 2) interest payments, 3) GDP growth, and 4) hedging costs. We provide quantitative comparative dynamic statements about debt capacity, debt-GDP ratio transition dynamics, and time to exhaust debt capacity.

Keywords: sovereign debt; default; limited commitment; debt capacity; convenience yield

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1 Introduction

What opportunities and constraints determine a government's maximum sustainable debt-to-GDP ratio? At what rate should a government approach it? How costly is it for a government to service its debt and how do today's costs depend on today's debt-GDP ratio? Should a government borrow more when, as until recently has been true for the U.S., interest rates on government debt are lower than prospective GDP growth rates? Under an optimal policy, how much will tax rates eventually have to rise as the debt-GDP ratio gradually approaches its limit?

To answer these questions, we analyze contending forces that shape a country's debt/GDP dynamics in a normative model of taxes and government debt management that combines **A**rrow's one-period-ahead securities and/or the GDP insurance advocated by Shiller (1994), **B**arro's (1979) tax-smoothing motives and impediments, **C**redit constraints induced by default options like those in Eaton and Gersovitz (1981), and extra **D**iscounting due coming from a convenience yield on risk-free government debt like Krishnamurthy and Vissing-Jorgensen (2012). We can solve our **ABCD** model mostly by hand. To explain how it works, we activate its four components sequentially.

We start by describing sources of randomness and an exogenous stochastic discount factor process that determines risk-return tradeoffs and prices a complete set of Arrow securities. Our model **A** incorporates risk and risk premia in a setting with Ricardian equivalence.

Model **AB** incorporates Barro's (1979) tax-distortion deadweight cost, versions of which Calvo (1978) and Chang (1998) adopted, into model **A**.[1] We derive the loss function that Barro imputed to the government from the indirect utility functional of a benevolent planner who maximizes the welfare of a representative household with standard preferences augmented by deadweight losses from taxing.

By allowing the government to default in the tradition of Eaton and Gersovitz (1981), Worrall (1990), Kehoe and Levine (1993), Kocherlakota (1996), Zhang (1997),[2] model **ABC** adds credit restrictions. That our government's primary surplus process exhibits geometric growth leads to an upper bound on the government's debt/GDP ratio that we call its debt capacity. Exercising an option to default sends the government into period-by-period primary budget balance, a consequence that pins down its debt capacity.

[1] In model **AB**, we retain a constant exogenous risk-free interest rate, thereby preventing the government from being able to manipulate bond prices, a principal focus of Lucas and Stokey (1983), Zhu (1992), Chari et al. (1994), and others, and continue to assume that GDP growth and government expenditure growth are geometric as in Barro (1979).

[2] Alvarez and Jermann (2000, 2001) study the quantitative effects of limited enforcement on asset prices.

Finally, our **ABCD** model follows Krishnamurthy and Vissing-Jorgensen (2012) by incorporating extra discounting associated with a "convenience yield" on the government's risk-free debt. Introducing a convenience yield makes the debt-GDP ratio b_t approach its limit from below, in contrast to the time-invariant debt-GDP ratio prescribed by Barro (1979) that is shared by our model **ABC** .

Working in continuous time lets us characterize debt dynamics explicitly. Two conditions determine a maximum sustainable risk-free debt-to-GDP ratio $\bar{b}$: 1.) the default option induces an indifference condition between defaulting and servicing debt, and 2.) a Gordon growth valuation formula at a steady state $\bar{b}$ implies a zero-drift condition for b_t at debt capacity $\bar{b}$. If we withhold the component **C** default option, we retrieve a stochastic version of Barro's 1979 model that shares his commitment-to-repay assumption. This model predicts debt capacities that seem implausibly high, e.g., 10-15 times GDP.

Four forces drive optimal debt-GDP ratio dynamics in our **ABCD** model:

$$\text{change of } b_t = \text{primary deficit} + \text{interest cost} - \text{growth effect} + \text{hedging cost}. \qquad (1)$$

Blanchard (2019) and Mehrotra and Sergeyev (2021) discuss the first three components. A fourth term appears in (1) because the government optimally hedges its GDP process in a way that ends up making b evolve deterministically, and has to pay for doing that. Hedging costs equal b times the risk premium on an asset whose payoff is GDP.

The presence of a convenience yield generates a backloaded tax schedule that prescribes a tax rate that increases over time. Fiscal deficits scaled by GDP decrease over time and eventually turn into surpluses. The debt-GDP ratio b_t approaches a steady state that attains a maximum sustainable level $\bar{b}$. If a government starts with small enough debt, it immediately issues enough debt to reach a lower bound $\underline{b} > 0$ on the debt-GDP ration at which the marginal cost of servicing debt equals one. Such a one-time jump in debt looks like a Blanchard (2019) "debt is cheap" response on steroids. Thus, optimal debt-GDP dynamics reside in three disjoint regions: 1.) a lump-sum debt issuance and payout region in which $b < \underline{b}$, which we refer to as a Blanchard region; 2.) a default region in which debt is unsustainable ($b > \bar{b}$); and 3.) an interior region in which $b \in [\underline{b}, \bar{b}]$. In the $b \in [\underline{b}, \bar{b}]$ interval, Barro tax-smoothing prevails. When $b > \bar{b}$, the government defaults immediately and thereafter balances its budget period by period.

An optimal policy is described by 1) a nonlinear first-order ordinary differential equation (ODE) for the government's (scaled) value $p(b)$; 2) a first-order condition for the optimal tax rate $\tau(b)$; 3) a zero-drift condition and an indifference condition between defaulting and not defaulting that characterize a steady state at which debt is at the maximum sustainable level $\overline{b}$; 4) value-matching and smooth-pasting conditions that characterize the lump-sum debt issuance and payout boundary $\underline{b}$. The upper debt-capacity boundary $\overline{b}$ is an absorbing state and the lower lump-sum debt issuance boundary $\underline{b}$ is a reflecting barrier. These two boundaries embody economic forces on the government's maximum sustainable debt and a rule for an initial lumpy debt-financed payout to the representative household. We characterize the two boundaries in ways that highlight how our continuous-time setting allows us to represent underlying economic forces concisely. We believe that we are the first to derive a zero-drift condition that pins down an endogenous debt capacity.

In calling it a p theory of taxes and government debt, we invoke an analogy with a q theory of investment. A convex tax distortion cost from Barro (1979) serves as a counterpart to the convex capital adjustment cost in a q theory of investment, e.g., Lucas and Prescott (1971), Hayashi (1982), and Abel and Eberly (1994).[3] The government's marginal cost of servicing debt, $-p'(b)$, measures how much the household's value decreases when government debt increases by one unit. In a q theory, marginal q, the marginal value of capital, measures how much the firm's value increases with its investment. In our model, tax distortions and limited commitment make $-p'(b)$ exceed one; $-p'(b)$ appears both in the first-order condition for an optimal tax rate and in an equation that restricts the government's optimal value function.

We describe components of our **ABCD** model in Section 2, then assemble them and compute an optimal fiscal policy in Section 3. In Section 4 we calibrate some key parameters to US debt-GDP ratio for 1980-2020 and use them to offer quantitative illustrations of our **ABCD** model in ways designed to illustrate the balance of forces that shape the government's optimal fiscal plan. We study how transitions path toward debt capacity depend on prevailing interest rates and convenience yields on government debt.

Section 5 extends our **ABCD** model by adding interest rate and spending ratio shocks in the form of jumps at random times. Anticipations of higher rates reduce the government's debt capacity and lower the rate at which the government increases its debt-GDP ratio. Anticipated increases in government spending (scaled by GDP) induce the government to increase the tax rate in order to exploit the convenience yield and smooth tax distortions.

In Section 6, we extend the **ABCD** model in another way by introducing uninsurable jump shocks to output. Unlike outcomes in the **ABCD** model, now the government defaults whenever its debt-GDP ratio b_t jumps above its debt capacity $\overline{b}$. Consequently, the government pays a credit spread. The Section 6 model brings a new region that acts as a "cushion"

[3] Jermann (1998) integrates a q theory of investment into an general equilibrium model to study asset-pricing implications. Dai, Giroud, Jiang, and Wang (2024) study a q theory for the internal capital markets.

between the tax smoothing and default regions by delaying inefficient defaults. In this new region, the drift of the debt-GDP ratio b_t is zero when output doesn't jump; the optimal tax rate is chosen accordingly. Section 7 summarizes our findings, discusses how normative models like ours have sometimes been deployed to organize monetary-fiscal histories, and indicates how our **ABCD** model opens fruitful avenues for future research.

Related Literature. Our model assembles building blocks from Lucas and Stokey (1983) (complete state-contingent debt) and Barro (1979) (tax distortion costs) in a tractable continuous-time framework with an exogenously specified SDF along lines of Black and Scholes (1973), Merton (1973), and Harrison and Kreps (1979). The exogenous stochastic discount factor process in our model distinguishes it from Lucas and Stokey (1983), where a government's tax and borrowing strategy affects the stochastic discount factor process. That motivates their government to manipulate equilibrium debt prices by altering distorting taxes. As in Lucas and Stokey (1983), the presence of complete financial markets allows the government to issue fully state contingent debt.[4] By staying within the Barro tradition of an exogenous SDF process, we remove dynamic inconsistencies that arise from bond-price-manipulation motives central to models in the Lucas-Stokey tradition.[5] We focus on implications of limited commitment for debt capacity and debt dynamics. In Section 5 we describe a setting in which an optimal tax rate is discontinuous even when shocks are hedgeable; in this setting smoothing taxes in the fashion recommended by Barro's 1979 model is just too costly.

Bohn (1995) valued government debt with an SDF like that of Lucas (1978). Bohn (1990) studied how hedging with financial instruments shapes optimal fiscal policy of a risk-neutral government in a stochastic reformulation of Barro (1979).[6] Unlike Bohn (1990), hedging costs play a key role in debt-GDP dynamics in our model. We extend Bohn's insights by incorporating effects of default opportunities on debt dynamics and sustainability. Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) analyze how the covariance between an intertemporal marginal rate of substitution and a primary government surplus ought to affect the value of government debt.

Brunnermeier, Merkel, and Sannikov (2020, 2022) incorporate a bubble within a fiscal

[4]Our complete financial spanning setting eases analysis and exposition. We leave important extensions to incomplete markets settings along the line of Aiyagari, Marcet, Sargent, and Seppälä (2002) for subsequent research.

[5]The Barro (1979) model is deterministic, so his SDF is an exponential function that decays at a risk-free rate per unit of time.

[6]Bohn (1998) described measures that the US took in response to the accumulation of debt during the 1970s and 1980s that are broadly consistent with dynamics prescribed by our model.

theory of the price level, develop a model of safe assets with a negative beta in an incomplete-markets setting, and analyze implications for debt sustainability. Kocherlakota (2021) develops a model of government debt bubbles associated with tail risk in a heterogeneous-agent incomplete-markets Aiyagari-Bewley-Huggett model. Reis (2021) studies debt capacity in a related model with a bubble on government debt. D'Erasmo, Mendoza, and Zhang (2016) review the literature on government debt sustainability. Abel, Mankiw, Summers, and Zeckhauser (1989) and Abel and Panageas (2022) analyze maximum budget-feasible government debt in overlapping generations models with perpetually zero primary budget surpluses. Chernov, Schmid, and Schneider (2020) synthesize insights from the macro and finance literatures in a quantitative framework in which a government can tax output and issue nominal debt to finance its expenditures. They show how credit default swap (CDS) premia reflect risk-adjusted probabilities of government default. Elenev et al. (2021) construct a New Keynesian model that includes financial intermediation, risk premia, production, fiscal policies, and monetary policies.[7]

Our work is related to papers in international macro that use limited enforcement as a source of financial imperfections. Kehoe and Perri (2002) use limited enforceability and consequent endogenous debt capacity in an international real business cycle model that extends Backus et al. (1992). Bai and Zhang (2010) introduce limited spanning into a Kehoe and Perri (2002) framework. Aguiar and Gopinath (2006) and Arellano (2008) provide quantitative analyses of sovereign debt in the tradition of Eaton and Gersovitz (1981). Tourre (2017) develops a continuous-time model to study sovereign debt spreads in which, because debt has a finite term, default can be triggered by continuous, diffusion shocks. In our baseline **ABCD** model with only diffusion shocks, complete (dynamic) financial spanning allow government debt to be risk free. But when unhedgeable downward output jumps are large enough, default occurs in our section 6 jump-diffusion extension of our **ABCD** model. This outcome is related to ones in Bornstein (2020), who studies sovereign default in a model in which output follows a Poisson process in a continuous-time version of Arellano (2008). In all of the papers discussed above, the government solves a representative household's problem. In contrast, we follow the tradition of Lucas and Stokey (1983) by explicitly analyzing a benevolent government's optimal fiscal plan. We jointly characterize debt capacity and debt dynamics that optimally serve a tax smoothing motive.

DeMarzo, He, and Tourre (2023) construct a continuous-time sovereign-debt model that

[7]For other discussions of '$r - g$' and debt sustainability, see Barro (2020), Van Wijnbergen, Olijslagers, and de Vette (2020), Aguiar, Amador, and Arellano (2021), Mian, Straub, and Sufi (2022), Reis (2021), and Liu, Schmid, and Yaron (2021).

generates debt ratcheting. Rebelo, Wang, and Yang (2022) construct a continuous-time sovereign-debt model in which a country's degree of financial development, defined as how easily it can issue debt denominated in domestic currency in international capital markets, generates "debt intolerance" in the sense of Rogoff, Reinhart, and Savastano (2003).

Because its fiscal policy exposes it to net-of-interest deficit risks, the government in our model follows Shiller's (1994) and recommendation to insure those risks by trading shares of a GDP-indexed Arrow security.[8] In the language of Jiang et al. (2019, 2020), our government uses that Shiller security to "manufacture" the risk-free debt that is part of its optimal debt management package. Costs of trading the Shiller security appear in the government's budget constraint and add a risk-premium to the "$r-g$" debt dynamics featured in Blanchard (2019). An alternative interpretation of these Arrow-Shiller type of insurance/hedging contractual payments is that the government increases its spending in good times and decreases it in bad times. This interpretation would have the government adopt a spending rule that makes its debt co-move GDP in a way that makes it become risk-free, allowing the debt-GDP ratio b_t to evolve deterministically.

2 Benchmark Setting

We construct an optimal fiscal policy for the government of a country that finances an exogenous random expenditure process and that is small in the sense that asset prices are determined outside the country. Time $t \in [0, +\infty]$. The government chooses a moment $T^{\mathcal{D}} \in [0, +\infty]$ at which it defaults on its contractual agreements. When $t < T^{\mathcal{D}}$, the government is in a "no-default regime" and has more opportunities to borrow and insure than it does in the "balanced budget regime" that prevails when $t \geqslant T^{\mathcal{D}}$.

The country's (potential) GDP $\{Y_t; t \geqslant 0\}$ is determined outside of our model and follows a geometric Brownian motion (GBM) process

$$\frac{dY_t}{Y_t} = gdt + \sigma d\mathcal{Z}_t^Y , \tag{2}$$

where g is expected GDP growth, $\sigma > 0$ is growth volatility, $\mathcal{Z}_t^Y$ is a standard Brownian motion, and $Y_0 > 0$. In the no-default regime that prevails before stopping time $T^{\mathcal{D}}$, GDP equals Y_t. But in the post-$T^{\mathcal{D}}$ default regime, GDP is systematically less than Y_t, as equation (11) below asserts. Except for risk-free debt that the government of this country issues, all

[8] Jiang et al. (2024) verifies that the presence of a complete set of Arrow history-contingent securities implies the existence of Shiller's macro security.

income streams are priced by investors who live outside the model and use a shared stochastic discount factor process (SDF) $\{\mathbb{M}_t\}$ with multiplicative increments described by

$$\frac{d\mathbb{M}_t}{\mathbb{M}_t} = -rdt - \eta d\mathcal{Z}_t^m\,, \quad \mathbb{M}_0 = 1\,, \tag{3}$$

where $\mathcal{Z}_t^m$ is a standard Brownian motion that represents an aggregate/systematic shock in (world) capital markets and r is the risk-free rate. Let ρ denote the constant correlation coefficient between the GDP shock $d\mathcal{Z}_t^Y$ and $d\mathcal{Z}_t^m$. Absence of arbitrage opportunities requires that the drift of $d\mathbb{M}_t/\mathbb{M}_t$ equals $-r$ (see Duffie, 2001). The diffusion coefficient of $d\mathbb{M}_t/\mathbb{M}_t$ equals $-\eta$, where η is the market price of risk in the sense of Black and Scholes (1973) and Merton (1973).[9]

The government finances an exogenous government spending process $\{\Gamma_t; t \geqslant 0\}$ that does not appear in the utility functional of a representative household. When $t < T^{\mathcal{D}}$ so that the no-default regime prevails, Γ_t varies with contemporaneous output Y_t according to

$$\Gamma_t = \gamma_t Y_t\,, \tag{4}$$

where $\gamma_t = \gamma \in (0,1)$ so that government spending is proportional to GDP.

Let $\{\mathcal{T}_t; t \geqslant 0\}$ denote the government's tax revenue process. When the government collects tax revenue $\mathcal{T}_t$ and GDP is Y_t, there are deadweight losses from taxation measured in lost consumption goods that are described by a function $\Theta_t = \Theta(\mathcal{T}_t, Y_t)$. Following Barro (1979), we assume that $\Theta(\mathcal{T}_t, Y_t)$, is homogeneous of degree one in GDP and tax revenue:

$$\Theta_t = \Theta(\mathcal{T}_t, Y_t) = \theta(\tau_t) Y_t\,, \tag{5}$$

where $\tau_t = \mathcal{T}_t/Y_t$ and scaled deadweight loss $\theta(\tau)$ is increasing, convex, and smooth. We assume that

$$\tau_t \leqslant \overline{\tau}\,, \tag{6}$$

where $\overline{\tau} \leqslant 1-\gamma$ is a maximal politically feasible tax rate on GDP in the no-default regime. Keynes (1923, pp.56–62) and Keynes (1931) described on how an upper bound $\overline{\tau}$ was shaped by political considerations and used it to infer limits on a country's debt-GDP ratio.

In the no-default regime, the government can finance its spending Γ_t partly by issuing

[9] The $[\mathcal{Z}_t^m, \mathcal{Z}_t^Y]^\top$ process is a bivariate Brownian motion with a covariance matrix $\left[\begin{smallmatrix} 1 & \rho \\ \rho & 1 \end{smallmatrix}\right] t$. If $\rho = 1$, the country's GDP shock is systematic. If $\rho = 0$, the country's GDP shock is idiosyncratic.

risk-free debt. Let $\{B_t; t \geqslant 0\}$ denote the government's risk-free debt process, with B_0 a given initial condition. The government can manage its exposure to expenditure and tax collection risks by dynamically trading a Shiller (1994) macro security whose payouts equal the country's (potential) GDP $\{Y_t\}$. The time t price S_t of the Shiller's macro security is

$$S_t = \mathbb{E}_t \left[\int_t^\infty \frac{\mathbb{M}_s}{\mathbb{M}_t} Y_s ds \right] , \tag{7}$$

where $\mathbb{M}_t$ is the SDF given in (3). Using Ito's lemma, the Shiller security's cum-dividend return is:

$$dR_t \equiv \frac{dS_t + Y_t dt}{S_t} = (r + \lambda)\, dt + \sigma d\mathcal{Z}_t^Y , \tag{8}$$

where $\lambda = \rho\eta\sigma$ is a risk premium.

In the spirit of Krishnamurthy and Vissing-Jorgensen (2012), we assume that risk-free government debt enjoys a convenience yield $\delta > 0$ that creates a wedge between the ordinary market risk-free rate r and the rate $r - \delta$ that the government pays on its risk-free debt. In the no-default regime, the government's risk-free debt B_t evolves according to[10]

$$dB_t = \underbrace{(\Gamma_t - \mathcal{T}_t)dt}_{\textbf{primary deficit}} + rB_t dt - \underbrace{\delta B_t dt}_{\textbf{convenience yield}} + \underbrace{dH_t}_{\textbf{debt issue}} \underbrace{-\Phi_t\,(dR_t - rdt)}_{\textbf{hedging net exposure}} . \tag{9}$$

The first term is the government's primary deficit. The second term would be the government's interest payment on its debt if it paid the risk-free rate r. The third term captures the convenience yield δ on the government's risk-free debt. The fourth term $dH_t \geqslant 0$ is a (lumpy) issue of government risk-free debt.[11] The last term describes how the government's purchase Φ_t of the Shiller security affects dB_t.

Integrating debt dynamics (9) forward and imposing a balanced budget if the government defaults, we conclude that the government budget constraint at $t = 0$ is

$$B_0 \leqslant \mathbb{E}_0 \left[\int_0^{T^{\mathcal{D}}} e^{\delta t} \mathbb{M}_t \left[(\mathcal{T}_t - \Gamma_t)\, dt - dH_t \right] \right] \tag{10}$$

where B_0 is the government's risk-free debt at $t = 0$. The right side of (10) is the present value of the government's primary surplus. It takes into account benefits of debt financing

[10] To construct an optimal fiscal plan, our government uses both singular control (lump-sum debt issuance and payout to the household) and convex control (tax smoothing). The US government's 2020 and 2021 covid stimulus checks and related transfers might be interpreted as examples of such payouts financed by lump-sum debt issuances.

