Understanding European Unemployment with a Representative Family Model

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July 16, 2007

Abstract

A representative family model with indivisible labor and employment lotteries has no labor market frictions and complete markets. Nevertheless, its aggregate responses to an increase in government supplied unemployment insurance (UI) and to an increase in microeconomic turbulence are qualitatively similar to those in two macro models with labor market frictions and incomplete markets, namely, the matching and search-island models in Ljungqvist and Sargent (2007c). Because there is no frictional unemployment in the representative family model, an increase in employment protection (EP) *decreases* aggregate work because the representative family substitutes leisure for work, an effect opposite to what occurs in matching and search-island models. Heterogeneity among workers highlights the economy-wide coordination in labor supply and consumption sharing that employment lotteries and complete markets achieve in the representative family model. A high disutility of labor makes generous UI cause very low employment levels.

Key words: Job, lotteries, skills, turbulence, unemployment, unemployment insurance, employment protection, representative family.

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1 Introduction

Macroeconomists today are working along several fruitful lines to bring labor market frictions and workers in heterogeneous situations into dynamic general equilibrium models in ways that make contact with both macro and micro data. Leading approaches differ in the ways that they model labor market frictions, the types of heterogeneity they envision, and the market arrangements they use to reconcile individual choices. For example, Mortensen and Pissarides (1999), Hall (2005), Shimer (2005), and Gertler and Trigari (2006) work with structures featuring a labor market friction that imposes congestion externalities captured by a matching function. In contrast, Lucas and Prescott (1974), Alvarez and Veracierto (2001), Gomes et al. (2001), and Ljungqvist and Sargent (2007b) use search-theoretic structures that purposefully exclude congestion externalities and instead features costly search by unemployed workers as the essential friction. Matching and search models both contain people who occasionally devote part of their time to an activity called unemployment and have equilibrium outcomes in which frictional unemployment is a well defined quantity. In contrast, Prescott (2002) and Rogerson (2007), use the Hansen-Rogerson employment lottery model to justify a representative agent model that has no meaningful individual activity distinct from leisure that can be usefully called unemployment. These alternative approaches incorporate different 'aggregation theories,' to use Prescott's (2002) term. All of this ongoing work in macro labor is sufficiently exciting and promising, and the alternative lines being pursued are sufficiently different, that it seems timely and worthwhile to compare alternative structures and see what makes them tick.

That is what we try to do in this and our companion paper Ljungqvist and Sargent (2007c). As a way of highlighting the different economic mechanisms at work in these alternative structures, our idea is to put them through their paces by creating a common testing ground on which to analyze an important practical problem, then to listen to what the models and the agents within them tell us. For the practical problem on which we ask the models to speak, we choose the longstanding discrepancies observed by U.S. and European levels of unemployment. Our companion paper studies two models that have equilibrium frictional unemployment, while the present paper studies a model that does not, namely, employment-lotteries representative family model.

Ljungqvist and Sargent (2007c) constructed some matching models and a search-island model that succeed in explaining the following facts about European and U.S. unemployment rates during the last half century: (1) in the 1950s and 1960s, unemployment rates were persistently *lower* in Europe than in the U.S. The difference was accounted for by a *higher* inflow rate into unemployment in the U.S.; (2) unemployment became persistently *higher* in Europe after the 1970s; (3) within both Europe and the U.S., inflow rates into unemployment were roughly constant across periods; (4) in Europe, average durations of unemployment were low in the 1950s and 1960s, but became high after the 1970s. Average duration in the U.S. stayed low; and (5) since the 1970s, in Europe hazard rates of leaving unemployment fall with increases in the duration of unemployment. To distinguish Europe from the U.S., Ljungqvist and Sargent (2007c) took as fixed the following two institutional features: (1) in both periods, government supplied unemployment insurance (UI in the language of Mortensen and Pissarides (1999)) were generous with long durations in Europe, while they were stingy with short durations in the U.S.; and (2) employment protection provisions (EP in the language of Mortensen and Pissarides (1999)) were stronger in Europe throughout both periods. Given these unchanging institutional backgrounds, we injected into both models an exogenous increase in microeconomic *turbulence*, meaning an increased risk of destruction of valuable job-specific human capital after involuntary job dissolution.

The matching and search-island models in Ljungqvist and Sargent (2007c) feature distinct labor market frictions that impede willing workers from finding jobs immediately. This paper formulates a model without any such labor market friction, namely, an employment-lotteries representative family model. The model has the same stochastic skill accumulation and deterioration technologies as those in Ljungqvist and Sargent (2007c) and the same specifications for UI and EP.[1] We use the model to study the same issues as Ljungqvist and Sargent (2007c), namely, how cross-country differences in UI and EP interact with an increase in the probability of skill deterioration after involuntary layoffs to generate outcomes, and whether those outcomes resemble what we have observed. We believe that comparing what different structures have to say about such observations is a good way to deepen our understandings of model properties and to guide model selection and improvement. As we shall see, while the labor-market-friction-free employment-lotteries representative family model shares some important aggregate outcomes with the two models that have labor market frictions (e.g., generous UI still interacts with increased turbulence to cause labor inactivity to increase), its different response in other dimensions (the response to EP is reversed because there is no frictional unemployment and inactivity is too responsive to generous UI benefits) prevents us from being able to use it as a vehicle to tell the same story about the facts that we can with the other models. To us, this is a disappointing feature of the representative family model, but we realize that it might not disappoint others who are less fond of the theory of the European unemployment experience advocated by Ljungqvist and Sargent (1998, 2007b). Regardless of one's reaction, the experiment has the virtue of exposing the economic mechanisms at work in different frameworks.

Our representative family model features indivisible labor and employment lotteries in conjunction with comprehensive *private* insurance arrangements in the tradition of Hansen (1985), Rogerson (1988), and Prescott (2002). Risk averse agents have access to complete markets in history-contingent consumption claims. The choice problem of firms and the structure of labor contracts are identical with those in the search-island model. An indivisi-

[1] How to introduce government-supplied UI into the representative family model, and whether it should be introduced at all, is controversial. Because the Hansen-Rogerson model has no theory of jobs, Lucas (1987, p. 53, footnote 4) indicated that he regarded it to be an inappropriate tool for analyzing policy questions concerning UI. Nevertheless, after Ljungqvist and Sargent (2006) challenged Prescott (2002) for ignoring the generous welfare systems in Europe, Prescott (2006) responded by including benefits in a representative agent analysis of low European employment. But in introducing UI, Prescott managed to keep the tax-benefit wedge used in his 2002 paper unchanged by positing a meager European replacement rate of 29% and cutting his earlier estimate of a 50% marginal labor tax in Europe (Prescott 2004) to 30% (Prescott 2006).

bility in the choice set of each worker requires him to work either a fixed number of hours or not at all. An economy-wide representative family uses lotteries to convexify its production opportunity set by assigning fractions of its members to specialize in work or leisure. The model yields a high *aggregate* labor supply elasticity that makes the fraction assigned to work respond sensitively both to tax wedges *and* to government-supplied UI benefits.[2]

The absence of market frictions in the representative family lotteries model and their presence in the models in Ljungqvist and Sargent (2007c) create different avenues through which UI and EP influence outcomes, not just for unemployment in the aggregate, but for individual workers' consumption and labor market experiences, including the incidence of long-term unemployment.[3]

1.1 Organization

Section 2 describes features of the environment that are common with the models in Ljungqvist and Sargent (2007c): (1) two transition matrices for workers' skill levels, one for workers whose jobs continue or end voluntarily, another for workers whose jobs terminate involuntarily; (2) a probability distribution for productivity levels of new workers and transition matrices for the productivity levels of workers whose jobs continue; and (3) parameters that define a replacement rate for UI and a layoff tax for EP. In addition to these common features, section 3 describes the special features of the representative family model. Firms face the same problem that they do in the search-island model of Ljungqvist and Sargent (2007c). But complete markets allow smoothing consumption across states and time. An infinitely lived representative family consists of a continuum of lineages of workers who value leisure, consumption, and their offsprings' welfare. Section 4 justifies our calibration. Section 5 describes quantitative outcomes. Against the backdrop of the section 5 outcomes, in the spirit of Lucas (1987, p. 56), section 6 interviews an unemployed European worker. Section 7 concludes. An appendix describes how to compute an equilibrium.

2 Common features with other models

Figure 1 shows the within-period timing of our model. It is identical with the timing for the search-island model of Ljungqvist and Sargent (2007c). Each of a continuum of eligible

[2]Ljungqvist and Sargent (2006) explore the role played by employment lotteries and complete markets, i.e., the "aggregation theory" emphasized by Prescott (2002). It is not these features but rather the indivisible labor and high disutility of working that are important for obtaining a high aggregate labor supply elasticity.

