

Roles of Rational Expectations

Consistency, Inference, and Policy in Economic Models

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Abstract

Rational expectations is a property of a model, not of real people. A rational expectations equilibrium requires the subjective statistical models of agents inside a model to equal the objective statistical model that their decisions generate. I describe that requirement as a fixed point of a mapping from subjective to objective models and trace its roles throughout modern economics. The resulting communism of models disciplines beliefs, supplies cross-equation restrictions that help identify structural parameters, turns models into machines that compute how probability distributions of outcomes respond to policies, and settles whose probabilities welfare comparisons should use. Often under other names, the restriction pervades general equilibrium theory, macroeconomics, finance, game theory, industrial organization, labor economics, public finance, and international economics, and laboratory experiments test its predictions. Equilibria in which agents share but distrust an approximating model keep a communism of approximating models and add ambiguity and misspecification components to equilibrium prices. The restriction strains for rare and off-path events, for unprecedented policies, and in models whose states include large cross-section distributions. Agents in such models need to forecast only the prices that enter their decision problems. An accounting of what disciplined alternatives keep and lose shows that discipline on beliefs, identification, and welfare comparisons are most at risk. I point to fuller treatments of those alternatives.

Keywords: Rational expectations; cross-equation restrictions; robustness; ambiguity; misspecification; uncertainty prices; Kullback–Leibler divergence; restricted perceptions; self-

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confirming equilibrium; heterogeneous agent models; forward guidance; fiscal theory of the price level; least squares learning.

1 Introduction

An economic model in which people act on forecasts has to describe two things: how forecasts affect decisions, and how decisions affect the outcomes being forecast. Specifying only the first leaves part of the model open. Rational expectations closes that part by requiring the forecasts that agents use to be consistent with the probability law generated by their decisions and the model's other assumptions.

This paper is about that restriction, which economists impose on their models, sometimes without naming it. Muth (1961) proposed that the subjective probability distributions that agents inside a model use to forecast should equal the objective probability distributions that the model itself generates. Lucas (1987) called that proposal a consistency axiom for economic models. Over six decades, the restriction has become the default in applications of general equilibrium, macroeconomics, finance, game theory, industrial organization, labor economics, public finance, and international economics, and in the econometric methods that estimate models in all of those fields. Experimental economists test its predictions in laboratory markets. I describe what the restriction is, what it buys, and how thoroughly it is used.

To make Lucas's consistency axiom precise, I organize the paper around a mapping T from the subjective statistical models of agents inside an equilibrium Markov model to the objective statistical model that their decisions generate. A rational expectations equilibrium is a fixed point of T at which subjective and objective models coincide. I call that a communism of models. Agents share a joint probability distribution but need not share information (section 5). Each agent needs to forecast only the variables that enter his or her own decision problem (section 11). The communism of models disciplines beliefs by eliminating free parameters that would otherwise describe them. It supplies cross-equation restrictions that help identify structural parameters. It lets a model builder compute how people inside a model respond to proposed policies, and it opens doors to analyzing time inconsistency, credibility, reputations, and information aggregation. It settles whose probabilities welfare comparisons should use. Section 13 collects these and other roles.

Current challenges make this a good time to take stock. Survey data on expectations display disagreement and predictable forecast errors (Coibion and Gorodnichenko, 2015; Bordalo et al., 2020). In heterogeneous agent models with aggregate risk, rational expectations

requires agents to forecast prices that depend on cross-section distributions, and Moll (2026) recommends replacing rational expectations about those prices. Evaluating such proposals requires an appreciation of what the restriction does and an accounting of what would and would not be lost by abandoning it. Parts I–III provide the appreciation, including where the restriction strains; Part IV provides the accounting. Those proposals prompted this paper, but I intend it as a reference for students and researchers. The paper draws on Sargent (2024) and Sargent (2025).

The paper has four parts. Part I (sections 2–5) describes what rational expectations is: words, statistical models, equilibrium Markov models, and the restriction as a fixed point of T . Part II (sections 6–9) describes what the restriction does in econometrics, across economics, and in policy analysis, and how economists respond when models that impose it fail. Part III (sections 10–13) describes rational expectations equilibria with agents who distrust their models, what the restriction asks of people and four defenses of asking it, and where it strains, and it closes by collecting the restriction’s roles. Part IV (section 14) describes disciplined alternatives and what they keep and lose. Section 15 concludes.

Part I. What rational expectations is

2 Words

Economics is what economists say about the economy. Economists use ordinary words in peculiar ways. Peculiar does not necessarily mean strange or incorrect, but it can. Lucas (1987, p. 13) complained that the phrase “rational expectations” often gets misused:

The term ‘rational expectations’, as Muth used it, refers to a consistency axiom for economic models, so it can be given precise meaning only in the context of specific models. I think this is why attempts to define rational expectations in a model-free way tend to come out either vacuous (‘People do the best they can with the information they have’) or silly (‘People know the true structure of the world they live in’).

The widely used acronym FIRE—“full-information rational expectations”—provides another example of a peculiar use of words. It contains two misleading modifiers and omits a noun. The misleading modifiers are “full” and “information.” Some rational expectations equilibrium models contain agents who have different information sets, i.e., different filtrations (Townsend, 1983; Huo and Pedroni, 2020; Hellwig and Venkateswaran, 2025). Their predictive densities differ. Suppose that z equals 0 or 1 with equal probabilities. An agent who observes

z forecasts z ; an agent who does not forecasts $1/2$. Both forecasts are correct. In such contexts, I do not know what “full” means.¹ The missing noun is “equilibrium” or “model.” A *model* either does or does not impose the rational expectations equilibrium restriction (4) to be described below. Real people do not “have” or “not have” rational expectations. Neither, one at a time, do the artificial people who live inside a model. Rational expectations is a property of an equilibrium: a relation between agents’ beliefs and the outcomes that their decisions produce.²

Optimization and correct forecasts are different restrictions: an agent can optimize under a misspecified model. Correct forecasts and full information are different too: a correctly specified model can give an agent little information. Internal consistency and empirical accuracy are different again: an internally consistent model can fit data poorly.

