Hicks-Arrow Prices for US Federal Debt 1791-1930

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Abstract

We enlist Turing.jl, Bayes' Law, Hamiltonian Monte Carlo, and a parameteric statistical model of Hicks-Arrow date-contingency prices to approximate nominal (meaning *gold*-dollar) yield curves for the US from 1791 to 1930. Posterior probability coverage intervals for yield curves indicate more uncertainty during periods in which data are especially sparse (e.g., during the administration of Andrew Jackson who, unlike his admirer President Donald Trump, paid off all US federal debt). We compare our approximate yield curves with standard historical series on yields on US federal debt and find substantial discrepancies especially during war time surges in government expenditures that were accompanied by units of account ambiguities. We use our approximate yield curves to study how long it took to achieve Alexander Hamilton's goal of reducing default premia in US yields by building a reputation for paying as promised on time.

Key words: Big data, default premia, nominal yield curve, units of account, gold standard, government debt, learning, Julia, Turing.jl, V.V. Chari's "tail wags dog" hypothesis, pricing errors, specification analysis.

1 Introduction

US politicians today care and talk about interest rates on Federal debt. They always have. This paper drills down into an historical big data set and takes a careful look at what they were talking about before 1930. We extract term structures of nominal *yields* from data that Hall et al. (2018) assembled on prices and quantities of US Federal bonds that promised to pay sequences of US gold dollars from 1791 to 1930, shortly before Franklin Roosevelt and Irving Fisher took the US off the gold standard that George Washington and Alexander Hamilton had set in place in 1790 and 1791.[1] We apply the asset price theory of John R. Hicks and Kenneth Arrow to organize how we "un-bundle" prices of the date- and state-contingent payments from those bonds to form the "yields" or "interest rates" that appear in modern macroeconomic theory. Economists at the Federal Reserve Board and other research institutions use a version of the same Hicks-Arrow pricing *theory* that we do, but in un-bundling a yield curve from observed prices and quantities they face and solve a technically different inference problem than we do. Because they have a superabundance of *cross-section* data on prices and quantities at each date, they solve an *overdetermined* inference problem when they fit yield curves at each date. While we too have a "big data" set, it is impoverished along exactly the dimensions that would require *more* cross-section data at each date if we had wanted to use even a *just-identified* of the Federal Reserve Board's procedure. To confront this challenge, we enlist *prejudice* in the form of a statistical model that views our data through the

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[1]Edwards (2018) describes how the US defaulted on its promises to pay gold dollars. Rothbard (2002) describes how Irving Fisher's advocacy shaped Roosevelt's decision to leave the gold standard.

lens of a profligately parameterized statistical model of a *panel* with many missing observations. The statistical model encodes our prejudice and tells us how, by computing probabilities conditional on the data, to learn about parameters that tie down posterior probability distributions of yield curves at all dates in our panel. We approximate that posterior probability distribution by deploying Hamiltonian Monte Carlo and No U-Turn sampling.[2] Our data and statistical model tell us how much *smoothing* across time we should do.

Why do we restrict ourselves to the period before 1930? Because, with only temporary suspensions, the US was on a gold standard throughout that period, while after 1930 it was not. Merging the period *after* the big default of 1933 with the gold standard period that we focus on would substantially complicate how we approach default risk. Why do we want these yield curves? Because "interest rates" paid on US government bonds have preoccupied politicians and economists from "time 0" for the US and ever after. In his 1790 *Report on Public Credit*, 34 year old Alexander Hamilton argued that the US could *lower* interest costs by restructuring political institutions to sustain tax, spending, and debt-servicing policies that would promote *expectations* among foreign and domestic lenders (i.e., the Federal government's *reputation*) that the US federal government would pay its bills on time. Hamilton and other framers of the US constitution were in the business of designing a mechanism to reduce what they knew were substantial *default-risk* premia that actions by earlier US Continental Congress and US state had earned during and after the War of Independence. But the same words can mean different things to different people and at different *times* (a distinction between *ex ante* versus *ex post* strikes again). Thus, when after the US Civil War, President Andrew Johnson and a majority of the Democratic party proposed to reduce debt servicing costs, they explained that they would reduce real interest payments simply by redefining the unit of account from gold to inconvertible paper dollars (greenbacks) that were then trading at a substantial discount relative to gold. This unit-of-account issue was contested in the 1868 election. In 1868, the Republican party and its candidate General Grant promised to sustain the system of promises to pay in gold coins Alexander Hamilton had started. General Grant was elected. Here is what President Grant said about the issue at his inauguration:

> A great debt has been contracted in securing to us and our posterity the Union. The payment of this, principal and interest, as well as the return to a specie basis as soon as it can be accomplished without material detriment to the debtor class or to the country at large, must be provided for. To protect the national honor, every dollar of Government indebtedness should be paid in gold, unless otherwise expressly stipulated in the contract. Let it be understood that no repudiator of one farthing of our public debt will be trusted in public place, and it will go far toward strengthening a credit which ought to be the best in the world, and will ultimately enable us to replace the debt with bonds bearing less interest than we now pay. U. S. Grant, March 1869

To add quantitative content to stories like those told in the previous paragraphs, we want numbers and measures of how much we should trust them. That is what this paper is about. As it speaks to us through our economic-statistical model, we use our data to spot patterns in yield curves during the 19th century that set the stage for comparisons with 20th and 21st century comparisons to come. Some of our findings are:

- Especially during some periods, our estimates of yields differ substantially from "standard" and widely used ones.
- While yield curves usually sloped upward, yield curve "inversions" occurred during major wars, the late 1820s, the mid 1890s, and before the Great Contraction that began in 1929.

[2]Hamiltonian Monte Carlo is named after mathematician and physicist William R. Hamilton, not US Secretary of Treasury Alexander Hamilton.

- Yields have trended downward since the beginning of the Republic.
- Comparing initially higher US yields with yields on British consols shows that by 1880 the spread had vanished. We interpret long-run change with a story about how it took decades and discipline to realize Hamilton's and Grant's aspirations of low default premia on US government bonds.
- Our yield curves and associated Hicks-Arrow prices allow us to approximate the market value of marketable federal debt and improve what seem to be standard estimates of the debt-to-GDP ratio in the 19th century.
- We can sort through data difficulties and ambiguities associated with temporary suspensions of convertibility into gold that occurred during the War of 1812 and the Civil War.
- Visual inspection of our yield curves during the nineteenth century belies a conclusion from widely cited regression studies of historical US time series that surges of government expenditures and deficits associated with big wars did not bring higher yields on US government debt.

Related Work

Homer and Sylla (2004) construct series for the yield to maturity on 10-year Federal treasuries, New England Municipal Bonds, and Corporate Bonds.[3] The closest counterpart to ours is their yield to maturity on 10-year treasuries, which they calculate as the coupon rate on US federal bonds that are both trading close to par and have approximately 10 years to maturity. Relative to their work, our contribution is to estimate the yield curve at all maturities and in all months. This allows us to discuss term spreads. It also allows us to fill in the Homer and Sylla (2004) 10-year yield series during the periods, such as the Civil War, when prices deviated significantly from par. Remarkably, Homer and Sylla (2004)'s 10-year Federal treasury series is not the long-term US bond series that is commonly used in the economic history literature. Instead, researchers (e.g. Officer and Williamson (2021), Shiller (2015), Jordà et al. (2019), and Hamilton et al. (2016)) have typically used a 'composite series' that combines the Homer and Sylla (2004) estimates for the period from 1798-1861 with the yield-to-maturity on the New England Municipal bond for the period 1862-1899 and the yield-to-maturity on corporate bonds for the period 1900-1940. We argue that this series underestimates the cost of government financing during the Civil War and overestimates the cost of government financing post war because it calculates the yield on high grade state or corporate bonds rather than the yield on US Federal debt.

Section 2 outlines a theory of zero-coupon bond yields. Section 3 tells sources of our data. Section 4 details our econometric strategy. Section 5 discusses our statistical inferences. Section 6 concludes.

2 The Problem

At a given date, a term structure of interest rates is a list of yields on zero-coupon bonds of maturities $j = 1, 2, \ldots, J$. We want to approximate term structures of interest rates on US Federal securities from 1790 to 1930. Because the US government typically did not issue zero-coupon securities that span the full maturity spectrum, the zero-coupon yield curve on US Treasury debt is not directly observable. Instead, it must be approximated indirectly from observed prices of a limited sets of securities having differing coupons, par values, and maturity dates that the US government issued at different dates. Inference is especially challenging before World War I because the Treasury issued bonds infrequently; those bonds varied in their default risks, whether they promised to pay gold dollars, silver dollars, or paper greenbacks that floated relative to gold and silver

[3]A yield to maturity is also called an internal rate of return.

dollars, their tax exemptions, their buyer redemption options, and government call options. To make progress, we study a gold denominated zero-coupon yield curve and stop our analysis shortly before the US left the gold standard in the early 1930s.[4] We use a mutated version of a statistical model proposed by Nelson and Siegel (1987). Our model imposes no arbitrage, common default risks across different bonds at a given date, and a parametric specification of yield curves.

