Efficiency, Insurance, and Redistribution Effects of Government Policies

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Abstract

We decompose welfare effects of switching from government policy A to policy B into three components: *aggregate efficiency* – gains from changes in total resources; *redistribution* – gains from altered consumption shares that *ex-ante* heterogeneous agents can expect to receive; and *insurance* – gains from changes in individuals' consumption risks. Our decomposition applies to a broad class of multi-person, multi-good, multi-period economies with diverse specifications of preferences, shocks, and sources of heterogeneity. It has the desirable property that it attributes to the insurance component all welfare effects that arise purely from mean preserving spreads in consumption. We compare our decomposition to earlier ones developed by Benabou (2002) and Floden (2001) and show that those approaches attribute welfare effects from such spreads to insurance only under special conditions.

1 Introduction

We want to understand sources of changes in social welfare induced by alternative government policies. When households are heterogeneous, welfare depends on how efficiently goods and services are produced as well as how they are allocated across households. Welfare changes from reallocations are influenced by Pareto weights and curvature of a social welfare function, so it is natural to isolate the contribution from redistribution.

We propose an approach that attributes a welfare change from moving from policy A to policy B to three components that we call *aggregate efficiency*, *redistribution*, and *insurance*. Aggregate efficiency captures consequences of changes in aggregate resources, for each agent holding fixed both an *ex-ante* expected consumption share and *ex-post* dispersion in consumption. Redistribution captures changes in *ex-ante* consumption shares of agents, holding fixed both the level of aggregate resources and the consumption dispersions for each agent. Insurance captures changes in the distribution of consumption, holding fixed their levels of expected consumption. Up to a second order of approximation, the sum of these three components equals the total welfare effect that arises from changing the allocation induced by policy A to the allocation induced by policy B. Our decomposition is *reflexive* in the sense that each component of a welfare change from policy A to policy B is equal in magnitude and of opposite sign to its counterpart for moving from policy B to policy A.

A popular way to partition welfare changes is to apply a decomposition developed by Benabou (2002) and Floden (2001). Their approach computes a certainty equivalent consumption for each agent and then averages it across agents. Their decomposition attributes sets the aggregate efficiency and insurance component equal to a change in welfare that would occur in a hypothetical economy in which all agents receive that average consumption equivalent and attributes the residual to redistribution.

The Benabou-Floden approach has limitations in some settings. Consider a policy that reduces dispersions in consumptions without changing the resources that each agent expects to receive. When agents are risk averse, such a policy raises welfare. It would be natural to attribute 100% of this welfare increase to improved insurance. Yet for such a policy change the decompositions of Benabou and Floden can attribute anything between 0% and $+\infty$% of a welfare change to insurance and anything between $-\infty$% and 100% to redistribution. Their decomposition would attribute 100% to insurance only when (i) agents have identical preferences, (ii) agents face uncertainty about a single consumption good, and (iii) each agent's variance of consumption is reduced in proportion to her expected consumption level. These conditions are satisfied in the setting studied by Benabou but not in that studied by Floden.

In contrast, our decomposition applies to a much larger class of economies. It imposes few restrictions on preferences, shocks, and sources of heterogeneity. To a second order of approximation, our decomposition coincides with the decomposition of Benabou and Floden when the environment satisfies properties (i), (ii), and (iii).

The rest of this paper is organized as follows. Section 2 describes the environment. Section 3 presents notions of aggregate efficiency, redistribution, and insurance and then describes our decomposition. Section 4 analyzes the Benabou-Floden decomposition. Section 5 concludes. Proofs and technical details appear in the Appendix.

2 Environment

A unit measure of households are *ex-ante* heterogeneous and are subject to risk *ex-post*. We refer to all the households who are *ex-ante* identical as a *family* and use index i to denote preferences and allocations for the households in a particular family.

A household in family i derives utility $U_i\left(\{c_{k,i}\}_k\right)$ from a bundle of goods $\{c_{k,i}\}_k$. The bundle can be either finite or infinite. Each $c_{k,i}$ is stochastic and drawn from a distribution that can differ across families. We use $\mathbb{E}_i$ to denote a mathematical expectation conditional on the identity of the family.[1] Expected utility of a person in family i is $\mathbb{E}_i U_i\left(\{c_{k,i}\}_k\right)$. We assume that U_i is twice differentiable and denote its first and second derivatives by $U_{k,i}, U_{km,i}$ for goods k, m. We impose no further structure on preferences. Consequently, this setup can include dynamic models in which consumptions in different periods are appropriately indexed. We assume that the joint distribution of random variables $\{c_{k,i}\}_{i,k}$ has finite second moments and that expected utilities are well-defined. Our setup allows for aggregate shocks that are common across families as well as idiosyncratic shocks that affect only households in a particular family.

The mass of households within families and the distribution of families is denoted by $\mu(di)$. Welfare is evaluated using Pareto weights $\{\alpha_i\}_i$ that satisfy $\alpha_i \geq 0$ and $\int \alpha_i d\mu = 1$. Welfare is denoted by $\mathcal{W}$ and is given by

$$\mathcal{W} = \int \alpha_i \mathbb{E}_i U_i\left(\{c_{k,i}\}_k\right) d\mu \equiv \mathbb{E}\alpha_i U\left(\{c_{k,i}\}_k\right),$$

where $\mathbb{E}$ denotes the mathematical expectation with respect to the measure $\mu(di)$.

An *allocation* under a government *policy* is a collection of stochastic processes $\{c_{k,i}\}_{k,i}$ that assign consumptions of all goods in all families. Switching from government policy A to B results in an altered allocation and consequent welfare change $\mathcal{W}^B - \mathcal{W}^A$. We use superscripts

[1] From the definition of family, this expectation will be over the *ex-post* risk.

$j \in \{A, B\}$ to denote all variables under the respective policies. Our goal is to partition a welfare change $\mathcal{W}^B - \mathcal{W}^A$. into economically interpretable components.

3 A decomposition

We partition welfare changes into aggregate efficiency, redistribution, and insurance components. We start with a single good economy. Then section 3.2 extends the decomposition to a setting with multiple goods.

3.1 Single good economy

When there is a single good, subscripts k are redundant. We can use U_c, U_{cc} to denote first and second derivatives of the utility function. Without loss of generality, a household's consumption is the product of three terms

$$c_i = \mathbb{E}c_i \times \frac{\mathbb{E}_i c_i}{\mathbb{E}c_i} \times \frac{c_i}{\mathbb{E}_i c_i} \equiv C \times w_i \times (1 + \varepsilon_i)\,. \tag{1}$$

The first term, C, is the aggregate consumption that measures the size of an aggregate "pie" available for consumption. The second, w_i, is the share of that pie that households in family i expects to receive. Finally, ε_i captures the uncertainty that households face in terms of their ultimate consumption relative to what they expected after conditioning on the identity of their family. We always have $\mathbb{E}_i \varepsilon_i = 0$ and $\mathbb{E}w_i = 1$.

Identity (1) motives us to decompose a welfare change across two consumption allocations into three components that measure *aggregate efficiency, redistribution,* and *insurance*. Before presenting our decomposition, it is useful to describe what we view as desirable properties for these components.

Property a. A welfare change from a policy that affects aggregate consumption C but not $\{w_i, \varepsilon_i\}_i$ should be attributed solely to aggregate efficiency;

Property b. A welfare change from a policy that affects expected shares $\{w_i\}_i$ but not C and $\{\varepsilon_i\}_i$ should be attributed solely to redistribution;

Property c. A welfare change from a policy that affects the stochastic process for $\{\varepsilon_i\}_i$ but not C and $\{w_i\}_i$ should be attributed solely to insurance.