3 Statistical models

I want to use words carefully, so I’ll start with a dictionary. By “economic model” in the above quote from Lucas, I understand that he meant a type of “statistical model” with parameters describing objective functions and constraint sets. A *statistical model* is a joint probability distribution $f(Y | \theta)$ for a vector stochastic process $Y = \{y_t\}$, indexed by a vector of parameters θ . A *manifold of statistical models* $\{f(Y | \theta)\}_{\theta \in \Theta}$ emerges as θ sweeps through a set Θ of possible parameter vectors.

Some notation. I write $f(Y | \theta)$ and $f_\theta(Y)$ interchangeably for the same joint density. The first notation reads it as a likelihood, a function of θ for given data. The second treats it as a point in a space of probability distributions. That is how I use it when I compare distributions with KL divergences and information projections (sections 5 and 6). Similarly, agent i ’s subjective model $f_i(Y | \theta_i)$ is also written $f_{i,\theta_i}(Y)$. When Y is an infinite sequence, KL divergences between two processes are to be read as per-period rates, or as divergences between one-step-ahead conditional densities such as $f_\theta(x_{t+1} | x_t)$, either state by state or averaged over the stationary distribution of the Markov state x_t .³

¹Throughout, I use *information* to mean data, i.e., the history or filtration on which a forecast conditions, and *knowledge* to mean knowing the structural parameters θ_s of a model (section 3). A rational expectations econometrician and the agents inside the model that the econometrician estimates can have the same information but different knowledge: the agents know θ_s and the econometrician does not (section 6.2). “Full information” conflates information with knowledge.

²Hansen (2014) distinguishes people inside a model from people outside it when he discusses sources of risk and model uncertainty.

³These readings have limits. A zero per-period rate does not imply equal initial distributions, and an average over a stationary distribution tests one-step conditional densities only where that distribution puts weight. For those reasons, section 5 states the rational expectations restriction as an equality of conditional distributions given each agent’s information.

In saying “economic model,” Lucas was using a long-standing distinction between two types of statistical models. In a *descriptive* model, the parameters and the functional form of the probability distribution are data compression devices. Vector autoregressions and dynamic factor models are descriptive models. Christopher Sims called them “economically uninterpreted” models. In a *structural* model, an economic theorist can interpret the parameters. They describe preferences, technologies, information sets, and government strategies. Sims called such models “fully interpreted.”

Koopmans (1947) interpreted descriptive models as data compression devices. He viewed them as necessary inputs into estimating the parameters of structural models, not as ends in themselves. He illustrated distinct roles of descriptive and structural models with examples from the history of physics: Tycho Brahe and Kepler produced descriptive models, the “Kepler stage”; Newton used them, together with Galileo’s experiments, to construct a structural model, the “Newton stage.”

I denote parameters of a structural model by θ_s and parameters of a descriptive model by θ_d . An equilibrium Markov model with structural parameters θ_s induces an objective statistical model $f(Y | \theta)$ (section 4). Without a subscript, θ denotes the parameters of that objective law of motion. *Objective* means generated by the model. Whether the model’s law also generates actual data is an empirical question. Under rational expectations, θ is a function $\theta(\theta_s)$ of the structural parameters (section 5). Section 6 describes how the Cowles Commission related parameters of a structural model to parameters of a descriptive model.

Three kinds of people appear in this paper (cf. Hansen, 2014). *Agents* live inside a model. The *model builder* designs the model and computes its equilibria. The *econometrician* lives outside the model and uses it to interpret data. One person often plays the last two roles.

4 Equilibrium Markov models

Lucas and Prescott (1971) and Prescott and Mehra (1980) showed how to impose rational expectations in models that share a Markov structure outlined informally by Lucas (1987, sec. I). I describe its components and how they fit together.

A Markov decision problem (MDP) consists of a set of states X , a set of actions U , a controlled transition law $P(x' | x, u)$ that gives the probability that next period’s state is x' when the current state is x and the action is u , a reward function $r(x, u, x')$, and a discount factor $\beta \in [0, 1)$. Its Bellman equation is

$$V(x) = \max_{u \in U} E [r(x, u, x') + \beta V(x') | x, u], \quad (1)$$

where the expectation is taken with respect to $P(x' | x, u)$.

Definition 4.1. *An equilibrium Markov model contains (1) a collection of decision makers, (2) associated MDPs defined over a common state space, and (3) budget and resource constraints that bind decision makers' MDPs together (Lucas, 1987, sec. I; Sargent, 2025). An equilibrium Markov process is the stochastic process of outcomes that such a model generates.*

Each MDP consists of a *subjective* probability distribution represented as a controlled Markov process together with an objective function. An equilibrium concept organizes those parts into a coherent set of MDPs, one for each agent. It need not impose a rational expectations restriction. As section 3 noted, an equilibrium Markov model with structural parameters θ_s induces an *objective* statistical model $f(Y | \theta)$ for a vector stochastic process Y . Each agent $i = 1, \dots, I$ inside the model has a subjective model $f_i(Y | \theta_i)$.⁴ Agents' subjective distributions describe two kinds of outcomes: outcomes determined *outside* the model, i.e., exogenous shock processes; and outcomes determined *inside* the model, i.e., prices and quantities.

Each MDP contains its own theory of cause and effect: the decision maker regards the controlled transition law P as “causal” in the sense that it is invariant across admissible choices of his or her actions. An equilibrium Markov model thus contains as many assumptions about causality as there are decision makers (Sargent, 2025).

Prescott and Mehra (1980) formulated an equilibrium Markov model as a recursive competitive equilibrium. They replaced a search for equilibrium sequences of history-contingent functions with a search for equilibrium decision rules. A decision rule expresses a current action as a function of a small number of state variables. In their model, households form a continuum of measure one. The state of household i is a triple (k_i, K, ζ) : the household's own capital k_i ; aggregate capital $K = \int k_i di$, which also equals capital per household; and a Markov shock ζ . In a symmetric equilibrium, $k_i = K$ for every i . To pose its decision problem, each household takes the law of motion of K as given and must be free to choose $k_i \neq K$. This is the “Big K , little k ” device of Lucas and Prescott (1971). Prescott and Mehra described what a household needs to know:

The consumer does not have a well defined decision problem, until the equilibrium law of motion, $K' = H(K, \zeta)$, and the pricing function, $p(K, \zeta)$, are specified. Knowledge of these elements along with $F(\zeta' | \zeta)$ is sufficient for forecasting future prices (actually forming predictive probability distributions of future prices) and

⁴Notice that while Y is common across agents' models, functional forms, the f_i 's, and their parameters θ_i can differ across agents i .

selection of optimal current actions. (Prescott and Mehra, 1980, p. 1371)⁵

The functions H and p are components of a household’s subjective model. Section 5 describes how the rational expectations equilibrium restrictions determine them.