2.1 Zero-Coupon Bonds

Suppose that a particular j-maturity zero-coupon bond, indexed by i, promises $\bar{c}_{t+j}^{(i)}$ gold dollars at period $t+j$. To acknowledge default risk, we denote the actual (possibly random) payment of gold dollars at period $t+j$ by $c_{t+j}^{(i)}$. Let $p_t^{(i,j)}$ denote the price of such a bond. We make three assumptions.

Assumption 1. No arbitrage.

This assumption implies that there exists a stochastic discount factor (SDF) process, S that verifies

$$p_t^{(i,j)} = \mathbb{E}_t \left[\left(\frac{S_{t+j}}{S_t} \right) c_{t+j}^{(i)} \right] = \mathbb{E}_t \left[\left(\frac{S_{t+j}}{S_t} \right) \left(\frac{c_{t+j}^{(i)}}{\bar{c}_{t+j}^{(i)}} \right) \right] \bar{c}_{t+j}^{(i)}$$

where $\mathbb{E}_t$ denotes bond holders' period t expectation about the realizations of future outcomes affecting the value of $c_{t+j}^{(i)}$ and we have taken $\bar{c}_{t+j}^{(i)}$ outside the expectation operator because it is state independent. For convenience, we define the payment-risk fraction $\xi_{t+j}^{(i)} := c_{t+j}^{(i)}/\bar{c}_{t+j}^{(i)} \in [0,1]$ and the price:

$$q_t^{(i,j)} := \mathbb{E}_t \left[\left(\frac{S_{t+j}}{S_t} \right) \xi_{t+j}^{(i)} \right] \tag{2.1}$$

so that the bond price is:

$$p_t^{(i,j)} := q_t^{(i,j)} \bar{c}_{t+j}^{(i)}.$$

Assumption 2. Under bond holders' beliefs, the fraction $\xi_t^{(i)}$ is independent of i at any time t. We write $\xi_t^{(i)} = \xi_t$, for all bonds i and times t.

This assumption says that, within a time period, there is no cross-sectional variation in default risk. This implies that the government would impose the same haircut on all bonds outstanding at the time of default, rather than imposing greater proportional losses on holders of particular types of bonds. As a result, the zero-coupon price is independent of i and so is denoted by $q_t^{(j)}$. The zero-coupon bond price can then be expressed as:

$$p_t^{(i,j)} := q_t^{(j)} \bar{c}_{t+j}^{(i)}.$$

Assumption 3. Let $\mathsf{y}_t := \{y_t^{(j)}\}_{j=0}^{\infty}$ denote the yield curve in period t, where the j-th element is defined as:

$$y_t^{(j)} := -\frac{\log q_t^{(j)}}{j} \tag{2.2}$$

[4] As is discussed in section 3, for most of the sample, US dollars were defined in terms of gold (and silver). However, from 1862-1879 the US federal government also issued a paper currency called the greenback dollar that was not convertible into gold dollars. During this period, we select only bonds making coupon and principal payments in gold and convert bond prices to gold dollars. In principle, we could also also include bonds making payments in greenbacks and then estimate both gold and greenback denominated yield curves. However, in practice, we have very few price observations for greenback denominated bonds.

and takes the following parametric form (with parameters $(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_t)$):

$$y_t^{(j)} = \beta_{0,t} + (\beta_{1,t} + \beta_{2,t}) \left[1 - \exp\left(-\frac{j}{\tau_t}\right)\right] / \left(\frac{j}{\tau_t}\right) - \beta_{2,t} \exp\left(-\frac{j}{\tau_t}\right) \tag{2.3}$$

This parametric specification follows Nelson and Siegel (1987) and has a number of desirable features. First, it is flexible enough to generate "typical yield curve shapes" (i.e., monotonic, humped, and S-shaped curves). Second, it ensures that yields converge as the horizon goes to $+\infty$, with $\beta_{0,t}$ parameterizing the asymptote. Third, separate parameters shape different parts of the yield curve: $\beta_{1,t}$ for the short end of the yield curve, $(\tau_t, \beta_{2,t})$ for medium-term yields. Fourth, it is consistent with recent estimates of recent yield curves.[5] Finally, it implies convenient formulas for the forward rate curve:

$$f_t^{(j)} = \beta_{0,t} + \beta_{1,t} \exp\left(-\frac{j}{\tau_t}\right) + \beta_{2,t} \exp\left(-\frac{j}{\tau_t}\right)\left(\frac{j}{\tau_t}\right)$$

2.2 Connection to Hicks-Arrow Prices

Let the price $\tilde{q}_t^{(j)}$ denote the number of units of gold at time t that buys a risk-free claim to a unit of gold j periods ahead and let $\tilde{y}_t^{(j)}$ denote the j-horizon risk-free yield:

$$\tilde{q}_t^{(j)} := \mathbb{E}_t\left[\frac{S_{t+j}}{S_t}\right], \qquad \tilde{y}_t^{(j)} := -\frac{\log \tilde{q}_t^{(j)}}{j} \tag{2.4}$$

We refer to $\tilde{q}_t^{(j)}$ as the Hicks-Arrow price. Using (2.1) and the definition of covariance we can write:

$$q_t^{(j)} = \mathbb{E}_t\left[\frac{S_{t+j}}{S_t}\right] \mathbb{E}_t\left[\xi_{t+j}^{(i)}\right] + \mathrm{cov}_t\left(\frac{S_{t+j}}{S_t}, \xi_{t+j}^{(i)}\right)$$

Rearranging terms and using (2.2) and (2.4) result in an expression for the yield differential:

$$y_t^{(j)} - \tilde{y}_t^{(j)} = \underbrace{\frac{-1}{j} \log \mathbb{E}_t\left[\xi_{t+j}^{(i)}\right]}_{\text{default probability}} + \underbrace{\frac{-1}{j} \log\left(1 + \frac{\mathrm{cov}_t\left(\frac{S_{t+j}}{S_t},\ \xi_{t+j}^{(i)}\right)}{\mathbb{E}_t\left[\frac{S_{t+j}}{S_t}\right] \mathbb{E}_t\left[\xi_{t+j}^{(i)}\right]}\right)}_{\text{risk premium}}$$

Assuming that events with $\xi_{t+j} < 1$ correspond to high marginal utility states, i.e., the covariance term in the above formula is non-positive, both terms on the right hand side are non-negative, hence, $y_t^{(j)} \geq \tilde{y}_t^{(j)}$.

Special case: Suppose that government default is governed by a two-state Markov Chain with default as an absorbing state. Let $\mathfrak{p}_t$ be the bondholders' perceived probability of default in period t and assume that they use the two-state Markov Chain to forecast future cash-flows. For simplicity, suppose that in the event of default government bonds pay 0. These assumptions imply $\mathbb{E}_t[\xi_{t+j}^{(i)}] = (1 - \mathfrak{p}_t)^j$. Finally, suppose that bondholders' are risk-neutral implying $\mathrm{cov}_t\left(\frac{S_{t+j}}{S_t},\ \xi_{t+j}^{(i)}\right) = 0$ and so

$$y_t^{(j)} - \tilde{y}_t^{(j)} = -\log(1 - \mathfrak{p}_t) \approx \mathfrak{p}_t.$$

[5] For example, Gürkaynak et al. (2007) use this form for the period 1961-1980. For more recent dates (from 1980), they use an extended version proposed by Svensson (1994) to allow for a second hump in the yield curve attributable to a "convexity effect".

2.3 Deploying Hicks-Arrow Pricing Theory

Suppose that at time t we observe error-ridden prices on M_t coupon-bearing government bonds. Let $p_t^{(i)}$ denote the price of bond $i \in \{1, \ldots, M_t\}$ and let $\bar{\mathsf{c}}_t^{(i)} := \{\bar{c}_{t+j}^{(i)}\}_{j=1}^{\infty}$ denote the promised payments stream associated with that bond. Using this notation, we allow $\bar{c}_{t+j}^{(i)}$ potentially to be zero. Most bonds have finite maturities so we let $J_t^{(i)}$ denote the remaining number of periods with non-zero payments.[6] To account for differences in maturities and coupons, we view each coupon-bearing bond i as a basket of zero-coupon securities and use standard no-arbitrage arguments to establish the following price formula:

$$p_t^{(i)} = \sum_{j=1}^{\infty} q_t^{(j)} \bar{c}_{t+j}^{(i)} \tag{2.5}$$

We have information on $p_t^{(i)}$ and $\bar{\mathsf{c}}_t^{(i)}$, so we can use equation (2.5) to infer the values of $\mathsf{q}_t = \{q_t^{(j)}\}_{j=0}^{\infty}$ or, more precisely the parameters $(\beta_{0,t}, \beta_{1,t}, \beta_{2,t}, \tau_t)$ that characterize the yield curve.