[11] The $\{H_t; t \geqslant 0\}$ process is nondecreasing and describes a non-negative net debt issuance dH_t for any dt.

coming from the convenience yield $\delta > 0$. Lumpy debt issuance $dH_t > 0$ is netted out from $(\mathcal{T}_t - \Gamma_t)\,dt$. The convenience yield δ multiplies the SDF $\mathbb{M}_t$ in (10).

Financing opportunities are more limited in the balanced budget regime that prevails when $t \geqslant T^{\mathcal{D}}$. Let $\widehat{Y}_t$ denote GDP in the balanced-budget regime. We assume that when it defaults the government repudiates all of its debts, that GDP then immediately drops from the pre-default level $Y_{T^{\mathcal{D}}-} = \lim_{s\uparrow T^{\mathcal{D}}-} Y_s$ to $\widehat{Y}_{T^{\mathcal{D}}} = \alpha Y_{T^{\mathcal{D}}-}$, where $\alpha \in (0,1)$,[12] and that the government is excluded from markets for the Shiller security and for risk-free debt.[13] For $t \geqslant T^{\mathcal{D}}$, GDP $\widehat{Y}_t$ follows the downward scaled version (11) of the process (2):

$$\widehat{Y}_t = \alpha Y_t\,, \quad t \geqslant T^{\mathcal{D}}\,. \tag{11}$$

Consequently, the government confronts budget constraints

$$\widehat{\mathcal{T}}_t = \Gamma_t = \gamma_t Y_t\,, \quad t \geqslant T^{\mathcal{D}}\,, \tag{12}$$

where $\widehat{\mathcal{T}}_t$ is the government's tax revenue in the balanced-budget regime. The government's spending $\{\Gamma_t; t \geqslant 0\}$ is not affected by its decision to default. When the government collects tax revenues $\widehat{\mathcal{T}}_t$ and output is $\widehat{Y}_t$, deadweight loss $\widehat{\Theta}_t = \widehat{\Theta}(\widehat{\mathcal{T}}_t, \widehat{Y}_t)$, where

$$\widehat{\Theta}(\widehat{\mathcal{T}}_t, \widehat{Y}_t) = \widehat{\theta}(\widehat{\tau}_t)\widehat{Y}_t\,, \tag{13}$$

$\widehat{\tau}_t = \widehat{\mathcal{T}}_t/\widehat{Y}_t$, and $\widehat{\theta}(\widehat{\tau})$ is increasing, convex, and smooth. Deadweight loss functions in the two regimes are related by

$$\widehat{\theta}(\,\cdot\,) = \kappa\,\theta(\,\cdot\,)\,, \tag{14}$$

where $\kappa \geqslant 1$ measures how much more costly taxation is in the balanced-budget regime. We require $\widehat{\mathcal{T}}_t \leqslant \overline{\tau}\,\widehat{Y}_t$, which is equivalent to the following constraint on the tax rate $\widehat{\tau}_t$ in the balanced-budget regime:

$$\widehat{\tau}_t \leqslant \overline{\tau}\,, \quad t \geqslant T^{\mathcal{D}}\,, \tag{15}$$

where $\overline{\tau}$ is the same maximum politically feasible tax rate described above.

[12] Many other studies of sovereign debt make this assumption, for example, Aguiar and Gopinath (2006) and Rebelo, Wang, and Yang (2022).

[13] The government has limited commitment with respect to *all* liabilities. When it defaults, it defaults on all of them, including promised repayment on short positions in the Shiller macro security. The government has one budget constraint. Gains and losses from its state-contingent contracts appear in the law of motion (9) of its debt. Reneging on any contractual agreement causes the country to lose access to international capital markets and casts it into the balanced-budget regime.

The government designs tax and debt management policies to maximize the utility functional of a representative consumer who receives an after-tax, after-tax-distortion income flows of N_t in the no-default regime and $\widehat{N}_t$ and in the balanced-budget regime, where

$$N_t = Y_t - \mathcal{T}_t - \Theta_t \quad \text{and} \quad \widehat{N}_t = \widehat{Y}_t - \widehat{\mathcal{T}}_t - \widehat{\Theta}_t\,. \tag{16}$$

In the no-default regime $t \in [0, T^{\mathcal{D}})$, the household occasionally receives non-negative lump sum government transfers financed by lumpy debt issuance dH_t, so that total payments are $dH_t + N_t dt$.[14]

Where $U(C,t)$ is increasing in C, the representative household chooses consumption $\{C_t\}$ and state-contingent assets to maximize lifetime utility $\mathbb{E}_0\left[\int_0^\infty U(C_t,t)dt\right]$ subject to the present-value budget constraint:

$$\mathbb{E}_0\left[\int_0^\infty \mathbb{M}_t C_t dt\right] \leqslant P_0, \tag{17}$$

where P_0 is the time 0 value of P_t, the time t market value of continuation GDP after taxes and tax distortions have been deducted:

$$P_t = \mathbb{E}_t\left[\int_t^{T^{\mathcal{D}}} \frac{\mathbb{M}_s}{\mathbb{M}_t}(dH_s + N_s ds) + \frac{\mathbb{M}_{T^{\mathcal{D}}}}{\mathbb{M}_t}\widehat{P}_{T^{\mathcal{D}}}\right], \quad t < T^{\mathcal{D}}\,, \tag{18}$$

and $\widehat{P}_t$ is the value of prospective GDP after taxes and tax distortions when the government defaults at $t = T^{\mathcal{D}}$:

$$\widehat{P}_t = \mathbb{E}_t\left[\int_t^\infty \frac{\mathbb{M}_s}{\mathbb{M}_t}\widehat{N}_s ds\right], \quad t \geqslant T^{\mathcal{D}}\,. \tag{19}$$

Because the value function for the consumer's problem is a function that is increasing in the consumer's P_0,[15] the government can design a tax and debt management policy to maximize the utility functional of the representative consumer by choosing a policy that maximizes P_0 subject to the budget constraints (10) and (12) as well as other the other implementability restrictions it faces. Thus, given the government expenditure process $\{\Gamma_t; t \geqslant 0\}$ and the

[14]Later we show that the government chooses to set $dH_t > 0$ is only at $t = 0$.

[15]Assume for example that $U(C,t) = e^{-\varrho t}\frac{C^{1-\omega}}{1-\omega}$, where $\omega \geqslant 0$ is a coefficient of relative risk aversion and ϱ is a discount rate. By adapting the Lagrangian martingale method developed by Cox and Huang (1989) (also see(Duffie, 2001, ch. 9)), We show that for a given CRRA utility the indirect utility function is $\frac{(\iota P_0)^{1-\omega}}{1-\omega}$, where $\iota = \left((1-\frac{1}{\omega})r + \frac{\varrho}{\omega} + \frac{1}{2}(1-\frac{1}{\omega})\frac{1}{\omega}\eta^2\right)^{-\frac{\omega}{1-\omega}}$. Appendix A analyzes a general case where the representative household is endowed with an initial level of wealth and a share of the aggregate flow payments.

stochastic discount factor process $\{\mathbb{M}_t\}$, at $t = 0$ the government chooses a default time $T^{\mathcal{D}}$, risk-free debt processes $\{dH_t, B_t\}$, taxes $\{\mathcal{T}_t, \widehat{\mathcal{T}}_t\}$, and Shiller's security holdings $\{\Phi_t\}$ to maximize P_0 defined in (18) subject to risk-free debt dynamics (9), budget constraints constraints (10) and (12), and constraints (6) and (15) on tax rates.

3 Optimal Fiscal Policy

We first study the government's problem in the balanced-budget regime, then in the no-default regime. Government debt is always zero in the balanced-budget regime, so the value function $\widehat{P}_t = \widehat{P}(\widehat{Y}_t)$ depends on only contemporaneous GDP $\widehat{Y}_t = \alpha Y_t$ and satisfies

$$r\widehat{P}(\widehat{Y}) = \left(\widehat{Y} - \Gamma - \widehat{\Theta}(\Gamma, \widehat{Y})\right) + (g - \rho\eta\sigma)\widehat{Y}\widehat{P}'(\widehat{Y}) + \frac{\sigma^2\widehat{Y}^2}{2}\widehat{P}''(\widehat{Y})\,. \tag{20}$$

The first term on the right side of (20) is the household's net income. Since the government neither borrows nor lends, tax revenues $\widehat{\mathcal{T}}_t$ equal government spending Γ_t. The second and third terms capture effects of the risk-adjusted drift and volatility of output on $\widehat{P}(\widehat{Y})$. To ensure that $\widehat{P}(\widehat{Y}) \geqslant 0$, we impose:

Assumption 3.1. $r - \delta + \lambda > g$, $\kappa \geqslant 1$, $\alpha \leqslant 1$, *and* $1 - \gamma/\alpha - \kappa\theta(\gamma/\alpha) \geqslant 0$.

Let $\widehat{p}_t = \widehat{P}(\widehat{Y}_t)/\widehat{Y}_t$. We'll soon show that $\widehat{p}_t$ is constant for $t \geqslant T^{\mathcal{D}}$.

Its access to the lump sum transfer process $\{dH_t\}$ means that under an optimal government policy there is a lower bound $\underline{B}_t$ on government debt for $t > 0$. We'll soon indicate how $\underline{B}_t$ is determined jointly with dH_0 and note how it is optimal to set $dH_t = 0$ whenever $B_t \geqslant \underline{B}_t$. Before doing that, we'll describe optimal government policy in an interior region in which $B_t \geqslant \underline{B}_t$.

When $\underline{B}_t \leqslant B$ and $t \leqslant T^{\mathcal{D}}$, the government sets $dH_t = 0$ and relies exclusively on risk management and taxation to shape its risk-free debt dynamics. The government chooses tax revenue $\mathcal{T}$ and allocates Φ to Shiller security purchases. The optimal value function $P(B, Y)$ solves the following Hamilton-Jacobi-Bellman (HJB) equation:

$$\begin{aligned} rP(B,Y) = \max_{\mathcal{T},\,\Phi}\ & (Y - \mathcal{T} - \Theta(\mathcal{T}, Y)) + [(r-\delta)B + \Gamma - \mathcal{T}\,]\,P_B(B,Y) + \frac{\sigma^2\Phi^2}{2}P_{BB}(B,Y) \\ & + (g - \rho\eta\sigma)\,Y P_Y(B,Y) + \frac{\sigma^2 Y^2}{2}P_{YY}(B,Y) - \sigma^2\Phi Y P_{BY}(B,Y)\,. \end{aligned} \tag{21}$$

The convenience yield δ and trades Φ of the Shiller security both influence the risk-free debt dynamics. The first term on the right side of (21) is the after-tax income flowing to the representative household. The second and third terms are drift and diffusion volatility effects of increasing debt B on $P(B,Y)$. The fourth and fifth terms express effects of drift and volatility of GDP on $P(B,Y)$. The sixth term describes effects of intertemporal hedging via holdings Φ of the Shiller security and the cross partial P_{BY} on $P(B,Y)$.

The FOC for tax revenue $\mathcal{T}$ equates the marginal cost $1+\Theta_{\mathcal{T}}(\mathcal{T},Y)$ of taxing the household with the marginal benefit $-P_B(B,Y)>0$ of using taxes to reduce debt:

$$1+\Theta_{\mathcal{T}}(\mathcal{T},Y) = -P_B(B,Y)\,. \tag{22}$$

In the spirit of Merton (1971), purchases Φ of the Shiller security satisfy:

$$\Phi = \frac{Y P_{BY}(B,Y)}{P_{BB}(B,Y)}. \tag{23}$$

We shall soon verify that the FOC (23) implies hedging demand $\Phi_t = -B_t$. The excess return on the Shiller macro security is $dR_t - rdt = \lambda dt + \sigma d\mathcal{Z}_t^Y$, so we can express the hedging term contributing to dB_t in (9) as:

$$\underbrace{-\Phi_t\,(dR_t - rdt)}_{\textbf{hedging net exposure}} = \lambda B_t dt + \sigma B_t d\mathcal{Z}_t^Y\,. \tag{24}$$

By incurring hedging cost $\lambda B_t dt$ the government acquires exposure $\sigma B_t d\mathcal{Z}_t^Y$ to risk in increments to B_t. This is optimal because it decreases government debt due when the output shock $d\mathcal{Z}_t^Y$ is negative. Equation (24) describes how hedging cost $\lambda B_t dt$ and the exposure to $\sigma B_t d\mathcal{Z}_t^Y$ that hedging acquires combine to contribute to debt dynamics in equation (9).

FOCs (22) and (23) let us represent HJB equation (21) as

$$\begin{aligned} rP(B,Y) = \max_{\mathcal{T}\leqslant \overline{\tau} Y}\; & Y - \mathcal{T} - \Theta(\mathcal{T},Y) + \left[(r-\delta)B + \Gamma - \mathcal{T}\ \right] P_B(B,Y) \\ & + \widetilde{g}\, Y P_Y(B,Y) + \frac{\sigma^2 Y^2}{2} P_{YY}(B,Y) - \frac{\sigma^2 Y^2}{2}\frac{P_{BY}^2(B,Y)}{P_{BB}(B,Y)}\,, \end{aligned} \tag{25}$$

where $\widetilde{g} = g - \rho\eta\sigma$ is a risk-adjusted growth rate.[16] Because value function $P(B,Y)$ is

[16]Technically, it is the expected GDP growth rate under the risk-neutral measure $\widetilde{\mathbb{P}}$.

homogeneous of degree one in B and Y, it follows that[17]

$$P_{YY}(B,Y) = \frac{P^2_{BY}(B,Y)}{P_{BB}(B,Y)}\,. \tag{26}$$

By using (26) to simplify (25), we obtain the following first-order partial differential equation:

$$rP(B,Y) = \max_{\mathcal{T}\leqslant\overline{\tau}Y}\ (Y - \mathcal{T} - \Theta(\mathcal{T},Y)) + ((r-\delta)B + \Gamma - \mathcal{T})\,P_B + \widetilde{g}YP_Y\,. \tag{27}$$

The first term on the right side of (27) is after tax income $N = Y - \mathcal{T} - \Theta(\mathcal{T},Y)$ of the representative household. The second term captures effects of the gross government budget deficit $((r-\delta)B + \Gamma - \mathcal{T})$ on the value function $P(B,Y)$, while the last term describes the risk-adjusted growth effect of Y. Optimality implies that the sum of these three terms equals $rP(B,Y)$. By optimally managing its risk, the government makes its debt be risk free. That explains why no diffusion terms associated with P_{BB}, P_{YY}, or P_{BY} appear in (27).

Whenever $B_t < \underline{B}_t$, the government wants to issue a lump-sum amount of debt and to use it to finance a one-time payout to households. That will happen only at $t = 0$. In the interval $0 \leqslant B_t < \underline{B}_t$, issuing government debt is inexpensive in a sense related to Blanchard's (2019) analysis. When the debt-GDP ratio $b_t = B_t/Y_t$ is low enough, it is optimal for the government instantaneously to capitalize on its convenience yield debt by issuing enough risk-free debt and using it to finance a one-time payout to the representative household in the amount

$$dH_t = \max\left\{\underline{B}_t - B_t, 0\right\}\,. \tag{28}$$

Equation (28) implies the following value-matching condition when $B_t < \underline{B}_t$:

$$P(B_t,Y_t) = P(\underline{B}_t,Y_t) + \underline{B}_t - B_t\,. \tag{29}$$

By setting dH_t to maximize (29), the government attains a new debt level $\underline{B}_t \geqslant 0$ that solves

$$\max_{\underline{B}\geqslant 0}\quad P(\underline{B}_t,Y_t) + \underline{B}_t\,. \tag{30}$$

If the optimal $\underline{B}_t$ is interior (i.e., if $\underline{B}_t > 0$), it satisfies the FOC:

$$-P_B(\underline{B}_t,Y_t) = 1\,, \tag{31}$$

[17]Using the homogeneity property $P(B,Y) = p(b)Y$, we obtain $P_B = p'(b)$, $P_{BB} = p''(b)/Y$, $P_Y = p(b) - p'(b)b$, $P_{YY} = p''(b)bB/Y^2 = p''(b)b^2/Y$, and $P_{BY} = -p''(b)b/Y$. Therefore, we can verify $P_{BB}P_{YY} = (p''(b)b/Y)^2 = P^2_{BY}$.

so that the government's optimal level $\underline{B}_t$ of its initial risk-free debt sets the marginal cost $-P_B(\underline{B}_t, Y_t)$ of using future fiscal surpluses to service its debt equal to one, the marginal benefit of distributing to the representative household all of the proceeds from an immediate issue of new risk free debt. Otherwise, the government issues no lump-sum debt and $\underline{B}_t = 0$.

Thus, Blanchard's (2019) "debt is inexpensive" logic is valid in our setting only at $t = 0$ and when $\underline{B}_0 > 0$ and initial risk-free debt $B_0 < \underline{B}_0$. But when $B_t \geqslant \underline{B}_t$, the government's risk-free debt still presents a convenience yield, but b_t moves smoothly as the government takes advantage of its risk-free debt's convenience yield by gradually increasing both the risk-free debt-GDP ratio and the tax rate.

Turning now to the government's debt capacity $\overline{B}_t$ at $t \leqslant T^{\mathcal{D}}$, let P_t^* denote the maximal time-t market value P_t of the tail of the representative household's' after-tax income flow given in (18). The Markovian structure of the Y process lets us write $P_t^* = P(B_t, Y_t)$. For the government to choose not to default, it is necessary that

$$P(B_t, Y_t) \geqslant \widehat{P}(\widehat{Y}_t)\,. \tag{32}$$

When (32) binds, it determines the government's debt capacity $\overline{B}_t$ as a function of Y_t.

If government debt B_t ever exceeds debt capacity $\overline{B}_t$, the government defaults and stays ever after in the balanced-budget regime.[18] The value function $P(B_t, Y_t)$ at $B_t > \overline{B}_t$ satisfies

$$P(B_t, Y_t) = \widehat{P}(\widehat{Y}_t)\,. \tag{33}$$

The government's debt capacity $\overline{B}_t$ is shaped by 1) its incentive to renege on any larger risk-free debt; and 2) an exogenous constraint $\tau \leqslant \overline{\tau}$ (again see Keynes (1923, pp.56–62) and Keynes (1931)), leading to two possible situations. In one, (33) holds but $\tau \leqslant \overline{\tau}$ is slack at $\overline{B}_t$. Having reached its debt capacity $\overline{B}_t$, the government is indifferent between defaulting and servicing its risk-free debt with future taxes, so the following value-matching condition prevails:

$$P(\overline{B}_t, Y_t) = \widehat{P}(\widehat{Y}_t)\,, \tag{34}$$

where $\widehat{Y}_t = \alpha Y_{t-}$ and $\widehat{P}(\widehat{Y}_t)$ satisfies (20). Counterparts of this condition play key roles in models of Worrall (1990), Kehoe and Levine (1993), and Kocherlakota (1996).[19] In the other

[18] We generalize our model to allow for the possibility where the government probabilistically exits the balanced-budget regime and returns to the no-default regime in Appendix A.

[19] Our approach is related to those of Bolton, Wang, and Yang (2019) and Rebelo, Wang, and Yang (2022). They incorporate default options into continuous-time corporate and international finance models.

situation, constraint $\tau \leqslant \overline{\tau}$ binds and therefore

$$\mathcal{T}(\overline{B}_t, Y_t) = \overline{\tau} Y_t \,. \tag{35}$$

In this case, (32) is slack at $B_t = \overline{B}_t$.

Thus, the government's risk-free debt capacity equals the minimum of the distinct $\overline{B}_t$'s that solve (34) and (35). As we shall see, although the government chooses not to enter this balanced-budget region, its presence helps pin down $\overline{B}_t$.

The government's risk-free debt-output ratio b is its state variable. Let $dh_t = dH_t/Y_t$ be the scaled lump-sum transfer and $\overline{b}_t = \overline{B}_t/Y_t$ be the maximum feasible debt-GDP ratio. Substituting $P(B, Y) = p(b)Y$ into the FOC (22) for tax revenue $\mathcal{T}$, we obtain the following FOC for $\tau(b)$:[20]

$$1 + \theta'(\tau(b)) = -p'(b) \,. \tag{36}$$

Since tax distortion costs are convex ($\theta''(\,\cdot\,) > 0$), we can invert the marginal tax distortion cost function $\theta'(\,\cdot\,)$ to obtain an optimal tax rate $\tau(b)$. Simplifying FOC (23) for Φ_t, we deduce that optimal hedging demand is $\phi_t = \Phi_t/Y_t = \phi(b_t) = -b_t$.