5 The rational expectations restriction

Muth (1961, p. 316) stated his hypothesis this way:

... expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the ‘objective’ probability distributions of outcomes).

Muth stated his hypothesis for probability distributions but applied it to conditional means in linear models (Lucas and Prescott, 1971, p. 660, n. 4). Lucas and Prescott (1971, p. 664) applied it to entire distributions: they assumed that “firms know the true distribution of prices for all future periods” (section 11). The rest of this section formalizes the hypothesis.

Let $\theta_i \in \Theta_i$ be parameters of agent i ’s subjective model and $\theta \in \Theta$ be parameters of the objective statistical model that the equilibrium Markov model induces. Agents’ subjective distributions shape their decisions; their decisions shape the equilibrium joint distribution. Equilibrium conditions thereby define a mapping

$$\theta = \mathbb{T}(\theta_1, \theta_2, \dots, \theta_I; \theta_s) \quad (2)$$

from agents’ subjective joint distributions to an objective joint distribution. The mapping $\mathbb{T}$ encodes preferences, technologies, information, market clearing, and aggregation, all of which the structural parameters θ_s describe. It also depends on the functional forms of agents’ families $\mathcal{F}_i = \{f_i(\cdot \mid \theta_i)\}_{\theta_i \in \Theta_i}$, which can differ across agents. A more complete notation is $\mathbb{T}_{\mathcal{F}}$, where $\mathcal{F} = (\mathcal{F}_1, \dots, \mathcal{F}_I)$. I suppress θ_s and the subscript $\mathcal{F}$ when that causes no confusion.

The Kullback–Leibler (KL) divergence, or relative entropy, of the objective probability measure relative to agent i ’s subjective measure is

$$KL(f_\theta, f_{i,\theta_i}) = \int \log \left(\frac{f_\theta(Y)}{f_{i,\theta_i}(Y)} \right) f_\theta(Y) dY \geq 0. \quad (3)$$

Its first argument is the measure under which the expectation is taken. KL divergences are positive whenever subjective and objective models differ. For example, KL divergences are

⁵I have replaced Prescott and Mehra’s symbols for aggregate capital, its law of motion, and the shock with K , H , and ζ .

positive in models inhabited by the decision makers of Hansen and Sargent (2001, 2008), who have doubts about their statistical model and make decisions that are partly shaped by those doubts.

Muth (1961) recommended building models in which subjective and objective distributions coincide. When those probability distributions coincide, the θ_i 's on the right side of (2) become endogenous. A rational expectations equilibrium is a fixed point

$$\theta = T(\theta, \theta, \dots, \theta) \quad (4)$$

of the mapping from subjective distributions to an objective distribution. At the fixed point, all KL divergences (3) equal zero. When the fixed point is unique, it expresses θ as a function $\theta = \theta(\theta_s)$ of the structural parameters. The properties of that function are the cross-equation restrictions of rational expectations. When the fixed point is not unique, the model builder must select an equilibrium, as in section 8.1. In 1993 I put it this way: “When an equilibrium is unique, ‘beliefs’ contribute no free parameters to the model. When equilibria are not unique, ... some of the free parameters index beliefs” (Sargent, 1993, p. 13, n. 7).

Rational expectations restricts functional forms as well as parameters. It requires each agent's subjective model to equal the objective model, $f_i(\cdot | \theta_i) = f(\cdot | \theta)$. Without loss of generality, each agent's family can then be taken to be the objective family, $\mathcal{F}_i = \{f(\cdot | \theta)\}_{\theta \in \Theta}$, with $\theta_i = \theta$. Restriction (4) asserts a communism of statistical models: everyone inside the model shares a joint probability distribution with everyone else and with the model builder, but not necessarily the same information set. Agents who observe different signals draw different conditional distributions from the same joint distribution, and each forecast is correct given what its maker observes. Restriction (4) eliminates a huge number of disparate functional forms and free parameters, namely, all of the f_i 's and θ_i 's.

Stated conditionally, the restriction requires

$$f_i(z_{t+1} | \mathcal{I}_{it}; \theta_i) = f(z_{t+1} | \mathcal{I}_{it}; \theta) \quad (5)$$

for every agent i , where z_{t+1} is a future variable that agent i forecasts and $\mathcal{I}_{it}$ is the information that agent i has at t . Equation (5) states that requirement agent by agent; the example in section 2 illustrates it.

Prescott and Mehra (1980) completed their model with rational expectations: “the prices and price distributions on which the economic agents base their consumption-investment-production decisions are exactly the same as those that result as a consequence of their decisions through market clearing” (Prescott and Mehra, 1980, pp. 1370–71). That sentence describes fixed point (4). The functions H and p of section 4 are *outputs* of the equilibrium;

households use them as *inputs* to their MDPs.

The fixed point answers a question that troubled economists before Muth. Morgenstern (1928) argued that economic forecasting is impossible because people react to published forecasts in ways that can falsify them. Grunberg and Modigliani (1954) showed that when outcomes depend continuously on a published forecast and both lie in a bounded interval, there exists a forecast that remains correct after people react to it. A rational expectations equilibrium extends that fixed point from a single published forecast to the entire subjective probability distributions of all agents inside a model.