Aside: Connection to yield-to-maturity. Some economists have expressed historical long-term interest rates as yields-to-maturity rather than zero-coupon yields. It is useful to make the connection between the two concepts explicit. The yield-to-maturity (or internal rate of return) is the fixed discount rate, $\bar{y}^{(i)}$, that equates the bond price to the present discounted value of promised payments. Thus, the yield-to-maturity on bond i with maturity $J^{(i)}$ is the rate $\bar{y}_t^{(i)}$ that solves:

$$p_t^{(i)} = \sum_{j=1}^{J^{(i)}} \exp\left(-\bar{y}_t^{(i)}\right)^j \bar{c}_{t+j}^{(i)}$$

To compare to the zero-coupon prices, let $\bar{q}_t^{(i)} := \exp\left(-\bar{y}_t^{(i)}\right)$ and express the bond price in terms of $\bar{q}_t^{(i)}$ as:

$$p_t^{(i)} = \sum_{j=}^{J^{(i)}} \left(\bar{q}_t^{(i)}\right)^j \bar{c}_{t+j}^{(i)}. \tag{2.6}$$

The yield-to-maturity on a j-period zero-coupon bond coincides with the j-period zero-coupon yield, $y_t^{(j)}$, but this equivalence does not hold in general. By comparing (2.5) and (2.6) we can see that the yield-to-maturity on a coupon-bearing bond is a type of *weighted average* of the respective zero-coupon yields, with the cash-flow payments serving as weights.[7] Because the principal payment is typically significantly larger than the coupons, the maturity-related zero-coupon yield gets the largest weight in the average. As a result, a yield-to-maturity on a J-maturity bond can be a relatively good approximation to the J-period zero-coupon yield. However, the quality of that approximation depends on specific features of the bond and so yield-to-maturity cannot be easily

[6] In case of perpetual *consols*, $J_t^{(i)} = \infty$.

[7] To see this formally, use (2.5) to substitute for the bond price in (2.6) and rearrange to get:

$$0 = \sum_{j=1}^{J^{(i)}} \bar{c}_{t+j}^{(i)} \left(q_t^{(j)} - \left(\bar{q}_t^{(i)}\right)^j \right).$$

For the special case of a fixed coupon annuity ($J^i = \infty$ and $\bar{c}_{t+j}^{(i)} = \bar{c}$), this simplifies to:

$$\bar{q}_t^{(i)} = \sum_{j=1}^{J^{(i)}} q_t^{(j)} \Bigg/ \left(1 + \sum_{j=1}^{J^{(i)}} q_t^{(j)}\right)$$

generalized for other securities.

The Congress and the Treasury typically aimed to set coupon rates on new bonds to make them trade at par initially. That would make their yields-to-maturities equal their coupon rates. As we will see below, changes in market conditions frustrated this objective during important episodes in US history. In times of financial stress such as during the War of 1812 and the Civil War, Treasury debt sold at deep discounts. In times of disagreements between the President and the Congress, like the 1890s, the Treasury issued bonds with coupon rates exceeding current yields, so that bonds sold at a premium.[8]

In remarks from the floor at a 2010 Minneapolis Fed conference, Professor V.V. Chari offered an "accounting tail wags the dog" explanation of why Congresses often wanted to market new bonds that would sell "at par".[9] Chari's explanation was that Congress was stuck with Alexander Hamilton's peculiar accounting that measured total government debt by simply adding up *undiscounted* par values of all outstanding debts, ignoring coupon values. That accounting system had a chance of revealing approximately accurate estimates of the total value of debt only if bonds were initially sold at or near par values.

3 Price and Quantity Data

We have assembled prices, quantities, and descriptions of all securities issued by the US Treasury between 1776 and 1960. Figures 7 and 8 (at the end of the paper) summarise the issue size, duration, and coupon rates for all of securities. The full data set is available at the Github repository `https://github.com/jepayne/US-Federal-Debt-Public` and the construction methodology is explained in Hall et al. (2018). In this section, we highlight the two series that we need for estimation: bond prices and promised gold payments.

The price data are at a monthly frequency. When available, we use the closing price at the end of each month. However, if the closing price is not available, then we use the average, bid, or ask price (in that order of precedence)[10]. The primary sources for the price data from 1776 to 1839 are Razaghian (2002) and Sylla et al. (2006). The prices from 1840 to 1899 are from Razaghian (2002), the *Merchants' Magazine and Commercial Review*, and the *Commercial and Financial Chronicle*. The prices from 1900 to 1918 are from the *Commercial and Financial Chronicle* and US Treasury Circulars. When overlap occurred, data were taken from the US Treasury Circulars. The prices from 1919 to 1925 are from "United States Govt. Bonds" tables in the New York *Times*. Price data after 1925 is taken from the *CRSP US Treasury Database*.[11]

A limitation with the price data is that the government issued relatively few securities during the 19th century and so we have relatively few price observations. Between 1776 and World War I, the US Congress authorized the Treasury to issue a total of approximately 200 distinct securities, with no more than 8 distinct ones being authorized in any particular year. This reflects US government fiscal policy up until the 1920s, which primarily involved long term borrowing to finance wars and specific infrastructure projects and debt repayment otherwise. Figure 1 shows the monthly time series for the number of securities with observed prices and the time to maturity (in years) of all the outstanding bonds each period. As can be seen, there are typically fewer than 5 price observations in a given period and many of those observations are for bonds with long maturities.

[8] In 1895, after a run drained 40% of the Treasury's Gold Reserve Fund, President Grover Cleveland sought to issue debt to purchase the gold needed to replenish these reserves. However, proponents of bimetallism in Congress blocked authorizations of new loans. Following advice from J.P. Morgan's legal team, the Cleveland Administration bypassed Congress and used some Civil War-era legislation to issue 30-year bonds bearing 4 percent coupons, at a time when the 10-year zero-coupon yield was below 3 percent. The controversy surrounding the issuance of these bonds helped motivate William Jennings Bryan's "Cross of Gold Speech" at the Democratic Convention in 1896. See chapter 5 of Chernow (2001) for more about this episode.

[9] Chari was responding to the content of a draft version of Hall and Sargent (2011), which documented differences between the US government accounting scheme and a mark-to-market alternative scheme.

[10] The order of precedence is chosen based on data availability

[11] See `http://www.crsp.com/products/research-products/crsp-us-treasury-database`.

The gap in observed prices in the late 1830s occurs because there were no government securities outstanding during that period.

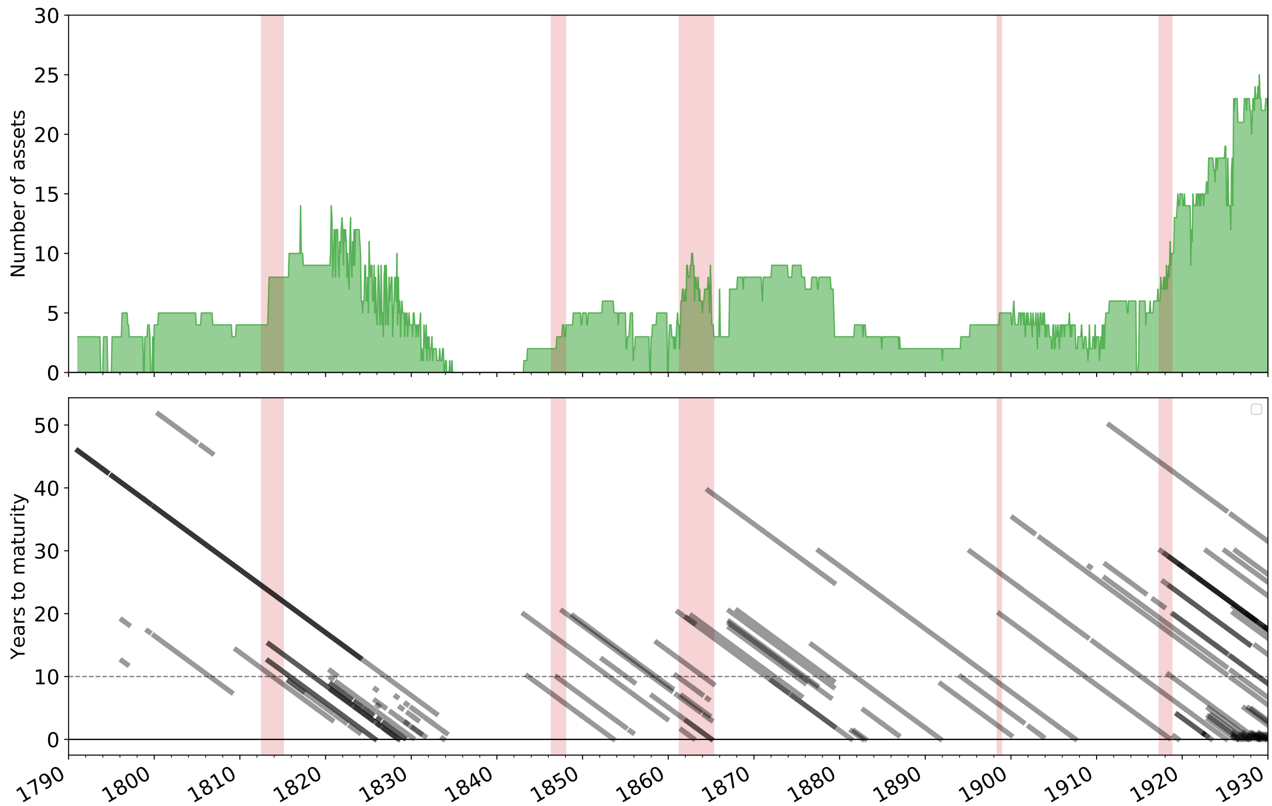

Figure 1: The top panel depicts the number of securities with observed prices each month. The bond panel depicts the maturity (in years) of observed securities in a given month. Darker lines are indicative of overlapping securities. The red bars correspond to wars.