[3]Our model shares some limitations with those in Ljungqvist and Sargent (2007c). First, it ignores the intensive margin of the labor supply decision by assuming that workers are either unemployed or employed full-time and working the same number of hours. Second, if it were not for the labor indivisibility, under laissez-faire everyone of working-age would be employed. Hence, the model ignores such non-market activities as education, child rearing, and other types of homework that explain why some people of working-age do not participate in the labor market. This observation prompts us to calibrate our employment-lotteries representative family model under laissez faire to explain an unemployment rate rather than the employment-to-population ratio that is more often taken as the target by users of this model.

workers faces a constant probability ρ of retiring and thereby being forbidden to work, and σ is the probability that a retired worker dies. To keep the total population and the shares of workers and retirees constant over time, people who die are replaced by newborn workers.

As in the search-island and matching models of Ljungqvist and Sargent (2007c), there are three other exogenous sources of uncertainty. First, an employed worker faces a probability π^o that his job terminates. Second, workers experience stochastic accumulation or deterioration of skills, conditional on employment status and instances of exogenous job terminations. Third, idiosyncratic shocks impinge on employed workers' productivities.

2.1 Skill dynamics

Two possible skill levels are indexed by $h \in \{0, H\}$. All newborn workers enter the labor force with the low skill index, $h = 0$. An employed worker with skill index h faces a probability $p^n(h, h')$ that his skill at the beginning of next period is h', conditional on no exogenous job termination. In the event of an exogenous job termination, a laid off worker with last period's skill h faces a probability $p^o(h, h')$ that his skill becomes h'. A worker's skill remains unchanged during an unemployment spell. The skill transition matrices are:

$$p^n = \begin{bmatrix} 1-\pi^u & \pi^u \\ 0 & 1 \end{bmatrix}, \tag{1}$$

$$p^o = \begin{bmatrix} 1 & 0 \\ \pi^d & 1-\pi^d \end{bmatrix}. \tag{2}$$

2.2 Firm formation and productivity

The process of uniting firms and workers has several common features with those in the search-island and matching models of Ljungqvist and Sargent (2007c). Firms incur a cost μ when posting a vacancy in the matching model or when creating a job in the other two frameworks. We model a new job opportunity as a draw of productivity z from a distribution $Q^o(z)$. The productivity of an ongoing job is governed by a Markov process: $Q(z, z')$ is the probability that next period's productivity is z', given current productivity z. For any two productivity levels z and $\hat{z} < z$, the conditional probability distribution $Q(z, z')$ first-order stochastically dominates $Q(\hat{z}, z')$, meaning that

$$\sum_{z' \leq \bar{z}} Q(z, z') < \sum_{z' \leq \bar{z}} Q(\hat{z}, z') \qquad \text{for all } \bar{z}. \tag{3}$$

An employed worker keeps his productivity from last period with probability $(1 - \pi)$ and draws a new productivity with probability π from the distribution $Q^o(z')$, so that new productivities on existing jobs are drawn from the same distribution as the productivities at the time of job creation. These assumptions about productivity processes are identical to those in the search-island model of Ljungqvist and Sargent (2007c).

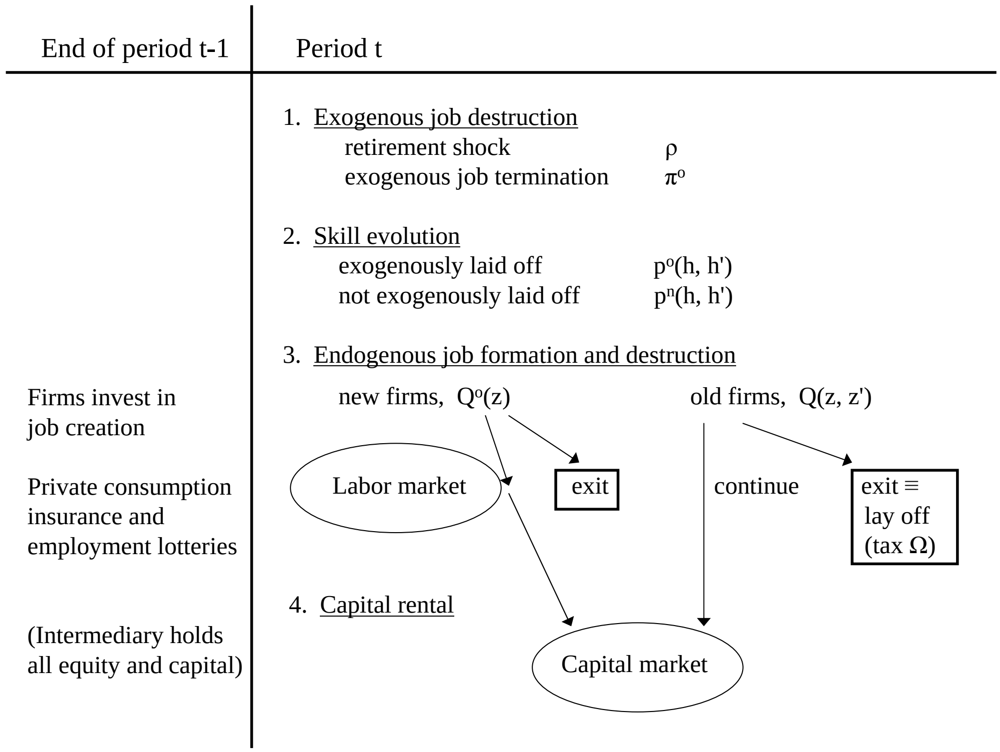


Figure 1: Timing of within period events.

2.3 Government mandated UI and EP

The government levies layoff taxes on job destruction and provides benefits to the unemployed. It imposes a layoff tax Ω on every endogenous job separation and on every exogenous job termination except retirement. The government pays unemployment benefits equal to a replacement rate η times a measure of past income. To determine his benefit entitlement, it suffices to remember a worker's skill in his last employment. Newborn workers are entitled to the lowest benefit level in the economy. The government finances unemployment benefits with revenues from the layoff tax and other model-specific taxes.

3 Representative family model

We make these changes to the search-island model of Ljungqvist and Sargent (2007c): (1) agents care about their offspring, so that each lineage of agents has an infinite-horizon utility function where the survival probability is equal to one; (2) a disutility of working replaces a disutility of search; (3) no search or other kinds of frictions impede workers from immediately finding the competitive labor market; and (4) agents have access to complete markets in history-contingent consumption claims. But along with the search-island and matching models of Ljungqvist and Sargent (2007c), we retain the assumption that there is no 'intensive' margin for an agent's choice of work, i.e., each working-age agent faces a $\{0, 1\}$ choice of working or not working an exogenously amount of time in any given period. Following Hansen (1985), and Rogerson (1988), we model the economy as a representative family that assigns a fraction of individuals to work and that uses employment lotteries to decide who works and who enjoys leisure.

A continuum of lineages are indexed by numbers in the unit interval. Each lineage consists of an infinite sequence of workers, only one of whom is alive at any time. Each worker retires with probability ρ, then dies with probability σ. Within each lineage, a newborn worker immediately replaces a deceased worker. The representative family has a utility function over consumption and the work of all of its lineage members:[4]

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t^j) - v(n_t^j) \Big] dj \ = \ \int_0^1 \sum_{t=0}^{\infty} \beta^t \Big[\log(c_t^j) - n_t^j B \Big] dj \ , \tag{4}$$

where c_t^j is lineage j's consumption at time t and n_t^j equals one if the current member of lineage j is working and equals zero otherwise. We assume that $v(n)$ increases with increases in n. The indivisibility of an individual worker's labor supply makes $v(0)$ and $v(1)$ the only pertinent values. As a normalization, we let $v(0) = 0$ and $v(1) = B$, so the parameter $B > 0$ captures the disutility of working.

A collective entity called 'the family' determines fractions R_t, U_{0t}, $U_{\triangle t}$, U_{Ht}, N_{0t}, N_{Ht} of workers who are retired, unemployed with low skills and low benefits, unemployed with low skills and high benefits, unemployed with high skills and high benefits, employed with low skills, and employed with high skills, respectively; $R_t + U_{0t} + U_{\triangle t} + U_{Ht} + N_{0t} + N_{Ht} = 1$. The family adjusts these fractions in response to wage rates, unemployment benefits, and taxes.[5] Each worker participates in an employment lottery that determines whether he works as his personal history of skill and benefit entitlement unfolds. Winners of the lottery do not work. As in the models of Hansen (1985) and Rogerson (1988), the employment lotteries enlarge the *aggregate* labor supply elasticity relative to what would be chosen by an individual worker with preferences under the integral sign on the left side of (4) who did *not* face the $\{0, 1\}$ restriction on his choice of n_t^j.[6]

Because (4) is additively separable, the family assigns the same consumption $\bar{c}_t$ to every family member. The family assigns an unretired person to work with a probability that depends on his or her skill and benefit entitlement.[7] The *unconditional* expected utility of a

[4]It is rewarding to compare the representative family model of Hansen (1985) and Rogerson (1988) with the household labor supply model of Chiappori (1992, 1997), and Browning and Chiappori (1998), and to recognize how differently they draw lines that separates families. In the representative family model, a family is an aggregate (national) economy. In applications of the household labor supply model, a family is what we usually think of as a nuclear family.