We turn now to an interior b interval where an optimal fiscal policy sets $dh_t = 0$ because the marginal benefit of financing an immediate payout to the representative household is smaller than the marginal cost of servicing risk-free government debt with future taxes and debt-management. Applying Ito's Lemma shows that in this region b_t evolves as

$$\dot{b}_t \equiv \mu_t^b = \mu^b(b_t) = \underbrace{\gamma - \tau(b_t)}_{\text{primary deficit}} + \underbrace{(r - \delta) \cdot b_t}_{\text{interest payment}} - \underbrace{g \cdot b_t}_{\text{growth}} + \underbrace{\lambda \cdot b_t}_{\text{hedging cost}} \,. \tag{37}$$

The first term on the right side of (37) is the scaled "primary" deficit $\gamma - \tau(b)$. The second term is the interest cost of servicing debt netting out the convenience yield. The sum of the first two terms is the scaled fiscal deficit, gross of interest payments. The third term is a reduction in growth of the debt-GDP ratio contributed by output growth. The last term captures the hedging cost due to the risk premium payment.[21] Although payouts from the government's net debt B_t are risk-free, the λb_t term appears because the source of funds for these payouts is a stochastic prospective primary surplus process that must be discounted at the rate of $r - \delta + \lambda$ in order to account for the government debt's convenience yield and for the cost of purchasing insurance by shorting the Shiller security in order to service the

[20] This condition holds regardless of whether the tax constraint (6) binds or not. The reason is that the tax constraint may bind only at $\overline{b}$. Tax smoothing implies that the FOC (36) holds also at the boundary $\overline{b}$.

[21] Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019) included this term in a related setting.

promised risk-free debt repayments.

That the debt GDP ratio b cannot exceed $\overline{b}$ implies $\mu^b(\overline{b}) \leqslant 0$. Furthermore, because $\delta \geqslant 0$, at the margin the government usually wants to postpone tax distortions, which implies that $\mu^b(\overline{b}) \geqslant 0$. Together these two inequalities imply that the drift of b is zero at $\overline{b}$:

$$\mu^b(\overline{b}) = 0. \tag{38}$$

Equation (38) is the free-boundary condition that pins down debt capacity $\overline{B}_t$ in (35).[22]

Substituting $\mu^b(\overline{b}) = 0$ into (37) yields

$$\overline{b} = \frac{\tau(\overline{b}) - \gamma}{r - \delta + \lambda - g} = \min\left\{\breve{b}\,, \frac{\overline{\tau} - \gamma}{r - \delta + \lambda - g}\right\}\,, \tag{39}$$

where $\breve{b}$ is the unique positive root of $p(\breve{b}) = \alpha\widehat{p}$ and $\widehat{p} = \frac{1-\gamma/\alpha-\kappa\theta(\gamma/\alpha)}{r+\lambda-g}$ is the scaled household's value in the balanced-budget regime.[23] The first equality in (39) asserts that the maximum sustainable debt-GDP ratio $\overline{b}$ equals the present value of the primary surplus $(\tau(\overline{b}) - \gamma)$ evaluated at the appropriate rate $r - \delta + \lambda$, because the optimal primary deficit is risky and bears an insurance premium of λ. The second equality states that debt capacity $\overline{b}$ equals $\breve{b}$ when the default option constraint (32) binds, or instead $\frac{\overline{\tau}-\gamma}{r-\delta+\lambda-g}$, when the tax-rate constraint is tighter and binds at debt capacity.

We can verify that the lump-sum debt issuance boundary $\underline{b}$ solves

$$\max_{\underline{b}\geqslant 0} \quad p(\underline{b}) + \underline{b}\,. \tag{40}$$

If the optimal $\underline{b}$ is interior (i.e., $\underline{b} > 0$), the marginal cost of debt issuance must be zero at $\underline{b}$ so that $p'(\underline{b}) = -1$. Otherwise, since $p'(\underline{b}) < -1$, the government issues no lumpy debt and $\underline{b} = 0$. Thus, an optimal lump-sum transfer policy satisfies

$$dh_t = \max\left\{\underline{b} - b_t, 0\right\}. \tag{41}$$

Different economic forces shape $\underline{b}$ and $\overline{b}$. The lower boundary $\underline{b}$ involves an instantaneous lump-sum transfer to the representative household financed by a lump-sum debt issue and is characterized by smooth-pasting and super-contact conditions. The upper boundary $\overline{b}$ is

[22]The zero-drift condition at $\overline{b}$ is an equilibrium argument based on local changes. The Gordon growth model at the steady state is a forward-looking present value calculation argument for the determination of $\overline{b}$. They are equivalent. A non-zero drift of b at $\overline{b}$ would be inconsistent with the notion of debt capacity.

[23]Simplifying $p(b) = \alpha\widehat{p}$ at $b = \overline{b}$, we obtain $1 - (r - \delta + \lambda - g)b - \theta((r - \delta + \lambda - g)b + \gamma) = \alpha - \alpha\kappa\theta(\gamma/\alpha)$.

the maximum sustainable level of debt per unit of GDP and can be approached only from the left.

If government debt were zero at $t = 0$ and if $\underline{b} > 0$, a government would immediately issue debt and use the proceeds to finance a lump-sum payment $dH_0 = \underline{b}Y_0$ to the representative household, thereby resetting b to equal $\underline{b}$; thereafter b_t would stay inside $[\underline{b}, \overline{b}]$, gradually approaching until $\overline{b}$ from below. As we show later, the optimal b_t process has a transition path towards its equilibrium debt capacity, if and only if the government debt enjoys a convenience yield. Moreover, the optimal initial debt-GDP ratio, b_0^*, is strictly positive only if the convenience yield is so large that the government is willing to make a lump-sum debt issuance at $t = 0$ to the households and then bear the consequence of incurring higher tax distortion costs over time.[24]

The following proposition pulls things together and characterizes an optimal tax and debt plan for our **ABCD** model.

Theorem 3.2. *Under Condition 3.1, the scaled value function $p(b)$ in the no-default regime satisfies the first-order nonlinear differential equation:*

$$[r + \lambda - g]\, p(b) = 1 - \tau(b) - \theta(\tau(b)) + [(r - \delta + \lambda - g)b + \gamma - \tau(b)]\, p'(b)\,, \tag{42}$$

for $b \in [0, \overline{b}]$ where $\overline{b}$ is the government's scaled debt capacity as determined by (39). The optimal lumpy debt issue policy is described by $\underline{b}$ in (40) and dh_t satisfies (41). The optimal tax rate policy $\tau(b)$ satisfies (36), optimal (scaled) holdings of the Shiller security is $\phi(b) = -b$, and the $\{b_t\}$ process satisfies differential equation(37).

Let V_t denote the sum of the household's value and creditors' value: $V_t = P_t + B_t$. Next, we use corollaries to summarize the results for the three important special cases of our **ABCD** model: **A**, **AB**, and **ABC**.

Corollary 3.3. (Ricardian Equivalence: Model A) *Under the $r + \lambda > g$ condition, the scaled total value, $v_t = V_t/Y_t$, is constant at all t and equals:*

$$v^{FB} = \frac{1 - \gamma}{r + \lambda - g}\,. \tag{43}$$

[24] Only at time 0 may the government be in either the lump-sum debt issuance and payout region or the balanced-budget region. If starting in the lump-sum debt issuance and payout region where $b < \underline{b}$, the government increases its debt so that its b instantly equals $\underline{b}$ after time 0 and then the b_t process is dictated by the law of motion in the no-default regime. If starting in the balanced-budget regime where $b > \overline{b}$, the government immediately defaults and sets the tax rate to its spending-output ratio γ so that its primary deficit is zero at all time.

Since tax and debt policies are irrelevant under Ricardian equivalence, the total value V_t equals the net present value of $(1-\gamma)Y_t$, the country's GDP netting out the government spending and the proper discount rate for the stream of $(1-\gamma)Y_t$ is $r+\lambda$, where λ is the risk premium of Shiller's GDP-indexed macro security. The superscript FB refers to the solution of Model **A** under Ricardian equivalence.

Corollary 3.4. (Stochastic Barro (1979): Model AB.) *Under Condition 3.1 and with no convenience yield ($\delta = 0$), the equilibrium debt capacity $\overline{b}$ in the **AB** model is given by*

$$\overline{b} = \frac{\overline{\tau} - \gamma}{r + \lambda - g} . \tag{44}$$

When $b_0 \in [0, \overline{b}]$, $b_t = b_0$, $\tau(b_t) = \tau(b_0) = (r + \lambda - g)b_0 + \gamma$,

$$p(b_t) = \frac{1 - \tau(b_0) - \theta(\tau(b_0))}{r + \lambda - g} , \quad \textit{and} \quad v_t = \frac{1 - \gamma - \theta(\tau(b_0))}{r + \lambda - g} . \tag{45}$$

Compared with the **A** model, we uniquely pin down the fiscal policy in the **AB** model. First, $b_t = b_0$ and the tax rate is constant for all t. The government does not exhaust its debt capacity (provided that $b_0 < \overline{b}$). The primary surplus $\tau(b_t) - \gamma = (r + \lambda - g)b_0 > 0$ if and only if $b_0 > 0$. The optimal tax rate is independent of the deadweight cost parameter κ because the government has to repay debt eventually and the (scaled) tax deadweight loss function $\theta(\,\cdot\,)$ is invariant over time. So long as the optimal tax rate is positive, the marginal cost of servicing debt exceeds one, $-p'(b) > 1$, as taxes are distortionary.[25] Second, complete tax smoothing implies that the scaled total value v_t is constant over time and lower than v^{FB} by $\theta(\tau(b_0))/(r + \lambda - g)$ due to tax-induced deadweight losses. Finally, if the initial debt b_0 is endogenously chosen, the optimal level would be $b_0 = 0$ and $\tau_t = \gamma$ in the **AB** model.

Next, we introduce our limited-commitment assumption into the **AB** model, we obtain the following new result relative to the **AB** model.

Corollary 3.5. (Stochastic Barro (1979) with Limited Commitment: Model ABC.) *When the limited-commitment constraint (32) is tighter than the Keynes tax constraint (6), the equilibrium debt capacity $\overline{b}$ equals $\check{b}$, where $\check{b}$ is the unique positive root of*

$$1 - (r + \lambda - g)b - \theta((r + \lambda - g)b + \gamma) = \alpha - \alpha\kappa\theta(\gamma/\alpha) . \tag{46}$$

[25] Consider the case the government savings is positive and moreover so large that it can fully cover all its future spending, in that $-b_0 > \gamma/(r + \lambda - g)$, then the government does not need to rely on taxes to pay for its spending. In this case, it optimally sets $\tau = 0$ and distributes the excess savings (i.e., if $(-B_0 - \Gamma_0/(r + \lambda - g)) > 0$), at $t = 0$ to the households.

*All other predictions are the same as in the **AB** model.*

Compared with the **AB** model, the government may have a tighter credit constraint due to the government's temptation to default. Quantitatively, the limited-commitment induced debt capacity is much tighter than the Keynes tax constraint as we show in our quantitative illustration in Section 4.

By optimally managing its risk exposures and taxing households, the government in model **ABC** makes its debt level, primary deficit, and the household's consumption evolve via GBM processes with the same drift g and volatility σ as the output process:

$$\frac{dB_t}{B_t} = \frac{d\mathcal{T}_t}{\mathcal{T}_t} = \frac{d(\mathcal{T}_t - \Gamma_t)}{(\mathcal{T}_t - \Gamma_t)} = \frac{dC_t}{C_t} = \frac{dY_t}{Y_t} = gdt + \sigma d\mathcal{Z}_t^Y \quad . \tag{47}$$

This behavior is in line with stochastic balanced-growth asset-pricing and business-cycle models as in Lucas (1978, 1987). Also note that debt is a $b_0(r+\lambda-g)$ share of an (unlevered) "equity" claim (the Shiller macro security) on the country's output. That is, public debt in effect is the country's equity.[26] Next, we turn to quantitative illustrations to bring out the role of the government debt's convenience yield in our **ABCD** model.

4 Quantitative Illustrations

To understand the forces that contribute to outcomes in our **ABCD** model, we begin by presenting the reasoning that shaped our choices of plausible values of free parameters. We follow Barro (1979) by assuming a quadratic deadweight loss function:

$$\theta(\tau) = \frac{\varphi}{2}\tau^2 \,, \tag{48}$$

where the parameter $\varphi > 0$ measures the deadweight cost caused by distortionary taxes. With this specification, we can obtain a formula for debt capacity.

Lemma 4.1. *Under Assumption 3.1 and when the deadweight loss function is quadratic so that $\theta(\tau) = \varphi\tau^2/2$ as given in (48), the equilibrium debt capacity $\overline{b}$ is given in (39). If the limited-commitment constraint binds, $\overline{b} = \breve{b} = \frac{\left(\sqrt{1+2\varphi(1-\alpha+\gamma+\varphi\kappa\gamma^2/\alpha/2)}-1\right)/\varphi-\gamma}{r-\delta+\lambda-g}$; so $\overline{b}$ rises with increases in expected growth g, convenience yield δ, default costs κ, and output losses $1-\alpha$; it falls with increases in tax distortion costs φ, the risk free rate r, and the Shiller macro security's risk premium λ.*

[26] This result no longer holds in our **ABCD** model. Because its debt enjoys a convenience yield, a government optimally increase its debt-GDP ratio b_t and its tax rate over time.

Table 1 summarizes our baseline parameter values. We set the mean of output growth to $g = 2\%$ per annum, the annual risk-free rate r to 1.5%, the risk premium λ to 3.5%, and the government spending/output ratio γ to 20%, in line with the estimates in Jiang, Lustig, Van Nieuwerburgh, and Xiaolan (2019, 2020).[27] Our choice of a 3.5% annual risk premium aligns with an equilibrium consumption CAPM analysis.[28] We set the upper bound $\overline{\tau}$ for a politically feasible tax rate at 50%.[29] By comparison, Denmark has the highest average tax-output ratio: 46.3% and the average tax rate in OECD countries is 33.8%. We set output recovery α in the balanced-budget regime to 0.93 in line with an estimate in Hébert and Schreger (2017) and a calculation in Rebelo, Wang, and Yang (2022).

Table 1: **Parameter Values for Model ABCD.** Where applicable, parameter values are continuously compounded and annualized.

Parameter	Symbol	Value
A. Calibration inputs		
risk-free rate	r	1.5%
risk premium	λ	3.5%
average output growth rate	g	2%
government spending to output ratio	γ	20%
output recovery in the balanced-budget regime	α	0.93
B. Calibration outputs		
convenience yield	δ	0.13%
default deadweight loss	κ	1.3
parameter in the quadratic (tax deadweight) loss function	φ	3.9

We inferred $\Omega = \{\delta, \kappa, \varphi\}$ from the US data over the 1980-2020 period (see Appendix B) and obtained 1.) a convenience yield of $\delta = 0.13\%$ per annum, which is on the conservative side of the estimates in the literature (Krishnamurthy and Vissing-Jorgensen, 2012); 2.) a default deadweight loss of $\kappa = 1.3$ in the balanced-budget regime; and 3.) a tax distortion parameter of $\varphi = 3.9$.

Figure 1 portrays how outcomes in the interior region of the state space $b \in [0, +\infty)$ vary as we include or withhold components of our **ABCD** model. The dotted black lines show Ricardian outcomes that prevail in our component-**A**-only model. Here, the total scaled value is constant: $(1-\gamma)/(r+\lambda-g) = 26.7$ and the tax rate and the dynamics

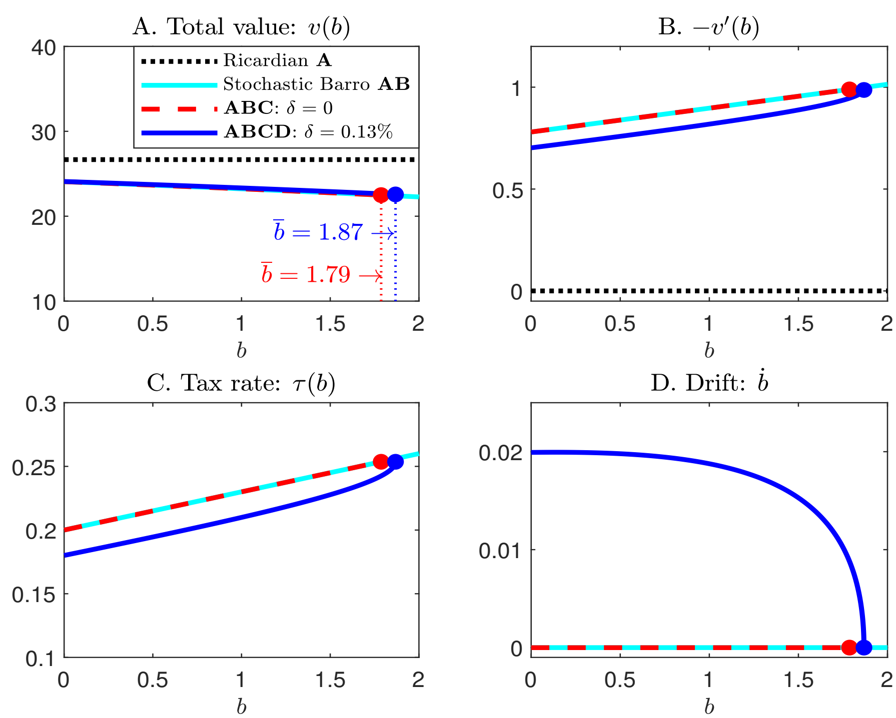
[27] We do not need to choose the value for output growth volatility σ once we calibrate risk premium λ.

[28] In a Lucas (1978) equilibrium asset pricing model in which the source of aggregate risk is the world stock market with a 6% annual market risk premium and the 'beta' of a financial claim (Shiller's security) on the US aggregate output proposed by Shiller (1994) around 0.6, which seems plausible in light of sizes of the US stock market and the US economy relative to the world's, we obtain a risk premium of $\lambda = 3.5\%$ for the financial claim on US output.

[29] Keynes (1931) guessed .25 for this parameter for France in 1926.

of b_t are indeterminate (and so absent from panels C and D). When we add the model **B** tax distortion component to get a Stochastic Barro model **AB**, the government optimally smooths tax distortions by hedging shocks to b_t with trades of the Shiller security and setting the drift of b_t to zero. That policy delivers constant debt-GDP ratios and tax rates, so $b_t = b_0$ and $\tau(b_t) = \tau(b_0) = \gamma + (r + \lambda - g)b_0$. A total welfare efficiency loss, measured by the wedge between the solid blue line (the solution for the stochastic Barro model **AB**) and the horizontal Ricardian model **A** line in panel A, increases with b. The maximum sustainable level of b is determined by the government's ability to tax households, so that $\overline{b} = \frac{\overline{\tau}-\gamma}{r+\lambda-g} = \frac{0.5-0.2}{1.5\%+3.5\%-2\%} = 10$, because we have assumed that the tax rate cannot exceed $\tau(\overline{b}) = \overline{\tau} = 50\%$. In model **AB**, the marginal deadweight cost of debt $-v'(b_t) = -v'(b_0) = \theta'(\tau_0) = \varphi(\gamma + (r + \lambda - g)b_0)$ is constant over time, increases linearly with b_0 and reaches the maximal value of $-v'(\overline{b}) = 1.95$ if $b_0 = \overline{b} = 10$.

Figure 1: Outcomes for a Ricardian model (**A**), a stochastic Barro model (**AB**), a credit limit (**ABC**) model with no convenience yield, and a credit limit (**ABCD**) model with a convenience yield $\delta = 0.13\%$. In the stochastic Barro (**AB**) model, debt capacity is $\overline{b} = 10$ with $\overline{\tau} = 0.5$. With credit limits, $\overline{b} = 1.79$ when $\delta = 0$ in **ABC** model and $\overline{b} = 1.87$ when $\delta = 0.13\%$ in **ABCD** model. Under Ricardian model (**A**) $v(b) = v^{FB} = 26.7$ and $v'(b) = 0$.



When we add the limited credit component **C** to obtain model **ABC**, maximum sustainable debt-to-GDP ratio $\bar{b}$ drops to a much lower value of $\bar{b} = 1.79$ from $\bar{b} = 10$ in the stochastic Barro model **AB**.[30] The 82% reduction in debt capacity comes from giving the government an option to default on its debt. The credit constraint binds at the debt capacity $\bar{b}$ in the **ABC** model, so that at $\bar{b}$ the government is indifferent between defaulting and not defaulting: $p(\bar{b}) = \alpha\widehat{p}$. It follows that

$$1 - \tau(\bar{b}) - \theta(\tau(\bar{b})) = \alpha(1 - \gamma/\alpha - \kappa\theta(\gamma/\alpha)), \tag{49}$$

which pins down a maximal tax rate $\tau(\bar{b}) = 25.4\%$, which depends on the default cost, e.g., output loss $1 - \alpha$ and tax distortions that are represented with κ and the $\theta(\,\cdot\,)$ function. Debt capacity solves $\bar{b} = \frac{\tau(\bar{b})-\gamma}{r+\lambda-g} = \frac{25.4\%-20\%}{1.5\%+3.5\%-2\%} = 1.79$.