Promised gold payments for each bond, $\bar{\mathsf{c}}_t^{(i)} := \{\bar{c}_{t+j}^{(i)}\}_{j=0}^{\infty}$, must be constructed from the terms of the bond contracts. There are two main difficulties. First, many 19th century bonds did not have a fixed maturity date. Instead, bond holders were given a window during which they could redeem the bond and receive their principal. For such cases, we use the following approach, which we find minimizes the pricing errors. For long term bonds (greater than 3 years) and/or bonds that pay regular coupons, we set the maturity date to the last date at which any of the bonds could be redeemed. For short term bonds (less than 3 years) that pay coupons at maturity, we set the maturity date to the last date at which bonds were issued plus the duration of the bond[12] and match total coupon payments with bond duration. For example, for a 1-year bond, this means we impose that only 1 year of coupons is paid at redemption, regardless of the date at which the bond is redeemed.

A second challenge is that during and after the Civil War (1862-1879), different bonds were denominated in different currencies. Between April 1792 and February 1862, the US dollar was was defined in terms of units of gold and silver.[13] Except for the activities of the monopoly First (1791-1811) and Second (1816-1836) Banks of the United States, the federal government did not issue bank notes, only gold and silver coins. But private state-charter states did issue paper notes convertible into gold on demand. After private banks stopped honoring their legal obligation to convert their bank notes into specie on demand in late 1861, in February 1862 the federal

[12]The last issue date plus the duration of the bond is the first date at which all the bonds issued can be redeemed.
[13]Prior to 1792, a dollar referred to a Spanish silver coin.

government began issuing legal tender notes, a paper currency known as greenbacks that the government did not promise immediately to exchange for gold dollars. But greenbacks could be used to purchase bonds from the government at their par values. So from from 1862 to January 1, 1879 there were two currencies – both called dollars – in circulation: paper notes ("greenbacks" or "lawful money") and gold coins ("gold" or "coin"). Figure 2 plots the greenback to gold exchange rate[14] and the prices of outstanding bonds. As can be seen, there was a significant devaluation of greenbacks during the Civil War. The greenback did not fully recover until January 1, 1879 when the US Treasury restored full convertibility of dollars into gold. Before then, different US Treasury bonds promised payments in different currencies. There were three distinct categories: bonds that made all payments in gold, bonds that paid coupons in gold but left ambiguous the denomination of the principal, and bonds that made all payments in greenbacks. In our baseline estimation, we drop bonds that paid in greenbacks (for which there are very few price observations) and impose that private agents believed that all other bonds would make all payments in gold. This can be interpreted as our imputing perfect foresight to private agents because bonds with ambiguously denominated payment streams eventually actually paid in gold.

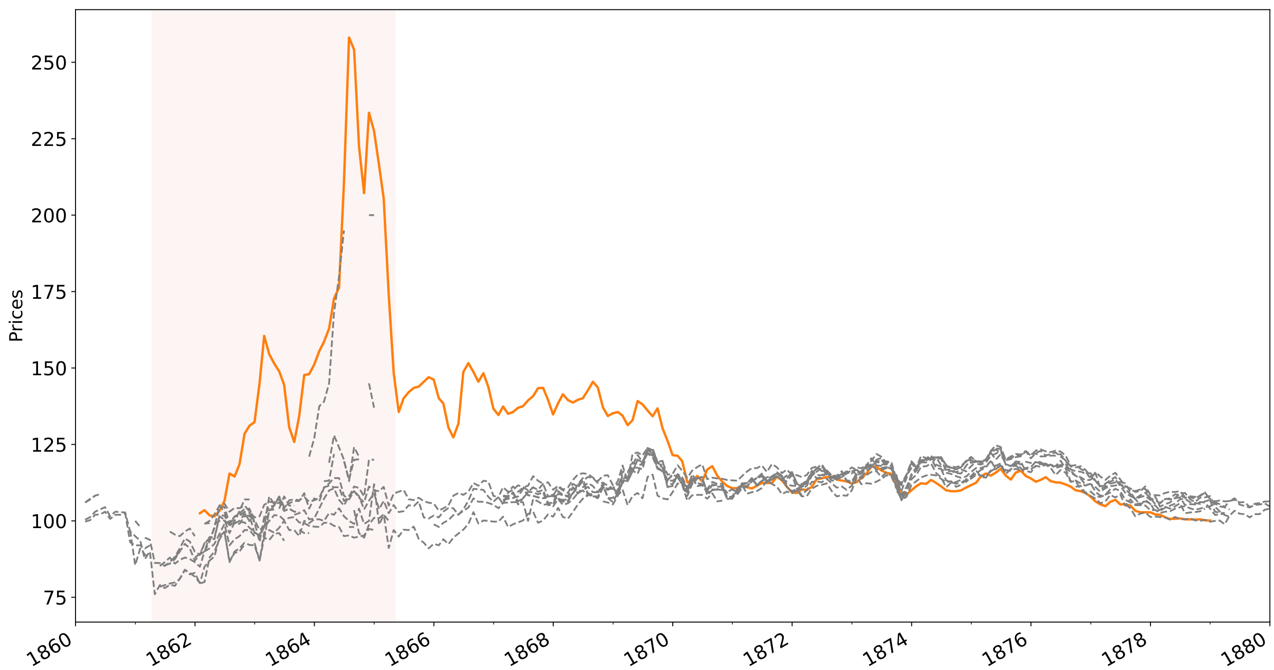

Figure 2: Gold and Bond Prices: 1860-1880.

The solid orange line depicts the greenback to gold exchange rate (expressed as the number of greenback dollars required to purchase 100 gold dollars). The grey lines depict observed prices (denominated in greenbacks) for the outstanding bonds. The light red interval depicts the Civil War.

4 Econometric Strategy

Taking full advantage of the abundance of data available for the post-WW2 period, Gürkaynak et al. (2007) estimate the four parameters in (2.3) with maximum likelihood *day-by-day*, i.e., they estimate a different

[14]The exchange rate is stated as the number of greenback dollars required to purchase 100 gold dollars.

$(\beta_0, \beta_1, \beta_2, \tau)$ for each t using only bond prices available at date t. As a result, nothing learned about the yield curve any one date informs their estimates for other dates no matter how close.

Unlike the post-WW2 period, prior to the First World War, the price data are sparse and coverage varies across time (see Figure 1). Consequently, when to estimate a time series of yield curves for the 19th century we in effect pool information over time. We want a statistical model that can learn simultaneously about all dates simultaneously. To this end, we will use a multilevel (a.k.a. an hierarchical) model. In particular, our model exhibits two key features:

1. Growth rates of short-, medium-, and long-term yields follow an i.i.d. Gaussian distribution with mean zero and covariance matrix Σ

2. Bond-specific measurement errors: the price of each bond i is observed with a Gaussian measurement error with mean zero and standard deviation $\sigma_m^{(i)}$

Parameter matrix Σ governs the amount of pooling over time: what we learn about the yield curve one date shapes inferences about other dates. The closer are two dates to each other, the more correlated are the associated yield curves, with Σ capturing what "close" means. The case $\Sigma \to 0$ corresponds to *complete pooling*: here the yield curve is assumed to be fixed over time, so each observation has an equal influence on all other dates in the sample. At the other end of the spectrum, $\Sigma \to \infty$ corresponds to *no pooling*: there is no relationship between adjacent parameter estimates, we use only period t information to estimate period t yield curve parameters similar to Gürkaynak et al. (2007). By inferring Σ from the data, we learn how much pooling across time should be used to improve estimates in light of imbalances in data availability over time.

We introduce bond-specific measurement errors as a device to detect failures of our bond pricing formula. If formula (2.5) has difficulty in pricing specific bonds, this formulation allows us to reduce the influence of those bonds on the yield curve estimates—by making their measurement errors larger— while still informing us about the magnitude of the specification error. Large bond-specific measurement error estimates are telltale signs of pricing errors for particular bonds.[15]

Aside: Time-varying measurement errors An alternative measurement error specification would assign the same $\sigma_{m,t}$ to all bonds available in period t. We refer to this as the time-varying measurement error model. Unlike our bond-specific measurement error model, this specification equalizes influences of different bond on the yield curve estimates—the likelihood does not let the yield curve price a subset of bonds well at the cost of yielding large pricing errors on the rest of the bonds. Large $\sigma_{m,t}$ estimates can be interpreted as indicating that our common payment-risk assumption (Assumption 2) is violated for the corresponding subperiod. In this case, it is advisable to divide bonds into subgroups according to their common characteristic (e.g., a call option) and re-estimate the yield curve on each group separately to see if they differ. The difference can be used to estimate the yield premium that arises from the specific characteristic.

4.1 Nonlinear state space model

Accompanied with our functional form assumption (2.3), the above two features give rise to a tractable nonlinear state space model. In order to guarantee that the yield curve is non-negative for all maturities, we impose the following constraints on the parameter space:[16]

$$\beta_0 > 0 \qquad \beta_0 + \beta_1 > 0 \qquad \beta_0 + \beta_2 > 0 \qquad \tau > 0$$

[15] See Hansen and Jagannathan (1997).