Our notion of aggregate efficiency posits that redistribution and insurance are unaffected if consumption of every household is multiplied by the same positive scalar. This seems consistent with common usages of these terms in the literature. More generally, it also implies that redistribution and insurance remain unchanged so long as the distribution of expected consumption shares, $\{w_i\}_i$ and dispersions of consumption relative to its mean, $\{std_i(c_i)/\mathbb{E}_i c_i\}_i$ are both unchanged. Once we agree on the notion pure aggregate efficiency, the other two notions follow naturally. By redistribution, we capture effects from reshuffling resources across families, that is, changes in expected consumption shares $\{w_i\}_i$. Thus, we attribute to redistribution changes that keep C and $\{\varepsilon_i\}_i$ constant and affect neither aggregate resources nor dispersions of individual consumptions. By insurance, we capture changes in the uncertainty that households face after they are assigned to a family. Thus, we attribute to the insurance component the welfare consequences of a policy-induced change in variances of shocks $\{\varepsilon_i\}_i$ that keeps aggregate resources and expected consumption shares constant (i.e., pure mean preserving spreads in consumption).

We use small-noise expansions to decompose welfare gains from switching from some policy A to policy B into the three components defined above. There are multiple ways to define a decomposition that satisfies Properties a, b, and c, and that are equivalent to the order of approximation we study. We choose a decomposition that is *reflexive*, which means that it satisfies

Property d. Each component of the welfare change from policy A to policy B equals in absolute value, but with the opposite sign, its counterpart for moving from policy B to policy A.

To obtain reflexivity, we consider our approximations relative to a "mid-point" between the allocations under the two policies. Specifically, we define $C^Z, \{w_i^Z, \bar{c}_i^Z\}_i$ as

$$\ln C^Z \equiv \frac{\ln C^B + \ln C^A}{2}, \ \ln w_i^Z \equiv \frac{\ln w_i^B + \ln w_i^A}{2}, \ \bar{c}_i^Z \equiv C^Z w_i^Z. \tag{2}$$

It is easy to verify that $C^Z, w_i^Z, \bar{c}_i^Z$ are simply geometric averages of C^A and C^B, of w_i^A and w_i^B, and of $\mathbb{E}_i c_i^A$ and $\mathbb{E}_i c_i^B$, respectively.

Our small-noise expansions are constructed as follows. Let $\Gamma^j \equiv \ln C^j - \ln C^Z$ and $\Delta_i^j \equiv \ln w_i^j - \ln w_i^Z$ and define a random variable $s_i^j \equiv \left(\Gamma^j, \Delta_i^j, \varepsilon_i^j\right)$. For an arbitrary scalar $\sigma \geq 0$, we construct functions $\mathrm{c}\left(\sigma s_i^j, \bar{c}_i^Z\right)$ and $\mathrm{W}\left(\left\{\sigma s_i^j, \bar{c}_i^Z\right\}\right)$ such that

$$\mathrm{c}\left(\sigma s_i^j, \bar{c}_i^Z\right) \equiv \exp\left(\sigma \Gamma^j\right) \exp\left(\sigma\left(\Delta_i^j\right)\right)\left(1 + \sigma \varepsilon_i^j\right) \bar{c}_i^Z$$

$$\mathrm{W}\left(\left\{\sigma s_i^j, \bar{c}_i^Z\right\}_i\right) \equiv \int \alpha_i U\left(\exp\left(\sigma\left(\Gamma^j + \Delta_i^j\right)\right)\left(1 + \sigma \varepsilon_i^j\right) \bar{c}_i^Z\right) \mu(di)$$

These constructions imply that $\mathrm{c}\left(s_i^j, \bar{c}_i^Z\right) = c_i^j$ and $\mathrm{c}\left(0, \bar{c}_i^Z\right) = \bar{c}_i^Z$. Thus, as we vary the parameter σ between 0 and 1, we sweep out allocations — starting from the allocation in which all households consume the average of their expected consumption shares across policies to the allocation in which they receive their respective consumptions prescribed by the policy. We show (see derivations in the appendix) that

$$\mathcal{W}^B - \mathcal{W}^A = \mathrm{W}\left(\left\{s_i^B, \bar{c}_i^Z\right\}\right) - \mathrm{W}\left(\left\{s_i^A, \bar{c}_i^Z\right\}\right) \simeq \underbrace{\mathbb{E}\phi_i\Gamma}_{\text{agg. efficiency}} + \underbrace{\mathbb{E}\phi_i\Delta_i}_{\text{redistribution}} + \underbrace{\mathbb{E}\phi_i\frac{\gamma_i}{2}\Lambda_i}_{\text{insurance}}, \quad (3)$$

where $\phi_i \equiv \alpha_i U_{c,i}\left(\bar{c}_i^Z\right)\bar{c}_i^Z$, $\gamma_i \equiv -U_{cc}\left(\bar{c}_i^Z\right)\bar{c}_i^Z / U_c\left(\bar{c}_i^Z\right)$,

$$\Gamma \equiv \ln C^B - \ln C^A,\ \Delta_i \equiv \ln w_i^B - \ln w_i^A,\ \Lambda_i \equiv -\left[var_i\left(\ln c_i^B\right) - var_i\left(\ln c_i^A\right)\right], \quad (4)$$

and where the symbol "$\simeq$" indicates that a relationship holds up to the order $O\left(\|\Gamma^j, \Delta_i^j, \varepsilon_i^j\|^3\right)$.

It is straightforward to verify that our definitions of aggregate efficiency, redistribution and insurance in equation (3) satisfy Properties a, b, c, and d. Decomposition (3) shows that the aggregate efficiency component depends on the change in aggregate consumption Γ; that the redistribution component depends on change in expected consumption shares $\{w_i\}_i$; and that the insurance component depends on the changes in the variance of log consumption and the coefficients of the relative risk aversion. The variance of log consumption appears in formula (3) because to our order of approximation it equals the square of the coefficient of variation $std_i\left(c_i\right) / \left(\mathbb{E}_i c_i\right)$. All three components are aggregated with quasi-weights $\{\phi_i\}_i$ that when defined as above assure that the total welfare change and its components are measured in the same units.

3.2 Decomposition in a multi-good economy

It is straightforward to extend our decomposition to general, multi-good settings. To decompose welfare gains into components, first compute points of approximation $\left\{c_{k,i}^Z\right\}_k$ for each good as in equation (2). Then extend definitions of Γ_k and $\Delta_{k,i}$ from equation (4) to every good k, and define $\Lambda_{km,i}$ for each pair of goods k, m as

$$\Lambda_{km,i} \equiv -\left[cov_i\left(\ln c_{k,i}^B, \ln c_{m,i}^B\right) - cov_i\left(\ln c_{k,i}^A, \ln c_{m,i}^A\right)\right].$$

Let $U_{k,i}, U_{km,i}$ be first and second derivatives of U_i evaluated at $\left\{\bar{c}_{k,i}^Z\right\}_k$ and let weights $\left\{\phi_{k,i}\right\}_k$ and cross-elasticities $\left\{\gamma_{km,i}\right\}_{k,m}$ be defined as

$$\phi_{k,i} \equiv \alpha_i U_{k,i}\bar{c}_i^Z, \quad \gamma_{km,i} \equiv -\frac{U_{km,i}\bar{c}_{m,i}^Z}{U_{k,i}}.$$

Using the same steps as in the one good prototype economy, we can show that

$$\mathcal{W}^B - \mathcal{W}^A \simeq \text{agg. efficiency} + \text{redistribution} + \text{insurance}, \quad (5)$$

where

$$\text{agg. efficiency} = \mathbb{E}\sum_k \phi_{k,i}\Gamma_k,$$
$$\text{redistribution} = \mathbb{E}\sum_k \phi_{k,i}\Delta_{k,i},$$
$$\text{insurance} = \frac{1}{2}\mathbb{E}\sum_k\sum_m \phi_{k,i}\gamma_{km,i}\Lambda_{km,i}.$$

When utility is separable across all goods, this decomposition amounts to first computing decomposition (3) separately for each good and then summing each respective components across all goods. When utility is not separable, proper accounting for the insurance component requires adding changes in the covariances in dispersions of different goods weighted with cross-elasticities $\gamma_{km,i}$.

4 Benabou-Floden decomposition

Motivated by the same questions as we are, Benabou (2002) and Floden (2001) developed a different decomposition of a policy-induced welfare change into efficiency, insurance and redistribution components. A substantial literature uses their decomposition to study consequences of alternative public policies. We first present their decomposition, then show that it satisfies property (c) only in special cases. More generally, their decomposition may attribute anything from 0% to ∞% of welfare gains to insurance (and anything from $-\infty$% to 100% to redistribution) from the policies the purely produce mean-preserving reduction in variances of agents' consumption.