[5]Alternatively, the family member responds to equilibrium prices. See Hansen (1985).

[6]The aggregate labor supply elasticity would be altered if we were to expand the number of points in the set of hours that an individual worker is allowed to work, thereby somewhat relaxing the severe indivisibility imposed in the original models of Hansen (1985) and Rogerson (1988). See the extension to handle straight-time and overtime work by Hansen and Sargent (1988).

[7]Relative to Hansen (1985) and Rogerson (1988), we extend the objects in the employment lottery to include entire histories (actually, it is better to call them '*futures*') of skills and benefit entitlements that a worker might experience. For example, a worker who experiences a skill upgrade will never again be assigned to enjoy leisure, at least as long as he does not suffer an exogenous layoff with skill loss. In the spirit of

lineage in the representative family is

$$\sum_{t=0}^{\infty} \beta^t \Big[\log(\bar{c}_t) - \bar{n}_t B\Big] , \tag{5}$$

where $\bar{n}_t$ is the unconditional probability that a member of the lineage works in period t, which equals the fraction of all family members of both skill levels sent to work, $\bar{n}_t = N_{0t} + N_{Ht}$. Reformulation (5) of the family's utility function (4) embeds the outcomes of both the work-sharing lottery and trades in a comprehensive set of markets in history contingent claims to consumption.

Given the steady-state wage rate w per unit of skill and the one-period net interest rate $i = (\beta^{-1} - 1)$, the present-value budget constraint of the representative family becomes

$$\sum_{t=0}^{\infty} (1+i)^{-t}\, \bar{c}_t \;\; \leq \;\; \sum_{t=0}^{\infty} (1+i)^{-t}\, (1-\tau) w \Big\{ N_{0t} + (1+H) N_{Ht} + \eta \Big[U_{0t} + (1+H)\left(U_{\triangle t} + U_{Ht}\right) \Big] \Big\} + (1+i) a_0 \, , \tag{6}$$

where τ is a flat-rate tax on labor earnings and unemployment benefits, and a_0 is the steady-state per capita asset level. As in the search-island model of Ljungqvist and Sargent (2007c), each worker has skill measured as $1+h$, where $h \in \{0, H\}$. Hence, a low-skilled worker earns w and a high-skilled worker earns $(1+H)w$ before taxes.

3.1 Firms

The production technology and choices open to a typical firm are identical with those in the search-island model of Ljungqvist and Sargent (2007c). Each firm employs one worker and rents physical capital. The firm's production function is

$$z_t k_t^{\alpha} (1+h_t)^{1-\alpha}, \qquad \text{with } \alpha \in (0,1), \tag{7}$$

where z_t is the current productivity level, $h_t \in \{0, H\}$ is the skill index of the firm's worker, and k_t is physical capital that depreciates at the rate δ. Output can be devoted to consumption, investment in physical capital, and startup costs. Incurring a startup cost μ at time t allows a firm to create a job opportunity at $t+1$ by drawing a productivity level z from the distribution $Q^o(z)$. After seeing z, a firm decides whether to hire a worker from the centralized labor market. Firms and workers first meet under a veil of ignorance about their partner's state vector: the firm hires a worker drawn randomly from a single pool of unemployed workers with a mix of low-skilled and high-skilled workers. Once hired, a firm observes a worker's skill, hires the appropriate physical capital, and pays the worker the market wage of w per unit of skill. A firm must retain a worker for at least one period.

Lucas (1987, p. 67, footnote 13) when he remarked that "in the Hansen and Rogerson papers, [unemployed] workers are happier than those who draw employment!," we can say that the real losers of our lotteries are the workers who have had successful careers with skill accumulation because they enjoy the same consumption as everyone else but none of the leisure. (For details, see appendix A.7.)

4 Calibrations

4.1 Unemployment versus an employment-population ratio

The issues about UI that interest us prompt us to calibrate the labor-leisure tradeoff in the representative-family employment-lotteries model to match an unemployment rate rather than the employment-to-population ratio that is more often the target in the real business cycle literature. In laissez-faire ($\tau = \eta = \Omega = 0$) versions of the model, these different calibration targets would have no substantial consequences other than to scale up or down the fraction of the population of working age that is employed. However, these differences do matter in the welfare-state versions of the model because we assume that those who are not employed are entitled to government supplied UI. The distortionary tax rate needed to finance the UI system is increasing in the number of people counted as unemployed. For that reason, our decision to take the unemployment rate as the calibration target seems to be appropriate for studying the issues about UI addressed in this paper and in Ljungqvist and Sargent (2007c).

4.2 Parameterizations that are common to other models

As far as possible, we retain common parameterizations with those adopted for the models in Ljungqvist and Sargent (2007c). This practice can be criticized because the same values of these parameters imply different outcomes in different models. For example, the same discount factor implies different outcomes for the risk-free interest rate in the search-island model of Ljungqvist and Sargent (2007c), where precautionary savings motives are in play, and in our representative family model, where there are complete insurance markets.

Our justification for keeping common parameters, including the discount factor and the variance of the productivity distribution, is that it serves our goal of isolating the economic forces at work in the alternative frameworks. Further, we can sometimes argue that the pertinent quantitative effects are so large that no reasonable change in parameter values can make a substantial difference. An example will be our finding that the representative-family employment-lotteries model delivers quantitatively unrealistic responses to government supplied UI so long as we keep disutility of labor parameter high enough to deliver the high aggregate labor supply elasticity featured in previous macroeconomic applications of the employment lotteries model, e.g., in the real business cycles literature (see Rogerson (1988) and Hansen (1985)).

The first panel of table 1 reports the part of the parameterization that is the same as in the search-island and matching models in Ljungqvist and Sargent (2007c), who in turn reuse parameter values from previous studies. Following Alvarez and Veracierto (2001), we set the model period equal to half a quarter, and specify a discount factor $\beta = 0.99425$ and a probability of retiring $\rho = 0.0031$. Hence, the annualized subjective discount rate is 4.7% and, on average, people spend 40 years in the labor force.

We set skill transition probabilities to make the average durations of skill acquisition and skill deterioration agree with those in Ljungqvist and Sargent (1998, 2007b), who let it

Parameters common to all models	
Discount factor β	0.99425
Retirement probability ρ	0.0031
Probability of upgrading skills, π^u	0.0125
Probability of exogenous breakup, π^o	0.005
Probability of productivity change, π	0.05
Productivity distribution	truncated $\mathcal{N}(1,1)$
Parameters shared with search-island model	
Probability of dying, σ	0.0083
Capital share parameter, α	0.333
Depreciation rate, δ	0.011
Job creation cost, μ	5.0
Low skill level	1.0
High skill level, $(1+H)$	2.0
Additional parameter in representative family model	
Disutility of working, B	1.01

Table 1: Parameter values (one period is half a quarter)

take a long time to acquire the highest skill level in order to match realistic shapes of wage-experience profiles. We set a semiquarterly probability of upgrading skills $\pi^u = 0.0125$, so that on average it takes 10 years to move from low to high skill, conditional on no job loss. Exogenous layoffs occur with probability $\pi^o = 0.005$, i.e., on average once every 25 years. The probability of a productivity switch on the job $\pi = 0.05$, so that a worker expects to retain a given productivity level for 2.5 years.

Another common assumption is that productivities are drawn from a normal distribution $\mathcal{N}(1,1)$ with mean 1.0 and standard deviation 1.0, which is truncated to the interval $[0,2]$ and then rescaled to integrate to one.

4.3 A new parameter in the representative family model

Comparing table 1 with the analogous table in Ljungqvist and Sargent (2007c) shows that our representative family model and the search-island model of Ljungqvist and Sargent (2007c) are calibrated alike, except for dropping the parameters pertaining to job search and adding the new parameter B that represents disutility of *working*. Setting $B = 1.01$ makes the laissez-faire unemployment rate be 4.7%.