[16] Appendix **??** describes how these inequality constraints imply $y_t^{(j)} \geq 0$ for all $j \geq 0$.

For simplicity, we define a vector, λ, that contains the logarithmic transformations of the left hand side terms in the four inequalities:

$$\lambda := \Big[\log\beta_0,\ \log(\beta_0+\beta_1),\ \log(\beta_0+\beta_2),\ \log\tau\Big]'$$

This formulation is convenient, because the entries of λ have an unconstrained domain.

Under Assumption 3, the sequence $\{q_t^{(j)}\}_{j=0}^{\infty}$ of zero-coupon prices in period t is a function only of λ_t. We denote this sequence by $\mathsf{q}(\lambda_t)$. That said, we can write our nonlinear state space model in the following compact form:

$$\begin{aligned} p_t^{(i)} &= \left\langle \mathsf{q}(\lambda_t), \bar{\mathsf{c}}_t^{(i)} \right\rangle + \sigma_m^{(i)}\varepsilon_t^{(i)} & \varepsilon_t^{(i)} &\sim \mathcal{N}(0,1) \ \ \forall i, \forall t \geq 1 \\ \lambda_t &= \lambda_{t-1} + \Sigma^{\frac{1}{2}}\eta_t & \eta_t &\sim \mathcal{N}\left(\mathbf{0}, \mathbb{I}_4\right), \ \forall t \geq 1 \end{aligned}$$

Having monthly price series for a more than 150-year period means that this model has more than $7,000$ parameters to be estimated. We tackle the high-dimensional parameter space by using Bayesian methods, namely, Hamiltonian Monte Carlo with a "No-U-Turn Sampler" of Hoffman and Gelman (2014), along with the latest developments as described in Betancourt (2018).

4.2 Priors

We decompose the covariance matrix Σ into the marginal variances and the correlation matrix:[17]

$$\Sigma = \underbrace{\begin{pmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & \sigma_4 \end{pmatrix}}_{=:\Xi} \underbrace{\begin{pmatrix} 1 & \omega_1 & \omega_2 & \omega_3 \\ \omega_1 & 1 & \omega_4 & \omega_5 \\ \omega_2 & \omega_4 & 1 & \omega_6 \\ \omega_3 & \omega_5 & \omega_6 & 1 \end{pmatrix}}_{=:\Omega} \begin{pmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 \\ 0 & 0 & 0 & \sigma_4 \end{pmatrix}$$

For numerical stability, we work with the Cholesky factor $L\Omega$ of the correlation matrix.

- We use weakly informative priors for the covariance matrix:
  - For the standard deviations in Ξ we use an exponential prior with the rate parameter tuned so that *a priori* the probability that $\sigma_i > 0.3$ is lower than 5%
  - For the correlation matrix Ω we use the LKJ prior with a concentration parameter $\eta = 5.0$, which is unimodal but fairly vague.[18]
- We use exponential priors on the standard deviation of measurement errors, $\sigma_m^{(i)}$, with the rate parameter tuned so that *a priori* the probability that $\sigma_m^{(i)} > 20$ is lower than 5%.
- We use (independent) Gaussian priors for the initial yield curve parameters λ_0:

$$\begin{aligned} \log\beta_0 &\sim \mathcal{N}(8,4) & \log(\beta_0+\beta_1) &\sim \mathcal{N}(7,5) \\ \log(\beta_0+\beta_2) &\sim \mathcal{N}(15,15) & \log\tau &\sim \mathcal{N}(60,60) \end{aligned}$$

[17] See Barnard et al. (2000). This decomposition implies $\sigma_i = \sqrt{\Sigma_{i,i}}$ and $\Omega_{i,j} = \frac{\Sigma_{i,j}}{\sigma_i\sigma_j}$ for $i,j \in \{1,2,3,4\}$.

[18] See Lewandowski et al. (2009). The LKJ distribution is defined by $p(\Omega|\eta) \propto \det(\Omega)^{\eta-1}$. For $\eta = 1$, this is a uniform distribution over correlation matrices. For $\eta > 1$, the density increasingly concentrates mass around the unit matrix, i.e., favoring less correlation.

5 Statistical Inferences

We now describe some salient features of our approximated nominal yield curves. We start by discussing our 10-year yield, since this yield has been frequently studied in the economic history literature. It is also the point on the yield curve for which there are, arguably, the best estimates of contemporary yields. This makes it a useful 'sense-check' on our results. We then discuss our approximation for the historical 1-year yield and the spread between the short and long term yields.

5.1 Yields on 10-Year Zero Coupon Bonds

Figure 3 depicts our estimate for the 10-year gold denominated zero-coupon yield series along with two reference series: the "Federal Government Bonds: Selected Market Yields" series of Homer and Sylla (2004) and the "UK Long-Term (contemporary) Yield" series from Officer and Williamson (2021). Homer and Sylla (2004) calculate their US long term market yield series as the coupon rate on US federal bonds that are both trading close to par and have approximately 10 years to maturity. Officer and Williamson (2021) calculate their long term yield series as the yield to maturity on 3% UK consols.

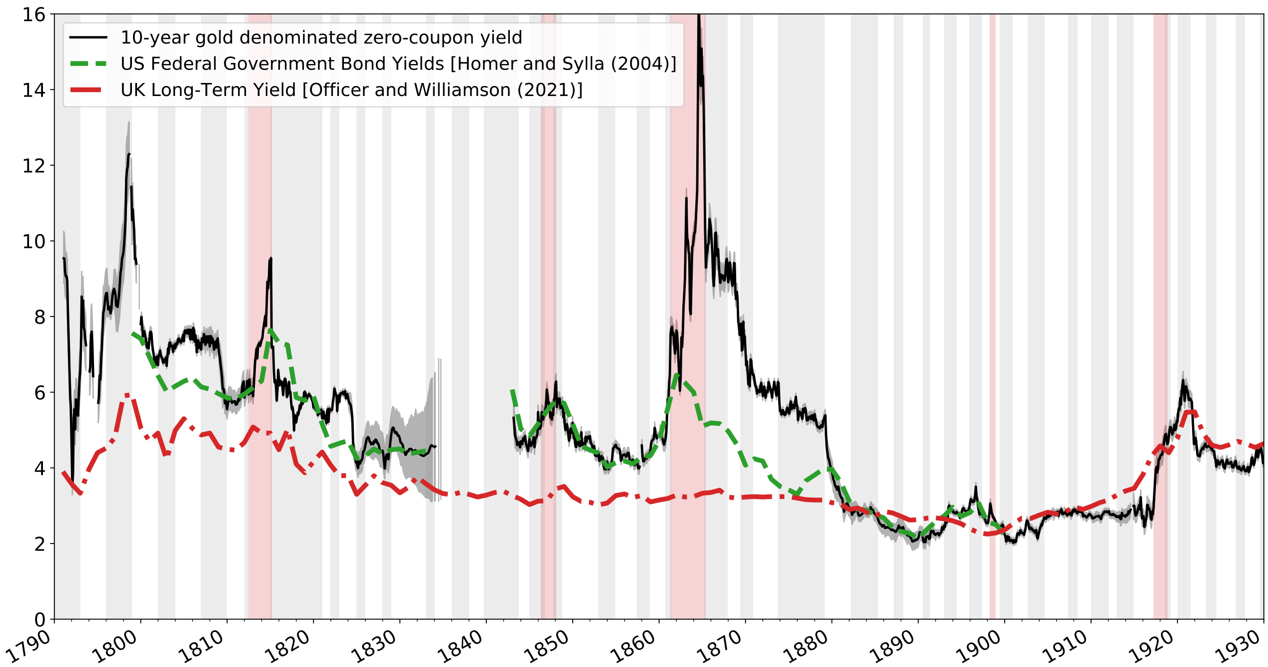

Figure 3: Long-Term Yield Estimates.

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The dashed green line depicts the 'Federal Government Bonds: Selected Market Yields' series from Table 38 of Homer and Sylla (2004). The red dashed and dotted line depicts the UK long-term yield (implied by the 3% consol price) from Officer and Williamson (2021). The light gray intervals depict NBER recessions. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

As can be seen in figure 3, our estimates typically follow the Homer and Sylla (2004) series, except during the War of 1812 and the Civil War where we estimate significantly higher yields. We find it reassuring that our estimate lines up with Homer and Sylla (2004) in "non-emergency" periods because there are good reasons to think that their estimates should be fairly accurate for the 10 year yield. Their approach calculates an average 'yield to maturity' for 10 year bonds, which should be similar to the 10 year zero-coupon yield if the yield curve is relatively flat over most of its domain. Moreover, away from the Civil War, the average duration of outstanding bonds is close to 10 years and the average market trading price is close to par so Homer and Sylla (2004) have a large available data set.[19] For these reasons, we consider the general congruence between our estimated 10-year yields and 'long-term federal government bond yields' in Homer and Sylla (2004) as a check on the plausibility of our findings.