It is easiest to present the Benabou-Floden decomposition in a single good economy in which all agents have identical preferences. Decompositions developed by Benabou (2002) and by Floden (2001) share several features and differ only slightly in details. They both start by computing the certainty equivalent level of consumption $c_i^{ce,j}$ for each family i under policy j. This level of consumption is solves

$$U\left(c_i^{ce,j}\right) = \mathbb{E}_i U\left(c_i^j\right). \tag{6}$$

The aggregate certainty equivalent is then defined as $C^{ce,j} \equiv \mathbb{E}c_i^{ce,j}$.

When applied to our single-good economy, Benabou's approach decomposes welfare differences between policies A and B according to

$$\mathcal{W}^B - \mathcal{W}^A = \underbrace{\left[U\left(C^B\right) - U\left(C^A\right)\right]}_{\text{agg. efficiency}} + \underbrace{\left[\left(\mathcal{W}^B - \mathcal{W}^A\right) - \left(U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right)\right]}_{\text{redistribution}}$$
$$+ \underbrace{\left[\left\{U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right\} - \left\{U\left(C^B\right) - U\left(C^A\right)\right\}\right]}_{\text{insurance}}. \quad (7)$$

Benabou interprets the three components as counterparts to our notions of aggregate efficiency, redistribution and insurance.[2]

Floden (2001) extended the framework by measuring welfare changes and its components in consumption units. The total welfare gain from policy B is denoted by ω_U and solves

$$\mathbb{E}\alpha_i U\left(c_i^B\right) = \mathbb{E}\alpha_i U\left(\left(1+\omega_U\right)c_i^A\right). \quad (8)$$

Floden then proceeds to decompose $(1+\omega_U)$ into three components as follows. For each policy j, define constants p^j_{unc} and p^j_{ine} according to

$$U\left(\left(1-p^j_{unc}\right)C^j\right) = U\left(C^{ce,j}\right), \quad U\left(\left(1-p^j_{ine}\right)C^{ce,j}\right) = \mathbb{E}\alpha_i U\left(c_i^{ce,j}\right), \quad (9)$$

and then define objects called uncertainty ω_{unc}, inequality ω_{ine}, and level ω_{lev} according to

$$1+\omega_{unc} \equiv \frac{1-p^B_{unc}}{1-p^A_{unc}}, \qquad 1+\omega_{ine} \equiv \frac{1-p^B_{ine}}{1-p^A_{ine}}, \qquad 1+\omega_{lev} \equiv \frac{C^B}{C^A}.$$

When the utility function assumes the constant relative risk aversion (CRRA) form $U(c) = \frac{c^{1-\gamma}}{1-\gamma}$ if $\gamma > 0,\ \gamma \neq 1$ and $U(c) = \ln c$ for $\gamma = 1$, the three ratios ω_U, ω_{lev}, ω_{unc} and ω_{ine} satisfy

$$\ln\left(1+\omega_U\right) = \ln\left(1+\omega_{lev}\right) + \ln\left(1+\omega_{ine}\right) + \ln\left(1+\omega_{unc}\right). \quad (10)$$

Floden then uses equation (10) to measure the contribution of aggregate efficiency, redistribution and insurance as $\frac{\ln(1+\omega_{lev})}{\ln(1+\omega_U)}$, $\frac{\ln(1+\omega_{ine})}{\ln(1+\omega_U)}$ and $\frac{\ln(1+\omega_{unc})}{\ln(1+\omega_U)}$, respectively.

The approach of Benabou and Floden extends naturally to multi-good economies only when agents have identical preferences that are separable across all goods. In this case, the single good decomposition presented above can be applied separately to each good and then aggregated across goods. When agents' utility functions differ, there is no natural way to compute a counterpart of $U\left(C^{ce,j}\right)$. When there are multiple goods and utility is non-separable, there is no natural definition of certainty equivalent. Floden (2001), who studies a model with

[2] We use different words than Benabou and Floden to refer to some of the same components. All of us call the second two components "redistribution" and "insurance". Benabou calls the first term "aggregate income", while Floden calls it "level". To avoid confusion, throughout this paper we apply our terminology to decompositions of all three sets of authors.

non-separable preferences over consumption and leisure, proposes to side-step this problem by fixing leisure to some *ad-hoc* level in order to compute a consumption certainty equivalent. This approach has limitations as it is easy to construct examples where consumption certainty equivalent does not exist if one follows Floden's procedure. In the economies where the certainty equivalent can be found, the decomposition results are sensitive to the level at which leisure is fixed.

4.1 Properties of the Benabou-Floden decomposition

We now study conditions under which a Benabou-Floden decomposition is consistent with Properties a, b, c. In light of the previous paragraph, we focus on a single good example. It turns out that a key condition under which these properties are satisfied is that dispersions in consumption across households change for all families by the same amounts. Formally, we can state the condition as

Condition 1 *There exists a* Λ *such that*

$$var_i\left(\ln c_i^A\right) - var_i\left(\ln c_i^B\right) = \Lambda \text{ for all } i.$$

Condition 1 narrows the set of policies $j \in \{A, B\}$ substantially. For example, consider a policy reform that increases the progressivity of a tax on labor income. For Condition 1 to hold, it has to be true that dispersion in post-tax labor income plus transfers and income from assets has to scale equally for extremely wealthy and extremely poor (and possibly borrowing-constrained) households. In most standard Aiyagari-Hugget-Bewley models, few interesting policy changes will satisfy this requirement.

We now state the main result of this section.

Lemma 1 *Consider a one-good economy.*

(B). Suppose U *is CRRA with* $\gamma = 1$*. Then Benabou's decomposition satisfies*

$$U\left(C^B\right) - U\left(C^A\right) \simeq \text{agg. efficiency} \simeq \Gamma.$$

Benabou's decomposition violates Property c for generic mean-preserving changes in the spreads of consumption. It satisfies Property c if Condition 1 holds, in which case

$$\left\{U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right\} - \left\{U\left(C^B\right) - U\left(C^A\right)\right\} \simeq \text{insurance} \simeq \frac{1}{2}\Lambda.$$

(F). Suppose U *is CRRA. Then Floden's decomposition satisfies*

$$\frac{\ln\left(1+\omega_{lev}\right)}{\ln\left(1+\omega_U\right)} \simeq \frac{\text{agg. efficiency}}{\mathcal{W}^B - \mathcal{W}^A}.$$

Floden's decomposition violates Property c for generic mean-preserving changes in spreads of consumption. It satisfies Property c if Condition 1 holds, in which case

$$\frac{\ln(1+\omega_{unc})}{\ln(1+\omega_U)} \simeq \frac{insurance}{\mathcal{W}^B - \mathcal{W}^A}.$$

Part (B) focuses on Benabou's decomposition. It shows that his approach satisfies Properties a, b, and c if utility is logarithmic and Condition 1 holds. Part (F) asserts that the same conclusion extends to Floden's decomposition for CRRA preferences. As we explain in the next section, these requirements were indeed satisfied in the model considered by Benabou, but in generic economies Condition 1 will be violated. We interpret the violation of Properties a, b, and c by those decompositions as decomposition errors. The explanation for why such errors emerge is this: while utility of individual consumption certainty equivalent, $U\left(c_i^{ce,j}\right)$, is a good measure of how welfare of an *individual* agent depends on risk, the utility of the aggregate consumption certainty equivalent, $U\left(\mathbb{E}c_i^{ce,j}\right)$, is not necessarily a good measure of how welfare of all agents collectively is affected by uncertainty.