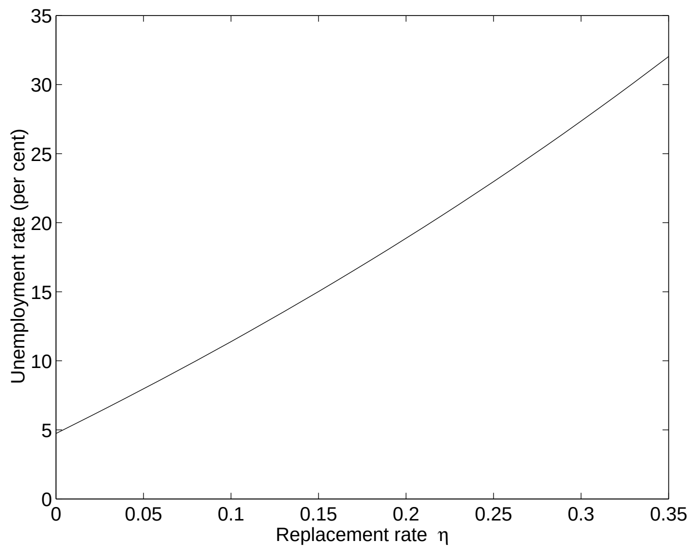


Figure 2: (Representative family model) Unemployment rates for different replacement rates η, given tranquil economic times and no layoff taxes.

5 Outcomes

The representative family model with employment lotteries reproduces the responses to high UI and high turbulence found by Ljungqvist and Sargent (1998), but not the response to higher layoff taxes featured in Ljungqvist and Sargent (2007b). Figure 2 shows that in tranquil times and with no layoff taxes, increasing the UI replacement rate η has a powerful effect that increases the unemployment rate. This outcome agrees with those from the matching models and the search-island model, but the mechanism is different. Here there are neither the matching models' congestion effects nor the search-island model's search costs. Therefore, the change in unemployment is not an adjustment of the level of frictional unemployment that serves to alter firms' waiting times, thereby compensating them for their costs of posting vacancies, as in the matching models. Nor does it reflect a longer duration of unemployment coming from workers' diminished search activity, as in the search-island model. Instead, a higher UI replacement rate makes leisure a more attractive use of the representative family's time.

Figure 3 reaffirms the findings of Hopenhayn and Rogerson (1993) and Ljungqvist (2002) that with zero turbulence in a representative family model with lotteries, an increase in the layoff tax causes the unemployment rate to *rise*, reversing the outcomes in Ljungqvist and Sargent (2007b) and in the matching and search-island models of Ljungqvist and Sargent (2007c). In the representative family model, anyone the family wants to put to work gets a job immediately, so the layoff tax does not have the effect of suppressing frictional unemployment that it does in those other models. Instead, an increase in the layoff tax decreases the equilibrium wage, prompting the family to substitute toward leisure.

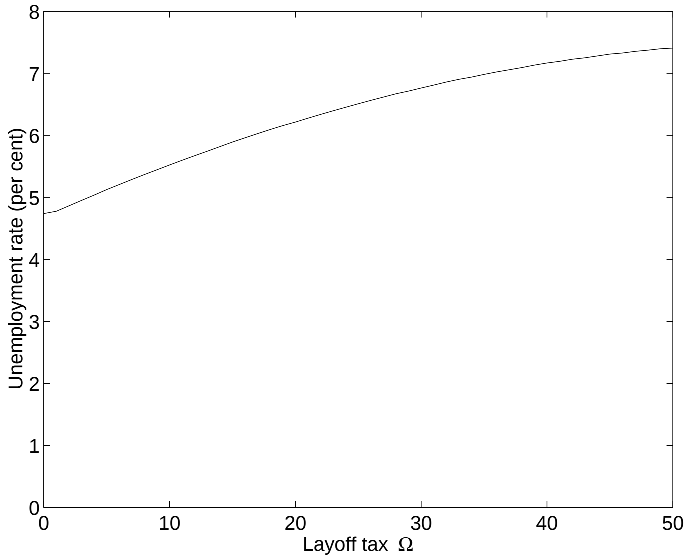
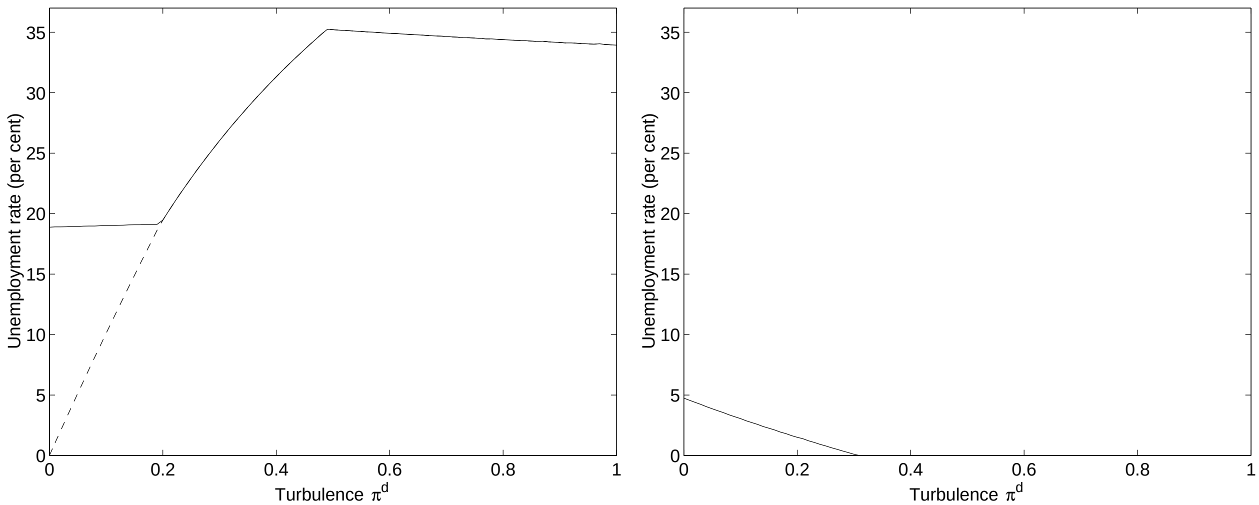


Figure 3: (Representative family model) Unemployment rates for different layoff taxes Ω, given tranquil times and no benefits. The magnitude of the layoff tax can be compared to a semiquarterly equilibrium wage of 6 per unit of skill in the laissez-faire economy, i.e., a layoff tax equal to 48 corresponds to one year of wage income for a low-skilled worker.

The model's sensitivity to UI prompts us to choose a low replacement rate, $\eta = 0.2$, in the benchmark parameterization of the welfare state. This is unrealistically low by European standards (see Martin (1996)). We set a low replacement rate to avoid further raising an already unrealistically high unemployment rate in the welfare state in tranquil times ($\pi^d = 0$). For the same reason, we set the layoff tax $\Omega = 0$, since it does not have the effect of suppressing unemployment that it does in the matching and search-island models. We thus admit defeat in our effort to use the representative family model to explain how welfare states can sustain relatively low unemployment rates when $\pi^d = 0$, and proceed to ask whether the model predicts that unemployment erupts in the welfare state but not in the laissez-faire economy when turbulence π^d increases. Figure 4 provides an affirmative answer to this question: the two panels show how an increase in turbulence has very different effects in the welfare state $(\eta, \Omega) = (.2, 0)$ and the laissez faire $(\eta, \Omega) = (0, 0)$ versions of this model. In the welfare state, unemployment increases with turbulence until turbulence reaches about .5, then unemployment falls slightly with further increases, while under laissez faire, increases in turbulence push unemployment down to zero.

To shed light on why the laissez-faire unemployment rate falls to zero as turbulence increases in panel b of figure 4, it is useful to consider a simplified version of our representative family model that abstracts from retirement, startup costs for new firms, and shocks to firms' productivity. Let z denote a deterministic productivity level of all firms, which we assume takes a value that induces the family to choose an interior solution for employment. Given



(a) Welfare state (b) Laissez-faire economy

Figure 4: (Representative family model) Unemployment rates in the welfare state (panel a) and the laissez-faire economy (panel b). The solid line is total unemployment. In the welfare state, the policy is $(\eta, \Omega) = (0.2, 0)$ and the dashed line shows the unemployed who have suffered skill loss (which is not a uniquely determined quantity in the laissez-faire economy and is therefore left out from panel b).