Evidently, yields were quite volatile during the 1790s when secondary markets in Treasury securities began. Yields fell steadily from January 1791 to March 1792 when a financial panic caused sharp drops in bonds prices and corresponding increases in yields. Ten-year zero-coupon yields remained high for the remainder of the decade and spiked in August 1798 at 12.3% at the outbreak of the Quasi-War with France and one month after the Congress authorized a 15-year loan paying an 8% coupon to cover increased military spending. Yields trend downward thereafter and by 1803, the US government was able to issue at par a $11.25 million 15-year loan with a 6% percent coupon to finance the Louisiana Purchase.

A strength of our approach is that we are able to approximate the 10-year yield not only during "non-emergency" periods. We can also estimate the 10-year yield during periods that include the War of 1812 and Civil War when prices were very volatile and deviated significantly from par. As reported in figure 3, during the War of 1812, the 10-year zero-coupon yield spikes to over 9 percent. As the main source of funds for this war, the Treasury issued five long-term loans with a total face value of $66 million. Resistance to the war mainly from Federalists in the Northeast and the failure to replace lost customs revenue with internal taxes forced the Treasury to sell these bonds at deep discounts. As Bayley (1882) reports, two of these loans were sold at a 12% discount, and a third was sold at a 20% discount. Further, these officially-stated discounts understate the true discounts on these loans since the Treasury accepted bank notes at face value, whose market value themselves were much less than par, as payment for these loans.

The Treasury faced more troubles in selling its debt at par during the Civil War, leading to sharply increasing yields.[20] Our 10-year yield estimate reaches a peak of 16% near the end of the Civil War, which is significantly higher than the Homer and Sylla (2004) series peak of 6% at the start of the war. A back-of-the-envelope calculation suggests that our estimate is plausible. Starting in 1862, US Treasury bonds could be purchased with greenback dollars regardless of whether the coupons and principal payments were to be paid in greenbacks or gold. Since the value of the greenback fluctuated with battlefield and political news, Treasury bond prices deviated significantly from par (both when denominated in gold and when denominated in greenbacks). In particular, during the summer of 1864 when the re-election of President Abraham Lincoln was very much in doubt, 100 greenback dollars could be purchased with as few as 40 gold dollars. Consequently, during that time Treasury bonds that promised to pay 6 percent coupons in gold dollars could be purchased for 40 percent of par implying long-term yields in excess of 15 percent.

Figure 3 also illustrates several patterns in movements of long-term yields during the nineteenth century. First, the long-term yield on US government debt exhibits a downward trend, falling from approximately 10% at the beginning of the nineteenth century to almost 2% at the end of the century. Second, there are large spikes in long-term yields during big wars, the largest spike coming during the Civil War. Finally, the 10-year US yield was persistently higher than the UK long-term yield until the 1880s when the two series converge. We have to be cautious in how we interpret this comparison because the UK long term yield is calculated as

[19] Bonds typically traded close to par because the government designed coupons to try ensure an issue price of par.

[20] Homer and Sylla (2004) themselves caution against using their estimates for the Civil War period stating on page 303, "...the tables of bond yields for the years 1863 to 1870 do not provide a reliable picture of long-term interest rates." This is because there were no federal bonds trading with a gold price of par and so they are forced to estimate the yield as the gold coupon rate for bonds trading with a greenback price of par. We can capture greater variation in the yield curve because we use the universe of US Treasury bonds at monthly frequency whereas Homer and Sylla (2004) use the subset of these bonds that are trading at par.

the yield to maturity on consol bonds, not the yield on a 10-year zero-coupon bond. Nevertheless, we consider this to be tentative evidence that US federal debt traded with a risk premium until the late nineteenth century when it became an alternative 'safe-asset' to UK consols.

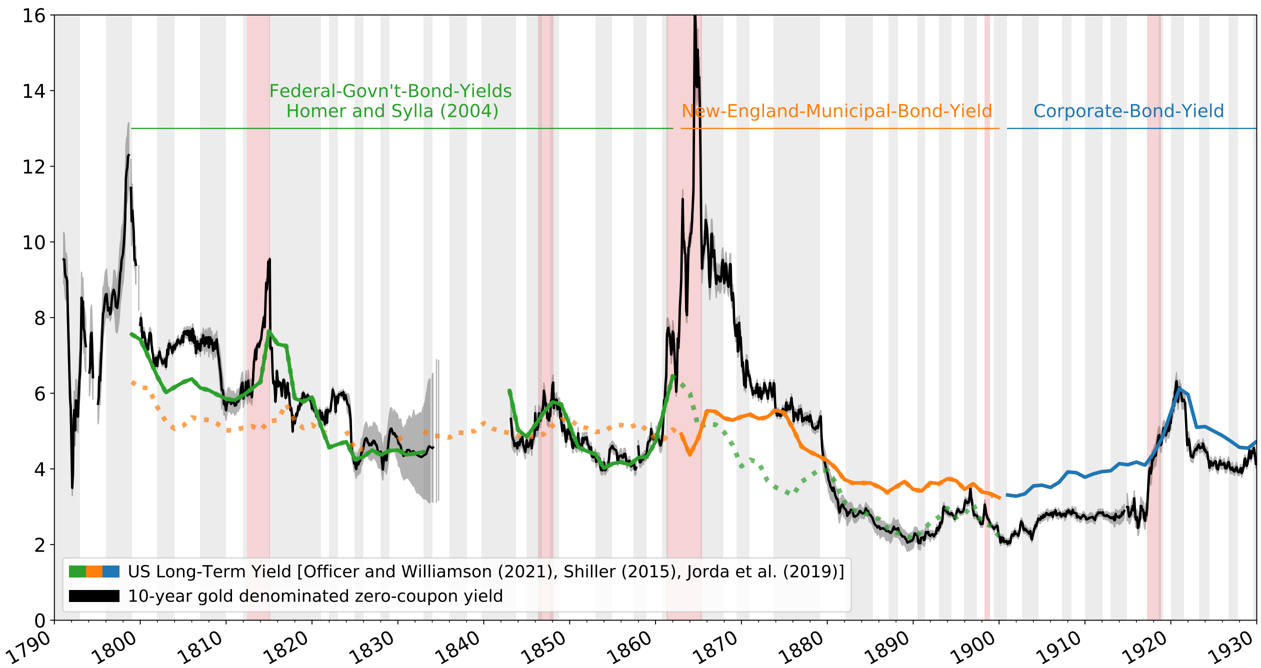

Figure 4: Alternative Long-Term Yield Estimates.

The solid black line depicts the mean of our posterior estimate for the 10-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green line (bold and dotted) depicts the 'US Government Bond Yield' series from Homer and Sylla (2004). The orange line (bold and dotted) depicts the New England Municipal Bond Yield reported by Homer and Sylla (2004). The blue line depicts the Corporate Bond Yield reported by Homer and Sylla (2004). The bold green-orange-blue line depicts the 'composite' bond series used by Officer and Williamson (2021). The light gray intervals depict NBER recessions. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

The Homer and Sylla (2004) series depicted in figure 3 is not the long-term US bond series that is commonly used in the economic history literature. Instead, researchers[21] have typically used a 'composite series' that combines the Homer and Sylla (2004) estimates for the period from 1798-1861 with the yield-to-maturity on the New England Municipal bond for the period 1862-1899 and the yield-to-maturity on corporate bonds for the period 1900-1940.[22] We plot this composite series alongside our 10-year yield series in Figure 4. Evidently, our estimates diverge post 1861 when the composite series stops using US federal debt prices. We estimate a much higher long-term yield during the war and a lower long-term yield in the late nineteenth century. A possible explanation for this divergence is that federal debt carried a greater default risk during the Civil War but after the wam National Banking Era protocols stimulated demands for a big set of federal bonds as required backing for National Bank Notes.

[21]For example, Officer and Williamson (2021), Shiller (2015), Jordà et al. (2019), and Hamilton et al. (2016).

[22]It is not obvious that during the 19th century municipal debt was a safer investment than federal debt. Until the 1934 Gold Reserve Act, the federal government had never defaulted. In contrast, eight states and one territory defaulted in 1830s and 1840s and ten states defaulted in 1870s and 1880s. These state defaults are discussed in McGrane (1935) and English (1996).

5.2 Short Term Yields

Figure 5 depicts our estimate for the 1-year gold denominated zero-coupon yield series alongside the short term yield series used by Officer and Williamson (2021) and Jordà et al. (2019).[23] As can be seen, our short term yield series significantly departs from current reference series, especially during the Civil War, where we estimate significantly higher yields peaking at approximately 44% in July 1864. Although it is not reflected in any of the currently used series, there is anecdotal evidence that the Union government paid very high yields during the Civil War. For example, Homer and Sylla (2004) reports that, in 1860, the treasury issued one-year notes at rates of 10-12% and was rejecting bids ranging from 15-36%[24].