Adding the convenience yield component **D** gives our **ABCD** model. Here both b_t and τ_t are time varying. The existence of the convenience yield of its risk-free debt lets the government use a lower tax rate $\tau(b)$ than prevails in the **ABC** model (see panel C). Now the government issues more debt than what was needed for the pure tax-smoothing purpose present in the **ABC** model. Drift $\{\dot{b}_t\}$ starts at its highest level $\dot{b}_0$, decreases with t and eventually declines to zero, as debt capacity $\bar{b}$ is an absorbing state (see panel D). Introducing a convenience yield increases $\bar{b}$ to 1.87 in **ABCD** model from 1.79 in **ABC** model. This follows from a formula for debt capacity:

$$\bar{b} = \frac{\tau(\bar{b}) - \gamma}{r - \delta + \lambda - g} \tag{50}$$

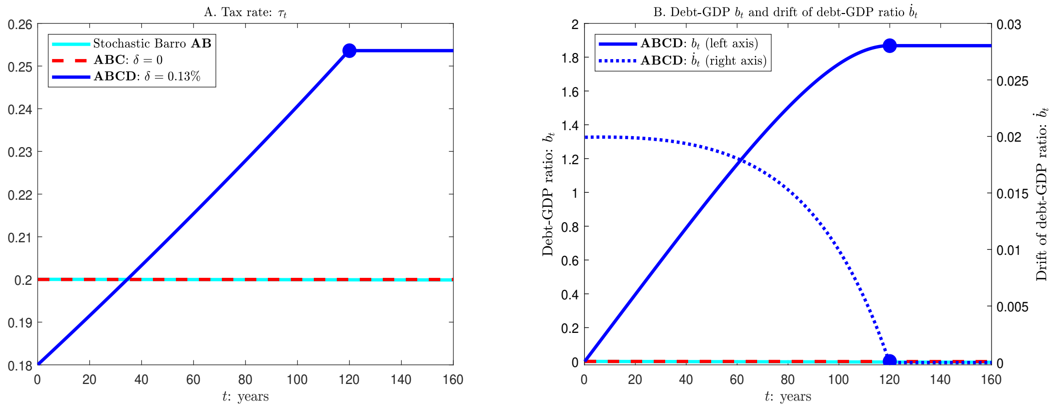
and from the optimal tax rate at debt capacity, $\tau(\,\cdot\,)$ when $b = \bar{b}$ in model **ABCD** equaling its value in model **ABC**: $\tau(\bar{b}) = 25.4\%$, an outcome that prevails even though the values of $\bar{b}$ in the two models are different.

We can characterize optimal fiscal policy in model **ABCD** by using the FOC (36) for $\tau(b)$ and the first-order nonlinear ODE (42) for $p(b)$. These present an initial-value problem with the boundary conditions: $\tau(\bar{b})$ given in (49) and $p(\bar{b}) = \frac{1-\tau(\bar{b})-\theta(\tau(\bar{b}))}{r-\delta+\lambda-g}$ when $b = \bar{b}$ at $\bar{b}$ given in (50).

The government's capacity debt-GDP ratio $\bar{b}$ is reached in finite time in model **ABCD** but not in model and **ABC** for $b_0 < \bar{b}$. The debt-output ratio b_t evolves deterministically at

[30] For all levels of b_0 up to $\bar{b} = 1.79$, $\tau(b_t) = \tau(b_0)$, $b_t = b_0$, and moreover, the solution for our **ABC** is the same as in our Stochastic Barro model (**AB**) with commitment. This is why the cyan-colored lines and the red lines overlap with each other in Figure 1.

Figure 2: **Transition of optimal tax rates τ_t and debt-GDP dynamics b_t.** The initial level of b is set at $b_0 = 0$. All other parameter values are reported in Table 1.



the rate of $\dot{b}_t = \mu^b(b_t)$ described by (37). For a given b_0, the time it takes to reach its debt capacity $\bar{b}$ is

$$\int_{b_0}^{\bar{b}} \frac{db_t}{\dot{b}_t} = \int_{b_0}^{\bar{b}} \frac{1}{(r - \delta + \lambda - g)b_t + \gamma - \tau(b_t)} db_t. \tag{51}$$

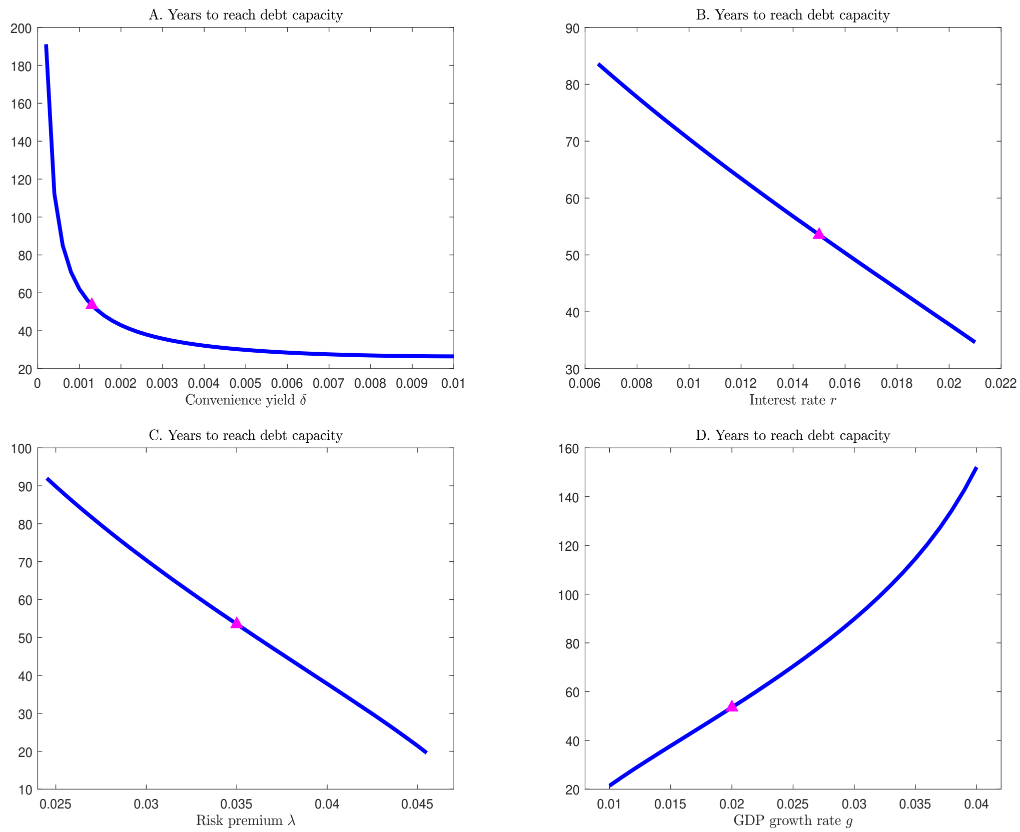
Starting from $b_0 = 0$, panels A and B of Figure 2 plot how the tax rate τ_t, debt-GDP ratio b_t, and drift $\dot{b}_t$ evolve. In models **AB** and **ABC**, $\tau_t = \gamma = 20\%$ and $b_t = 0$ for all $t \geqslant 0$. In the **ABCD** model, the government optimally increases its borrowing in order to enjoy the convenience yield ($\delta = 0.13\%$), lowers its tax rate to $\tau_0 = 18\%$ from 20%, smoothly increases its tax rate to 20% after 35 years, and then continues to increase its tax rate until it reaches its debt capacity $\bar{b} = 1.87$ in 120 years and permanently taxes households at rate $\tau(\bar{b}) = 25.4\%$.[31] Drift of the debt-GDP ratio starts high at $\dot{b}_0 = 0.02$ and gradually decreases it over time.

Figure 3 shows how the time to reach debt capacity varies with the convenience yield δ, interest rate r, risk premium λ, and GDP growth g. Panel A shows that as convenience yield δ increases, the time it takes exhaust debt capacity decreases. This happens because the government borrows faster to take advantage of its ability to issue risk-free debt cheaply. Effects on time to reach debt capacity are large even for a small increase of convenience yield δ. If $\delta = 0.13\%$, then starting from debt-GDP ratio of $b = 126.1\%$, it takes about 54 years to reach debt capacity (the pink triangle for our baseline model). It takes less than 27 years to reach the debt limit if δ were instead to be 0.73%, an estimate reported in Krishnamurthy

[31]Mian, Straub, and Sufi (2022) also predict a debt capacity around 200%, although their notion of debt capacity differs from ours. The risk premium on the Shiller macro security affects debt capacity in our model, but it is absent from theirs. Consequently, a zero primary deficit characterizes an equilibrium debt capacity in their model, but not in ours.

and Vissing-Jorgensen (2012).

Figure 3: **Time to Reach Debt Capacity as Function of Convenience Yield δ, Interest Rate r, Risk Premium λ, and GDP Growth Rate g.** For all panels, $b_0 = 126.1\%$. All other parameter values are reported in Table 1.



Panel B plots time it takes to reach its debt capacity as a function of the world risk-free rate r, holding the convenience yield fixed at $\delta = 0.13\%$. When it faces a lower interest rate, the government can finance its debt repayments with a lower tax rate $\tau(b)$, so debt capacity $\bar{b}$ is higher. The lower is r, the more valuable is a 13 basis-point interest cost reduction contributed by that convenience yield. So the government borrows more, causing its debt-GDP ratio to drift upward at a faster rate $\dot{b}_t$. Overall, it still takes longer for the government to exhaust its debt capacity when the interest rate is lower (panel B). Starting from the current US debt level of $b = 126.1\%$, it takes about 79 years to reach the debt limit if $r = 0.75\%$, but takes about 54 years to reach the debt limit if $r = 1.5\%$ (the triangle for our baseline case). This pattern aligns with reasoning by Blanchard (2019) and Furman and Summers (2020).

Panel C of Figure 3 plots time to reach debt capacity $\bar{b}$ as we vary the risk premium λ.

A higher risk premium λ makes risk management more costly, lowering debt capacity $\bar{b}$, and requiring a higher tax rate $\tau(b)$ to service debt. In response, the government lowering the drift of its debt-GDP ratio. The government reaches its debt capacity quicker the higher its risk premium λ (panel C). Across economies, variations in the risk premium have large consequences. Starting from $b = 126.1\%$, it would take 21 years to reach debt capacity if we were to increase λ to 4.5%.

GDP growth has big effects on borrowing capacity. Panel D plots times to reach debt capacity as we vary the GDP growth rate g. A higher GDP growth rate g increases a government's ability to service its debt increases its debt capacity $\bar{b}$. In response, the government increases the drift of its debt-GDP ratio b_t. The government reaches its debt capacity later when its GDP growth g is higher. Starting from $b = 126.1\%$, the time it takes to reach borrowing capacity increases 90 years when g permanently increases to 3% from $g = 2\%$.

Across all of the parameter combinations portrayed in the four panels of Figure 3, $\tau(\bar{b}) = 25.4\%$ even though $\bar{b}$ varies substantially. Since the primary surplus ratio $\tau(\bar{b}) - \gamma = 5.4\%$ at debt capacity $\bar{b}$, times to reach debt capacity depend sensitively on the convenience yield δ, the risk-free rate r, the risk premium λ, and the GDP growth rate g. Notice how these four parameters jointly determine the denominator of debt-capacity formula: $\bar{b} = \frac{\tau(\bar{b})-\gamma}{r-\delta+\lambda-g}$.

5 Stochastic Interest Rate or Spending Ratio

By specifying the interest rate and the government spending ratio as fixed parameters, in early sections we characterized a government's value function and optimal fiscal and debt-management policies with a single ODE that helped us isolate and understand our baseline model's mechanics. In this section, we study how stochastic interest rates and/or government spending GDP ratios affect optimal fiscal plans. In particular, we use a Markov state variable s_t to govern stochastic variations of the interest rate r_t and the government spending ratio γ_t. We allow the government to hedge those risks.

The state s_t can take one of two possible values, $\{\mathcal{L}, \mathcal{H}\}$. The economy starts with $s_0 = \mathcal{L}$ and moves from $\mathcal{L}$ to the absorbing state $\mathcal{H}$ at a constant rate of $\xi > 0$ per unit of time. We turn on either interest rate risk or spending-ratio risk by setting either $r_t = r^{s_t}$ or $\gamma_t = \gamma^{s_t}$.

We can represent output shock $d\mathcal{Z}_t^Y$ over dt under physical measure $\mathbb{P}$ as $d\mathcal{Z}_t^Y = \sqrt{1-\rho^2}\, d\mathcal{Z}_t^h + \rho\, d\mathcal{Z}_t^m$, where standard Brownian motion $\mathcal{Z}_t^h$ represents the idiosyncratic shock, standard Brownian motion $\mathcal{Z}_t^m$ represents the systemic shock, and ρ is the constant correlation coefficient between output shock $d\mathcal{Z}_t^Y$ and aggregate (market) shock $d\mathcal{Z}_t^m$.[32]

[32]We also refer to the systematic shock $d\mathcal{Z}_t^m$ as the market shock. For mnemonic purposes, we use

At time $t-$, the government can hedge state-transition risk in the following way. Over a small time interval dt, the government pays a unit insurance premium $\widetilde{\xi}dt$ to a counterparty in exchange for a unit payment that is made at time t if and only if the state transition occurs so that $s_t \neq s_{t-}$. This contract earns the counterparty a risk premium.[33]

Let Ψ_{t-} denote the units of this state-transition hedging contract that the government purchases at $t-$ and let $\psi_{t-} = \Psi_{t-}/Y_{t-}$. Before the state transitions to $\mathcal{H}$, the government's debt evolves according to

$$\begin{aligned} dB_t = &((r_{t-} - \delta)B_{t-} + \Gamma_{t-} - \mathcal{T}_{t-})\,dt + dH_t \\ &- \Phi^h_{t-}\sqrt{1-\rho^2}\sigma d\mathcal{Z}^h_t - \Phi^m_{t-}\rho\sigma\,(\eta dt + d\mathcal{Z}^m_t) - \widetilde{\xi}\Psi_{t-}dt + \Psi_{t-}d\mathcal{N}_t\,, \end{aligned} \tag{52}$$

where $-\Phi^h_{t-}\sqrt{1-\rho^2}\sigma$ and $-\Phi^m_{t-}\rho\sigma$ are holdings of the idiosyncratic-risk hedging asset and the systematic (diffusion) risk hedging asset, respectively, $\widetilde{\xi}\Psi_{t-}$ is the insurance premium payment for the state-transition, and $d\mathcal{N}_t = 1$ if and only if the state $\{s_t\}$ moves from $\mathcal{L}$ to $\mathcal{H}$. By setting $\Psi_{t-} < 0$, the government hedges against stochastic transition of s_t by the counterparty at the rate of $-\widetilde{\xi}\Psi_{t-} > 0$. While increasing the government's risk-free debt before the state transitions, this hedging payment lowers the government's risk-free debt if the state transitions to $\mathcal{H}$. The remaining terms in (52) are the same as those in (9) for our baseline model.

The value function $P(B_t, Y_t; \mathcal{L})$ in state $\mathcal{L}$ satisfies the HJB equation:

$$\begin{aligned} r^{\mathcal{L}}P(B,Y;\mathcal{L}) = \max_{\mathcal{T}\leqslant\overline{\tau}Y,\Phi^h,\Phi^m,\Psi}\; & Y - \mathcal{T} - \Theta(\mathcal{T},Y) + \left[\left(r^{\mathcal{L}}-\delta\right)B + \gamma^{\mathcal{L}}Y - \mathcal{T} - \widetilde{\xi}\Psi\right]P_B(B,Y;\mathcal{L}) \\ & + \frac{\sigma^2\left((1-\rho^2)(\Phi^h)^2 + \rho^2(\Phi^m)^2\right)}{2}P_{BB}(B,Y;\mathcal{L}) + \widetilde{g}YP_Y(B,Y;\mathcal{L}) \\ & + \frac{\sigma^2Y^2}{2}P_{YY}(B,Y;\mathcal{L}) - \sigma^2\left((1-\rho^2)\Phi^h + \rho^2\Phi^m\right)YP_{BY}(B,Y;\mathcal{L}) \\ & + \widetilde{\xi}\left[P(B+\Psi,Y;\mathcal{H}) - P(B,Y;\mathcal{L})\right]\,, \end{aligned} \tag{53}$$

where $\widetilde{g} = g - \lambda$ is the risk-adjusted growth rate. Because $\mathcal{H}$ is an absorbing state, the value function $P(B,Y;\mathcal{H})$ satisfies HJB equation (21). In Appendix D, we calculate $P(B,Y;\mathcal{H})$ for state $\mathcal{H}$ and value functions $\widehat{P}(Y;\mathcal{H})$ and $\widehat{P}(Y;\mathcal{L})$ in the balanced-budget regime. Exploiting homogeneity again, it is enough to compute $p(b_t;s_t) = P(B_t,Y_t;s_t)/Y_t$

superscript m to refer to the *market* shock and the superscript h to refer to the *hedgeable* idiosyncratic shock.

[33]The ratio $\widetilde{\xi}/\xi$ captures the risk premium associated with the state s_t transition (see, e.g., Duffie (2001), Chen (2010), and Bolton, Chen, and Wang (2013) for details.)

and $\widehat{p}(s_t) = \widehat{P}(Y_t; s_t)/Y_t$.

For $s_t = \mathcal{L}, \mathcal{H}$, let $\tau(b_t; s_t)$, $\phi^h(b_t; s_t)$, $\phi^m(b_t; s_t)$, $\overline{b}(s_t)$, and $\underline{b}(s_t)$ denote the optimal tax rate, optimal idosyncratic risk hedging demand, optimal systematic risk hedging demand, debt capacity, and the scaled lumpy debt issuance boundary, respectively. FOCs for taxes $\tau(b; s)$ and diffusion hedging demand $\phi(b; s)$ are

$$1 + \theta'(\tau(b; s)) = p'(b; s) \quad \text{and} \quad \phi^h(b; s) = \phi^m(b; s) = -b \tag{54}$$

in state $s = \mathcal{L}, \mathcal{H}$. We focus on the FOC for the jump hedging demand $\psi(b)$ in state $\mathcal{L}$, a new part of the model relative to our baseline fixed parameter model. When the post-transition level of the debt-GDP ratio $b + \psi(b) \in [0, \overline{b}(\mathcal{H})]$ so that $b + \psi(b)$ can be said to be admissible, the government chooses $\psi(b)$ to keep its marginal cost $-p'(b; s)$ of servicing debt constant when the state s transitions from $\mathcal{L}$ to $\mathcal{H}$:

$$p'(b; \mathcal{L}) = p'(b + \psi(b); \mathcal{H}) \,. \tag{55}$$

Condition (55) uniquely pins down $\psi(b)$, the scaled demand for hedging against state transitions. Otherwise, the post-s_t-transition level of b is at the corner of the admissible set so that either $b + \psi(b) = \overline{b}(\mathcal{H})$ or $b + \psi(b) = 0$ holds, which we discuss below.

The debt-GDP ratio limit $\overline{b}(\mathcal{L})$ for state $\mathcal{L}$ satisfies

$$\overline{b}(\mathcal{L}) \;=\; \frac{\tau(\overline{b}(\mathcal{L}); \mathcal{L}) - \gamma^{\mathcal{L}} + \widetilde{\xi}\psi(\overline{b}(\mathcal{L}))}{r^{\mathcal{L}} - \delta + \lambda - g} = \min\left\{\breve{b}(\mathcal{L})\,, \frac{\overline{\tau} - \gamma^{\mathcal{L}} + \widetilde{\xi}\psi(\overline{b}(\mathcal{L}))}{r^{\mathcal{L}} - \delta + \lambda - g}\right\}, \tag{56}$$

where $\breve{b}(\mathcal{L})$ is the unique positive root of $p(\breve{b}(\mathcal{L})) = \alpha\widehat{p}(\mathcal{L})$ and $\widehat{p}(\mathcal{L})$ is the scaled fiscal planner's value in the balanced-budget regime for state $\mathcal{L}$ given in Appendix D.