5.1 An example of $\mathbb{T}$

A stochastic version of the Cagan (1956) model studied by Sargent and Wallace (1973) and Sargent (2024) exhibits $\mathbb{T}$. Let m_t be the log of the money supply, p_t the log of the price level, $\mu_t = m_{t+1} - m_t$ the rate of money creation, $\pi_t = p_{t+1} - p_t$ the rate of inflation, and π_t^* the public's expected rate of inflation between t and $t + 1$. The demand for real balances is

$$m_t - p_t = -\alpha\pi_t^*, \quad \alpha > 0. \quad (6)$$

Money creation is governed by

$$x_{t+1} = Ax_t + C\epsilon_{t+1}, \quad \mu_t = Gx_t, \quad (7)$$

where x_t is an $n \times 1$ vector that carries information about future rates of money creation and ϵ_{t+1} is an i.i.d. shock. To exhibit $\mathbb{T}$, suppose that the public forecasts inflation with a linear rule $\pi_t^* = \tilde{h}x_t$, where the $1 \times n$ vector $\tilde{h}$ parameterizes its subjective model. Equation (6) then implies $p_t = m_t + \alpha\tilde{h}x_t$, so actual inflation is $\pi_t = \mu_t + \alpha\tilde{h}(x_{t+1} - x_t)$. Its conditional expectation under the objective model is

$$E_t\pi_t = \mathbb{T}(\tilde{h}; \theta_s) x_t, \quad \mathbb{T}(\tilde{h}; \theta_s) = G + \alpha\tilde{h}(A - I), \quad (8)$$

where $\theta_s = (\alpha, A, G, C)$. The mapping $\mathbb{T}$ maps the public's subjective forecasting rule $\tilde{h}$ into the objective forecasting rule, a conditional-mean coefficient that summarizes the objective model. Restricting forecasts to rules that are linear in x_t rules out bubbles when x_t has no component that grows at the rate $(1 + \alpha)/\alpha$ at which a bubble grows. Rational expectations imposes the fixed point $h = \mathbb{T}(h)$, whose solution is

$$h : \quad \pi_t^* = hx_t, \quad h(\theta_s) = \frac{1}{1 + \alpha} G \left[I - \left(\frac{\alpha}{1 + \alpha} \right) A \right]^{-1}. \quad (9)$$

When $(1 + \alpha)I - \alpha A$ is invertible, the fixed point is unique among linear rules. When the eigenvalues of $\alpha A/(1 + \alpha)$ lie inside the unit circle, solving (6) forward and ruling out bubbles gives the same answer,

$$\pi_t^* = \frac{1}{1 + \alpha} \sum_{j=0}^{\infty} \left(\frac{\alpha}{1 + \alpha} \right)^j E_t \mu_{t+j}.$$

Equation (9) exhibits the hallmark cross-equation restrictions of rational expectations: the parameters (A, G) of the money creation process (7) appear in the rule h that describes expectations. A change in (A, G) changes h . With a scalar state and $A = \rho$, expected inflation is $\pi_t^* = \mu_t/(1 + \alpha - \alpha\rho)$, so more persistent money creation makes expected inflation respond more to current money creation. Lucas (1976) and Lucas and Sargent (1981) said that econometric practice should impose these restrictions.⁶

5.2 Several agents

The Cagan example has a single representative forecaster. The communism of models matters most when there are several. Consider a version of Muth's (1961) market with two groups of suppliers, with all variables measured as deviations from their means. Demand is $D_t = -ap_t$ and supply is $S_t = b[\omega p_{1t}^e + (1 - \omega)p_{2t}^e] + u_t$, where p_t is the price, p_{it}^e is the price that group i expects, at $t - 1$, to prevail at t , $\omega \in (0, 1)$ is the share of group 1, $a, b > 0$, and the supply shock obeys $u_t = \rho u_{t-1} + \varepsilon_t$ with ε_t i.i.d. Suppose that group i forecasts with a linear rule $p_{it}^e = h_i u_{t-1}$. Market clearing, $D_t = S_t$, then implies that the objective conditional expectation of the price is $E_{t-1} p_t = T(h_1, h_2; \theta_s) u_{t-1}$, where

$$T(h_1, h_2; \theta_s) = -\frac{b[\omega h_1 + (1 - \omega)h_2] + \rho}{a}, \quad \theta_s = (a, b, \omega, \rho). \quad (10)$$

Rational expectations imposes $h_1 = h_2 = h = T(h, h; \theta_s)$, whose solution $h(\theta_s) = -\rho/(a + b)$ is Muth's. The share ω drops out. Under the communism of models, how forecasters are distributed across groups does not matter, because there is only one forecasting rule. Suppose instead that group 2 uses a fixed rule h_2 that differs from $h(\theta_s)$, while group 1's forecasts equal the objective conditional expectation given group 2's rule. Then group 1's rule solves $h_1 = T(h_1, h_2; \theta_s)$, so that $h_1 = -[\rho + b(1 - \omega)h_2]/(a + b\omega)$. Now both ω and h_2 matter. The free parameters that the communism of models eliminated return as soon as one group leaves it.

⁶The shock loading C does not appear in h . That is an implication of the Simon (1956)–Theil (1957) certainty equivalence property of linear models.

5.3 Inference from prices

In the Cagan and cobweb examples, agents forecast prices. When some agents have private information, prices also carry information, and an agent who interprets a price needs a model of how it was generated. A one-auction version of Kyle (1985) exhibits $\mathbb{T}$ for that inference problem. An asset pays V . An informed trader observes V and submits the order νV . Noise traders submit the order w . The payoff V and the noise order w are independent, mean-zero normal random variables with standard deviations σ_V and σ_w . A competitive, risk-neutral market maker observes total order flow $q = \nu V + w$ and sets the price equal to the conditional expectation of the payoff. If the market maker conjectures that the informed trader's coefficient is $\tilde{\nu}$, the price is

$$p = E[V \mid q] = \kappa(\tilde{\nu}) q, \quad \kappa(\tilde{\nu}) = \frac{\tilde{\nu} \sigma_V^2}{\tilde{\nu}^2 \sigma_V^2 + \sigma_w^2}. \quad (11)$$

Facing that pricing rule, the informed trader chooses an order x to maximize expected profit $x[V - \kappa(\tilde{\nu})x]$, so the trader's actual coefficient is

$$\nu = \mathbb{T}(\tilde{\nu}) = \frac{1}{2\kappa(\tilde{\nu})} = \frac{\tilde{\nu}^2 \sigma_V^2 + \sigma_w^2}{2\tilde{\nu} \sigma_V^2}. \quad (12)$$

Rational expectations imposes $\nu = \mathbb{T}(\nu)$, whose positive solution is $\nu = \sigma_w / \sigma_V$, so that $\kappa = \sigma_V / (2\sigma_w)$. The market maker's inference rule and the informed trader's decision rule are solved together. How much a price reveals is an output of the fixed point. Section 7.2 describes richer models in which prices aggregate information dispersed across many traders (Radner, 1979; Grossman and Stiglitz, 1980).