4.2 Error bounds in the Benabou-Floden decomposition

In this section we evaluate the size of the discrepancy between the components of Benabou-Floden decomposition and the requirements implied by Properties a, b, and c. We refer to magnitude of these discrepancies as errors bounds. It is instructive to start with an economy in which these errors are zero, namely, the environment considered in Benabou (2002). In his economy households have preferences

$$\ln c_{t,i} - \frac{1}{1+\eta} l_{t,i}^{1+\eta}$$

over consumption good $c_{t,i}$ and labor $l_{t,i}$ in every period t. Households receive a stochastic productivity shock $\theta_{t,i}$, so that their labor income is $y_{t,i} = \theta_{t,i} l_{t,i}$. Productivity $\theta_{t,i}$ consists of two components, $\theta_{i,t} = \exp\left(e_i + \xi_{t,i}\right)$, where the permanent component e_i captures *ex-ante* heterogeneity and shock $\xi_{t,i}$ drives *ex-post* heterogeneity. Importantly, both shocks are drawn from the same distribution and independently for each family and follow $e_i \sim N\left(-\frac{\upsilon_e^2}{2}, \upsilon_e^2\right)$ and $\xi_{t,i} \sim N\left(-\frac{\upsilon_{\xi,t}^2}{2}, \upsilon_{\xi,t}^2\right)$. Households hold no assets and consume their after-tax labor incomes. After-tax income is $d_t y_{t,i}^{1-\tau}$, where τ is the degree of progressivity of the tax system and d_t is a parameter in the tax function chosen so that the net tax revenues are zero in each period t.

All households supply identical amount of labor

$$l_{t,i}(\tau) = L(\tau) = (1-\tau)^{\frac{1}{1+\eta}},$$

the aggregate consumption is $C(\tau) = (1-\tau)^{\frac{1}{1+\eta}}$ and household consumption is

$$c_{t,i}(\tau) = C(\tau) \times \underbrace{\exp\left((1-\tau) e_i + \frac{\tau(1-\tau)}{2} \upsilon_e^2\right)}_{\equiv w_i(\tau)} \times \left[1 + \underbrace{\exp\left((1-\tau)\xi_{t,i} + \frac{\tau(1-\tau)}{2} \upsilon_{\xi,t}^2\right) - 1}_{\equiv \varepsilon_{t,i}(\tau)}\right]. \tag{11}$$

It is easy to verify that

$$\mathbb{E} w_i(\tau) = 1, \; \mathbb{E}_i \varepsilon_{t,i}(\tau) = 0,$$

so that $C, w_i, \varepsilon_{t,i}$ map into the three components of equation (1).

Benabou's policy instrument is tax progressivity τ. It affects all three components, which motivated Benabou to develop his decomposition. A crucial property is that Benabou's assumptions imply that if the government changes the tax progressivity parameter from τ^A to τ^B then the dispersions of consumptions of all agents change by the same amount

$$var_i\left(\ln c_{t,i}^A\right) - var_i\left(\ln c_{t,i}^B\right) = \exp\left(\left(1-\tau^A\right)^2 \upsilon_{\xi,t}^2\right) - \exp\left(\left(1-\tau^B\right)^2 \upsilon_{\xi,t}^2\right) \text{ for all } i,$$

satisfying Condition 1. Other papers, e.g. Heathcote, Storesletten, and Violante (2017), that built on Benabou's economy to retain its tractability, also preserve this property.

To illustrate why this condition is necessary it is informative to directly apply the Benabou-Floden decomposition. Computing certainty equivalence yields

$$c_{t,i}^{ce}(\tau) = C(\tau) \times \exp\left((1-\tau) e_i + \frac{\tau(1-\tau)}{2} \upsilon_e^2\right) \times \underbrace{\exp\left(-\frac{(1-\tau)^2 \upsilon_{\xi,t}^2}{2}\right)}_{\pi_{t,i}}. \tag{12}$$

The last term in the expression, $\pi_{t,i}$, is the consumption price of risk and represents the fraction of consumption family i would be willing to pay to remove all risk at time t. Constant relative risk aversion and Condition 1 imply that this price is independent across agents. The price of risk automatically factors when aggregating certainty equivalence yielding $C_t^{ce} = C(\tau)\pi_t$. Comparing the utility from aggregate consumption to the utility from the aggregate certainty equivalence of consumptions allows the Benabou-Floden decomposition to isolate the cost of uncertainty in the economy. However, this only succeeds to the extent that $\pi_{t,i}$ is uncorrelated with e_i. Any correlation will necessarily confound redistribution with insurance.

Attaining this property depends sensitively on embracing suitable assumptions: preferences and budget constraints must be such that there is no heterogeneity in labor supply; uncertainty shocks must drawn from the same distribution by all agents; and taxes must affect all agents proportionally. Crucially, there must be no trading of financial assets, so that consumption volatility inherits the volatility of after-tax earnings.

Relaxing any of these assumptions will introduce errors in the decomposition. For instance, if agents have assets, then their consumption is no longer proportional to earnings and so Condition 1 will generally not hold. This is the case in most quantitative macro models, including the one considered by Floden (2001).

We now show that even relatively modest deviations from the benchmark assumptions of Benabou can produce big errors in the Benabou-Floden decomposition. To keep our analysis as close to Benabou's as possible, we maintain his assumptions about preferences, shock processes, and study a policy that reduces dispersions in individual earnings by the same amount. We focus on effects of introducing assets. To keep things simple, we study a static economy, assume inelastic labor supply and abstract from taxes. Under policy A, consumption of household i is

$$c_i^A = \bar{a}_i + \bar{y}_i \left(1 + \tilde{\varepsilon}_i\right),$$

where $\bar{a}_i$ are agents' assets and $\bar{y}_i \left(1 + \tilde{\varepsilon}_i\right)$ is stochastic labor income. Under policy A all agents are in autarky. Under policy B, the government allows agents to trade a full set of Arrow securities contingent on realizations of their idiosyncratic shocks. Since there is no aggregate uncertainty and each family has a continuum of agents, all insurance in this case is attained by trading assets within families. Therefore, consumption under policy B satisfies

$$c_i^B = \bar{a}_i + \bar{y}_i.$$

We impose that shocks $\tilde{\varepsilon}_i$ satisfy $\mathbb{E}_i \tilde{\varepsilon}_i = 0$; that $\tilde{\varepsilon}_i$ are bounded so that c_i^A is strictly positive; and that $var_i \left(\tilde{\varepsilon}_i\right)$ is the same for i. The last condition ensures that in the absence of assets ($\bar{a}_i = 0$ for all i) this economy satisfies Condition 1 as in Benabou (2002). The example is purposefully constructed to make all welfare gains come from better insurance. The next lemma evaluates error bounds for our and Benabou-Floden decompositions.

Lemma 2 *In the constructed example,*

(i) our decomposition (3) implies that $\frac{insurance}{\mathcal{W}^B - \mathcal{W}^A} = 1$;

(ii) insurance component in Benabou and Floden decompositions, namely,

$$\frac{\left\{U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right\} - \left\{U\left(C^B\right) - U\left(C^A\right)\right\}}{\mathcal{W}^B - \mathcal{W}^A}, \ \frac{\ln\left(1 + \omega_{unc}\right)}{\ln\left(1 + \omega_U\right)}$$

can take any values on the $(0, \infty)$ *interval depending on assumptions about the distribution of families and Pareto weights.*

The intuition for part (ii) can be understood as follows. In the proof of the Lemma 2, we show that the fraction of welfare change that Benabou-Floden decompositions will attribute to insurance is given by

$$\frac{\mathbb{E}\left(\bar{a}_i+\bar{y}_i\right)\left(\frac{\bar{y}_i}{\bar{a}_i+\bar{y}_i}\right)^2}{\mathbb{E}\left(\bar{a}_i+\bar{y}_i\right)\times\mathbb{E}\alpha_i\left(\frac{\bar{y}_i}{\bar{a}_i+\bar{y}_i}\right)^2}.$$

This insurance component depends on how the individual price of consumption risk (analogous to term $\pi_{t,i}$ in equation (12)), which is proportional to $\left(\frac{\bar{y}_i}{\bar{a}_i+\bar{y}_i}\right)^2$, covaries with consumption shares. Suppose that cash on hand is log linearly related to income, $\bar{a}_i+\bar{y}_i=\kappa\bar{y}_i^{\beta}$, and income is distributed according to a Pareto distribution with shape parameter α, then the fraction of welfare attributed to insurance simplifies to

$$\frac{\left(\alpha-\beta\right)\left(\alpha-\left(2-2\beta\right)\right)}{\alpha\left(\alpha-\left(2-\beta\right)\right)}.$$

Examining the expression, we see that there is an inflection point at $\beta=0$. When cash on hand and income are positively correlated, $\beta\in(0,1)$, consumption shares will be negatively correlated with individual prices of consumption risk. In this case, the fraction attributed to insurance is always greater than 100% and can be made arbitrarily large as $\alpha\to2-\beta$. Alternatively, when cash on hand and income are positively correlated, $\beta<0$, consumption shares positively correlate with the price of consumption risk, the fraction attributed to insurance is always less than 100% and can be made to approach 0% as $\alpha\to2-2\beta$.