The following proposition describes optimal fiscal policy.[34]

Proposition 5.1. *Under Assumption 3.1 for $s = \mathcal{L}, \mathcal{H}$, the scaled value function $p(b; \mathcal{L})$ in state $s = \mathcal{L}$, satisfies the following nonlinear differential equation:*

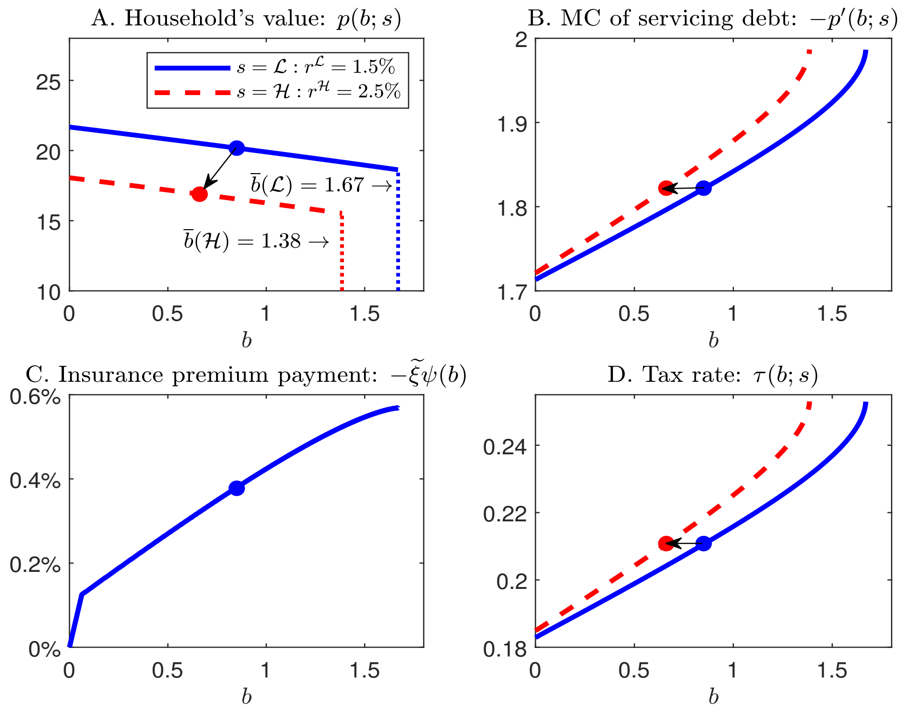
$$\begin{aligned}\left[r^{\mathcal{L}} + \lambda - g\right] p(b; \mathcal{L}) = {} & \left[(r^{\mathcal{L}} - \delta + \lambda - g)b + \left(\gamma^{\mathcal{L}} - \tau(b; \mathcal{L})\right) - \widetilde{\xi}\psi(b)\right]p'(b; \mathcal{L}) \\ & + 1 - \tau(b; \mathcal{L}) - \theta(\tau(b; \mathcal{L})) + \widetilde{\xi}[p(b + \psi(b); \mathcal{H}) - p(b; \mathcal{L})]\end{aligned} \tag{57}$$

for $b \in [0, \overline{b}(\mathcal{L})]$ where $\overline{b}(\mathcal{L})$ is debt capacity determined by (56). The optimal tax rate

[34] The scaled value $p(b; \mathcal{H})$, debt capacity $\overline{b}(\mathcal{H})$, debt issuance boundary $\underline{b}(\mathcal{H})$, and optimal tax rate policy $\tau(b; \mathcal{H})$ are the same as in our baseline model as $\mathcal{H}$ is an absorbing state.

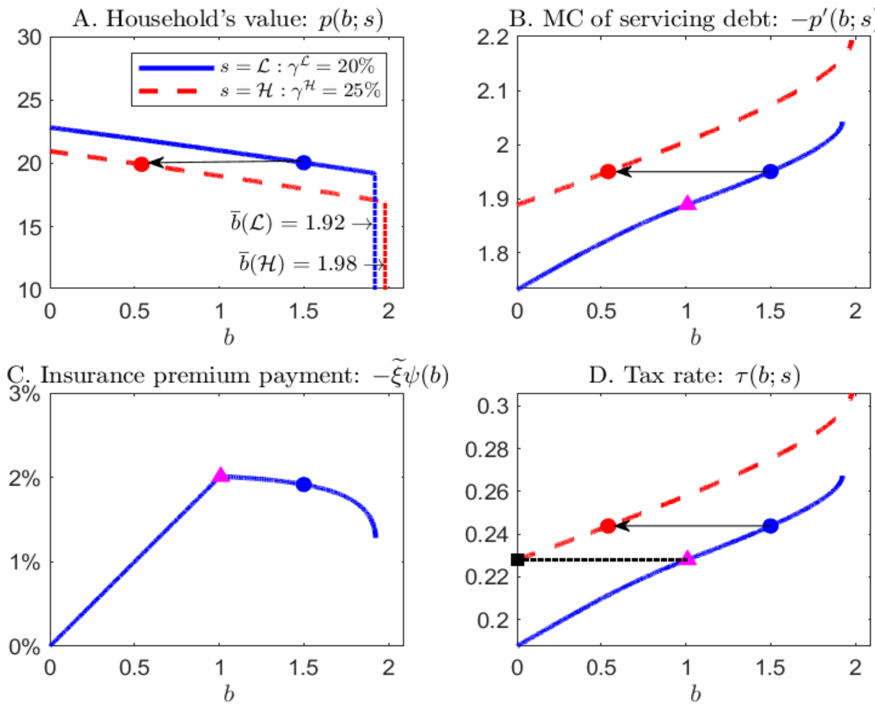
policy $\tau(b;\mathcal{L})$ *and risk hedging demand* $\phi^h(b;\mathcal{L})$ *and* $\phi^m(b;\mathcal{L})$ *satisfy (54). The lumpy debt issuance boundary* $\underline{b}(\mathcal{L})$ *is* $\arg\max_{\underline{b}\geqslant 0}\ p(\underline{b},\mathcal{L})+\underline{b}$ *and the optimal scaled lumpy debt issuance is* $dh_t=\max\{\underline{b}(\mathcal{L})-b_t,0\}$.

Figure 4: **Stochastic Interest Rate.** Interest rate r_t stochastically increases from $r^{\mathcal{L}}=1.5\%$ to $r^{\mathcal{H}}=2.5\%$. Other parameters are from Table 1.



To illustrate effects of a stochastic interest rate or spending ratio on the government's optimal fiscal plans, we set the risk-adjusted state transition rate from $\mathcal{L}$ to $\mathcal{H}$, $\widetilde{\xi}$, to 2% for per year. First, consider a situation in which the interest rate r_t stochastically increases from $r^{\mathcal{L}}=1.5\%$ to $r^{\mathcal{H}}=2.5\%$. Figure 4 plots an optimal fiscal plan. Anticipating that the interest rate will probabilistically increase, the government buys insurance and pays a time-varying insurance premium (given in panel C) as a function of b_{t-}. When the interest rate jumps to $r^{\mathcal{H}}=2.5\%$ at time t, the government receives a lump-sum transfer $\psi(b_{t-})Y_{t-}$ from its insurance counterparty, which lowers its debt balance in state $\mathcal{H}$ (see the arrow in panel A). This state-contingent hedging contract optimally keeps its tax rate $\tau(b)$ and the marginal cost of servicing debt $-p'(b)$ constant in the face of the interest rate shock (see the horizontal arrows in panels B and D.) The upward interest-rate shock lowers debt capacity in the $\mathcal{L}$ state (where $r_t = r^{\mathcal{L}} = 1.5\%$) to $\bar{b}(\mathcal{L}) = 1.67$ from $\bar{b} = 1.87$, debt capacity in the constant-interest-rate setting where $r_t = r^{\mathcal{L}} = 1.5\%$ at all t.

Figure 5: **Stochastic Government Spending.** The spending ratio γ_t stochastically increases from $\gamma^{\mathcal{L}} = 20\%$ to $\gamma^{\mathcal{H}} = 25\%$. All other parameters are given in Table 1.



Unlike outcomes in our **ABCD** diffusion model, an optimal tax rate can be discontinuous even when aggregate s_t state transition shocks are hedgeable. The stochastic-s_t version of our model thus modifies Barro's tax smoothing result substantially. Figure 5 illustrates this for a setting in which the government spending ratio stochastically increases from $\gamma^{\mathcal{L}} = 20\%$ to $\gamma^{\mathcal{H}} = 25\%$. The figure shows the optimal fiscal plan.[35] If $b_t \geqslant 1.01$, the government is able to keep its tax rate unchanged in response to its government spending shock γ_t. Thus, when $b_t = 1.01$. the optimal tax rate remains unchanged from its pre-jump level of 22.8% (see the horizontal dashed line between the black square and the pink triangle in panel D.) However, if $b_t < 1$, the government no longer keeps its tax rate unchanged because when the government's spending ratio γ_t jumps to $\gamma^{\mathcal{H}} = 25\%$, its debt capacity and the benefits/costs

[35] This new (jumpy tax rate) prediction is also present in the stochastic interest-rate model analyzed above (see Figure 4), but it is less visible and quantitatively less important.

of hedging s_t shock jump discretely, making it too costly to insulate the marginal cost $-p'(b)$ of servicing debt from the γ_t shock. Thus, when $b_t = 0.25$, it is optimal to increase the tax rate to 22.8% (see the square on the vertical axis in panel D) from 19.9% in response to the increase of γ_t. Anticipating prospective higher government spendings, it is optimal for the government to manage its γ_t shock before it arrives and raise the tax rate immediately when the shock arrives.

6 GDP Jumps and Defaultable Government Debt

We extend the **ABCD** model in Section 2 by adding downward discontinuous (jump) GDP shocks that cannot be hedged. This creates circumstances in which the government chooses to default and brings our tractable continuous-time formulation even closer to international macro papers such as Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008) in which governments default, a departure from limited-commitment models with a complete set of state-contingent securities, e.g., Kehoe and Levine (1993) and Kocherlakota (1996).

6.1 The Setting

GDP $\{Y_t; t \geqslant 0\}$ now evolves according to a geometric jump-diffusion process that is subject both to idiosyncratic shocks $\mathcal{Z}_t^h$ that bear no risk premium and to systematic shocks $\mathcal{Z}_t^m$ that do bear a risk premium:

$$\frac{dY_t}{Y_t} = gdt + \left(\sqrt{1-\rho^2}\,\sigma d\mathcal{Z}_t^h + \rho\sigma d\mathcal{Z}_t^m\right) - (1-Z)d\mathcal{J}_t\,. \tag{58}$$

Here $\mathcal{J}_t$ is a pure jump process with a constant arrival rate ζ.[36] If no jump occurs at date t ($d\mathcal{J}_t = 0$), $Y_t = Y_{t-}$, where $Y_{t-} \equiv \lim_{s\uparrow t} Y_s$ denotes the left limit of the output. If a jump does arrive at date t ($d\mathcal{J}_t = 1$), GDP changes from Y_{t-} to $Y_t = ZY_{t-}$, where $Z \in [0,1]$ is a random variable with cumulative distribution function $Q(\,\cdot\,)$. As in the rare-disaster literature (e.g., Barro (2006), Gabaix (2012), Pindyck and Wang (2013), and Wachter (2013)), we assume that recovery fraction Z is governed by a power law (Gabaix, 2009) so that:

$$Q(Z) = Z^{\beta}\,, \quad 0 \leqslant Z \leqslant 1\,. \tag{59}$$

[36]To capture a small open economy, we assume that GDP jump risk is idiosyncratic, unhedgeable and thus bears no risk premium.

The smaller is β, the more fat-tailed is the distribution of $(1-Z)$. Although diffusion shocks are hedgeable, jump shocks are not.

In the no-default regime, the government debt B_t evolves as

$$dB_t = \left[(r+\pi_{t-}-\delta)B_{t-} + (\Gamma_{t-} - \mathcal{T}_{t-})\right]dt + dH_t - \Phi^h_{t-}\sqrt{1-\rho^2}\sigma d\mathcal{Z}^h_t - \Phi^m_{t-}\rho\sigma\left(\eta dt + d\mathcal{Z}^m_t\right) , \tag{60}$$

where π_{t-} is the equilibrium credit spread and $-\Phi^h_{t-}\sqrt{1-\rho^2}\sigma$ and $-\Phi^m_{t-}\rho\sigma$ are holdings of the idiosyncratic risk hedging asset and the systematic risk hedging asset, respectively. As in discrete-time sovereign-debt models, e.g., Eaton and Gersovitz (1981), debt is priced in a symmetric Markov-perfect equilibrium. In Appendix E, we show that the equilibrium credit spread π_{t-} equals $\pi(b_{t-})$, where

$$\pi(b_{t-}) = \zeta Q(b_{t-}/\overline{b})\,, \tag{61}$$

ζ is the jump arrival rate,and the equilibrium debt capacity $\overline{b}$ is described by equation (66) to be discussed below. Equation (61) ties the equilibrium credit spread to the country's default strategy. For a unit of debt per unit of time, the left side of equation (61) is the compensation for bearing credit risk and the right side is the expected loss given default. The government defaults when $b_t > \overline{b}$. Because downward GDP jump shocks are uninsurable, the government sometimes chooses to default.

The household's value $P(B,Y)$ in the no-default regime satisfies the HJB equation:

$$\begin{aligned} rP(B,Y) = \max_{\mathcal{T},\,\Phi^h,\,\Phi^m} \; & (Y - \mathcal{T} - \Theta(\mathcal{T},Y)) + \left[(r+\pi-\delta)B + \Gamma - \mathcal{T}\,\right] P_B(B,Y) \\ & + \frac{\sigma^2\left((1-\rho^2)(\Phi^h)^2 + \rho^2(\Phi^m)^2\right)}{2} P_{BB}(B,Y) + \widetilde{g}\,Y P_Y(B,Y) + \frac{\sigma^2 Y^2}{2} P_{YY}(B,Y) \\ & - \sigma^2\left((1-\rho^2)\Phi^h + \rho^2\Phi^m\right) Y P_{BY}(B,Y) + \zeta\mathbb{E}\left[P(B,ZY) - P(B,Y)\right] , \end{aligned} \tag{62}$$

where $\widetilde{g} = g - \lambda$. Appendix E presents the household's value function $\widehat{P}(Y)$ in the balanced-budget regime and shows that $\widehat{p} = \widehat{P}(Y)/Y$ is a constant described by equation (A-34).

Applying the Ito's Lemma and the optimal hedging policy $\phi^h(b) = \phi^m(b) = -b$ lets us show that the debt-output ratio $\{b_t\}$ evolves according to

$$db_t = \left[(r+\pi(b_{t-}) - \delta + \lambda - g)b_{t-} + \gamma - \tau(b_{t-})\right]dt + \left(Z^{-1}-1\right) b_{t-} d\mathcal{J}_t\,. \tag{63}$$

Let $\breve{b}$ denote the root of:

$$1 + \theta'((r + \pi(\breve{b}) - \delta + \lambda - g)\breve{b} + \gamma) = -p'(\breve{b})\,. \tag{64}$$

In the region where $b \in (\breve{b}, \overline{b}]$, the government optimally sets its tax rate according to:

$$\tau(b) = (r + \pi(b) - \delta + \lambda - g)b + \gamma\,. \tag{65}$$

This policy ensures that the drift of b_t is zero in the absence of jumps. If a jump arrives at t, the debt-output ratio jumps from b_{t-} to $b_t = b_{t-}/Z$. If $b_t < \overline{b}$, the tax rate jumps to $\tau(b_t)$ given by (65) and the government then sets $b_s = b_t$ for $s \geqslant t$ until another jump arrives. However, if a jump arrives that causes $b_t = b_{t-}/Z$ to exceed $\overline{b}$, the government defaults.

In Appendix E, we show that $\overline{b}$ satisfies

$$\overline{b} = \frac{\tau(\overline{b}) - \gamma}{r + \zeta - \delta + \lambda - g} = \min\left\{\breve{b},\ \frac{\overline{\tau} - \gamma}{r + \zeta - \delta + \lambda - g}\right\}\,, \tag{66}$$

where $\breve{b}$ is the unique positive root of $p(\breve{b}) = \alpha\widehat{p}$. In our Section 2 **ABCD** model in which jumps are not present, the drift of b_t (absent jumps) equals zero only at debt capacity $\overline{b}$. Now with jump shocks present, the drift of b_t equals zero for a range of b_t values.

The equilibrium debt capacity b in our jump-diffusion model is lower than that in our baseline **ABCD** model and also the equilibrium credit spread is higher due to the jump risk.

Proposition 6.1. *Under conditions $r - \delta + \lambda > g - \zeta(1 - \mathbb{E}(Z))$, $\kappa \geqslant 1$, $\alpha \leqslant 1$, and $1 - \gamma/\alpha - \kappa\theta(\gamma/\alpha) \geqslant 0$, the scaled value function $p(b)$ in the no-default regime satisfies the nonlinear differential equation:*

$$\begin{aligned}[r + \lambda - g]\,p(b) \quad &= \quad 1 - \tau(b) - \theta(\tau(b)) + \left[(r + \pi(b) - \delta + \lambda - g)b + \gamma - \tau(b)\right]p'(b) \\ &\qquad + \zeta\mathbb{E}\left[Zp(b/Z) - p(b)\right]\,,\end{aligned} \tag{67}$$

for $b \in [0, \overline{b}\,]$, where $\overline{b}$ is debt capacity determined by (66).The lump-sum debt issue boundary $\underline{b}$ is described by (40), and the optimal lump-sum transfer policy dh_t is given by (41). The optimal tax rate policy $\tau(b)$ is described by (36) for $\underline{b} < b < \breve{b}$ and (65) for $\breve{b} \leqslant b \leqslant \overline{b}$, where $\breve{b}$ solves (64). The debt-output ratio $\{b_t\}$ process is described by (63).

6.2 Quantitative Illustration

To provide a quantitative illustration, we use the parameter values from our quantitative illustration of the baseline **ABCD** model in Section 4. The additional parameters for the jump-diffusion output process (58), we follow Rebelo, Wang, and Yang (2022) by setting the power-law parameter (β) to 6.3 and the jump arrival rate (ζ) to 0.073 per annum. To capture a small open economy, we assume that the jump risk is idiosyncratic and hence carries no risk premium.

Figure 6: **Optimal fiscal plan when output is subject to unhedgeable downward jump shocks and government debt is defaultable.**

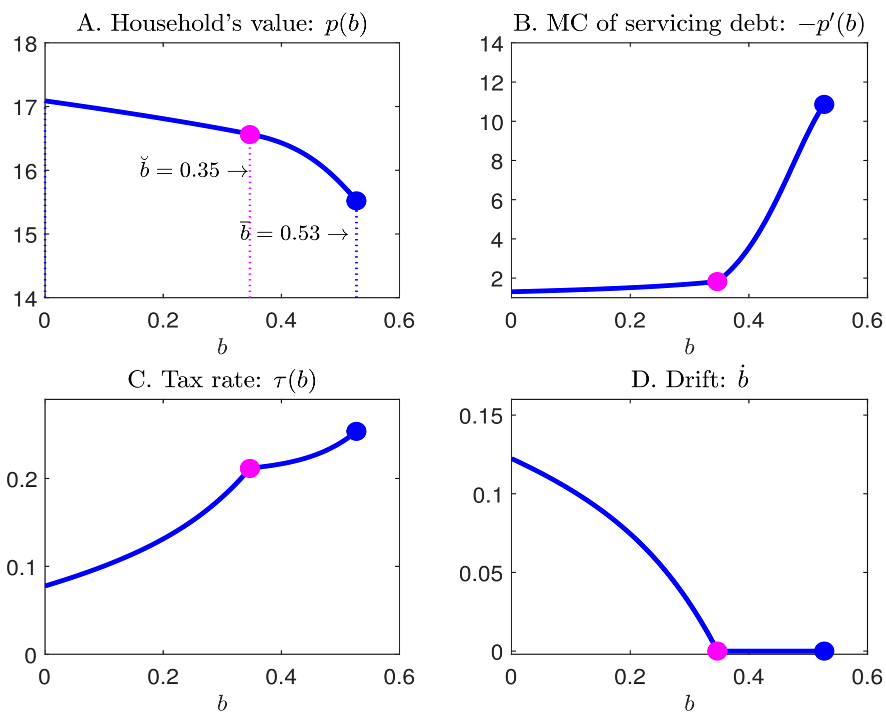


Figure 6 plots value and policy functions. Panels A and B show that the value function is decreasing and concave in b. Introducing unhedgeable jump shocks substantially lowers debt capacity to $\overline{b} = 0.53$ from 1.87 in our baseline model. When the debt balance is low, e.g., when $b \leqslant \breve{b} = 0.35$, the government optimally increases its tax rates smoothly over time in order to take advantage of its convenience yield. This is the tax-smoothing region.

When the debt-GDP ratio b is larger than the debt capacity $\overline{b} = 0.53$, the government defaults and enters the balanced-budget regime. The $b \geqslant \overline{b} = 0.53$ is the default region.

For b_t that lies between the tax-smoothing region and the default region, i.e., if b in the "cushion" region where $b \in [\breve{b}, \overline{b}] = [0.35, 0.53]$, the government keeps its b_t constant absent jumps in order to lower its probability of defaulting subsequently (see Panel D). However, when a jump arrives, the debt-GDP ratio increases to b_t/Z from b_t. Depending on the realized value of Z, the government either defaults (if $b_t/Z > \overline{b} = 0.53$) or continues to stay in this "cushion" region but with a higher debt-GDP ratio b_t/Z.

The zero-drift result for b in this cushion region is accompanied by the tax policy described by (65); notice how this policy differs from the policy (36) which is optimal in the $b \leqslant \breve{b}$ tax-smoothing region. Because tax policies differ in the $b \leqslant \breve{b}$ tax-smoothing and the $b \in (\breve{b}, \overline{b}]$ cushion regions, the tax policy $\tau(b)$ and the marginal cost of servicing debt $-p'(b)$, while continuous, are not smooth when $b = \breve{b}$.