5.4 Two ways to build a rational expectations model

Beware of theorists bearing free parameters. (Arthur Goldberger; R. E. Lucas, Jr.; see Sargent, 1993, ch. 7, p. 168)

Lucas and Sargent (1981, pp. xv–xvi) described what is wrong with an alternative that prevailed widely before Muth's idea took hold:

What is “wrong” with [Milton Friedman's adaptive expectations] is not [expressing] forecasts of future variables as distributed lags of current and lagged variables. The future must be forecast on the basis of the past, and it is surely acceptable to simplify things by modeling agents as using linear forecasting rules. . . . The difficulty lies not in postulating forecasts which are linear functions of history

but rather in introducing the coefficients in these linear functions as so many additional “free parameters,” unrestricted by theory.

There are two ways to build a model that equates subjective distributions to an objective distribution. One way is to compute a fixed point of (4), as in sections 5.1, 5.2, and 5.3. Another way is to start with a subjective forecasting rule and reverse engineer an environment for which it is optimal. Muth (1961) did it the first way. A year earlier, Muth (1960) had done it the second way.⁷

Muth (1960) started with Friedman’s (1957) adaptive expectations, i.e., geometric distributed lag, model of how a consumer forecasts permanent income. He reverse engineered a *univariate* stochastic process for income for which Friedman’s formula $(1 - \lambda) \sum_{j=0}^{\infty} \lambda^j y_{t-j}$, with $\lambda \in (0, 1)$, for a consumer’s “permanent income” coincides with linear least squares forecasts. Let income be

$$y_t = y_t^p + \sigma_\eta \eta_t, \quad y_t^p = y_{t-1}^p + \sigma_\varepsilon \varepsilon_t,$$

where $\{\eta_t\}$ and $\{\varepsilon_t\}$ are mutually orthogonal white noises with means of 0 and standard deviations of 1. Then the linear least squares forecast is

$$\hat{E} [y_{t+k} | y^t] = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j y_{t-j}, \quad \forall k \geq 1,$$

where $\lambda \in (0, 1)$ solves $\sigma_\varepsilon^2 / \sigma_\eta^2 = (1 - \lambda)^2 / \lambda$, so that λ falls as the signal-to-noise ratio $\sigma_\varepsilon^2 / \sigma_\eta^2$ rises. With Gaussian shocks, $\hat{E}$ is also the conditional expectation. Furthermore $y_t - y_{t-1} = e_t - \lambda e_{t-1}$ where e_t is the innovation in y_t relative to the history y^{t-1} . Muth’s rationalization restricts the stochastic process for income. Before Lucas (1976), economists had not digested the implications of that finding for econometric practice (Sargent, 2024).⁸

Sargent (1977) performed another reverse engineering exercise in the spirit of Muth (1960), using the Cagan model that section 5.1 solved as a fixed point. He started with Cagan’s

⁷Constantinides and Duffie (1996) did it the second way for asset pricing. They started with a stochastic discount factor process that prices an arbitrary list of observed returns and reverse engineered an incomplete markets rational expectations economy whose equilibrium supports it. The key ingredient is a cross-section variance of idiosyncratic income growth that varies with aggregate consumption growth. Bhandari (2015) reverse engineered a different economy that supports the same stochastic discount factor. That economy has complete markets and heterogeneous beliefs, represented by a panel of likelihood ratio processes. Because agents’ subjective beliefs differ from the objective distribution, it leaves rational expectations. Bhandari (2015) disciplines that departure by mapping beliefs into the cross-section dispersion of return forecasts.

⁸However, Sargent (1971) and Lucas (1972a) had incorporated Muth’s cross-equation restrictions into econometric tests of the natural rate hypothesis.

(1956) adaptive expectations model of how people forecast inflation,

$$\pi_t^* = (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j \pi_{t-1-j}.$$

Because $\pi_t = p_{t+1} - p_t$ is not known at t , the forecast uses inflation rates through $t - 1$. He reverse engineered a *bivariate* stochastic process for money creation and inflation for which Cagan's forecasts coincide with linear least squares forecasts of inflation conditioned on histories of money creation *and* inflation. That puts a restriction on the Granger–Sims causality structure of the joint (μ_t, π_t) process: money creation fails to Granger-cause inflation, while inflation can Granger-cause money creation. That restriction fit the 1922–1923 German hyperinflation well.⁹

5.5 Computing a rational expectations equilibrium

The restriction also connects economics to numerical analysis and to the theory of stochastic approximation. Computing a rational expectations equilibrium means finding a fixed point θ of (4). The mapping T need not be a contraction. That calls for a relaxation algorithm, in which $\mathsf{T}(\theta)$ with one argument abbreviates $\mathsf{T}(\theta, \dots, \theta)$:

$$\theta^{k+1} = \theta^k + \gamma_k [\mathsf{T}(\theta^k) - \theta^k], \quad \gamma_k \in (0, 1]. \quad (13)$$

Iterations (13) are Euler's method for solving the ordinary differential equation (ODE)

$$\frac{d\theta}{d\tau} = \mathsf{T}(\theta) - \theta. \quad (14)$$

Rest points of (14) are fixed points of (4). Stable rest points are the ones that relaxation (13) can find. A rest point is stable when the eigenvalues of the derivative of T at it have real parts less than one. Relaxation cannot find other fixed points. For example, $\mathsf{T}(\theta) = 2\theta + 1$ has the fixed point $\theta = -1$, but each step of (13) multiplies the distance from it by $1 + \gamma_k$. Adding noise to (13) leads to stochastic approximation and quasi-stochastic approximation algorithms for finding a θ that solves (4) (Meyn, 2022). Four questions differ. Does an equilibrium exist? Is it unique? Can an algorithm compute it? Can a learning process find it? A yes to one question does not imply a yes to another. A determinate equilibrium (section 7.1) can be unstable under a particular learning rule. Section 14.3 describes how a literature of the 1980s and 1990s turned perturbations of these iterations into theories of how agents

⁹Lucas and Sargent (1981) reprint that analysis. The joint distribution that emerges from the equilibrium is not invariant to monetary-fiscal interventions that alter the money creation process.

might learn an equilibrium.