In the appendix, we show that one can also get unbounded errors by keeping constant the joint distribution of assets and income, but varying Pareto weights $\{\alpha_i\}$. This is possible because, generically, changing the Pareto weights will affect the welfare gains attributed to insurance and redistribution. This can be observed in a simple endowment economy with households of two types. Suppose that type 1 households have an endowment that moves with aggregate shocks while type 2 households receive a constant endowment. Consider a policy change from autarky (policy A) to introducing complete markets (policy B). The presence of Arrow securities allows households of type 2 to insure households of type 1 and, in return, receive a greater share of the aggregate endowment. As a result, Condition 1 is not satisfied since some households face an increase in uncertainty while for others uncertainty decreases. The policy change from A to B is a Pareto improvement so any choice of Pareto weights will result in an increase in welfare, but what components of welfare account for this increase will necessarily depend on the Pareto weights. If the planner places full weight on households of type 1, we would attribute the welfare gain to insurance and find a welfare loss from redistribution.[3] The reverse is true if the planner places full weight on households of type 2. It would be impossible to capture this reversal under the Benabou-Floden decomposition since their insurance term is always independent of their choice of Pareto weights. That independence is only correct when Condition 1 is satisfied so all households receive the same benefit from the change in uncertainty. This example also cautions against the idea that one can focus on the sum of aggregate efficiency and insurance components to isolate Pareto improvements.

Similar results emerge in calibrated quantitative economies. For example, in Bhandari, Evans, Golosov, and Sargent (2021) we considered a standard incomplete market heterogeneous agent (HA) New Keynesian economy. We assumed that utility is isoelastic and separable in consumption and leisure

$$\frac{c^{1-\nu}-1}{1-\nu}-\frac{l^{1+\gamma}}{1+\gamma}$$

over consumption and labor. The labor productivity shocks in our calibration were similar to those used by Benabou (2002), but agents also hold financial assets calibrated to the U.S. data. In our baseline setting, with $\nu = 3$, we showed that switching from the optimal policy prescribed by the textbook representative agent economy to the optimal policy in the HA economy increased welfare, and that 158% of that welfare gain could be attributed to insurance, -66% to aggregate efficiency and 8% to redistribution. This decomposition was also robust to changes on assumption preferences, redistributive objectives of the government, and other things. In contrast, when applied directly Benabou's decomposition attributed +800% of welfare gains to both the insurance and redistribution component. Moreover, computed values varied wildly even with modest changes in such parameters as degrees of agents' risk aversion or the planner's inequality aversion even though neither optimal policies nor allocations were very sensitive to them. The reason for these results can be traced back to the source isolated in the example studied in Lemma 2.[4]

5 Conclusion

We developed a decomposition of welfare changes following policies into three components: aggregate efficiency, that captures effects from changes in the aggregate quantity of resources;

[3] Since households of type 1 have a smaller share of consumption under policy B as compared to policy A.

[4] Floden's decomposition is not well-defined for the isoelastic separable functional form for the calibration used in Bhandari, Evans, Golosov, and Sargent (2021). The reason is that utility from consumption is bounded from above, and equation (9) that defines p^j_{ine} is not guaranteed to have a solution. We tried Floden's decomposition setting $\nu = 1$. With log utility of consumption, we found Floden's decomposition to be sensitive to the ad-hoc choice of labor used to compute certainty equivalents: the contribution of the redistribution component varied from -33% to 103% depending on if labor was set at the aggregate level or expected individual choice, respectively.

redistribution, that captures effects from changes in shares of resources that *ex-ante* heterogeneous agents expect to receive; and insurance, that captures effects of changes in the uncertainty that agents face. Our decomposition applies to a large class of multi-person, multi-good, multi-period economies with general specifications of few preferences and shocks and sources of heterogeneity. Our decomposition satisfies the desirable property that welfare effects arising from pure mean preserving spreads in consumption are always attributed to the insurance component. Commonly used decompositions correctly identify these components only under special conditions. When these conditions do not hold, big errors can occur.

References

BENABOU, R. (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, 70(2), 481–517.

BHANDARI, A., D. EVANS, M. GOLOSOV, AND T. SARGENT (2021): "Inequality, Business Cycles, and Monetary-Fiscal Policy," Working paper.

FLODEN, M. (2001): "The effectiveness of government debt and transfers as insurance," *Journal of Monetary Economics*, 48(1), 81–108.

HEATHCOTE, J., K. STORESLETTEN, AND G. VIOLANTE (2017): "Optimal Tax Progressivity: An Analytical Framework," *The Quarterly Journal of Economics*.

A Appendix

We first describe a few conventions and notation that we will use in this appendix. For our analysis, we impose a mild technical condition that $\|s_i^j\| < \bar{s}$. For some any variable $x\left(s_i^j\right)$, we construct a $x\left(\sigma s_i^j\right)$ as a function of σ. Our approximation uses a second-order Taylor-series expansions of functions with respect to σ around $\sigma = 0$. Let $\nabla x\left(0, \bar{c}_i^Z\right)$ and $\nabla^2 x\left(0, \bar{c}_i^Z\right)$ denote the first- and second derivatives with respect to the argument σs_i^j. By Taylor theorem,

$$
\begin{aligned}
x\left(\sigma s_i^j\right) &= x(0) + \sigma s_i^j \cdot \nabla x\left(0, \bar{c}_i^Z\right) + \frac{\sigma^2}{2} s_i^j \cdot \nabla^2 x\left(0, \bar{c}_i^Z\right) \cdot s_i^j + \mathcal{O}\left((\sigma \bar{s})^3\right) \\
&= \bar{x} + \sigma x_\sigma\left(s_i^j\right) + \frac{\sigma^2}{2} x_{\sigma\sigma}\left(s_i^j\right) + \mathcal{O}\left((\sigma \bar{s})^3\right) \\
&\simeq \bar{x} + \sigma x_\sigma\left(s_i^j\right) + \frac{\sigma^2}{2} x_{\sigma\sigma}\left(s_i^j\right)
\end{aligned}
$$

For expressions that take the same value for policy $j = A$ and $j = B$ we occasionally use a shorthand "t.i.p.", i.e. terms independent of policy.

By the Law of Iterated Expectations (LIE), we mean the property that for any deterministic function x_i of i, we have $\mathbb{E} x_i \varepsilon_i^j = \int \mathbb{E}_i x_i \varepsilon_i^j d\mu = \int x_i \mathbb{E}_i \varepsilon_i^j d\mu = 0$ since $\mathbb{E}_i \varepsilon_i^j = 0$, and, similarly, $\mathbb{E} x_i \left(\varepsilon_i^j\right)^2 = \mathbb{E} x_i var_i\left(\varepsilon_i^j\right)$.

We use shorthand $U_i, U_{c,i}, U_{cc,i}$ for $U_i\left(\bar{c}_i^Z\right), U_{c,i}\left(\bar{c}_i^Z\right), U_{cc,i}\left(\bar{c}_i^Z\right)$ in the one-good economy, and $U_i, U_{k,i}, U_{km,i}$ for $U_i\left(\left\{\bar{c}_{k,i}^Z\right\}_k\right), U_{k,i}\left(\left\{\bar{c}_{k,i}^Z\right\}_k\right), U_{km,i}\left(\left\{\bar{c}_{k,i}^Z\right\}_k\right)$ in the multi-good economy.