7 Concluding Remarks

We have studied convergence to a maximal sustainable government's debt-to-GDP ratio together the associated optimal tax and debt management policy. Because we have cast it in continuous time, our model is tractable and solvable mostly by hand. It includes three types of agents: 1.) a representative household that ranks outcomes according to the expected value of intertemporal utility; 2.) investors in competitive financial markets; and 3) a benevolent government that finances an exogenous government expenditure path with taxes that inflict deadweight losses described by the convex function used by Barro (1979). As exogenous stochastic processes, the government confronts: a discount factor process that prices state-contingent securities; a GDP process with idiosyncratic (i.e., country-specific) and systemic components; and government spending that is a fraction of GDP. The government can default on its debt, an option that imposes endogenous credit constraints on it. The government issues risk-free debt that carries a convenience yield. The benevolent planner's value function p depends on a single state variable, a debt-to-GDP ratio b_t, whose dynamics reflect (i) primary deficit, (ii) interest payment, (iii) GDP growth, and (iv) costs for hedging systemic shocks.

Value function $p(b)$ measures the maximum present value of after-tax income net of deadweight losses from taxes that a budget-feasible tax and debt management plan can deliver to a representative household. Barro's (1979) convex tax distortion cost function shapes the marginal cost $-p'(b)$ of increasing the debt-GDP ratio and also, when risk-free government debt carries a convenience yield, the rate at which the government chooses gradually to approach that limit from below. The presence of deadweight taxation costs and the government's option to default on its debt combine to make $-p'(b)$ exceed one.

We actually walk through *four* p theories as we successively activate the **A**, **B**, **C**, and **D** components that together form the **ABCD** model portrayed in Figure 1. The figure depicts two endogenous debt-to-GDP thresholds ($\underline{b}$ and $\overline{b}$) that characterize the **ABCD** model. If $b < \underline{b}$, the government immediately jumps its b to $\underline{b}$. If $b \in [\underline{b}, \overline{b}]$, the b_t level moves smoothly over time toward the debt capacity $\overline{b}$. After reaching the debt capacity, the economy stays there forever. We calculated that introducing limited commitment and a convenience yield substantially can lower debt-GDP capacity from the 10 associated with an arbitrary 50% Keynes constraint (6) on the tax rate to an empirically more plausible level of 1.87.

Section 5 extends our baseline **ABCD** model to include stochastic interest rates and government spending-output ratios. Section 6 then extends **ABCD** model by including unhedgeable output jumps. When a negative jump shock pushes the country's debt-GDP ratio too high, the government optimally defaults.[37] Introducing unhedgeable jumps generates a fourth "cushion" region that lies between the "tax smoothing" and "default" region, but is absent in our **ABCD** diffusion model. In this fourth region, the government keeps its b_t invariant absent jumps to reduce the cost of inefficient default.

We have called our model "normative" because, being cast as an optimal fiscal policy problem, it studies "what should be?" rather than the "what is?" questions that concern a positive analysis. Nevertheless, our exposition has occasionally strayed across the normative-positive line, for example, when we calculated the marked drop in the sustainable debt-GDP ratio from 10 to 1.87 that accompanies introduction of the default option component **C** in a numerical illustration of our **ABCD** model.[38] Applied macroeconomists including ourselves have used contending normative models to interpret observed monetary and fiscal policies.[39]

It would be fruitful to explore other possible sources for key forces in our model. For example, a similar but not observationally equivalent way to generate a positive drift in the debt-GDP dynamics would be to make the household whose welfare the government

[37]The mechanics here resemble ones operating in parts of the sovereign debt literature, e.g., Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), and Arellano (2008).

[38]Many economists have productively crossed the thin line separating "normative" and "positive" models. Thus, theoretical work on "rationalizable" decision rules starts from observed behavior patterns and reverse engineers normative models of purposes and constraints that explain them. Bernheim (1984), Pearce (1984), and Muth (1960) provide instructive examples. Lucas (1987, Sec. II) discussed pros and cons of normative and positive uses of models in applied macroeconomics.

[39]For example, Hall and Sargent (2014, 2021) applied both the Barro (1979) and the Lucas and Stokey (1983) to interpret patterns observed in US monetary-fiscal history, while Sargent and Velde (1995) used the same two models to organize observations about French fiscal policies before and during the Revolution (1789-1799). The Barro (1979) model guided actions of Secretary of Treasury Albert Gallatin during the Jefferson and Madison administrations (see Gallatin (1837)), while Figure 2 of Sargent and Velde (1995) depicting 18th century British fiscal policy resembles a simulation of the Barro (1979) model.

maximizes be more impatient than the government's creditors (Amador (2004) and Aguiar and Amador (2021)). A subtle difference in outcomes emerges *vis a vis* the convenience yield specification in our **ABCD** model: the representative household's impatience wouldn't affect the government's debt capacity because households are equally impatient before and after the government's defaults in the alternative specification.

To construct streamlined formulas that isolate forces that shape optimal fiscal policy, debt capacity, and debt dynamics, we purposefully chose to work with a limited-commitment model in which a shock to GDP growth is the sole aggregate shock. For some purposes it would be useful to introduce additional aggregate shocks, for example, the monetary shock that Lucas (1987, Sec. VI) stressed as an independent source of both price level fluctuations and business cycles. In subsequent work, we plan to extend our model to include a quantity theory of money and an inflation tax as an additional source of government revenues. That will force us to confront some monetary-fiscal policy coordination issues described by Sargent and Wallace (1981) and Bassetto and Sargent (2020).

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Appendices

A Optimal Fiscal Plan for ABCD Model

We provide technical details for the optimal plan that appeared in Section 3 for the dynamic debt management problem defined in Section 2, discuss special cases of our ABCD model, then extend our ABCD model to a setting in which a government that has defaulted regains access to international capital markets.

A.1 Technical Details for the ABCD Model Solution

HJB equation for $P(B, Y)$. Consider the tax-smoothing region where $dH_t = 0$. Using Ito's formula, we obtain the following SDF-adjusted dynamics for the household's value function $P(B_t, Y_t)$:

$$d(\mathbb{M}_t P(B_t, Y_t)) = \mathbb{M}_t dP(B_t, Y_t) + P(B_t, Y_t) d\mathbb{M}_t + < d\mathbb{M}_t, dP(B_t, Y_t) >, \tag{A-1}$$

where the SDF $\{\mathbb{M}_t; t \geqslant 0\}$ is given in (3) and

$$\begin{aligned} dP(B_t, Y_t) &= P_B dB_t + \frac{P_{BB}}{2} < dB_t, dB_t > + P_Y dY_t + \frac{P_{YY}}{2} < dY_t, dY_t > + P_{BY} < dB_t, dY_t > \\ &= \left[\left((r - \delta) B_t + (\Gamma_t - \mathcal{T}_t) - \lambda \Phi_t \right) P_B + g Y_t P_Y \right] dt - \sigma \Phi_t P_B d\mathcal{Z}_t^Y + \sigma Y_t P_Y d\mathcal{Z}_t^Y \\ &\quad + \left[\frac{\sigma^2 Y_t^2 P_{YY}}{2} + \frac{\sigma^2 \Phi_t^2 P_{BB}}{2} - \sigma^2 \Phi_t Y_t P_{BY} \right] dt. \end{aligned} \tag{A-2}$$

The process defined by

$$\int_0^t \mathbb{M}_s \left(Y_s - \mathcal{T}_s - \Theta(\mathcal{T}_s, Y_s) \right) ds + \mathbb{M}_s dH_s + \mathbb{M}_t P(B_t, Y_t)$$

is a martingale under physical measure $\mathbb{P}$ so its drift under $\mathbb{P}$ is zero:

$$\mathbb{E}_t \left[d \left(\mathbb{M}_t P(B_t, Y_t) \right) \right] + \mathbb{M}_t \left(Y_t - \mathcal{T}_t - \Theta(\mathcal{T}_t, Y_t) \right) dt = 0. \tag{A-3}$$

Simplifying (A-3) gives HJB equation (21) for value function $P(B_t, Y_t)$. First-order conditions (FOCs) for tax and risk management policies, respectively, are given in (22) and (23).

Debt-GDP ratio b_t dynamics. Using the government's optimal risk management policy $\phi_t = -b_t$ implied by (23) and applying the Ito's Lemma to $b_t = B_t / Y_t$, where B_t is given in (9) and Y_t is given in (2), we obtain the following b_t process in the tax-smoothing region:

$$db_t = \mu_t^b \, dt + \sigma_t^b \, d\mathcal{Z}_t^Y = \mu^b(b_t) dt \,, \tag{A-4}$$

where the volatility of b_t is zero, $\sigma_t^b = -\sigma\phi_t + \sigma b_t = 0$, and the drift of b_t is

$$\mu_t^b = \mu^b(b_t) = (r - \delta + \lambda - g)b_t + \gamma - \tau(b_t)\,. \tag{A-5}$$

Lemma A.1. *Under Condition 3.1, the equilibrium debt capacity $\overline{b}$ exists and is uniquely determined by (39).*

Proof. Equations (38) and (42) imply

$$p(\overline{b}) = \frac{1 - \tau(\overline{b}) - \theta(\tau(\overline{b}))}{r - \delta + \lambda - g}\,, \tag{A-6}$$

where $\tau(\overline{b}) = (r - \delta + \lambda - g)\overline{b} + \gamma$. Debt capacity $\overline{b}$ solves one of the following two equations

$$p(\overline{b}) = \alpha\widehat{p}\,, \quad \text{when the tax rate constraint (6) does not bind}\,; \tag{A-7}$$

$$\tau(\overline{b}) = \overline{\tau}\,, \quad \text{when the tax rate constraint (6) binds}\,. \tag{A-8}$$

If tax constraint (6) binds, debt capacity $\overline{b}$ is the unique solution of (A-8): $\overline{b} = \frac{\overline{\tau}-\gamma}{r-\delta+\lambda-g}$. If tax constraint (6) does not bind, we can show that debt capacity is the solution of (A-7), that it exists, and that it is unique. First, (A-6) implies that the left side of (A-7) is decreasing $\overline{b}$. Second, the left side of (A-7) when $\overline{b} = 0$ equals $\frac{1-\gamma-\theta(\gamma)}{r-\delta+\lambda-g}$, which is strictly larger than the right side of (A-7), given that the deadweight loss function $\theta(\,\cdot\,)$ is increasing and convex (in addition to the $\kappa \geqslant 1$ and $\alpha \leqslant 1$ conditions). Third, the left side of (A-7) approaches negative infinity as $\overline{b} \to \infty$. Therefore, there exists a unique value of $\overline{b} > 0$ at which (A-7) holds with equality. Let $\check{b} > 0$ denote the unique positive root of (A-7), which can be simplified as $1 - (r - \delta + \lambda - g)b - \theta((r - \delta + \lambda - g)b + \gamma) = \alpha - \alpha\kappa\theta(\gamma/\alpha)$. Thus, debt capacity exists and is uniquely determined by (39). □

A.2 Special Cases: Model A, Model AB, and Model ABC

Model A (Ricardian equivalence). We revisit the Ricardian equivalence logic of Barro (1974) via model **A** that includes a complete set of Arrow's one-period ahead securities. The household's value (18) at time 0 becomes

$$P_0 = \mathbb{E}_0 \int_0^\infty \mathbb{M}_t \left[dH_t + (Y_t - \mathcal{T}_t)\,dt\right] \tag{A-9}$$

and the government's budget constraint holds with equality:

$$B_0 = \mathbb{E}_0 \int_0^\infty \mathbb{M}_t \left[(\mathcal{T}_t - \Gamma_t)\,dt - dH_t\right]\,. \tag{A-10}$$

Combining (A-9) and (A-10), we obtain a single budget constraint for the household:

$$P_0 + B_0 = \mathbb{E}_0 \left[\int_0^{\infty} \mathbb{M}_t \left(Y_t - \Gamma_t \right) dt \right] . \tag{A-11}$$

Expression (A-11) states that the total value $V_0 = P_0 + B_0$ is independent of policies $\{H_t, \mathcal{T}_t; t \geqslant 0\}$. Corollary 3.3 summarizes this Ricardian equivalence for the **A** model.

Model AB (A Stochastic Version of Barro (1979)). Our **AB** model excludes the **C** and **D** features of our **ABCD** model but includes Barro's tax distortions and a complete set of Arrow's one-period-ahead securities. The government maximizes

$$\mathbb{E}_0 \left[\int_0^{\infty} \mathbb{M}_t \left[dH_t + \left(Y_t - \left(\mathcal{T}_t + \Theta_t \right) \right) dt \right] \right] \tag{A-12}$$

subject to its budget constraint (A-10) and the Keynes constraint (6) on the tax rate. Using (A-10), we rewrite the government's objective (A-12) as:

$$\mathbb{E}_0 \left[\int_0^{\infty} \mathbb{M}_t \left(Y_t - \Gamma_t - \Theta_t \right) dt \right] - B_0 . \tag{A-13}$$

Choosing $\{\mathcal{T}_t; t \geqslant 0\}$ to maximize (A-13) is equivalent to minimizing the value of deadweight losses $\mathbb{E}_0 \left[\int_0^{\infty} \mathbb{M}_t \Theta_t dt \right]$ subject to the constraint of honoring an initial debt B_0 that satisfies (A-10). This was Barro's justification for recasting the government's value maximization problem as a deadweight loss minimization problem. However, that equivalence does not hold in our **ABCD** model because the government's option to default induces an endogenous distortion cost. Therefore, the government in our **ABCD** model maximizes household's value function.

Model ABC (Stochastic Barro (1979) with Default Option). The government can default so its debt capacity is smaller in our **ABC** model whenever the Keynes tax constraint (6) does not bind. Because government risk-free debt carries no extra convenience yield, the tax rate and debt-GDP ratio are both constant over time.

A.3 Extension with Finite Balanced-budget Regime Duration

Our baseline **ABCD** model assumes that the government stays in the balanced-budget regime permanently after reneging on its liability. We extend that model by letting the government to regain its access to international capital markets with probability χ per unit of time after entering into the balanced-budget regime. This realistic assumption appears in Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008), and much of the international macro literature.

Let T^{ε} denote the government's stochastic exogenous exit time from the balanced-budget regime. Upon returning to the no-default regime at T^{ε}, the household's value function is

$P(0, Y_{T^\varepsilon})$, where output is continuous at T^ε in the sense that $Y_{T^\varepsilon} = \widehat{Y}_{T^\varepsilon}$. The household's value function in the balanced-budget regime $\widehat{P}(\widehat{Y})$ satisfies

$$(r+\chi)\widehat{P}(\widehat{Y}) = \widehat{Y} - \Gamma - \widehat{\Theta}(\Gamma, \widehat{Y}) + (g - \rho\eta\sigma)\widehat{Y}\widehat{P}'(\widehat{Y}) + \frac{\sigma^2 \widehat{Y}^2}{2}\widehat{P}''(\widehat{Y}) + \chi P(0, \widehat{Y}) . \quad \text{(A-14)}$$

The scaled value $\widehat{p}$ in the balanced-budget regime is

$$\widehat{p} \;=\; \frac{1 - \gamma/\alpha - \kappa\theta(\gamma/\alpha) + \chi p(0)}{r + \lambda - g + \chi} . \quad \text{(A-15)}$$

B More Illustrations for Section 4

We use the annual debt-output ratio from 1980 to 2020 for the US. US debt and GDP data are from FRED provided by St. Louis Fed: https://fred.stlouisfed.org. Let $\Omega = \{\delta, \kappa, \varphi\}$. Our model asserts that the government debt-GDP ratio b_t grows deterministically at rate $\dot{b}_t \equiv \mu^b(b_t)$ given in (37). Let $\mu^b(b_t; \Omega)$ denote the drift of b given Ω. To account for measurement errors, we introduce a noise term into the law of motion (37) for b_t and discretize the b_t process:

$$b_{t_{i+1}} = b_{t_i} + \mu^b(b_{t_i}; \Omega)(t_{i+1} - t_i) + \varepsilon_{i+1} , \quad i = 1, 2, \cdots , \quad \text{(A-16)}$$

where ε_{i+1} is a random variable that captures the effect of measurement errors. Let $f(\varepsilon_{i+1})$ denote the density function of ε_{i+1}:

$$f\left(b_{t_{i+1}} - b_{t_i} + \mu^b(b_{t_i}; \Omega)(t_{i+1} - t_i)\right) . \quad \text{(A-17)}$$

Let $\{\widehat{b}_{t_i}, i = 1, \cdots, 41\}$, where $t_i = 1979 + i$, denote the annual US debt-to-GDP ratio from 1980 to 2020. We calibrate Ω by minimizing the sum of squared differences between one-step-ahead model-predicted b_t and the realized b_t:

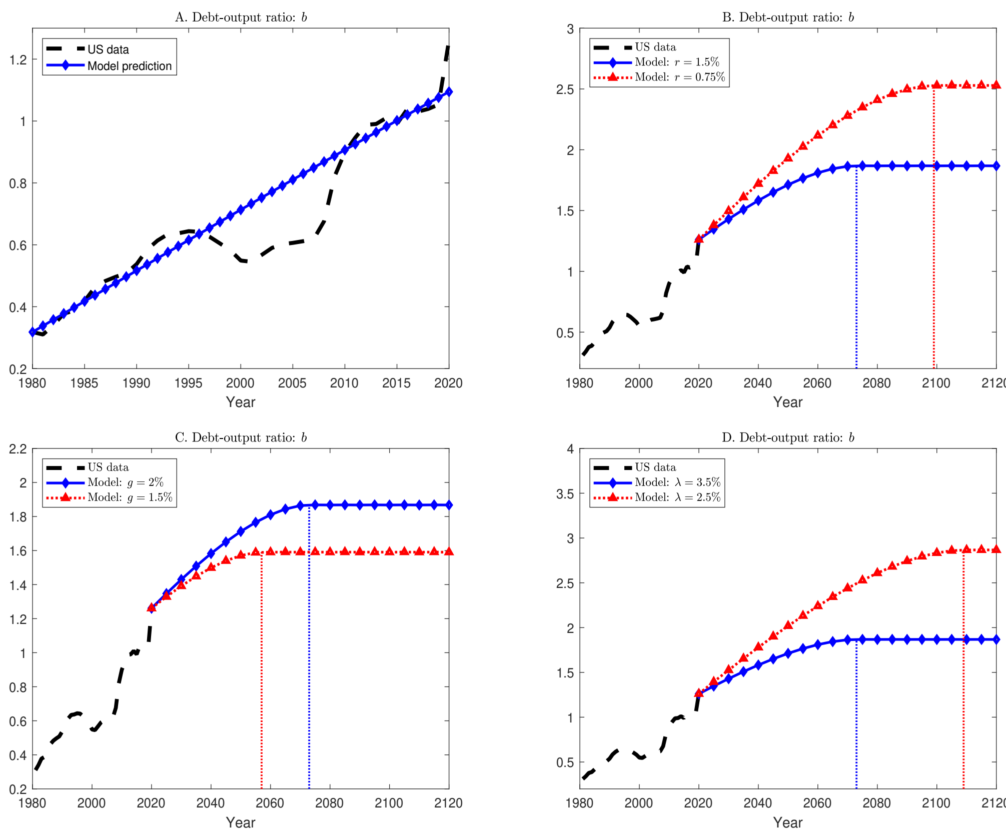
$$\widehat{\Omega} = \arg\max_{\Omega} \sum_{i=1}^{40} \ln f\left(\widehat{b}_{t_{i+1}} - \widehat{b}_{t_i} + \mu^b(\widehat{b}_{t_i}; \Omega)\right) . \quad \text{(A-18)}$$

In Panel A of Figure A-1, we plot the model-implied debt-GDP ratio dynamics using the parameter values $\widehat{\Omega}$ obtained from the above procedure using the 1980-2020 US data. The diamond blue solid lines in panels B, C, and D of Figure A-1 plot the same outcomes using the parameter values described in the main body of this paper.

Next, we perturb a set of key parameters one at a time and display effects on outcomes.

Effects of convenience yield δ. A larger parameter δ has quantitatively important effects on taxes and value functions. Figure A-2 compares outcomes from the case ($\delta = 0.13\%$) with those from a $\delta = 1.6\%$ case, where in both cases we fix risk-free rate $r = 2\%$ and risk premium $\lambda = 2.5\%$. As δ increases from 0.13% to 1.6%, the total value $p(b)$ increases by about one tenth at all admissible levels of b (panel A.) This outcome mostly reflects a

Figure A-1: **Prospective Debt-GDP Ratio Dynamics.** The diamond blue solid line in panel A calibrates to the average growth of the US debt-output ratios from 1980 to 2020. The black dashed lines in all four panels depict the US debt-output ratios from 1980 to 2020. The diamond blue solid lines in panels B, C, and D are projected debt-output ratios from 2020 to 2120 for our baseline case using parameters in Table 1. The triangle red solid lines in panels B, C, and D are projected debt-output ratios from 2020 to 2120 as we modify our baseline case with a single change as follows: 1.) decreasing r to $r = 0.75\%$, 2.) decreasing g to $g = 1.5\%$, and 3.) decreasing risk premium λ to $\lambda = 2.5\%$, respectively.