Part II. What rational expectations does

6 Rational expectations econometrics

Lucas (1987, p. 5) thought that econometric implications and applications “may well be the most lasting contribution of John F. Muth’s idea of rational expectations.” Rational expectations econometricians are vitally interested in computational economics because they routinely want to solve two interconnected problems (Sargent, 2025). A *direct problem* takes a vector of known structural parameters θ_s and computes an equilibrium Markov process. A solution of the direct problem lets us draw random samples from $f(Y \mid \theta(\theta_s))$. An *inverse problem* takes an observed data set as known and infers unknown structural parameters θ_s .

Rational expectations econometrics assigns different knowledge to two types of people:

- agents *inside* the model know the model, i.e., they know θ_s , which equilibrium is selected, and hence $\theta = \theta(\theta_s)$;
- an econometrician *outside* the model knows the manifold $\{f(Y \mid \theta(\theta_s))\}_{\theta_s \in \Theta_s}$ but not θ_s .

These objects are in play:

- structural parameters $\theta_s \in \Theta_s$;
- hypothetical data $\{y_t\}_{t=0}^N$ generated by sampling from $f(Y \mid \theta(\theta_s))$;
- an estimator $\hat{\theta}_{s,t} = \hat{\theta}_s(y^t)$, where y^t is the history $y_t, y_{t-1}, \dots, y_0$;
- regularity conditions under which $\hat{\theta}_{s,t} \rightarrow \theta_s$ in some sense;
- a parameter-estimation algorithm, for example, a recursion

$$\begin{bmatrix} \hat{\psi}_{t+1} \\ R_{t+1} \end{bmatrix} = \mathcal{G} \left(\begin{bmatrix} \hat{\psi}_t \\ R_t \end{bmatrix}, z_t \right), \quad (15)$$

where $\hat{\psi}_t$ is an estimate of a parameter vector ψ , R_t is an empirical second-moment matrix that co-moves with $\hat{\psi}_t$, and z_t is a vector of observations. The econometrician applies such a recursion with $\psi = \theta_s$. Learning agents inside a model apply it with $\psi = \delta_i$ (section 14.3).

To solve the inverse problem, an econometrician computes, i.e., numerically approximates, an equilibrium for each candidate θ_s and assumes that nature had drawn samples from

it. Estimators include maximum likelihood (Hansen and Sargent, 1980), indirect inference (Gouriéroux et al., 1993), the efficient method of moments (Gallant and Tauchen, 1996), and the generalized method of moments (Hansen, 1982), which section 6.3 shows can also be applied without computing an equilibrium. An econometrician can pretend to be a frequentist and use maximum likelihood. An econometrician can instead pretend to be a Bayesian, put a prior over θ_s , and approximate a posterior.

Maximum likelihood minimizes a sample counterpart of (3). For a family of models $\{g_\delta\}_{\delta \in \Delta}$, $KL(f_\theta, g_\delta) = E \log f_\theta(Y) - E \log g_\delta(Y)$, where E is an expectation under the objective model f_θ , the first argument of (3). Minimizing over δ therefore maximizes the expected log likelihood $E \log g_\delta(Y)$. Maximum likelihood replaces that expectation with a sample average. When the family contains f_θ , estimates converge to the true parameter. When it does not, they converge to the member that minimizes (3), an information projection (section 14.2; White, 1982). Bayesian posteriors concentrate on the same minimizers (Berk, 1966).

6.1 Identification

Let θ_d be parameters of a descriptive model. The Cowles Commission constructed a special descriptive model called the reduced form of a structural model. Population moments implied by the structural model determine θ_d through a function

$$\theta_d = \varphi(\theta_s). \tag{16}$$

Structural parameters θ_s are (globally) identified when φ has a well defined inverse $\theta_s = \varphi^{-1}(\theta_d)$ (Koopmans, 1950; Hood and Koopmans, 1953).¹⁰ Sims (1980) argued that rational expectations makes many Cowles Commission exclusion restrictions incredible. A rational expectations equilibrium restriction replaces them with cross-equation restrictions like (9). Under rational expectations, φ is a composition of two maps: the map $\theta_s \mapsto \theta(\theta_s)$ defined by fixed point (4), followed by a map from the objective model to the descriptive model. That second map delivers the parameters of the Cowles Commission reduced form, or those of an information projection when the descriptive model is misspecified (sections 8.4 and 14.2). Cross-equation restrictions help make φ invertible (Sargent, 2025). In the market of section 5.2, the forecast coefficient $h = -\rho/(a + b)$ pins down the ratio $\rho/(a + b)$. Separating a from b requires other observations.

¹⁰The classic Cowles Commission reduced form differs from the contemporary debased “reduced form” that denotes a descriptive model not tied explicitly to a structural model (Sargent, 2024).

6.2 Who is learning?

Statistical learning is always *about* a parameter vector. A rational expectations econometrician outside the model is learning about θ_s , for example via recursion (15). Agents inside the model are not learning about θ_s . They know it. In rational expectations models whose agents are Bayesian learners, agents know a prior over θ_s and the model that generates data from it (section 14.3).

That asymmetry makes predictive densities differ. An agent inside the model who has observed y^t forecasts with $f(y_{t+1} | y^t; \theta)$. A Bayesian econometrician who has observed the same y^t forecasts with

$$\int f(y_{t+1} | y^t; \theta(\tilde{\theta}_s)) p(\tilde{\theta}_s | y^t) d\tilde{\theta}_s,$$

where $p(\tilde{\theta}_s | y^t)$ is a posterior. Agents inside the model have the same *information* as the econometrician, namely, y^t , but more *knowledge*: they know θ_s . Their predictive density is the true one. The expected KL divergence (3) of the true predictive density relative to the econometrician's is generally positive for $t < +\infty$ and, under regularity conditions, vanishes as $t \rightarrow +\infty$. Defining a “behavioral bias” requires comparing predictive distributions associated with two statistical models. The comparison above shows that the choice of benchmark matters.