A.1 Derivations of equation (3)

Let $\Gamma^j \equiv \ln C^j - \ln C^Z$ and $\Delta_i^j \equiv \ln w_i^j - \ln w_i^Z$. Observe that by construction we have

$$
\Gamma^B - \Gamma^A = \Gamma,\ \Delta_i^B - \Delta_i^A = \Delta_i,\ \Gamma^B = -\Gamma^A,\ \Delta_i^B = -\Delta_i^A, \tag{13}
$$

and therefore

$$
\left(\Gamma^j\right)^2, \left(\Delta_i^j\right)^2, \Gamma^j \Delta_i^j \text{ are t.i.p. for all } i.
$$

From the definitions of $c_i^j(\sigma)$ from the main text we have

$$
c_i^j(\sigma) \quad \equiv \quad \exp\left(\sigma\left(\underbrace{\ln C^j - \ln C^Z}_{\Gamma^j}\right)\right) \exp\left(\sigma\left(\underbrace{\ln w_i^j - \ln w_i^Z}_{\Delta_i^j}\right)\right)\left(1 + \sigma \varepsilon_i^j\right) \bar{c}_i^Z,
$$

We can therefore write welfare $\mathcal{W}^j(\sigma)$ as

$$
\mathcal{W}^j(\sigma) = \mathbb{E} \alpha_i U\left(\exp\left(\sigma\left(\Gamma^j + \Delta_i^j\right)\right)\left(1 + \sigma \varepsilon_i^j\right) \bar{c}_i^Z\right).
$$

Taking the second order expansion and applying the LIE we obtain

$$\begin{aligned}\mathcal{W}^j(\sigma) &\simeq \mathbb{E}\alpha_i U_i + \sigma \mathbb{E}\alpha_i U_{c,i}\bar{c}_i^Z\left(\Gamma^j + \Delta_i^j\right) + \frac{\sigma^2}{2}\mathbb{E}\alpha_i U_{c,i}\bar{c}_i^Z\left(\Gamma^j + \Delta_i^j\right)^2 \\ &\quad + \frac{\sigma^2}{2}\mathbb{E}\alpha_i U_{cc,i} \times \left(\bar{c}_i^Z\right)^2\left[\left(\Gamma^j + \Delta_i^j\right)^2 + \left(\varepsilon_i^j\right)^2\right] \\ &= \sigma \mathbb{E}\alpha_i U_{c,i}\bar{c}_i^Z\left(\Gamma^j + \Delta_i^j\right) + \frac{\sigma^2}{2}\mathbb{E}\alpha_i U_{cc,i} \times \left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right)^2 + \text{t.i.p.} \qquad (14)\end{aligned}$$

Since $\mathcal{W}^j = \mathcal{W}^j(1)$, we have

$$\mathcal{W}^B - \mathcal{W}^A \simeq \mathbb{E}\underbrace{\alpha_i U_{c,i}\bar{c}_i^Z}_{\equiv\phi_i}\Gamma + \mathbb{E}\underbrace{\alpha_i U_{c,i}\bar{c}_i^Z}_{\equiv\phi_i}\Delta_i + \frac{1}{2}\mathbb{E}\underbrace{\alpha_i U_{c,i}\bar{c}_i^Z}_{\equiv\phi_i}\underbrace{\frac{U_{cc,i}\bar{c}_i^Z}{U_{c,i}}}_{\equiv-\gamma_i}\left[\mathbb{E}_i\left(\varepsilon_i^B\right)^2 - \mathbb{E}_i\left(\varepsilon_i^A\right)^2\right]. \qquad (15)$$

Finally, observe that

$$var_i\left(\ln c_i^j\right) = \mathbb{E}_i\left(\ln\left(1+\varepsilon_i^j\right) - \mathbb{E}_i\ln\left(1+\varepsilon_i^j\right)\right)^2 \simeq \mathbb{E}_i\left(\varepsilon_i^j\right)^2. \qquad (16)$$

Substitute this relationship and definitions of γ_i, ϕ_i into (15) to obtain (3).

A.2 Derivations of equation (5)

Similarly to section A.1, we define $\Gamma_k^j \equiv \ln C_k^j - \ln C_k^Z$ and $\Delta_{k,i}^j \equiv \ln w_{k,i}^j - \ln w_{k,i}^Z$, and observe that they satisfy $\Gamma_k^A = -\Gamma_k^B,\ \Delta_{k,i}^A = -\Delta_{k,i}^B$ and therefore

$$\Gamma_k^j\Gamma_m^j, \Gamma_k^j\Delta_{m,i}^j, \Delta_{k,i}^j\Delta_{m,i}^j \text{ are t.i.p. for all } k, m, i.$$

We have

$$\mathcal{W}^j(\sigma) = \mathbb{E}\alpha_i U\left(\left\{\exp\left(\sigma\left(\Gamma_k^j + \Delta_{k.i}^j\right)\right)\left(1+\sigma\varepsilon_{k,i}^j\right)\bar{c}_{k,i}^Z\right\}_k\right),$$

and therefore its second order expansion can be written, using the LIE, as

$$\mathcal{W}^j(\sigma) \simeq \sigma\mathbb{E}\sum_k \alpha_i U_{k,i}\bar{c}_{k,i}^Z\left(\Gamma^j + \Delta_{k,i}^j\right) + \frac{\sigma^2}{2}\mathbb{E}\sum_k\sum_m \alpha_i U_{km,i}\bar{c}_{k,i}^Z\bar{c}_{m,i}^Z\mathbb{E}_i\left(\varepsilon_{k,i}^j\varepsilon_{m,i}^j\right) + \text{t.i.p.}$$

Since $\mathcal{W}^j = \mathcal{W}^j(1)$, we have

$$\begin{aligned}\mathcal{W}^B - \mathcal{W}^A &\simeq \mathbb{E}\sum_k \alpha_i U_{k,i}\bar{c}_{k,i}^Z\Gamma_k + \mathbb{E}\sum_k \alpha_i U_{k,i}\bar{c}_{k,i}^Z\Delta_{k,i} \\ &\quad + \frac{1}{2}\mathbb{E}\sum_k \alpha_i U_{k,i}\bar{c}_{k,i}^Z\sum_m \frac{U_{km,i}\bar{c}_{m,i}^Z}{U_{k,i}}\left[\mathbb{E}_i\left(\varepsilon_{k,i}^B\varepsilon_{m,i}^B\right) - \mathbb{E}_i\left(\varepsilon_{k,i}^A\varepsilon_{m,i}^A\right)\right].\end{aligned}$$

Use the relationship

$$cov_i\left(\ln c_{k,i}^j, \ln c_{m,i}^j\right) \simeq \mathbb{E}_i\left(\varepsilon_{k,i}^j\varepsilon_{m,i}^j\right),$$

together with definitions of $\phi_{k,i}, \gamma_{km,i}$ to obtain (5).

A.3 Derivations of equation (10)

Floden derives (10) for the case of utilitarian planner, but we show here that it holds generally. We have

$$\begin{aligned}\mathbb{E}\alpha_i U\left(c_i^B\right) &= \mathbb{E}\alpha_i U\left(c_i^{ce,B}\right) = \left(1-p_{ine}^B\right)^{1-\gamma} U\left(C^{ce,B}\right) = \left(1-p_{ine}^B\right)^{1-\gamma}\left(1-p_{unc}^B\right)^{1-\gamma} U\left(C^B\right) \\ &= \left(1-p_{ine}^B\right)^{1-\gamma}\left(1-p_{unc}^B\right)^{1-\gamma}\left(1+\omega_{lev}\right)^{1-\gamma} U\left(C^A\right)\end{aligned}$$

and

$$\mathbb{E}\alpha_i U\left(\left(1+\omega_U\right)c_i^A\right) = \left(1+\omega_U\right)^{1-\gamma}\left(1-p_{ine}^A\right)^{1-\gamma}\left(1-p_{unc}^A\right)^{1-\gamma} U\left(C^A\right)$$

Therefore

$$\left(1+\omega_U\right)^{1-\gamma} = \left(\frac{1-p_{ine}^B}{1-p_{ine}^A}\right)^{1-\gamma}\left(\frac{1-p_{unc}^B}{1-p_{ine}^A}\right)^{1-\gamma}\left(1+\omega_{lev}\right)^{1-\gamma},$$

or

$$\left(1+\omega_U\right) = \left(1+\omega_{lev}\right)\left(1+\omega_{ine}\right)\left(1+\omega_{unc}\right).$$

Take logs to get (10).