$\pi^u > 0$ and $\pi^d > 0$, the family chooses strictly positive steady-state fractions U_0, N_0, N_H of family members who are unemployed with low skills, employed with low skills, and employed with high skills, respectively. The representative family's Euler equations and the equilibrium conditions that state that the family supplies the economy's labor and holds the aggregate capital stock imply the following steady-state values for k and n:

$$\begin{aligned}\bar{k} &= \left[\frac{i+\delta}{\alpha z}\right]^{\frac{1}{\alpha-1}} \left[N_0 + (1+H)N_H\right], \\ \bar{n} &= N_0 + N_H = \frac{(1-\alpha)(i+\delta)(\pi^d+\pi^u)\left[1+\beta\pi^u(1+H)-\beta(1-\pi^d)\right]}{\left[i+\delta(1-\alpha)\right]\left[\pi^d+(1+H)\pi^u\right]\left[1+\beta\pi^u-\beta(1-\pi^d)\right]B},\end{aligned}$$

where $i = (\beta^{-1} - 1)$ is the stationary net interest rate. Note that employment $\bar{n}$ does not depend on the productivity level z, as in real business cycle specifications with logarithmic preferences. But the employment effect of an increase in turbulence is strictly positive and given by

$$\frac{\partial \bar{n}}{\partial \pi^d} = \frac{(1-\alpha)(i+\delta)\pi^u(1-\beta)\left[1-\beta+2\beta(\pi^u+\pi^d)+\beta\pi^u H\right]}{\left[i+\delta(1-\alpha)\right]\left[\pi^d+(1+H)\pi^u\right]^2\left[1+\beta\pi^u-\beta(1-\pi^d)\right]^2 B}\, H > 0\,.$$

Changes in the parameter π^d give rise to an equilibrium response in the representative family's behavior that can be understood in terms of substitution and wealth effects. An increase in π^d means that the return to working falls, which should reduce the family's labor supply because of the substitution effect, on one hand, and increase the family's labor supply due to the wealth effect from lower labor income, on the other hand. Evidently, the latter effect is stronger, since unemployment falls in response to increases in turbulence.

To understand the unemployment effects of increased turbulence in the welfare state, we divide the range of turbulence in panel a of figure 4 into three regions: (i) the positive but relatively flat segment in region $\pi^d \in [0, .2]$, (ii) the dramatic surge in region $\pi^d \in [.2, .5]$, and (iii) the mildly downward-sloping segment in region $\pi^d \in [.5, 1]$. Recall that our specification of the skill deterioration technology (2) implies that at zero turbulence, there are no low-skilled workers entitled to high benefits. The low-skilled workers are either newborns or people who have not yet experienced a skill upgrade. At zero turbulence, the family decides to furlough so many of these workers into leisure that nearly 19% of the working-age population is unemployed.[8]

For low levels of turbulence, workers who suffer skill loss after layoffs are inframarginal in the family's leisure decision. Their high benefit entitlements make it optimal to tell these workers to specialize in leisure. They share the same potential for future skill accumulation as low-skilled workers who are entitled to low benefits. They remain inframarginal workers in the representative family's leisure decision so long as the family chooses unemployment for any low-skill, low-benefit workers. Hence, the family's marginal condition for assigning an additional worker to specialize in leisure at low levels of turbulence pertains to a low-skilled worker who is entitled to *low* benefits: all low-skilled workers with *high* benefits have already been furloughed into unemployment. This marginal condition, which pertains to the entire region (i), yields an almost constant unemployment rate of approximately 19%. Thus, as turbulence π^d rises until it approaches .2, the family leaves the total amount leisure roughly constant and just changes the mix of types of workers who enjoy leisure. Low-skilled workers with low benefits are replaced by low-skilled workers with high benefits whom the family furloughs into leisure for the rest of their lives. The slightly positive relationship between turbulence and unemployment in region (i) can be attributed to an increasing tax wedge on labor supply in response to the government's need to balance a budget with higher benefit expenditures (since the number of unemployed people who collect high benefits increases with turbulence). The higher tax rate on labor has only a substitution effect because the wealth effect is neutralized when the government pays out the tax proceeds as unemployment benefits to the representative family.

At a critical level of turbulence of approximately $\pi^d = 0.2$, the marginal condition for assigning a low-skilled worker who is entitled to low benefits to specialize in leisure holds with equality when the family assigns no one of that category to enjoy leisure. At that point, turbulence has reached the critical level at which the last available unemployed low-skilled worker with low benefits has been replaced by a low-skilled worker entitled to high benefits

[8]As discussed in appendix A.1, we have restricted parameters to guarantee that benefit policies are not so generous that they would induce families to accumulate skills simply to furlough high-skilled workers into unemployment and then forgo earning wages in order to collect UI. Hence, there is no steady state with unemployment among high-skilled workers.

– turbulence has entered region (ii). Since the marginal condition holds with equality at that critical level of turbulence, it follows that the family's first-order condition for letting an additional low-skilled worker with high benefits specialize in leisure is a strict inequality when a further increase in turbulence causes that additional worker to 'materialize'. Thus, for levels of turbulence in region (ii), the family strictly prefers to furlough into leisure *all* low-skilled workers who are entitled to high benefits but to furlough *none* of the low-skilled workers who are entitled to low benefits. As a result, the unemployment rate surges dramatically with increases in turbulence in region (ii) because unemployment then increases one-for-one with the growing number of skill losers whom the family furloughs into leisure for the rest of their lives.

When turbulence climbs to another critical level of approximately $\pi^d = 0.5$, the family's marginal condition for assigning a low-skilled worker who is entitled to high benefits to specialize in leisure starts to hold with equality – turbulence has entered region (iii). The representative family now sends some of the skill losers back to work, and it forgoes their high benefits in exchange for their after-tax wages and their potential for renewed skill accumulation. In region (iii), unemployment falls mildly with increases in turbulence because of the wealth effect from turbulence, an effect that we described for the case of the laissez-faire economy above. But one might then ask why the equilibrium response to the wealth effect is muted in the welfare state as compared to the laissez-faire economy in panel b of figure 4, where unemployment plummets in response to turbulence. The reason is that the representative family of the welfare state faces a very different opportunity set in which the labor income tax needed to finance the UI system is about 11.4% in region (iii), and the unemployed workers collect an *effective* replacement rate of 40% on current earnings potential rather than the stipulated rate of 20%. (Recall that the stipulated replacement rate $\eta = 0.2$ is applied to their past earnings, which are twice as high as their current earnings potential.) It follows that unemployment remains a relatively attractive option for the family in the welfare state as compared to the family in the laissez-faire economy in which neither taxes nor benefits distort its labor supply.

6 Another view of unemployed European workers

In Ljungqvist and Sargent (2007c), we talk with inactive workers in a search-island model and a matching model. Although an adverse interaction between high UI and high turbulence transcends the model of this paper and those in Ljungqvist and Sargent (2007c), the unemployed workers in these models have different alibis for why they aren't working. Unemployed workers are discouraged in the search-island model, while in the matching models they are victims of high congestion costs and their own high bargaining power. Now let's listen to what the inactive workers in the employment lotteries representative family model have to say and think about whether they remind us, in the words of Lucas (1987, p.56), of 'talking to an unemployed [European] friend'.[9]

[9] We have added 'European' to Lucas's phrase.

In response to the question 'why aren't you working?', an unemployed worker in the employment-lotteries representative family model would tell us that he has had the good luck to win a lottery prize that entitles him to specialize in enjoying leisure.[10] In response to 'but how will you eat?', the unemployed worker would tell us that the Europe he lives in functions like one big happy European-wide family that awards everyone the same consumption stream independently of his luck in the employment lottery. If he were well read, the unemployed European worker might detect an Anglo-Saxon drift to our questions and tell us that we are thinking incorrectly about unemployed European workers. He would remind us that in the Europe of the employment-lotteries representative family model, individual workers are not isolated decision makers who are left to protect themselves as best they can against the random shocks that economic life throws at them.[11] Perhaps, to our surprise, he would credit this outcome to *private* financial markets – the complete contingent claims markets that enable sharing the labor income risk associated with the employment lotteries – not to the activities of his government. Seen from a private perspective, government-provided UI might seem to enrich the collective of workers who participate in the employment lotteries, but from a social perspective, the associated adverse incentive effects impoverish the economy. Even modest amounts of government-provided UI threaten to send the economy into an abyss.

The adverse welfare state dynamics in the employment lotteries representative family model are driven by that model's high aggregate labor supply elasticity. Prescott (2004) vividly demonstrated the potency of that high labor supply elasticity in his time series analysis of the American-European labor market divide. After observing similar numbers in the early 1970s, Prescott attributed the dramatic decline of 20 to 30 percent in hours worked per person of working age in Germany and France in the 1990s to tax increases of 7 and 10 percentage points, respectively.[12] Prescott incorporated neither government-provided UI nor any other welfare benefits. In light of the large equilibrium responses to the tax increases studied by Prescott, think about what would happen if we were to introduce European-style social insurance into Prescott's framework: figure 2 shows how economic activity in the representative family model shuts down with generous government supplied UI. Evidently, the high labor supply elasticity in the representative family model implies that generous government welfare programs would lead to the ultimate 'abuse' of such social insurance. Hence, a shortcoming of that model for explaining the European employment experience is that the model predicts far too low a labor supply for realistically calibrated levels of government-provided UI, a theme we pursue in Ljungqvist and Sargent (2006). With realistic levels of government-provided UI, the puzzle for that model becomes, why do Europeans work at all?