6.3 Incomplete rational expectations models

A widespread empirical use of the rational expectations restriction imposes it only partially. A less ambitious agenda learns some parameters without learning all of them. Hansen's (1982) generalized method of moments estimates parameters that appear in Euler equations without specifying complete laws of motion for agents' information variables. An agent who optimizes under subjective probabilities Q satisfies the Euler equation

$$E_t^Q [m_{t+1}(\gamma)R_{t+1} - 1] = 0, \quad m_{t+1}(\gamma) = \beta \frac{u'_\gamma(c_{t+1})}{u'_\gamma(c_t)}, \quad (17)$$

where R_{t+1} is a gross return and γ indexes preferences. Optimization alone delivers (17). Rational expectations replaces the subjective expectation E_t^Q with the objective expectation E_t . The pricing error $m_{t+1}(\gamma)R_{t+1} - 1$ is then orthogonal to every variable z_t in agents' information set:

$$E [z_t (m_{t+1}(\gamma)R_{t+1} - 1)] = 0. \quad (18)$$

Hansen and Singleton (1982) used such conditions to estimate preference parameters without solving for an equilibrium. Semi-parametric efficiency bounds (Hansen and Singleton, 1991)

partition parameters into parameters of interest and nuisance parameters. Hansen and Sargent (1993) and Sims (1993) constructed examples in which seasonal adjustment improves estimates of preference and technology parameters, the parameters of interest, while degrading nuisance parameters that describe the evolution of information variables in agents' uncontrollable subspaces (Sargent, 2025). This agenda is the econometrician's counterpart to robustness (section 10.1). The econometrician learns some parameters without committing to a complete model.

7 What the restriction buys and where it is used

7.1 What the restriction buys

The rational expectations restriction across subjective models

- eliminates free parameters that describe subjective distributions. It converts agents' subjective conditional distributions of prices and quantities from *inputs* into *outputs* of theorizing;
- sets KL divergences (3) between subjective distributions and the objective distribution to zero. That renders the model immune to Blume and Easley's (2006) question "If you're so smart, why aren't you rich?" because, when beliefs are correct, market selection has no differences in beliefs to act on, although it still acts on differences in discount factors (section 11);
- respects and strengthens Harsanyi's (1967) common prior doctrine. Rational expectations implies a common prior. It also requires that common prior to equal the objective distribution. A common prior can be commonly wrong.

The rational expectations communism of models delivers coherence and simplicity. A coherent system of conditional distributions opens doors to framing new problems. Rational expectations equilibria set the stage for modern macroeconomics and applied dynamic modeling. Section 7.2 gives examples from many fields.

Rational expectations coherence makes these tools and analyses applicable:

- recursive representations of equilibria, including nested Bellman equations for Ramsey and credibility problems (section 8.2);
- applications of Wiener–Kolmogorov optimal prediction theory to construct equilibria;¹¹
- analysis of counterfactuals;

¹¹See Whiteman (1983), Kasa (2000), Walker (2007), Nimark (2017), Huo and Pedroni (2020), and Huo and Takayama (2025).

- explicit models of “expectations management” through policy choices;
- models of runs, e.g., Diamond and Dybvig (1983);
- statistical inference about parameters. Stationarity and ergodicity (after suitable scaling) deliver laws of large numbers; cross-equation restrictions like (9) help deliver identification.

In a rational expectations equilibrium, nobody misunderstands the consequences of his or her choices.

Policy evaluation. Rational expectations made macroeconomic models operational. Lucas (1980, pp. 709–10) described the task this way:

Our task, as I see it ... is to write a FORTRAN program that will accept specific economic policy rules as ‘input’ and will generate as ‘output’ statistics describing the operating characteristics of time series we care about, which are predicted to result from these policies.

Rational expectations is what makes such a program possible. It lets the program compute agents’ beliefs from the policy rule instead of asking its user to supply them.

Welfare comparisons. The restriction also settles whose probabilities welfare comparisons should use. Let E^{Q_i} denote expectation under agent i ’s subjective model. Under rational expectations, agent i ’s expected utility $U_i(c)$ from an allocation c satisfies

$$E^{Q_i} [U_i(c) \mid \mathcal{I}_{it}] = E [U_i(c) \mid \mathcal{I}_{it}],$$

so a model builder can rank policies by agents’ own criteria. The equality holds at a given date and information set. Welfare weights and the choice between ex ante and interim comparisons remain for the model builder to specify. When beliefs differ from the objective model, a welfare criterion must take a stand on whose probabilities to use. Brunnermeier et al. (2014) propose judging an allocation inefficient only if it is inefficient under every convex combination of agents’ beliefs.

Determinacy. The restriction also makes determinacy of equilibrium a criterion for policy design. Blanchard and Kahn (1980) gave conditions under which a linear rational expectations model has a unique stable solution. Monetary economists judge interest rate rules partly by whether they deliver a unique, or determinate, rational expectations equilibrium; the Taylor principle, which requires the nominal interest rate to move more than one for one with

inflation, is a condition of that kind (Woodford, 2003). Sections 7.3 and 8.1 describe how the criterion organizes debates about monetary and fiscal policy and how a model builder or a Ramsey planner can rule out bubbles.

Benchmark. Finally, rational expectations equilibria serve as benchmarks for models that do not impose the restriction. Optimism and pessimism are optimism and pessimism *relative to* something. Rational expectations equilibria are the benchmarks against which “behavioral biases” are defined (section 6.2). They are benchmarks for analyzing model misspecification and robustness concerns of agents inside the model, of the model builder, and of the econometrician estimating the model. They supply equilibrium computation algorithms that can be converted into models of non-rational learning (sections 5.5 and 14.3).