A.4 Proof of Lemma 1

Throughout this proof, U, U_c, U_{cc} are understood to have the argument $\mathbb{E}\bar{c}_i^Z$. As before, $U_i, U_{c,i}, U_{cc,i}$ have the argument $\bar{c}_i^Z$.

First observe that under CRRA preferences, we have from decomposition (3)

$$\begin{aligned}\text{agg efficiency} &= \mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\Gamma, &(17)\\ \text{insurance} &= \frac{\gamma}{2}\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)_i^{1-\gamma}\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right], &(18)\\ \mathcal{W}^B-\mathcal{W}^A &\simeq \mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\Gamma+\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\Delta_i &(19)\\ &\quad+\frac{\gamma}{2}\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right].\end{aligned}$$

Part (B). When preferences are logarithmic, we immediately have

$$U\left(C^B\right)-U\left(C^A\right)=\ln C^B-\ln C^A=\Gamma.$$

In this case, from (17), agg. efficiency = Γ since $\mathbb{E}\alpha_i = 1$. Therefore, with logarithmic preference $U\left(C^B\right)-U\left(C^A\right)$ = agg. efficiency.

Consumption certainty equivalent for arbitrary σ is defined by

$$U\left(c_i^{ce,j}(\sigma)\right)=\mathbb{E}_i U\left(\exp\left(\sigma\left(\Gamma^j+\Delta_i^j\right)\right)\left(1+\sigma\varepsilon_i^j\right)\bar{c}_i^Z\right).$$

The right and left hand sides of this expression are

$$\begin{aligned} RHS(\sigma) &\simeq U_i + \sigma U_{c,i}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{\sigma^2}{2}U_{cc,i}\times\left(\bar{c}_i^Z\right)^2\left[\left(\Gamma^j+\Delta_i^j\right)^2 + var_i\left(\varepsilon_i^j\right)\right] \\ &+\frac{\sigma^2}{2}U_{c,i}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)^2, \end{aligned}$$

and

$$LHS(\sigma) = U\left(\bar{c}_i^{ce,j}\right) + \sigma U_c\left(\bar{c}_i^{ce,j}\right)c_{i,\sigma}^{ce,j} + \frac{\sigma^2}{2}\left[U_{cc}\left(\bar{c}_i^{ce,j}\right)\left(c_{i,\sigma}^{ce,j}\right)^2 + U_c\left(\bar{c}_i^{ce,j}\right)\times c_{i,\sigma\sigma}^{ce,j}\right].$$

Since $RHS(\sigma) = LHS(\sigma)$ for all σ, this implies

$$\bar{c}_i^{ce,j} = \bar{c}_i^Z,\ c_{i,\sigma}^{ce,j} = \bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right),\ \ c_{i,\sigma\sigma}^{ce,j} = \frac{U_{cc,i}}{U_{c,i}}\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right) + \bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)^2.$$

The aggregate consumption certainty equivalent is

$$\begin{aligned} C^{ce,j}(\sigma) &= \mathbb{E}c_i^{ce,j}(\sigma) \simeq \mathbb{E}\bar{c}_i^Z + \sigma\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{\sigma^2}{2}\mathbb{E}\frac{U_{cc,i}}{U_{c,i}}\times\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right) + \frac{\sigma^2}{2}\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)^2 \\ &= \sigma\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{\sigma^2}{2}\mathbb{E}\frac{U_{cc,i}}{U_{c,i}}\times\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right) + \text{t.i.p.}, \end{aligned}$$

and therefore

$$\begin{aligned} U\left(C^{ce,j}(\sigma)\right) &\simeq U + \sigma U_c\times\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{\sigma^2}{2}U_{cc}\times\left(\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)\right)^2 \\ &+\frac{\sigma^2}{2}U_c\times\left[\mathbb{E}\frac{U_{cc,i}}{U_{c,i}}\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right) + \mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)^2\right] \\ &= \sigma U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{\sigma^2}{2}U_c\mathbb{E}\frac{U_{cc,i}}{U_{c,i}}\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right) + \text{t.i.p.} \end{aligned} \tag{20}$$

Since $C^{ce,j} = C^{ce,j}(1)$, this equation implies

$$U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right) \simeq U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma+\Delta_i\right) + \frac{1}{2}U_c\mathbb{E}\bar{c}_i^Z\gamma_i\left[var_i\left(\varepsilon_i^A\right) - var_i\left(\varepsilon_i^B\right)\right]. \tag{21}$$

Observe that

$$\begin{aligned} U\left(C^j\right) &= U\left(\mathbb{E}c_i^j\right) = U\left(\mathbb{E}\exp\left(\Gamma^j+\Delta_i^j\right)\bar{c}_i^Z\right) \\ &\simeq U + U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \frac{1}{2}U_{cc}\left[\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)\right]^2 + \frac{1}{2}U_c\mathbb{E}\left[\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)\right]^2 \\ &= U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right) + \text{t.i.p.} \end{aligned} \tag{22}$$

Therefore

$$\begin{aligned}\left\{U\left(C^{ce,B}\right)-U\left(C^{ce,A}\right)\right\}-\left\{U\left(C^{B}\right)-U\left(C^{A}\right)\right\} &\simeq \frac{1}{2}U_c\mathbb{E}\bar{c}_i^Z\gamma_i\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]\\ &= \frac{1}{2}\frac{\mathbb{E}\bar{c}_i^Z\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\bar{c}_i^Z}, \qquad (23)\end{aligned}$$

where the last relationship holds when preferences are logarithmic and so we can substitute $U_c = \frac{1}{\mathbb{E}\bar{c}_i^Z}$ and $\gamma_i = 1$.

By construction, $var_i\left(\varepsilon_i^A\right) - var_i\left(\varepsilon_i^B\right)$ is always a mean preserving spread. Therefore Property c holds if any policy B that induces an arbitrary change in $\left\{var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right\}$ while keeping $\Gamma = \Delta_i = 0$ for all i, satisfies

$$\left\{U\left(C^{ce,B}\right)-U\left(C^{ce,A}\right)\right\}-\left\{U\left(C^{B}\right)-U\left(C^{A}\right)\right\} = \mathcal{W}^B - \mathcal{W}^A$$

Using equation (23) and (19) the previous condition can be written as

$$\frac{\mathbb{E}\bar{c}_i^Z\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\bar{c}_i^Z} = \mathbb{E}\alpha_i\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right] \qquad (24)$$

For generic $\left\{var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right\}_i$ this condition does not hold. If Condition 1 is satisfied, then $\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right] \simeq \Lambda$ for all i. This implies that equation (24) holds, and

$$\left\{U\left(C^{ce,B}\right)-U\left(C^{ce,A}\right)\right\}-\left\{U\left(C^{B}\right)-U\left(C^{A}\right)\right\} \simeq \frac{\Lambda}{2}.$$

Part (F).

From equation (8), the term $\ln\left(1+\omega_U\right)$ in Floden decomposition satisfies

$$(1-\gamma)\ln\left(1+\omega_U\right) = \ln\mathcal{W}^B - \ln\mathcal{W}^A. \qquad (25)$$

Observe that for any variable x,

$$\ln x(\sigma) \simeq \ln\bar{x} + \sigma\frac{x_\sigma}{\bar{x}} + \frac{\sigma^2}{2}\left[\frac{x_{\sigma\sigma}}{\bar{x}} - \left(\frac{x_\sigma}{\bar{x}}\right)^2\right]. \qquad (26)$$

Apply this to (14)

$$\begin{aligned}\ln\mathcal{W}^j(\sigma) &\simeq \frac{\sigma\mathbb{E}\alpha_i U_{c,i}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)}{\mathbb{E}\alpha_i U\left(\bar{c}_i^Z\right)} + \frac{\sigma^2}{2}\frac{\mathbb{E}\alpha_i U_{cc,i}\times\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right)^2}{\mathbb{E}\alpha_i U\left(\bar{c}_i^Z\right)} + \text{t.i.p.}\\ \ln\mathcal{W}^j &= (1-\gamma)\frac{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\left(\Gamma^j+\Delta_i^j\right) - \frac{\gamma}{2}\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}var_i\left(\varepsilon_i^j\right)}{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}} + \text{t.i.p.},\end{aligned}$$

where in the second line we used the assumption that U is CRRA and $\ln \mathcal{W}^j = \ln \mathcal{W}^j(1)$. Therefore

$$\begin{aligned}\ln(1+\omega_U) &\simeq \frac{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}(\Gamma+\Delta_i)+\frac{\gamma}{2}\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}} \\ &\simeq \frac{\mathcal{W}^B-\mathcal{W}^A}{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}},\end{aligned} \tag{27}$$

where in the last line we used (19).