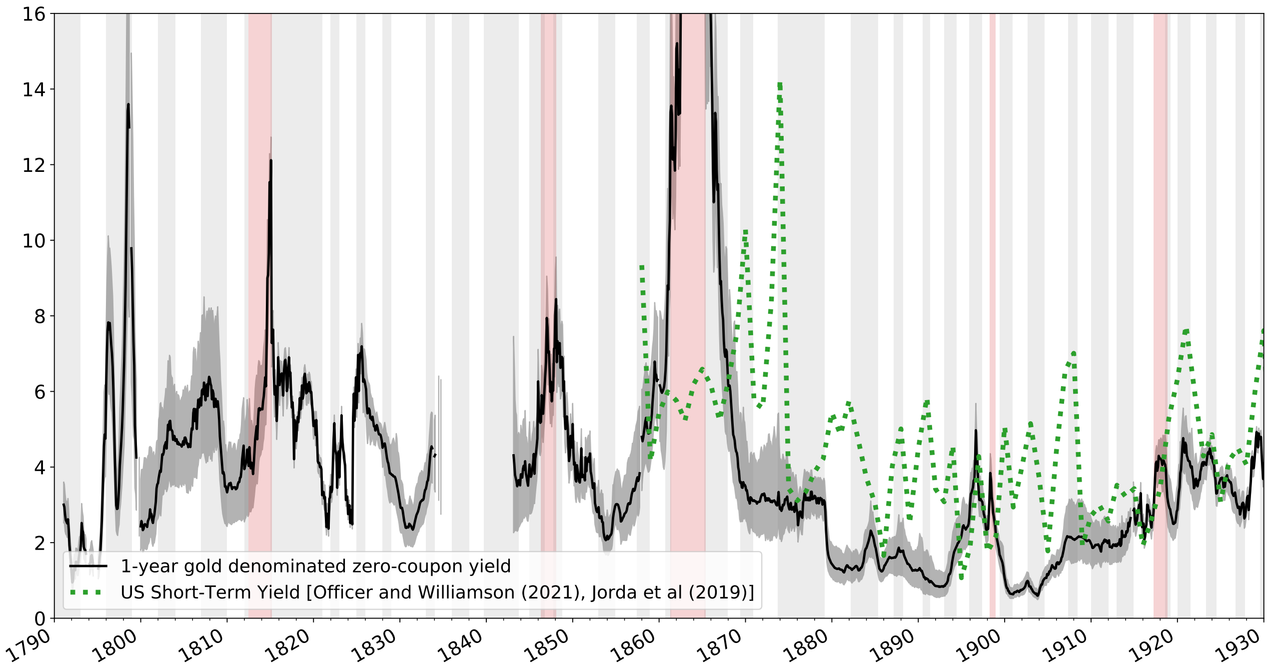

Figure 5: Short-Term Yield Estimates.

The solid black line depicts the mean of our posterior estimate for the 1-year, gold denominated, zero coupon yield. The grey bands around the posterior mean depict the 95% interquantile range. The green dotted line depicts the US short term yield series (surplus funds, contemporary) used by Officer and Williamson (2021) and Jordà et al. (2019). The light gray intervals depict NBER recessions. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

5.3 The Term Spread

Figure 6 depicts the yield on 5-year government bonds minus the yield on 1-year government bonds. We refer to this as the term spread on government debt. A positive term spread indicates an upward sloping yield curve (i.e., longer maturity bonds have higher rates). A negative term spread indicates an inverted yield curve (i.e., shorter maturity bonds have higher rates). As can be seen, the yield curve was typically upward sloping throughout the nineteenth century with major inversions during the War of 1812, the early 1830s, the Mexican-American War, the Civil War, and the late 1890s.

[23]The figure depicts the series labeled as "Surplus Funds (Contemporary Series)". The Series involves the short-term lending or borrowing of surplus funds, that is, funds that are considered excess by the lending institution and are required for immediate temporary use by the borrowing entity.

[24]Homer and Sylla (2004), page 302

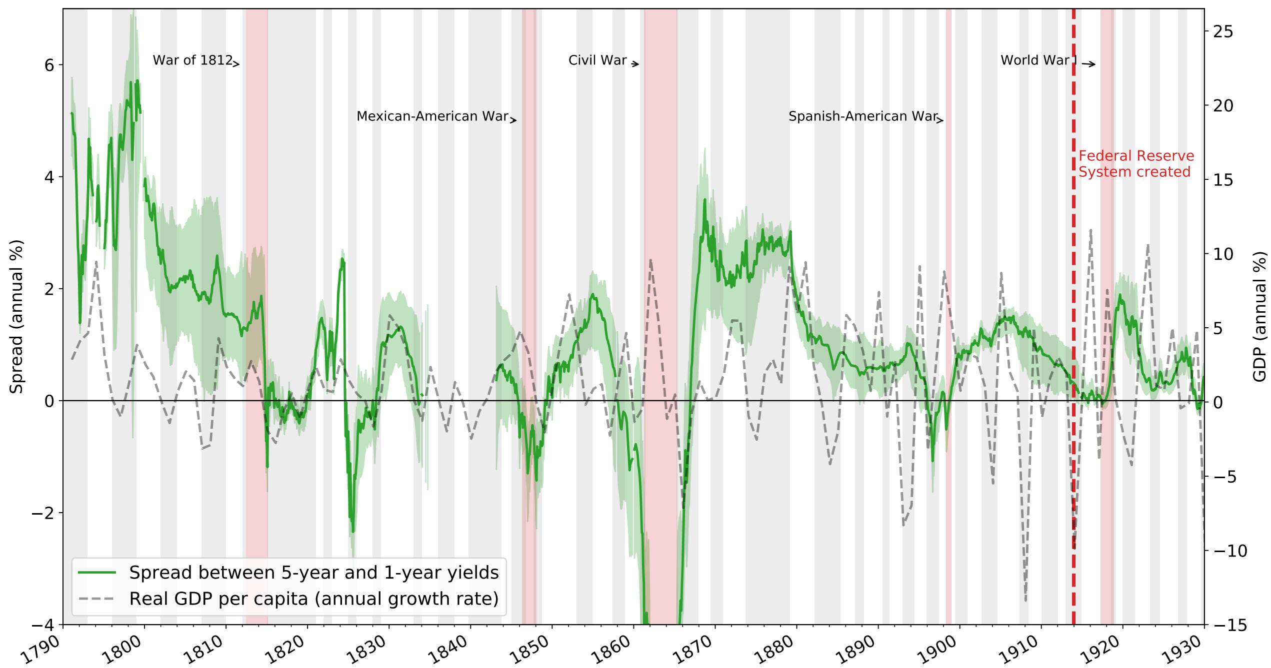

Figure 6: The Spread Between 5 Year and 1 Year Yields.

The green line depicts the yield on 5-year government bonds minus the yield on 1-year government bonds. The light gray intervals depict NBER recessions. The light red intervals depict wars (from left to right: the War of 1812, the Mexican-American War, the Civil War, the Spanish-American War, and World War I).

A large literature has used yields to help predict real GDP growth.[25] Our yield curve estimates open the way to extend such work back into the nineteenth century. As a preliminary step, table 1 replicates table 2 from Ang et al. (2006). It reports the coefficient $\beta_k^{(j)}$ and adjusted R^2 for the regression:

$$g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)}\left(y_t^{(j)} - y_t^{(1)}\right) + \varepsilon_{t+k,k}^{(j)}$$

where g_{t+k} is the annual percentage growth of real GDP per capita over the next k years and $y_t^{(j)}$ denotes the annualized j-year zero coupon yield. As can be seen, the coefficient for the 5-year spread at the 2-year horizon, $\beta_2^{(5)}$, is 0.59 for the period 1790-1860 and 0.36 for the period 1866-1930 (albeit with high standard errors). As a reference point, Ang et al. (2006) report a coefficient of 0.73 for the 1964-2001. We find it intriguing that an upward sloping yield curve appears to be positively correlated with future economic growth during the 19th century even though there existed no central bank to engage in "active" monetary policy.[26]

6 Concluding remarks

Our research here is partly a successful "proof of concept": we have used modern Bayesian computations to approximate posterior probabilities of a profligately parameterized but theoretically highly restricted statistical model. The model blends Hicks-Arrow price asset pricing theory with a statistical model of how quickly yield curves move over time. We have used the model's pricing errors to diagnose measurement errors and conceptual

[25] See Stock and Watson (2003) for a literature review and discussion.

[26] However, from 1897 until 1913, Republican Secretaries of the Treasury more and more violated the letter of the 1844 Independent Treasury Act and *de facto* conducted open market operations designed to lean against the wind.

	1790-1930				1790-1860				1866-1930			
	Term spread maturity											
Horizon	5 year - 1 year		10 year - 1 year		5 year - 1 year		10 year - 1 year		5 year - 1 year		10 year - 1 year	
k-years	β_k^5	R^2	β_k^{10}	R^2	β_k^5	R^2	β_k^{10}	R^2	β_k^5	R^2	β_k^{10}	R^2
1-year	0.10	0.005	0.09	0.007	0.37	0.048	0.35	0.068	0.20	0.001	0.08	0.000
	(0.09)		(0.08)		(0.23)		(0.20)		(0.59)		(0.45)	
2-year	0.29	0.018	0.25	0.021	0.59	0.042	0.52	0.051	0.76	0.009	0.36	0.003
	(0.17)		(0.14)		(0.40)		(0.36)		(1.14)		(0.79)	
3-year	0.57	0.043	0.44	0.041	0.58	0.025	0.45	0.024	1.99	0.038	1.08	0.018
	(0.18)		(0.14)		(0.59)		(0.54)		(1.66)		(1.11)	

Table 1: Forecasts of Real GDP growth per capita from term spreads (1790-1930)

The table reports the coefficient $\beta_k^{(j)}$ and R^2 for the regression $g_{t+k} = \alpha_k^{(j)} + \beta_k^{(j)} \left(y_t^{(j)} - y_t^{(1)} \right) + \varepsilon_{t+k,k}^{(j)}$ where g_{t+k} is the annual percentage growth of Real GDP per capita over the next k years and $y_t^{(j)}$ denotes the annualized j-year zero coupon yield. Newey and West heteroskedasticity- and autocorrelation-consistent standard errors with lag order one in parentheses.

problems involving units of accounts. Here we have just shown the tip of an iceberg. Offline, we have used the model to compose "bond biographies" of some classic bonds beloved of US financial historians such as the Stocks of 1790s, the Civil War era 5-20s, and the World War I Liberty and Victory Loans.