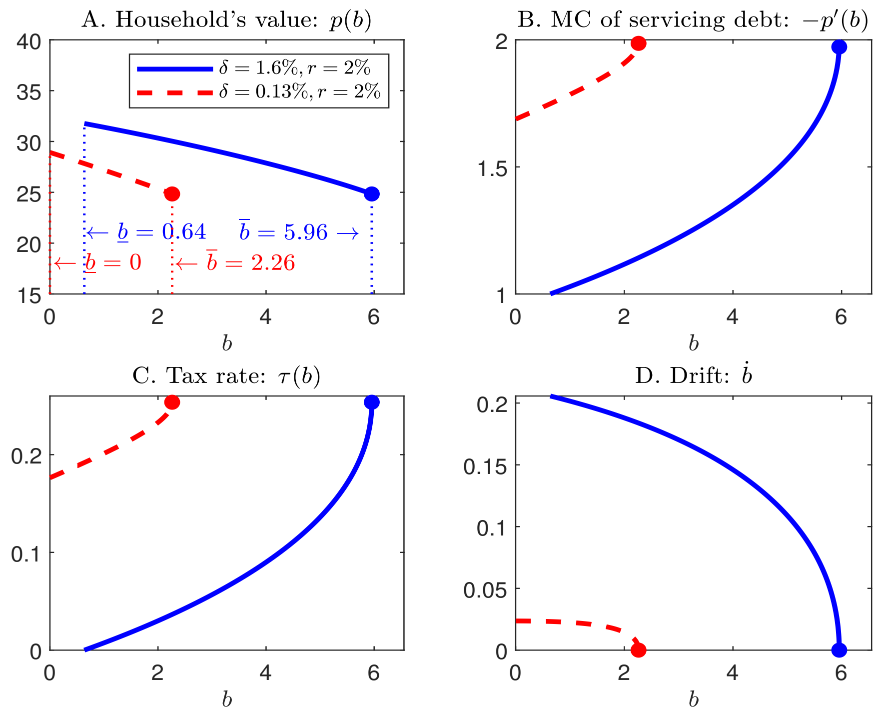


typical discounting channel. Debt capacity $\bar{b}$ substantially increases from 2.26 to 5.96. More interesting to us is that the marginal cost of debt ($-p'(b)$) and the optimal tax rate ($\tau(b)$) both decrease substantially for most values of b (panels B and C). This happens because it becomes much less costly for the government to defer taxes. As a result, the marginal cost $-p'(b)$ of debt at $b = 0.64$ is one when $\delta = 1.6\%$ but equals 1.69 dollars in our baseline $\delta = 0.13\%$ case. The optimal tax rate $\tau(b)$ at $b = 0.64$ is zero when $\delta = 1.6\%$ but equals 17.6% in our baseline $\delta = 0.13\%$ case.

Over time, as b increases, the tax rate $\tau(b)$ and the marginal cost of debt increase until debt reaches debt capacity. Increasing δ substantially increases the drift of the debt-GDP ratio $\mu^b(b)$, which in turn affects the time it takes for a government to reach its debt capacity, as we describe in Section 4.

Effects of risk-free rate r. Figure A-3 compares outcomes in our baseline ($r = 1.5\%$)

Figure A-2: **Effects of Convenience Yield δ.** We choose $r = 2\%$ and $\lambda = 2.5\%$. All other parameter values other are reported in Table 1.

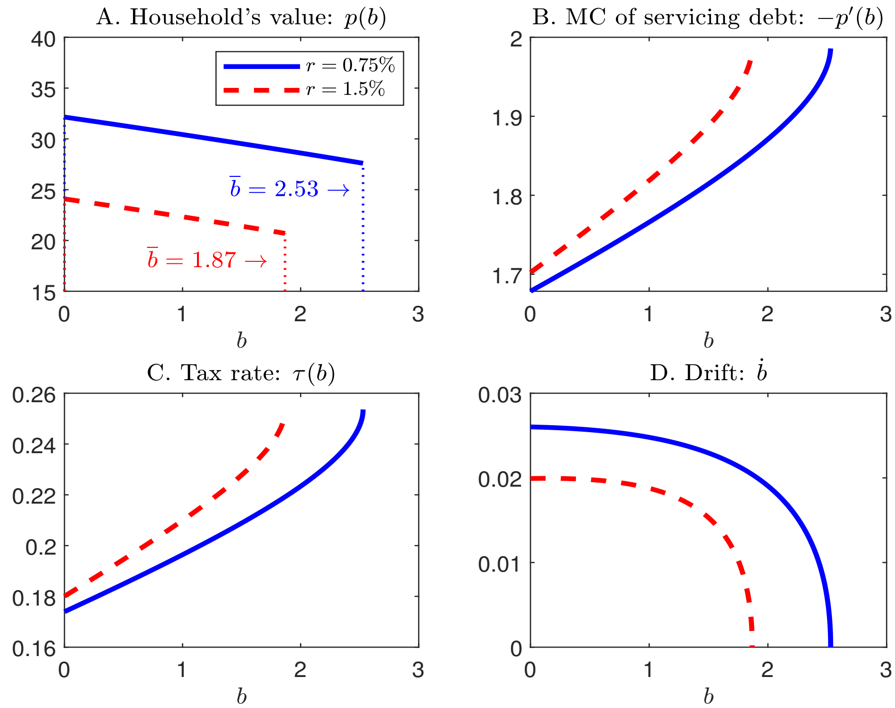


case with those when $r = 0.75\%$. When r decreases across economies from 1.5% to 0.75%, a government's debt capacity $\overline{b}$ increases substantially from 1.87 to 2.53. The marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both fall substantially in the lower r economy. Because interest payments are smaller, debt service smaller and tax distortions are also smaller. Consequently, a government is more willing to borrow, increasing the drift of the debt-GDP ratio $\mu^b(b)$ at all levels of b (panel D).

Effects of risk premium λ. Figure A-4 compares outcomes under our baseline ($\lambda = 3.5\%$) case with those when $\lambda = 2.5\%$. When λ decreases from 3.5% to 2.5% across economies, a government's debt capacity $\overline{b}$ doubles from 1.87 to 2.87. The marginal cost $-p'(b)$ of debt and the tax rate $\tau(b)$ both decrease markedly. Because systematic risk management costs are smaller, debt service and tax distortions are smaller. Consequently, a government is more willing to borrow, so the drift $\dot{b}_t = \mu^b(b)$ of the debt-GDP ratio increases at all levels of b (panel D).

Effects of Output Growth Rate g. Figure A-5 compares outcomes under our baseline ($g = 2\%$) case with those in a $g = 1.5\%$ economy. When the growth rate across economies decreases from 2% to 1.5%, a government's debt capacity $\overline{b}$ decreases by about one third, from 1.87 to 1.59. The marginal cost $-p'(b)$ of debt and the tax rate $\tau(b)$ both increase substantially.With slower growth, a government is less willing to borrow, causing drift $\dot{b}_t =$

Figure A-3: **Effects of Interest Rate r.** All parameter values other than r are reported in Table 1.

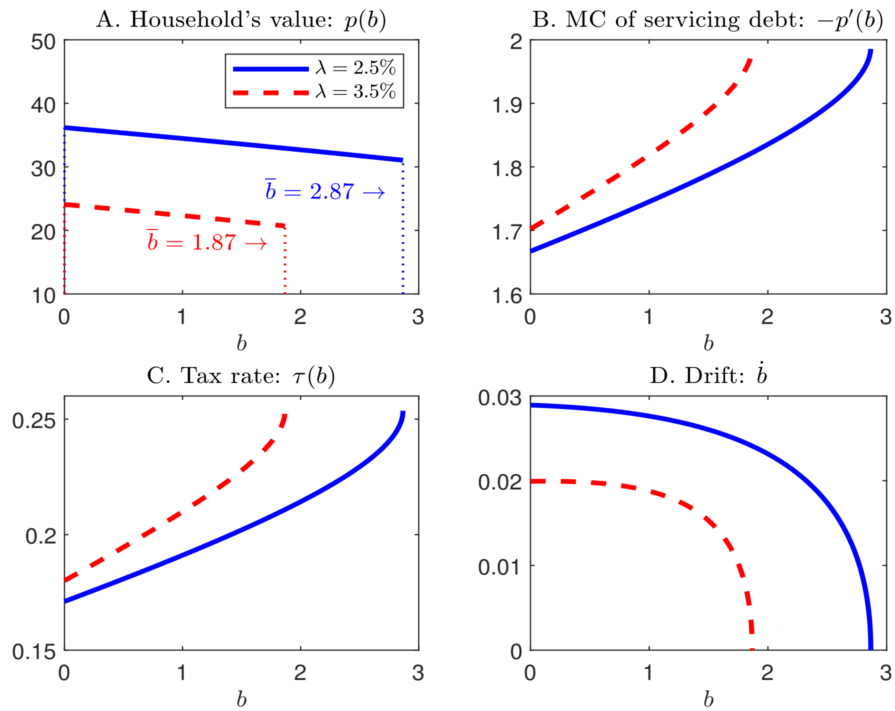


$\mu^b(b)$ of the debt-GDP ratio to fall at all levels of b (panel D). That affects the time it takes to reach a debt limit.

Effects of tax distortion cost φ. The parameter φ governs tax distortions in the deadweight loss function $\theta(\cdot)$. Figure A-6 compares outcomes under our baseline ($\varphi = 3.9$) with those when $\varphi = 0.1$. When φ decreases from 3.9 to 0.1, a government's debt capacity $\bar{b}$ increases substantially from 1.87 to 2.41 and the household's value function $p(b)$ increases. The marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both decrease. When taxes are less distortionary, a government is more willing to borrow, causing the lump-sum debt issuance threshold $\underline{b}$ to increase from 0 to 0.12 (panel A), and the drift of the debt-GDP ratio $\dot{b}_t = \mu^b(b_t)$ to increase at all levels of b_t (panel D).

Effects of default costs: (increasing tax distortion costs $\kappa \geqslant 1$). The parameter κ measures how much more distortionary taxes are in the balanced-budget regime than in the service-debt regime. Figure A-7 compares outcomes under our baseline ($\kappa = 1.3$) case with those under a $\kappa = 1.5$ case. When across economies κ increases from 1.3 to 1.5, a government's debt capacity $\bar{b}$ *increases* from 1.87 to 2.16 and the household's value function $p(b)$ increases slightly. The marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both decrease because a higher default cost makes a government is more willing to repay debt, so it can borrow more. As κ increases across economies, the drift of the debt-GDP ratio $\dot{b}_t = \mu^b(b_t)$

Figure A-4: **Effects of Risk Premium λ.** All parameter values other than λ are reported in Table 1.



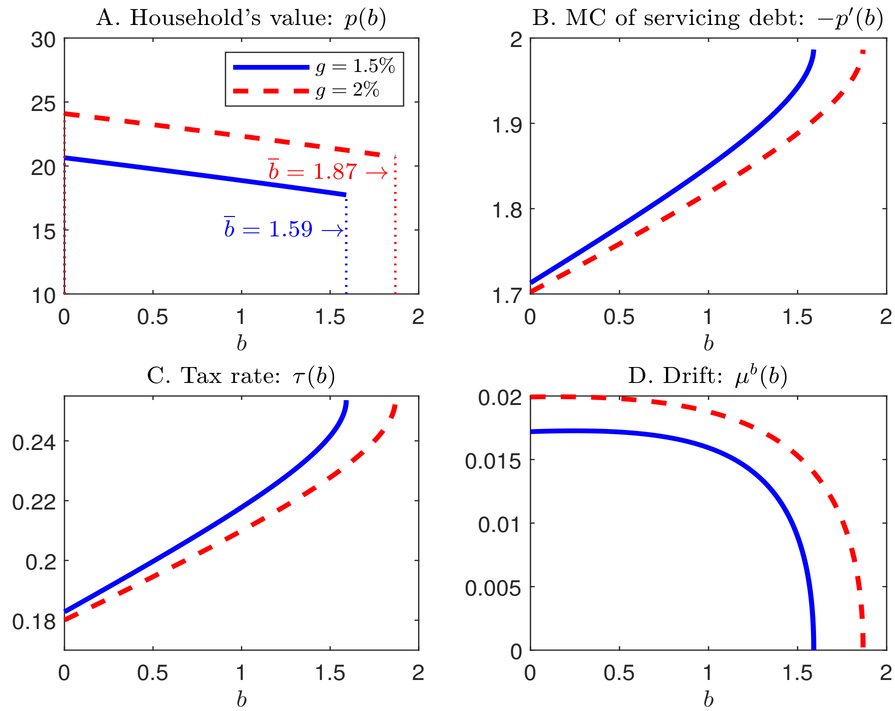
is higher at all levels of b (panel D).

Effects of default costs: output loss $(1 - \alpha)$. The parameter α measures the recovery of output in the default regime. Figure A-8 compares outcomes under our baseline ($\alpha = 0.93$) case with those when $\alpha = 0.90$. When across economies output loss $(1 - \alpha)$ increases from 7% to 10%, a government's debt capacity $\bar{b}$ increases markedly from 1.87 to 2.45, but the household's value function $p(b)$ increases only slightly. The marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both decrease. When default is more costly, the government is more willing to repay debt and hence can borrow more. Finally, for higher output loss $(1 - \alpha)$, the drift of the debt-GDP ratio $\dot{b}_t = \mu^b(b_t)$ is higher at all levels of b (panel D).

Comparative results with respect to $(1 - \alpha)$ and κ are similar because increasing $(1 - \alpha)$ directionally has the same effect as increasing κ. Both make default more costly, improving incentives to repay and consequently debt capacity.

Effects of government spending-GDP ratio γ. The parameter γ measures government spending as a fraction of output. Figure A-9 compares outcomes under our baseline ($\gamma = 0.2$) case with those when $\gamma = 0.25$. When across economies government spending γ increases from 0.2 to 0.25, a government's debt capacity $\bar{b}$ *increases* slightly from 1.87 to 1.98, but the household's value function $p(b)$ decreases markedly. The marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both increase substantially. When the government spending fraction is higher,

Figure A-5: **Effect of Average Output Growth Rate g.** All parameter values other than g are reported in Table 1.

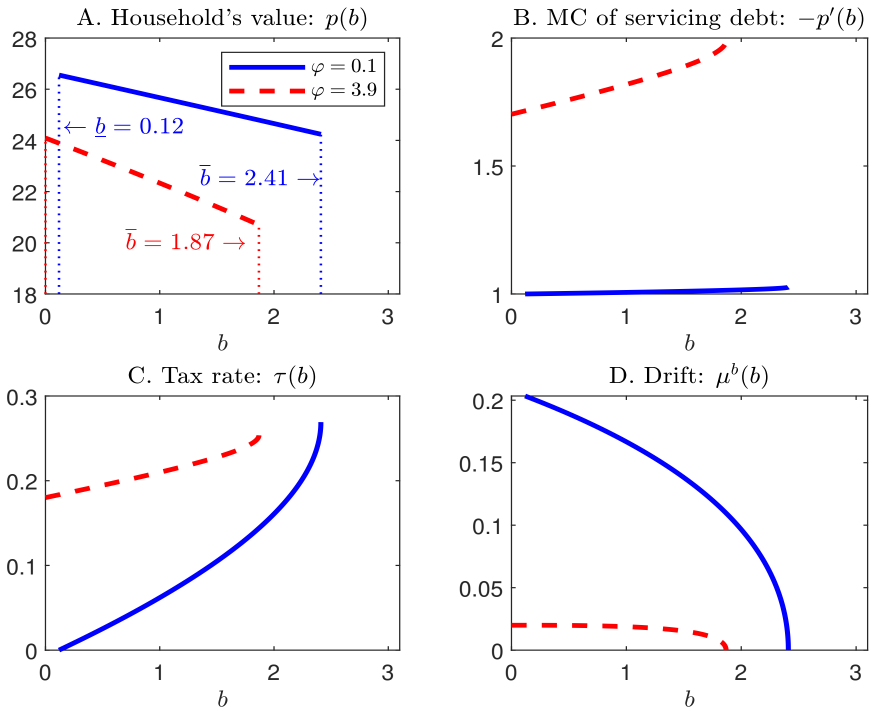


a household's value in the balanced-budget regime becomes lower, making a government be more willing to tax more in order to repay its debt. That enables it to borrow more.

Effects of expected balanced-budget-regime duration $1/\chi$. In our baseline Section 2 model, the government permanently stays in the balanced-budget regime once it enters. In practice, sovereign that default eventually regain access to capital markets. To capture a finite stochastic duration of staying in the balanced-budget regime, we assume that a government exits that regime at a constant (annual) rate, denoted by χ. We set $\chi = 1/5$ per annum to capture that after default a sovereign on average can't access international capital markets for four or five years (e.g., see the estimate in Aguiar and Gopinath, 2006). In Figure A-10, we compare a $\chi = 0.2$ case with our baseline $\chi = 0$ case in which the balanced-budget regime is an absorbing state.

As we decrease the expected duration of being in the balanced-budget regime $1/\chi$ from ∞ to five years, debt capacity $\bar{b}$ decreases from 1.87 to 1.06 while the marginal cost of debt $-p'(b)$ and the tax rate $\tau(b)$ both increase. With a lower debt capacity (for the $\chi = 0.2$ case), the government has less room to smooth taxes and hence has to tax more in order to honor its debt. Because higher taxes cause more distortions, the government's marginal cost of debt is higher. As a result of higher taxes, the government pays back its debt at a faster rate (for all admissible levels of b) causing the drift of its debt-GDP ratio $\dot{b}_t = \mu^b(b_t)$ to be

Figure A-6: **Effect of Tax Distortion Cost φ.** All parameter values other than φ are reported in Table 1.



lower for the $\chi = 0.2$ case than for our baseline $\chi = 0$ case (panel D).

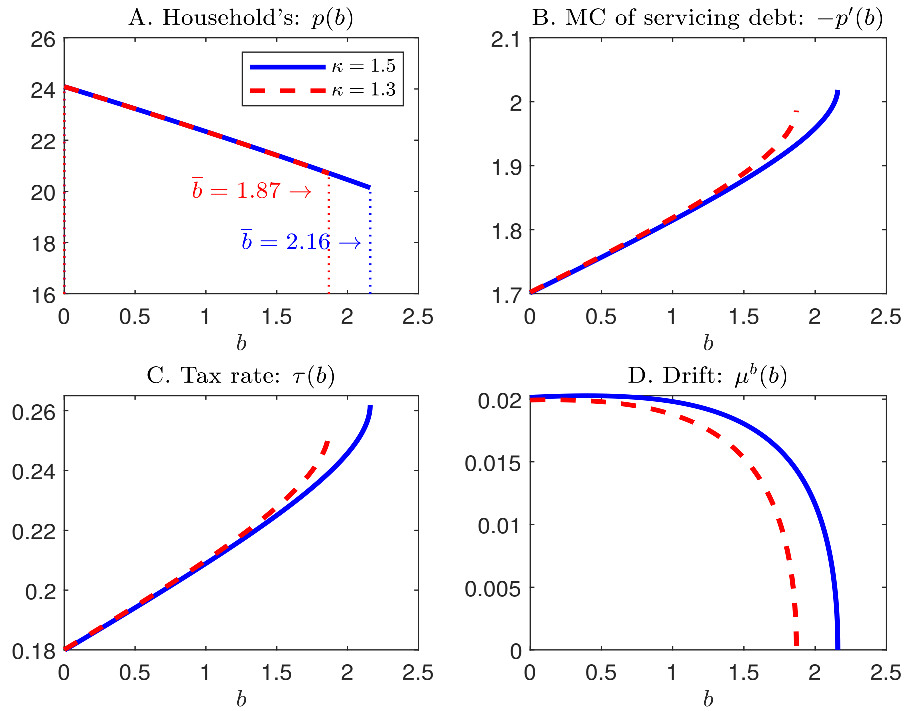
C Households' and Government's Optimality

We provide technical details about the household's and the government's optimization problems. 1.) The representative household maximizes lifetime utility taking the government's policies as given; 2.) The government chooses tax and debt management policies to maximize the representative household's lifetime utility. 3.) The household, government, and investors take the stochastic discount factor (SDF) process as given. The first two assumptions are made by Lucas and Stokey (1983), while the third disarms Lucas and Stokey's government's ability to manipulate the SDF process.

Below we show that the government's optimal policy boils down to maximizing the net present value of the net transfers to the representative household. We start with the household's problem.

Households' optimization problem. Taking the government's tax policies $\{\mathcal{T}_t, \widehat{\mathcal{T}}_t\}$ in the no-default and balanced-budget regimes, lumpy transfer policies $\{dH_t\}$, and default timing $T^{\mathcal{D}}$ as given, the representative household is endowed with an initial wealth W_0 and share

Figure A-7: **Effects of Default Costs: (Increasing Tax Distortion Costs $\kappa \geqslant 1$).** All parameter values other than κ are reported in Table 1.