We obviously have $\ln(1+\omega_{lev}) = \Gamma$ from the definitions of ω_{lev} and Γ. Therefore, the second order approximations give us

$$\frac{\ln(1+\omega_{lev})}{\ln(1+\omega_U)} = \frac{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}\Gamma}{\mathcal{W}^B-\mathcal{W}^A} = \frac{\text{agg efficiency}}{\mathcal{W}^B-\mathcal{W}^A},$$

where the last equation follows from (17).

Term $\ln(1+\omega_{unc})$ can written as

$$(1-\gamma)\ln(1+\omega_{unc}) = \left[\ln U\left(C^{ce,B}\right)-\ln U\left(C^{ce,A}\right)\right]-\left[\ln U\left(C^B\right)-\ln U\left(C^A\right)\right].$$

Apply (26) to equations (20) and (22) to show that

$$\begin{aligned}\ln U\left(C^{ce,j}\right) &\simeq \frac{1}{U}\left[U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)+\frac{1}{2}U_c\mathbb{E}\frac{U_{cc,i}}{U_{c,i}}\left(\bar{c}_i^Z\right)^2 var_i\left(\varepsilon_i^j\right)\right]+\text{t.i.p.}, \\ \ln U\left(C^j\right) &\simeq \frac{1}{U}\left[U_c\mathbb{E}\bar{c}_i^Z\left(\Gamma^j+\Delta_i^j\right)\right]+\text{t.i.p.}\end{aligned}$$

Therefore

$$(1-\gamma)\ln(1+\omega_{unc}) \simeq \frac{\gamma}{2}\frac{U_c}{U}\mathbb{E}\bar{c}_i^Z\Lambda_i = \frac{\gamma(1-\gamma)}{2}\frac{\mathbb{E}\bar{c}_i^Z\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\bar{c}_i^Z}. \tag{28}$$

Using (27) and (28), we get

$$\frac{\ln(1+\omega_{unc})}{\ln(1+\omega_U)} = \frac{\gamma}{2}\frac{\mathbb{E}\bar{c}_i^Z\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\bar{c}_i^Z}\frac{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}}{\mathcal{W}^B-\mathcal{W}^A}.$$

Property c holds if for all $\left\{var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right\}_i$ we have

$$\frac{\mathbb{E}\bar{c}_i^Z\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right]}{\mathbb{E}\bar{c}_i^Z}\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma} = \mathbb{E}\alpha_i\left(c_i^Z\right)^{1-\gamma}\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right].$$

For generic $\left\{var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right\}_i$ this condition does not hold. If Condition 1 is satisfied, then $\left[var_i\left(\varepsilon_i^A\right)-var_i\left(\varepsilon_i^B\right)\right] \simeq \Lambda$ for all i, this equation holds and

$$\frac{\ln(1+\omega_{unc})}{\ln(1+\omega_U)} = \frac{\gamma}{2}\Lambda\frac{\mathbb{E}\alpha_i\left(\bar{c}_i^Z\right)^{1-\gamma}}{\mathcal{W}^B-\mathcal{W}^A} = \frac{\text{insurance}}{\mathcal{W}^B-\mathcal{W}^A}.$$

A.5 Proof of Lemma 2

Part (i). Let $\bar{A} \equiv \mathbb{E}a_i$ and $\bar{Y} \equiv \mathbb{E}y_i$. We then can write

$$c_i^A = \left(\bar{A} + \bar{Y}\right) \times \frac{\bar{a}_i + \bar{y}_i}{\bar{A} + \bar{Y}} \times \left(1 + \underbrace{\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\tilde{\varepsilon}_i}_{\equiv \varepsilon_i}\right).$$

and therefore

$$\Lambda_i = var_i\left(\varepsilon_i\right) = \left(\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\right)^2 var\left(\tilde{\varepsilon}\right). \tag{29}$$

It is immediate to see that $\Gamma = 0$ and $\Delta_i = 0$ for all i and, therefore, equation (3) implies when preferences are logarithmic that

$$\text{Insurance} = \mathcal{W}^B - \mathcal{W}^A = \frac{1}{2} var\left(\tilde{\varepsilon}\right) \mathbb{E}\alpha_i \left(\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\right)^2.$$

This establishes part (i).

Part (ii). Substitute (29) into (23) to calculate insurance component as

$$\left\{U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right\} - \left\{U\left(C^B\right) - U\left(C^A\right)\right\} \simeq \frac{1}{2} var\left(\tilde{\varepsilon}\right) \frac{\mathbb{E}\bar{c}_i^Z \left(\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\right)^2}{\mathbb{E}\bar{c}_i^Z}.$$

This implies that

$$\frac{\left\{U\left(C^{ce,B}\right) - U\left(C^{ce,A}\right)\right\} - \left\{U\left(C^B\right) - U\left(C^A\right)\right\}}{\mathcal{W}^B - \mathcal{W}^A} \simeq \frac{\mathbb{E}\bar{c}_i^Z \left(\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\right)^2}{\mathbb{E}\bar{c}_i^Z \times \mathbb{E}\alpha_i \left(\frac{\bar{y}_i}{\bar{a}_i + \bar{y}_i}\right)^2}. \tag{30}$$

We show the result in two steps. First, we keep Pareto weights fixed and vary the joint distribution of income and assets and then we keep the joint distribution of income and assets fixed, and vary Pareto weights.

Assume that the expected cash on hand (assets plus income) is log linear in income, so

$$\bar{a}_i + \bar{y}_i = \kappa \bar{y}_i^{\beta}.$$

Furthermore, we assume that $\bar{y}_i$ is distributed according to a Pareto distribution with shape parameter α. Noting that $\bar{c}_i^Z = \bar{a}_i + \bar{y}_i$, we find that the fraction of welfare gain attributed to insurance is

$$\frac{\mathbb{E}\bar{y}_i^{2-\beta}}{\mathbb{E}\bar{y}_i^{\beta}\mathbb{E}\bar{y}_i^{2-2\beta}}.$$

This fraction is well-defined as long as $\alpha > \max\{2-\beta, \beta, 2-2\beta\}$ and equal to

$$\frac{(\alpha-\beta)(\alpha-(2-2\beta))}{\alpha(\alpha-(2-\beta))}.$$

If income and cash on hand are positively related with $\beta \in (0,1)$ this fraction can become unboundedly large in the limit as $\alpha \to 2-\beta$. If income and cash on hand are negatively related with $\beta < 0$ this fraction can approach zero as $\alpha \to 2-2\beta$.

We can also get unbounded errors by varying Pareto weights. The easiest way is to consider Pareto weights that put weight 1 on one family ι. In this case, the right hand size of expression (30) becomes $(1+\bar{a}_\iota/\bar{y}_\iota)^2 \mathbb{E}\bar{c}_i^Z \left(\frac{\bar{y}_i}{\bar{a}_i+\bar{y}_i}\right)^2 / \mathbb{E}\bar{c}_i^Z$. If the distribution of families is such that $\bar{a}_i/\bar{y}_i$ has full support on $[-1+\epsilon,\infty)$ interval (with ϵ is chosen to ensure non-negative consumption for all agents under policy A), then the insurance component of Benabou decomposition can take any value on the interval $\left[\epsilon\mathbb{E}\bar{c}_i^Z \left(\frac{\bar{y}_i}{\bar{a}_i+\bar{y}_i}\right)^2 / \mathbb{E}\bar{c}_i^Z , +\infty\right)$. If the upper bound for shocks $|\tilde{\varepsilon}|$ is sufficiently low, ϵ can be made arbitrarily low, establishing the result of the lemma for Benabou's decomposition. The arguments for Floden decomposition work similarly.