The quality and plausibility of our approximate yield curves convince us that they qualify as plausible inputs to subsequent research that in the spirit of "factor models" of stochastic discount factors will use related macro series to refine understandings of forces that drive yield curves and forecasts of other macro time series.

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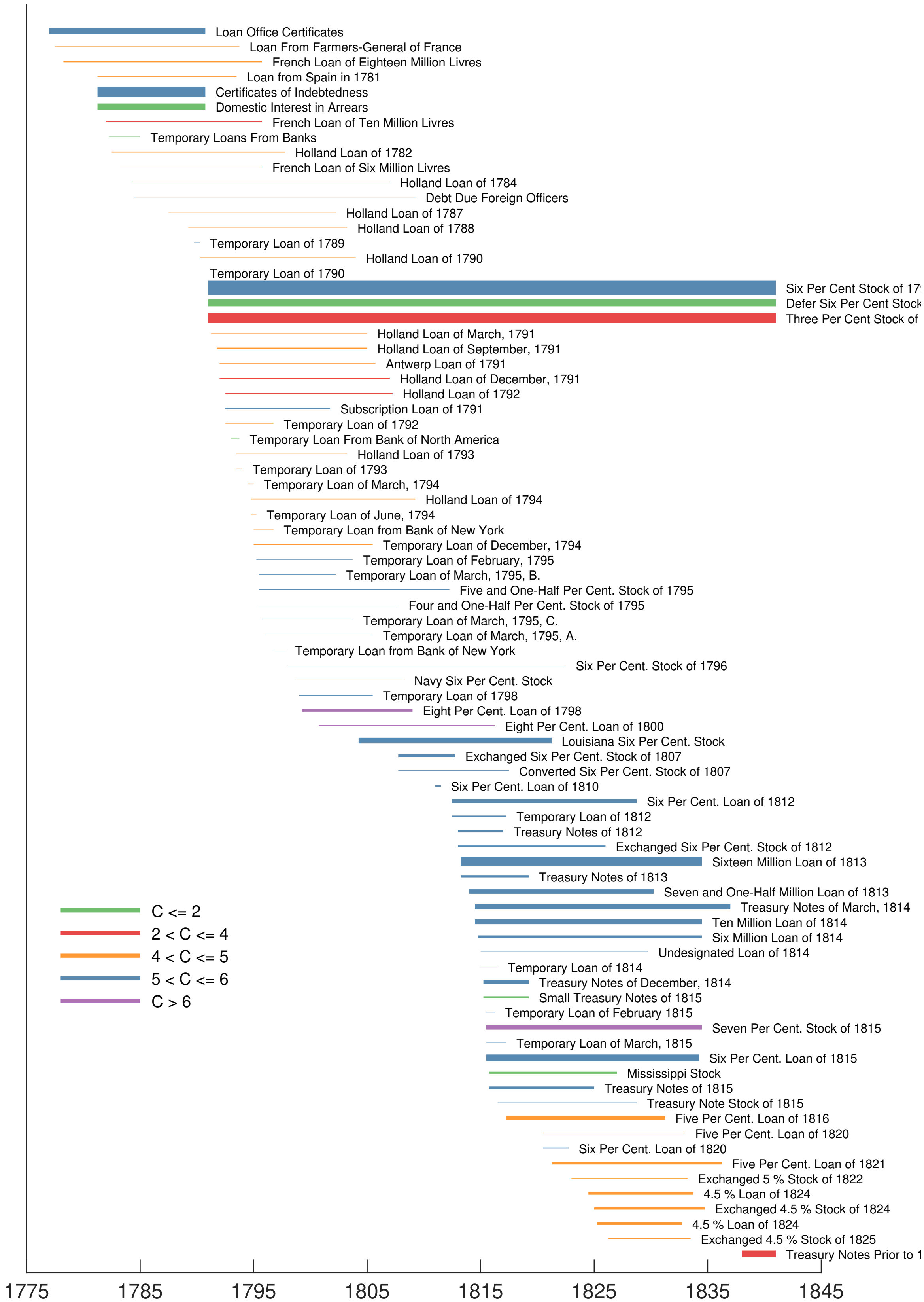

Figure 7: Treasury Bonds Issued from 1776 to 1840.

The span of each line corresponds to the period the security was outstanding. The width is proportional to the size of the issue, and the color denotes the coupon rate.

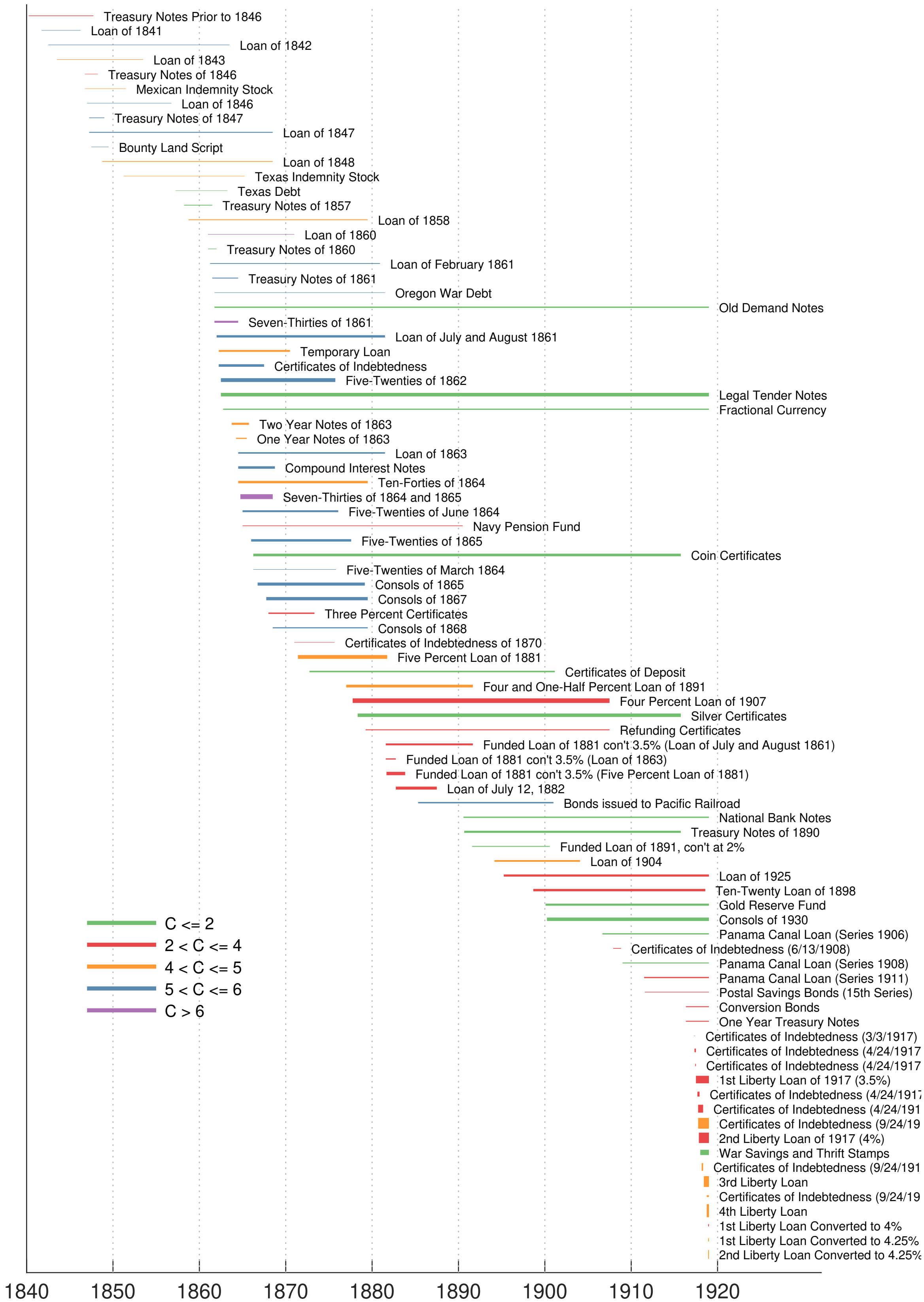

Figure 8: Treasury Bonds Issued from 1840 to 1918.

The span of each line corresponds to the period the security was outstanding. The width is proportional to the size of the issue, and the color denotes the coupon rate.