[10]If we were to ask him when he had last worked, he might tell us about a job loss and skill transition event that, according to the lottery ticket he had drawn, declared him a leisure-specializing winner of the lottery contingent on the history of shocks corresponding to that event. Recall our discussion of figure 4.

[11]And he might tell us that Chiappori (1992, 1997), and Browning and Chiappori (1998) mistakenly drew the boundaries of their families around nuclear instead of national families.

[12]Ljungqvist (2005) offers a qualification to this statement by showing that two thirds of the predicted employment effects in Prescott's analysis is due to the tax increases while one third is caused by movements in Prescott's estimate of a consumption to output ratio that determines the wealth effect of taxation.

7 Concluding remarks

The employment-lotteries representative family model has chalked up so many successes in applied dynamic macroeconomics that many researchers appropriately regard it as *the* bench mark model against which alternative models should be compared. In this paper and its companion (Ljungqvist and Sargent (2007c)), we have taken seriously both the labor-market-friction-free employment-lotteries representative family model and competing matching and search-island models with frictions by using them to understand how labor market outcomes differ across continents that have very different policies with respect to UI and EP, and how these policies influence the responses of unemployment to increased turbulence. Our inquiry serves both as a robustness check of the theory of the European unemployment experience advocated by Ljungqvist and Sargent (2007b), and as a vehicle for highlighting economic mechanisms at work in different frameworks.

7.1 No frictional unemployment

In the representative family lotteries model, people who are randomly selected to work face none of the matching or search frictions present in the matching and search-island models of Ljungqvist and Sargent (2007c). Everyone selected to work is immediately hired. The representative family chooses to allocate leisure among workers in different skill and benefit entitlement categories in a way that is privately efficient by sending workers who are entitled to the best deals from the UI benefit agency to specialize in leisure.

The employment effects of turbulence in the laissez-faire economy in panel b of figure 4 are reminders of the fact that the representative family model with employment lotteries has no frictional unemployment. Even though our preference specification is the same as that in many real business cycle analyses such as Hansen (1985), we have evidently lost the standard result that changes in productivity do not affect the steady-state employment-to-population ratio. The reason is that we have extended the notion of productivity beyond the standard multiplicative productivity level in the aggregate production function to entail a skill technology in which work experience can enhance individual workers' productivities. This technology introduces new intertemporal considerations into the representative family's labor supply decision that prevent substitution and wealth effects in response to steady-state changes in the productivity of that technology from canceling each other. This makes possible the outcome that unemployment vanishes in our representative family model when turbulence increases, something that is not possible in our matching and search-island models because these models embody frictional unemployment when new workers enter the labor market and old workers change jobs.

7.2 The high labor supply elasticity

The strong response of the representative family model to UI stems from a high labor supply elasticity, a feature that Hansen (1985) and Rogerson (1988) invoked to allow their models to explain business cycle fluctuations. A high labor supply elasticity that works so well to explain big labor supply fluctuations over a business cycle yields unrealistically large equilibrium responses to the replacement rate in the UI system, a finding that probably explains why researchers have refrained from incorporating UI benefits of European generosity into the representative family employment lotteries model.

The employment lotteries and complete markets are not the key ingredients that make it difficult to reconcile this structure with those generous European UI benefits. Rather, it is the high labor supply elasticity implied by the high disutility of work in the preference specification. To explore this possibility, Ljungqvist and Sargent (2006) extend the analysis in this paper to a model that retains a high disutility of labor and employment nonconvexity but lacks employment lotteries and complete markets. Long-lived workers cope with the indivisibility of labor by time-averaging between working and not working while smoothing consumption over time by borrowing and lending a single risk-free asset, in the style of Jones (1988) and Mulligan (2001). We find that this environment gives rise to large aggregate employment responses to taxes similar to those reported by Prescott (2002, 2004) and Rogerson (2007), so that so far as aggregate outcomes are concerned, time-averaging of employment is a good substitute for employment lotteries and complete markets in confronting the nonconvexity in the labor supply choice. As mentioned in footnote 2, under both employment lotteries with complete markets and the time-averaging arrangements, the assumptions of indivisible labor and a high disutility of working are the important ingredients for generating a high aggregate labor supply elasticity. Despite these similar aggregate outcomes, in Ljungqvist and Sargent (2007a) we point out that there are interesting differences in the identities of the nonemployed workers under complete markets and employment lotteries, on the one hand, and time-averaging and incomplete markets, on the other hand. There, we also present reasons to believe building in that high disutility of labor makes the complete markets employment lotteries model and the time-averaging incomplete markets model *both* become inconsistent with observed nonemployment outcomes in Europe.

7.3 Heterogeneous workers

A central feature of our analysis here and in Ljungqvist and Sargent (2007b,c) is that workers are *ex post* heterogeneous in skills and benefit entitlements. The workers in the search-island model can be indifferent to the fortunes of others because they are linked to them neither through labor market congestion externalities nor through complete risk-sharing arrangements. But the distribution of heterogeneous agents has direct consequences for individual workers in the matching model, through the impact of the composition of unemployed workers on the arguments of a matching function, and, in the representative family model, because their labor supplies and consumption rates are regulated in economy-wide insurance arrangements. We think that further explorations of worker heterogeneity promise to help us discriminate among alternative macro models of the labor market and to understand employment and unemployment.

A Computation of equilibrium

A.1 Permissible benefit policies

We assume benefit policies that are not so generous that they would induce families to accumulate skills simply to furlough high-skilled workers into unemployment and then forgo earning wages in order to acquire benefits from the government. This assumption implicitly generates a restriction on benefit policies that can be derived by taking a steady state in which the family initially enjoys leisure by keeping some of its low-skilled workers unemployed, then asking how the family's wealth would change were it to send an unemployed low-skilled family member to work with the intention of furloughing him into unemployment after he has attained the higher skill level. We impose that during the skill accumulation phase for that worker, the family keeps its leisure unchanged by temporarily furloughing an already high-skilled worker into unemployment. This strategy gives rise to stochastic streams of costs during the worker's skill accumulation phase and payoffs after the worker has attained the higher skill level. These can be exchanged for their expected present values evaluated at a stationary interest rate equal to $(1+i) = \beta^{-1}$. For notational simplicity, let $\hat{w}$ denote the after-tax wage rate per unit of skill, i.e., $\hat{w} = (1-\tau)w$.

During the accumulation phase, when the low-skilled worker replaces the high-skilled worker in the labor market, the family gains an amount $(1-\eta)\hat{w}$ per period from sending the low-skilled worker to work but loses an amount $(1-\eta)(1+H)\hat{w}$ from furloughing the high-skilled worker into unemployment. Thus, the impact on the family's disposable income during the accumulation phase is $-(1-\eta)H\hat{w}$ per period. This loss continues for another period with probability $(1-\pi^u)$, i.e., so long as the low-skilled worker does not experience an upgrade in skills.[13] But with probability π^u, the low-skilled worker attains the higher skill level. When that happens, the family sends the originally high-skilled worker back to work and furloughs the originally low-skilled but newly high-skilled worker into unemployment. That originally low-skilled worker is now entitled to benefits that exceed his earlier benefit level by $\eta H\hat{w}$. The family keeps this stream of a higher disposable income until the worker with the newly upgraded skill level retires.

Let κ_0^H be the capitalized value of this whole strategy on its inception, and let κ_H^H be the capitalized value of the higher benefit stream at the time when the low-skilled worker gains the higher skill level and is furloughed into unemployment. These capitalized values satisfy

$$\kappa_0^H = -(1-\eta)H\hat{w} + \beta\left[\pi^u\kappa_H^H + (1-\pi^u)\kappa_0^H\right], \tag{8}$$

$$\kappa_H^H = \eta H\hat{w} + \beta(1-\rho)\kappa_H^H. \tag{9}$$

After solving for κ_H^H from equation (9) and substituting into equation (8), we can solve for

[13] The retirement probability ρ does not enter these calculations, because if either the low-skilled or the high-skilled worker retires while the strategy is being executed, the family will just replace that worker with another person from his skill category.

the capitalized value associated with this strategy,

$$\kappa_0^H = \frac{-(1-\eta) + \dfrac{\beta\pi^u\eta}{1-\beta(1-\rho)}}{1-\beta(1-\pi^u)} \, H\hat{w}. \tag{10}$$

We require that a permissible benefit policy make this strategy unprofitable, so that $\kappa_0^H \leq 0$. This implies that

$$\beta\pi^u\eta \leq [1-\beta(1-\rho)](1-\eta). \tag{11}$$

This condition implies an upper bound on the generosity of the replacement rate η. Alternatively, for a given replacement rate η, expression (11) states that the probability π^u of experiencing an upgrade and the subjective discount factor β together must be sufficiently low that it is not worthwhile to accumulate skills just in order to collect benefits at the higher skill level. Thus, we set benefit levels so that it is in the representative family's interest to reap the returns from any skill accumulation that come from working.