ζ_0 of aggregate flow payments. The household chooses consumption policies $\{C_t\}$ to solve

$$\max\ \mathbb{E}_0\left[\int_0^{\infty} U(C_t, t)dt\right] \tag{A-19}$$

subject to the budget constraint:

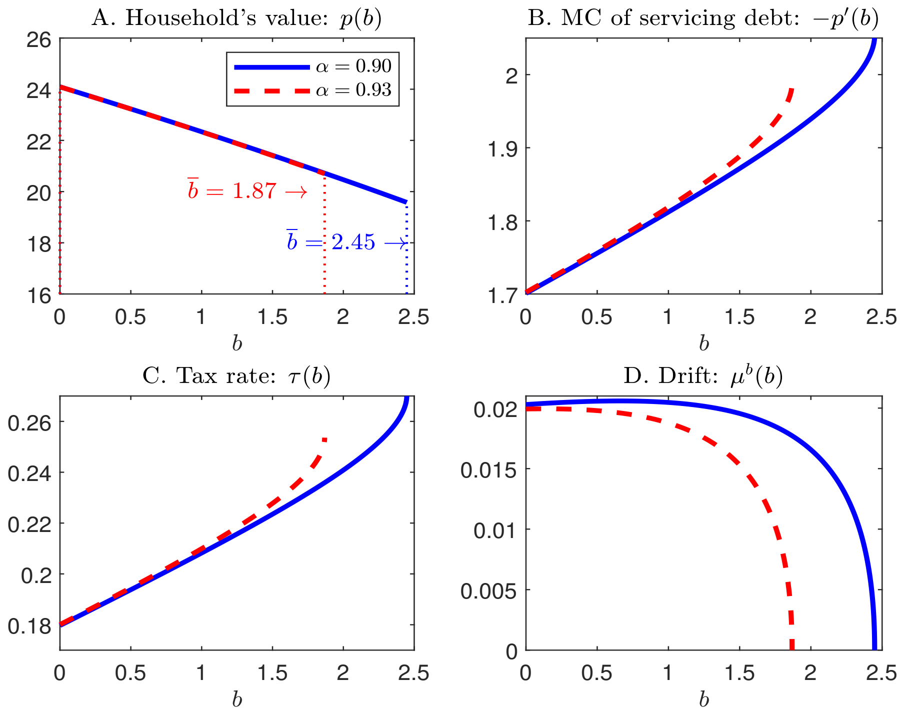
$$\mathbb{E}_0\left(\int_0^{\infty} (\mathbb{M}_t C_t)\, dt\right) \leqslant W_0 + \zeta_0 P_0\,. \tag{A-20}$$

In (A-20), $\mathbb{M}_t$ is the exogenously given SDF defined in (3) and P_0 is the present value of aggregate transfer payments to households:

$$P_0 = \mathbb{E}_0\left[\int_0^{T^{\mathcal{D}}} \mathbb{M}_t\,(dH_t + N_t dt) + \int_{T^{\mathcal{D}}}^{\infty} \mathbb{M}_t \widehat{N}_t dt\right], \tag{A-21}$$

where $N_t = Y_t - \mathcal{T}_t - \Theta_t$ for $t < T^{\mathcal{D}}$ in the no-default regime and $\widehat{N}_t = \widehat{Y}_t - \widehat{\mathcal{T}}_t - \widehat{\Theta}_t$ for

Figure A-8: **Effect of Default Costs: Output Recovery α.** All parameter values other than α are reported in Table 1.



$t \geqslant T^{\mathcal{D}}$ in the balanced-budget regime. The household is endowed with a constant-relative risk-averse (CRRA) utility and a subjective discount rate $\varrho > 0$: $U(C,t) = e^{-\varrho t}\frac{C^{1-\omega}}{1-\omega}$, where $\omega \geqslant 0$ is the coefficient of relative risk aversion.

A complete financial spanning assumption makes the time−0 budget constraint (A-20) appropriate. As in Cox and Huang (1989), we can pin down the household's consumption path and trading strategy at time 0. The Lagrangian for the household's problem is

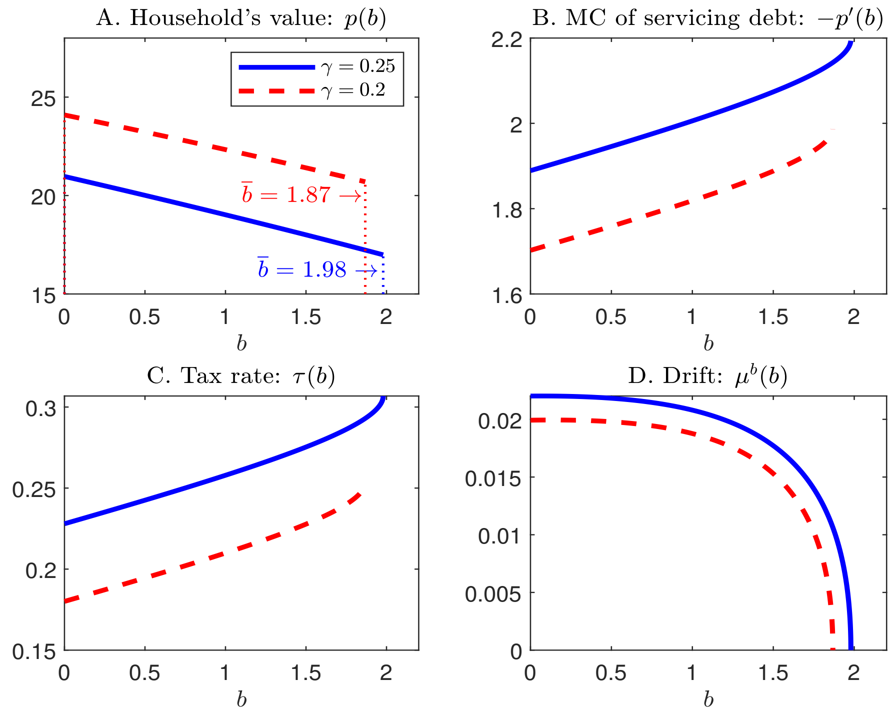
$$\begin{aligned}\mathcal{L} =& \mathbb{E}_0\left[\int_0^\infty U(C_t,t)dt\right] \\ &+ \vartheta\left[W_0 + \mathbb{E}_0\left(\zeta_0\int_0^{T^{\mathcal{D}}} \mathbb{M}_t\left(dH_t + N_t dt\right) + \zeta_0\int_{T^{\mathcal{D}}}^\infty \mathbb{M}_t\widehat{N}_t dt - \int_0^\infty \mathbb{M}_t C_t dt\right)\right],\end{aligned}$$

where ϑ is Lagrange multiplier. The FOC for consumption for all $t \geqslant 0$ is

$$U_C(C_t,t) = \vartheta\mathbb{M}_t\,, \tag{A-22}$$

which implies the standard optimal consumption rule: $C_t = \vartheta^{-1/\omega}\mathbb{M}_t^{-1/\omega}e^{-\varrho t/\omega}$.

Figure A-9: **Effect of Government Spending γ.** All parameter values other than γ are reported in Table 1.



Applying Ito's Lemma lets us obtain the following geometric Brownian motion for $\{C_t\}$:

$$\frac{dC_t}{C_t} = \left(\frac{r}{\omega} - \frac{\varrho}{\omega} + \frac{1}{2}\frac{(1+\omega)\eta^2}{\omega^2}\right) dt + \frac{\eta}{\omega} d\mathcal{Z}_t^m \,. \tag{A-23}$$

By substituting (A-23) into the budget constraint (A-20), we obtain the optimal consumption C_0 at time 0

$$C_0 = \left((1-\frac{1}{\omega})r + \frac{\varrho}{\omega} + \frac{1}{2}(1-\frac{1}{\omega})\frac{1}{\omega}\eta^2\right)(W_0 + \zeta_0 P_0)\,,$$

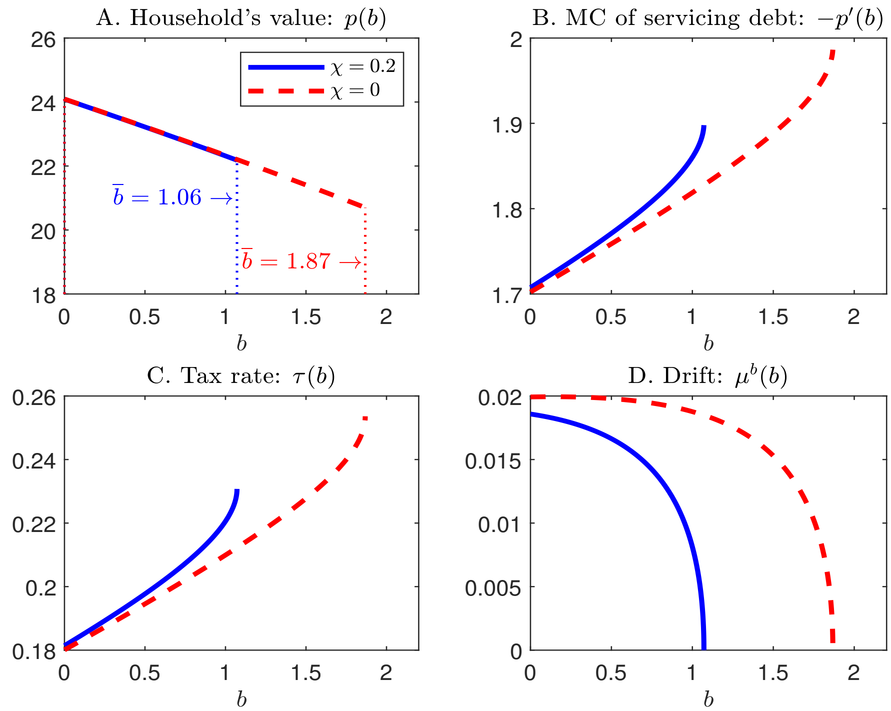
and the household's time-0 indirect utility functional:

$$\mathbb{E}_0\left[\int_0^\infty U(C_t,t)dt\right] = \frac{(\iota(W_0+\zeta_0 P_0))^{1-\omega}}{1-\omega}\,, \tag{A-24}$$

where $\iota = \left((1-\frac{1}{\omega})r + \frac{\varrho}{\omega} + \frac{1}{2}(1-\frac{1}{\omega})\frac{1}{\omega}\eta^2\right)^{-\frac{\omega}{1-\omega}}$.

Government's problem. The benevolent government chooses tax policies $\{\mathcal{T}_t, \widehat{\mathcal{T}}_t\}$, lumpy transfer policies $\{dH_t\}$, and default timing $T^{\mathcal{D}}$ to maximize the household's indirect utility

Figure A-10: **Effect of intensity to exit balanced-budget regime χ.** All parameter values other than χ are reported in Table 1.



functional given in (A-24). This optimization problem is equivalent to maximizing P_0 where

$$P_0 = \mathbb{E}_0\left[\int_0^{T^{\mathcal{D}}} \mathbb{M}_t\left(dH_t + N_t dt\right) + \int_{T^{\mathcal{D}}}^{\infty} \mathbb{M}_t \widehat{N}_t dt\right], \tag{A-25}$$

subject to the government's budget constraint:

$$B_0 \leqslant \mathbb{E}_0\left[\int_0^{T^{\mathcal{D}}} e^{\delta t}\mathbb{M}_t\left[(\mathcal{T}_t - \Gamma_t)dt - dH_t\right]\right], \tag{A-26}$$

where δ is the government debt's convenience yield and B_0 is the initial debt level. To make dependence of the present value of net payments to households on consumption and taxes explicit, we have substituted the expressions for N_t and $\widehat{N}_t$ given by (16) into (A-21).

D Technical Details for Section 5

We provide technical details about the extended model analyzed in Section 5. Because the $\mathcal{H}$ state is absorbing, the value function $P(B,Y;\mathcal{H})$ in this state for the no-default regime satisfies the HJB equation:

$$\begin{aligned} r^{\mathcal{H}}P(B,Y;\mathcal{H}) = \max_{\mathcal{T}\leqslant\overline{\tau}Y,\Phi^h,\Phi^m} \; & Y-\mathcal{T}-\Theta(\mathcal{T},Y)+\left[\left(r^{\mathcal{H}}-\delta\right)B+\gamma^{\mathcal{H}}Y-\mathcal{T}\right]P_B(B,Y;\mathcal{H}) \\ & +\frac{\sigma^2\left((1-\rho^2)(\Phi^h)^2+\rho^2(\Phi^m)^2\right)}{2}P_{BB}(B,Y;\mathcal{H})+\widetilde{g}YP_Y(B,Y;\mathcal{H}) \\ & +\frac{\sigma^2Y^2}{2}P_{YY}(B,Y;\mathcal{H})-\sigma^2\left((1-\rho^2)\Phi^h+\rho^2\Phi^m\right)YP_{BY}(B,Y;\mathcal{H})\,, \end{aligned} \tag{A-27}$$

where $\widetilde{g}=g-\lambda$. An HJB equation for $P(B,Y;\mathcal{L})$ and associated FOCs for the government's fiscal policies in the $\mathcal{L}$ state are described in Section 5.

Value functions in the balanced-budget regime, $\widehat{P}(Y;\mathcal{L})$ and $\widehat{P}(Y;\mathcal{H})$, satisfy the following differential equations:

$$r^{\mathcal{L}}\widehat{P}(Y;\mathcal{L})=Y-\mathcal{T}-\widehat{\Theta}(\gamma^{\mathcal{L}}Y,Y)+\widetilde{g}Y\widehat{P}_Y(Y;\mathcal{L})+\frac{\sigma^2Y^2}{2}\widehat{P}_{YY}(Y;\mathcal{L})+\widetilde{\xi}\left[\widehat{P}(Y;\mathcal{H})-\widehat{P}(Y;\mathcal{L})\right] \tag{A-28}$$

$$r^{\mathcal{H}}\widehat{P}(Y;\mathcal{H})=Y-\mathcal{T}-\widehat{\Theta}(\gamma^{\mathcal{H}}Y,Y)+\widetilde{g}Y\widehat{P}_Y(Y;\mathcal{H})+\frac{\sigma^2Y^2}{2}\widehat{P}_{YY}(Y;\mathcal{H})\,. \tag{A-29}$$

Using the homogeneity property, we obtain the scaled value functions, $\widehat{p}(\mathcal{L})$ and $\widehat{p}(\mathcal{H})$:

$$\begin{aligned} \widehat{p}(\mathcal{L}) &= \frac{1-\gamma^{\mathcal{L}}/\alpha-\kappa\theta(\gamma^{\mathcal{L}}/\alpha)}{r^{\mathcal{L}}+\lambda-g+\widetilde{\xi}}+\frac{\widetilde{\xi}}{r^{\mathcal{L}}+\lambda-g+\widetilde{\xi}}\frac{1-\gamma^{\mathcal{H}}/\alpha-\kappa\theta(\gamma^{\mathcal{H}}/\alpha)}{r^{\mathcal{H}}+\lambda-g}\,, \\ \widehat{p}(\mathcal{H}) &= \frac{1-\gamma^{\mathcal{H}}/\alpha-\kappa\theta(\gamma^{\mathcal{H}}/\alpha)}{r^{\mathcal{H}}+\lambda-g}\,. \end{aligned}$$

State $\mathcal{H}$ is absorbing, so the scaled value $p(b;\mathcal{H})$, debt capacity $\overline{b}(\mathcal{H})$, debt issuance boundary $\underline{b}(\mathcal{H})$, and optimal tax rate $\tau(b;\mathcal{H})$ are the same as in our baseline model. Thus, $p(b;\mathcal{H})$ solves

$$\begin{aligned} \left[r^{\mathcal{H}}+\lambda-g\right]p(b;\mathcal{H}) = & 1-\tau(b;\mathcal{H})-\theta(\tau(b;\mathcal{H}))+\left[(r^{\mathcal{H}}-\delta+\lambda-g)b\right. \\ & \left.+\left(\gamma^{\mathcal{H}}-\tau(b;\mathcal{H})\right)\right]p'(b;\mathcal{H})\,, \end{aligned} \tag{A-30}$$

subject to the following debt-sustainability condition:

$$\bar{b}(\mathcal{H}) \quad = \quad \frac{\tau(\bar{b}(\mathcal{H});\mathcal{H}) - \gamma^{\mathcal{H}}}{r^{\mathcal{H}} - \delta + \lambda - g} \,.$$

The debt-GDP ratio b_t in state $\mathcal{H}$ follows (A-4), as in our baseline model.

In state $\mathcal{L}$, the law of motion for scaled debt, b_t, in the no-default regime where $b \leqslant \bar{b}$ is

$$db_t \quad = \quad \mu^b_{t-}\, dt + \psi_{t-} d\mathcal{N}_t \ , \tag{A-31}$$

where

$$\mu^b_{t-} = (r_{t-} + \lambda - \delta - g)b_{t-} + (\gamma_{t-} - \tau_{t-}) - \widetilde{\xi}\psi_{t-} \,, \tag{A-32}$$

As in our **ABCD** diffusion model, the b_t process does not respond to diffusion shocks as $\sigma^b_{t-} = -(\sigma\phi_{t-} + \sigma b_{t-}) = 0$. But unlike in our baseline model, b_t discretely jumps as the state transitions.

E Technical Details for Section 6

We provide technical results for the jump-diffusion model of Section 6. First, we characterize the representative household's value in the no-default and balanced-budget regimes. The household's value in the balanced-budget regime $\widehat{P}(\widehat{Y})$ satisfies

$$r\widehat{P}(\widehat{Y}) = \left(\widehat{Y} - \Gamma - \widehat{\Theta}(\Gamma,\widehat{Y})\right) + (g-\lambda)\widehat{Y}\widehat{P}'(\widehat{Y}) + \frac{\sigma^2\widehat{Y}^2}{2}\widehat{P}''(\widehat{Y}) + \zeta\mathbb{E}\left[\widehat{P}(ZY) - \widehat{P}(Y)\right] \,, \tag{A-33}$$

where the last term captures jumps. To ensure that $\widehat{P}(\widehat{Y})$ is finite, we impose $r + \lambda > g - \zeta(1 - \mathbb{E}(Z))$. Using the homogeneity property, we obtain the following expression for the scaled value $\widehat{p}$ in the balanced-budget regime:

$$\widehat{p} \quad = \quad \frac{1 - \gamma/\alpha - \kappa\theta(\gamma/\alpha)}{r + \lambda - (g - \zeta(1 - \mathbb{E}(Z)))} \,. \tag{A-34}$$

Using the optimal diffusion hedging strategy, i.e., $\Phi^m = \Phi^h = YP_{BY}/P_{BB}$, we obtain the following first-order partial differential equation for the households' value $P(B,Y)$ in the no-default regime:

$$\begin{aligned} rP(B,Y) \quad = \quad & \max_{\mathcal{T} \leqslant \overline{\tau} Y} \ (Y - \mathcal{T} - \Theta(\mathcal{T},Y)) + ((r + \pi - \delta)B + \Gamma - \mathcal{T})\, P_B + (g - \lambda)YP_Y \\ & + \zeta\mathbb{E}\left[P(B,ZY) - P(B,Y)\right] \,. \end{aligned} \tag{A-35}$$

Second, we turn to the default boundary. Let $\overline{Z}_t$ denote the highest level of fractional recovery Z at t that satisfies $P(B_t, Y_t) \leqslant \widehat{P}(\alpha Y_t)$, where $Y_t = ZY_{t-}$. Using the homogeneity property, we express the default boundary $\overline{Z}_t$ as:

$$\overline{Z}_t = \overline{Z}(b_t) = b_t/\overline{b}\,, \quad \text{for } \ b_t \leqslant \overline{b}\,. \tag{A-36}$$

Third, we determine the equilibrium credit spread. To compensate investors for credit losses they bear when the government defaults at t, the credit spread π_{t-} must satisfy the following zero-profit condition for creditors:

$$B_{t-}(1 + rdt) = B_{t-}(1 + (r + \pi_{t-})dt)\left[1 - \zeta\, Q(\overline{Z}(b_{t-}))dt\right] + \zeta\, Q(\overline{Z}(b_{t-}))dt \times 0\,. \tag{A-37}$$

The first term on the right side of (A-37) is the expected total payment to investors, the product of the probability of repayment, $\left[1 - \zeta\, Q(\overline{Z}(b_{t-}))dt\right]$ and the cum-interest value of debt repayment, $B_{t-}(1 + (r + \pi_{t-})dt)$. The second term on the right side of (A-37) captures the zero payment to investors upon default. The left side of (A-37) is investors' total expected payoff at $t + dt$,including principal B_{t-}. Simplifying (A-37) gives credit spread (61).

Finally, at the debt capacity $\overline{b}$, $\pi(\overline{b}) = \zeta$ and b_t evolves as

$$\dot{b}_t = \mu^b(b_t) = (r + \zeta - \delta + \lambda - g)b_{t-} + \gamma - \tau(b_{t-})\,. \tag{A-38}$$

The debt-sustainability zero-drift (absent jumps) given in (39) continues to hold.

When the Keynes tax constraint (6) does not bind, the debt-sustainability condition implies $\overline{b} = \frac{\tau(\overline{b})-\gamma}{r+\zeta-\delta+\lambda-g} = \check{b}$, where $\check{b}$ solves $p(\check{b}) = \alpha\widehat{p}$. When the Keynes tax constraint (6) binds, the debt-sustainability condition implies that $\overline{b} = \frac{\overline{\tau}-\gamma}{r+\zeta-\delta+\lambda-g}$. Therefore, debt capacity satisfies (66).