A.2 Steady-state employment and population dynamics

We study an economy in a stochastic steady state. A representative family runs the household sector. In a steady state, the family's optimal policy is characterized by two flow rates into unemployment: a fraction e_0 of newborns that enter life-time unemployment, and a fraction $e_\triangle$ of all laid off workers with skill losses who enter unemployment for the rest of their lives.[14] When the benefit policy satisfies restriction (11), there is no unemployment among high-skilled workers.

At time t, let R_t be the fraction of a family's members who are retired. The remaining working-age members are divided into four categories. Let U_{0t}, $U_{\triangle t}$, N_{0t}, and N_{Ht} be the fractions of a family's members who are unemployed from birth, unemployed after suffering skill loss, employed with low skills, and employed with high skills, respectively. These fractions satisfy

$$R_t + U_{0t} + U_{\triangle t} + N_{0t} + N_{Ht} = 1. \tag{12}$$

For given flow rates $(e_0, e_\triangle)$, the laws of motion are

$$R_t = (1-\sigma)R_{t-1} + \rho\Big[U_{0t-1} + U_{\triangle t-1} + N_{0t-1} + N_{Ht-1}\Big], \tag{13}$$

$$U_{0t} = (1-\rho)U_{0t-1} + e_0\sigma R_{t-1}, \tag{14}$$

$$U_{\triangle t} = (1-\rho)\Big[U_{\triangle t-1} + \pi^o\pi^d e_\triangle N_{Ht-1}\Big], \tag{15}$$

$$N_{0t} = (1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]N_{0t-1} + \pi^o\pi^d(1-e_\triangle)N_{Ht-1}\Big\} + (1-e_0)\sigma R_{t-1}, \tag{16}$$

$$N_{Ht} = (1-\rho)\Big\{\Big[1-\pi^o\pi^d\Big]N_{Ht-1} + (1-\pi^o)\pi^u N_{0t-1}\Big\}. \tag{17}$$

[14] These flow rates into unemployment correspond to one particular design of the employment lottery, but there are many other designs that implement the same steady-state aggregate labor allocation and yield the same expected utility to workers. See section A.7.

We can use equations (12) and (13) to solve for the stationary fraction of retired members

$$R = \frac{\rho}{\sigma + \rho}, \tag{18}$$

which can be substituted into equation (14) to obtain the stationary fraction of family members who have been unemployed since birth

$$U_0 = \frac{e_0\,\sigma}{\sigma + \rho}. \tag{19}$$

To compute the stationary labor allocation, we start with equation (17) and express N_H in terms of N_0,

$$N_H = \frac{(1-\rho)(1-\pi^o)\pi^u}{1-(1-\rho)(1-\pi^o\pi^d)} N_0, \tag{20}$$

which can be substituted together with equation (18) into equation (16) and then solved for

$$N_0 = \frac{\Big[1-(1-\rho)(1-\pi^o\pi^d)\Big](1-e_0)\sigma\rho}{\chi^e(\sigma+\rho)}, \tag{21}$$

where

$$\chi^e \equiv 1-(1-\rho)\Big\{1+\rho\Big[1-\pi^o\pi^d-(1-\pi^o)\pi^u\Big]-(1-\rho)\pi^o\pi^d e_\triangle(1-\pi^o)\pi^u\Big\} > 0; \tag{22}$$

χ^e is strictly positive since

$$\chi^e \geq 1-(1-\rho)\Big\{1+\rho\Big[1-\pi^o\pi^d-(1-\pi^o)\pi^u\Big]\Big\} \geq 1-(1-\rho)(1+\rho) = \rho^2 > 0.$$

By using equations (20) and (21), we can solve for $U_\triangle$ from equation (15),

$$U_\triangle = \frac{(1-\rho)^2\pi^o\pi^d e_\triangle(1-\pi^o)\pi^u(1-e_0)\sigma}{\chi^e(\sigma+\rho)}. \tag{23}$$

It is interesting to note that the skill composition of employed workers is a function only of exogenous parameters and does not depend on the choice of flow rates $(e_0, e_\triangle)$. Use equation (20) to compute

$$\phi_N = \frac{N_H}{N_0+N_H} = \frac{(1-\rho)(1-\pi^o)\pi^u}{\rho+(1-\rho)\pi^o\pi^d+(1-\rho)(1-\pi^o)\pi^u} \in (0,1). \tag{24}$$

A.3 A perturbation of employment

Before turning to equilibrium labor dynamics in a steady state, we examine two perturbations from a steady-state labor allocation. We will use these perturbations to compute a steady state.

Suppose that the steady state is such that the representative family has a positive measure of unemployed workers who have suffered a skill loss. We can then ask: how would the family's wealth change if the set of unemployed workers who have suffered skill loss is permanently reduced by one worker? That is, the family considers sending one such worker to the labor market and, when he retires, replacing him with another unemployed worker who has suffered skill loss. Such a succession of workers will give rise to a stochastic stream of labor income that the family can immediately exchange for the expected present value of the stream discounted at the stationary interest rate $(1+i) = \beta^{-1}$.

Let $\kappa_0^{\triangle}$ be the capitalized value of the labor income associated with this strategy of reducing unemployment among workers who have suffered skill loss. Let $\kappa_h^{\triangle}$ be the capitalized value of the stream of labor income at a future time when this worker (or one of his successors) has attained high skills. These capitalized values satisfy

$$\kappa_0^{\triangle} = \hat{w} + \beta(1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]\kappa_0^{\triangle} + (1-\pi^o)\pi^u\kappa_H^{\triangle}\Big\} + \beta\rho\kappa_0^{\triangle}, \tag{25}$$

$$\kappa_H^{\triangle} = (1+H)\hat{w} + \beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^{\triangle} + \pi^o\pi^d\kappa_0^{\triangle}\Big\} + \beta\rho\kappa_0^{\triangle}, \tag{26}$$

where $\hat{w}$ is the market-clearing after-tax wage rate.

We can use equation (26) to solve for $\kappa_H^{\triangle}$,

$$\kappa_H^{\triangle} = \frac{(1+H)\hat{w} + \beta\Big[(1-\rho)\pi^o\pi^d + \rho\Big]\kappa_0^{\triangle}}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{27}$$

which can be substituted into equation (25),

$$\kappa_0^{\triangle} = \frac{1-\beta(1-\rho)(1-\pi^o\pi^d) + \beta(1-\rho)(1-\pi^o)\pi^u(1+H)}{\chi^0}\,\hat{w} > 0, \tag{28}$$

where

$$\begin{aligned}\chi^0 &\equiv \Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u\Big]-\beta\rho\Big\}\\ &\qquad -\beta(1-\rho)(1-\pi^o)\pi^u\beta\Big[(1-\rho)\pi^o\pi^d+\rho\Big]\\ &= (1-\beta)\Big\{1-\beta(1-\rho)\Big[1-(1-\pi^o)\pi^u-\pi^o\pi^d\Big]\Big\} > 0.\end{aligned} \tag{29}$$

A.4 A second perturbation of employment

Suppose that in the steady state that the representative family has a positive measure of unemployed workers who have never been employed. We ask: how would the family's wealth change if the set of unemployed workers who have never worked is permanently reduced by one worker? That is, the family considers sending one such worker to the labor market and, when he retires, replacing him with an unemployed worker who has never worked. This gives rise to a stochastic stream of labor income that the family can immediately exchange for the present value of the stream's expected value discounted at the stationary interest rate $(1+i) = \beta^{-1}$.

We add a twist to this strategy. Whenever the worker (or one of his successors) has become high-skilled and then loses those skills after an exogenous layoff, the strategy furloughs the worker into unemployment indefinitely and replaces him in the work force with another unemployed family member who has never worked. This switch of workers yields a gain to the family because a stream of low unemployment benefits becomes a stream of high unemployment benefits. The uncertainty associated with retirement makes the gain of benefits stochastic, but the associated stochastic stream of gains can be sold immediately for its expected present value, as given by κ_H^H in expression (9), so that $\kappa_H^H = \frac{\eta H \hat{w}}{1-\beta(1-\rho)}$.[15]

Let κ_0^0 be the capitalized value of the labor income associated with this strategy of reducing unemployment among the workers who have never been employed. Moreover, let κ_H^0 be the capitalized value of the stream of labor income at a future point in time when this worker (or one of his successors) has attained the high skill level. These capitalized values satisfy

$$\kappa_0^0 = \hat{w} + \beta(1-\rho)\Big\{\Big[1-(1-\pi^o)\pi^u\Big]\kappa_0^0 + (1-\pi^o)\pi^u\kappa_H^0\Big\} + \beta\rho\kappa_0^0, \tag{30}$$

$$\kappa_H^0 = (1+H)\hat{w} + \beta(1-\rho)\Big\{(1-\pi^o\pi^d)\kappa_H^0 + \pi^o\pi^d(\kappa_0^0+\kappa_H^H)\Big\} + \beta\rho\kappa_0^0. \tag{31}$$

Equation (31) implies that

$$\kappa_H^0 = \frac{(1+H)\hat{w} + \beta(1-\rho)\pi^o\pi^d\kappa_H^H + \beta\Big[(1-\rho)\pi^o\pi^d + \rho\Big]\kappa_0^0}{1-\beta(1-\rho)(1-\pi^o\pi^d)}, \tag{32}$$

which can be substituted into equation (30),

$$\begin{aligned}\kappa_0^0 &= \frac{\Big[1-\beta(1-\rho)(1-\pi^o\pi^d)\Big]\hat{w} + \beta(1-\rho)(1-\pi^o)\pi^u\Big[(1+H)\hat{w} + \beta(1-\rho)\pi^o\pi^d\kappa_H^H\Big]}{\chi^0} \\ &= \kappa_0^{\triangle} + \frac{\beta^2(1-\rho)^2(1-\pi^o)\pi^o\pi^u\pi^d\eta H\hat{w}}{\Big[1-\beta(1-\rho)\Big]\chi^0},\end{aligned} \tag{33}$$

where $\kappa_0^{\triangle}$ and χ^0 are given by equation (28) and (29), respectively.

A.5 Steady-state consumption

The representative family takes wages and interest rates as given. Since the utility function is additively separable in consumption and leisure, it is optimal for the family to assign equal consumption to each of its members. In a steady state with constant consumption,

[15] Recall that newborn workers are also entitled to the lower benefit level $\eta\hat{w}$.

the stationary interest rate must equal $1 + i = \beta^{-1}$ and the family must be content to hold a constant level of wealth in the form of physical capital and claims to firms.

Given the family's optimal labor decisions encoded in flow rates into unemployment $(e_0, e_\triangle)$, the representative family has fractions N_0 and N_H of its members employed with low skills and high skills, respectively, as determined by equations (20) and (21). The stationary production of consumption goods per worker c implies per-capita consumption

$$\bar{c} \;=\; \bar{n}c \tag{34}$$

where $\bar{n}$ is the fraction of employed workers among all members of the family,

$$\bar{n} \;=\; N_0 + N_H = \frac{\Big\{1 - (1-\rho)\Big[(1-\pi^o\pi^d) - (1-\pi^o)\pi^u\Big]\Big\}(1-e_0)\rho\sigma}{\chi^e(\sigma+\rho)}. \tag{35}$$

The representative family's utility in a steady state is

$$\int_0^1 \sum_{t=0}^{\infty} \beta^t u(c_t^j, n_t^j)dj \;=\; \frac{\log(\bar{c}) - \bar{n}B}{1-\beta}. \tag{36}$$

A.6 Steady-state labor dynamics

It remains to describe optimal labor decisions in a steady state. When the benefit policy satisfies restriction (11), all high-skilled workers will be employed in a steady state. Unemployment will be confined to workers who currently have low skills. There are two possibilities concerning steady-state outcomes:

1. $e_0 = 0$ and $e_\triangle \in [0, 1]$;
2. $e_0 \in (0, 1]$ and $e_\triangle = 1$.

If there is any unemployment among low-skilled workers with low benefits, all high-skilled workers who suffer skill losses must flow into unemployment, i.e., $e_\triangle = 1$. If that were not true, the family would be better off working a low-skilled worker with low benefits instead of a laid off high-skilled worker who has just suffered a skill loss. Both workers are equally productive, but the latter is entitled to higher unemployment compensation. Hence, the steady-state labor dynamics must fall into either class 1 or 2.

What is the optimal setting of the two flow rates into unemployment, $(e_0, e_\triangle)$? To check whether a candidate pair of flow rates constitutes a steady state, we consider the welfare effects of the perturbations to employment that we described above. If the candidate $(e_0, e_\triangle)$ falls into class 1, we examine the first type of perturbation in which the set of unemployed workers who have suffered skill loss is permanently reduced by one worker. That increases the family's labor income by a capitalized value equal to $\kappa_0^\triangle$. In a steady state with equilibrium gross interest rate β^{-1}, it would be optimal for the family to convert this capitalized value into an annuity flow of $(1-\beta)\kappa_0^\triangle$ and permanently to increase consumption by that amount.

The utility derived from this extra flow of consumption should be compared to the loss of benefits $\eta(1+H)\hat{w}$ and the loss of leisure. The condition for an interior optimum is

$$u_c(\bar{c},\,\bar{n})\Big[(1-\beta)\kappa_0^{\triangle} - \eta(1+H)\hat{w}\Big] + u_n(\bar{c},\,\bar{n}) = 0, \tag{37}$$

where the marginal utilities of consumption and leisure are evaluated at the candidate steady-state allocation. Given our particular utility function, which is additively separable in the logarithm of consumption and a linear disutility term for labor, the condition for an interior optimum in class 1 becomes

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^{\triangle} - \eta(1+H)\hat{w}\Big] = B. \tag{38}$$

If the candidate $(e_0, e_\triangle)$ falls into class 2, we consider the second type of perturbation in which the set of unemployed workers who have never worked is permanently reduced by one worker. Analyzing this perturbation leads to the following condition for an interior optimum:

$$u_c(\bar{c},\,\bar{n})\Big[(1-\beta)\kappa_0^{0} - \eta\hat{w}\Big] + u_n(\bar{c},\,\bar{n}) = 0, \tag{39}$$

which with our preference specification implies

$$\frac{1}{\bar{c}}\Big[(1-\beta)\kappa_0^{0} - \eta\hat{w}\Big] = B. \tag{40}$$

A.7 Employment lotteries

Although the aggregate allocation of labor is unique in an employment lotteries model, many possible lottery designs that randomly assign different tasks to individual workers can implement that allocation and yield the same expected utility to workers. In real business cycle models like the one of Hansen (1985), the identical workers could be randomizing over an arbitrary number of periods of working and leisure, possibly contingent on the phase of the business cycle. Alternatively, at the beginning of each period, there could just be an employment lottery for that period's labor supply. In our model with *ex post* heterogeneous workers, there are two kinds of multiplicity in the design of lotteries. First, the optimal lottery design is not unique with respect to the identity of low-skilled unemployed workers who are entitled to low benefits. For example, the workers would be indifferent between the device proposed above of randomly furloughing newborn workers into life-time unemployment and other devices that repeatedly randomize employment status among low-skilled workers who are entitled to low benefits. So long as the devices result in identical aggregate employment outcomes, workers would derive the same *ex ante* expected life-time utility. Second, there is nonuniqueness with respect to the identity of unemployed workers with skill losses whenever the optimal allocation requires some of these people to work. For example, workers would be indifferent between the above device of randomly furloughing a fraction of laid off workers with skill losses into unemployment for the rest of their lives and alternative devices with higher inflow rates but correspondingly shorter unemployment spells among laid off workers who experience skill losses.

The equilibrium conditions do restrict the multiplicity of lottery designs in one important respect. Since the representative family faces no frictions in the labor market and there is a single wage rate per unit of skill, the *family* is indifferent between, on the one hand, lotteries that include only the newly born and laid off old workers and, on the other hand, lotteries that include all working-age members and that entail furloughing some lottery winners who are employed into leisure. But *firms* are not indifferent to these alternative lottery designs because the latter would result in extra layoff taxes and the loss of firms' prior investments in job creation, at least if we assume that productivity processes are lost whenever there are worker separations, as we have assumed in the case of retirements. It follows that steady-state lottery designs cannot include employed workers because otherwise firms would have the incentive to offer 'back-loaded' wage payments – making the representative family strictly prefer to assign leisure to the new born and laid off old workers, rather than to furlough employed workers into leisure.

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