7.2 The restriction at work across economics

The rational expectations restriction is not confined to macroeconomics, and it often goes by other names: equilibrium price expectations, a common prior, correct beliefs about other players’ strategies, or knowledge of a transition law. Two distinctions help to organize its uses.¹²

The first distinction is between rational expectations about *exogenous* processes and rational expectations about *equilibrium* objects. An agent who knows the law of motion of an exogenous shock, such as the distribution from which a job searcher draws wage offers, has rational expectations about something that nobody’s beliefs can affect. Imposing that requires no fixed point, because the objective law of motion of an exogenous process does not depend on beliefs. Prices, job-finding rates, and other agents’ strategies are different. Their objective laws of motion depend on beliefs through the mapping T , so rational expectations about them requires a fixed point.

The second distinction is between rational expectations *on* the equilibrium path and *off* it. Some equilibrium concepts require beliefs to be correct only about events that occur in equilibrium. Others also require correct beliefs about what would happen after deviations that never occur. Game theory draws that distinction most sharply.

General equilibrium. Radner (1972) defined an equilibrium of plans, prices, and price expectations in a sequence of markets. It requires that the prices that agents expect in each

¹²Executable versions of several of the models described in this section appear in Sargent and Stachurski (2026b), including lectures titled “Rational Expectations Equilibrium,” “Investment Under Uncertainty,” “Markov Perfect Equilibrium,” “Job Search I: The McCall Search Model,” “Estimating Euler Equations by Generalized Method of Moments,” “Competitive Equilibria with Arrow Securities,” and “Heterogeneous Beliefs and Bubbles.”

future event be the prices that clear markets in that event. Agents need not agree about the probabilities of events, but they must agree about, and be right about, the price that will prevail in each event. Radner (1979) studied rational expectations equilibria in which prices also transmit private information, and Grossman and Stiglitz (1980) showed that when information is costly to acquire, prices cannot transmit it fully. In these models, what agents infer from prices is itself an output of the fixed point (section 5.3).

Macroeconomics. Lucas (1972b) characterized equilibrium prices as functions of the state of the economy and required agents' forecasts to use those functions. The real business cycle models that followed Kydland and Prescott (1982), the New Keynesian models of Woodford (2003), and estimated models such as Smets and Wouters (2007) all impose the restriction. So do heterogeneous agent models. In Aiyagari's (1994) stationary equilibrium, households know the constant interest rate that clears the market. In Krusell and Smith (1998), households facing aggregate risk approximate rational expectations by forecasting a few moments of the cross-section distribution of wealth, in practice its mean, from which they compute prices. Their algorithm fits the coefficients of those forecasting rules by least squares to data that the model generates, so it computes what section 14.2 calls a restricted perceptions equilibrium, which Krusell and Smith found to be close to a rational expectations equilibrium. Heterogeneous agent models such as Achdou et al. (2022) and Kaplan et al. (2018) descend from that work. Policy institutions use the restriction too. The Federal Reserve Board's FRB/US model was designed so that it can be simulated under either VAR-based or model-consistent, that is, rational, expectations (Brayton et al., 1997). Section 7.3 describes several roles of the restriction in current macroeconomic practice.

Consumption and asset pricing. Hall (1978) showed that under rational expectations and the life cycle–permanent income hypothesis, marginal utility is a martingale when the discount factor times the gross interest rate equals one: no variable in consumers' information set other than current marginal utility helps to predict future marginal utility. With quadratic utility, the same statement holds for consumption itself. Lucas (1978) made asset prices functions of the state and priced assets with a stochastic discount factor built from a representative consumer's marginal rates of substitution and the objective law of motion of endowments. Hansen and Singleton (1982) estimated the preference parameters in that stochastic discount factor from orthogonality conditions that rational expectations implies (section 6.3). The expectations theory of the term structure combines rational expectations with the assumption that term premia are constant. It makes long rates averages of expected future short rates plus those premia. Sargent (1979) estimated that theory by maximum likelihood, imposing

its cross-equation restrictions. Campbell and Shiller (1991) found that yield spreads forecast long-run movements in short rates in the direction that the theory predicts, but forecast short-run movements in long rates with the wrong sign. The equity premium and risk-free rate puzzles of section 9 are failures of models that impose the restriction.

Finance and market microstructure. Fama’s (1970) efficient markets hypothesis, that prices fully reflect available information, is a statement about equilibrium beliefs. Testing it requires a model of equilibrium expected returns, because predictable returns can reflect risk premia that vary over time. In Kyle (1985), a market maker sets prices knowing the equilibrium relation between order flow and an insider’s private information (section 5.3). Milgrom and Stokey (1982) showed that when agents share a common prior, start from a Pareto efficient allocation, and it is common knowledge that they are rational, private information alone cannot generate trade. Explaining the large volume of trade in actual markets therefore requires relaxing an assumption of the theorem: that the initial allocation is efficient, which leaves no hedging or liquidity motive to trade; that rationality is common knowledge; or that agents share a common prior. Harrison and Kreps (1978) showed how short-sale constraints and heterogeneity among traders’ statistical models can support speculative trade and asset prices that exceed what any trader would pay to hold the asset forever.

Game theory and mechanism design. A Nash equilibrium (Nash, 1950) can be read as a communism of beliefs about strategies: each player best responds to the strategies that the other players actually use. Conversely, a competitive rational expectations equilibrium with a continuum of agents can be read as a Nash equilibrium of a nonatomic game (Schmeidler, 1973), in which each player’s payoff depends on his or her own actions and on aggregate outcomes, such as prices, that no single player can affect. Lasry and Lions (2007) call such games mean field games and derive their equilibrium conditions as limits of Nash equilibria of games with N players as $N \rightarrow \infty$. Harsanyi (1967) closed games of incomplete information by giving players a common prior over types, and Aumann (1987) showed that correlated equilibrium expresses Bayesian rationality under a common prior. In an extensive-form game, a Nash equilibrium already requires each player to predict correctly how the others would play at information sets that are never reached in equilibrium. Subgame perfection adds that those off-path plans be optimal. Self-confirming equilibrium (Fudenberg and Levine, 1993) relaxes the requirement of correct beliefs off the equilibrium path (section 14.2). In mechanism design, Myerson’s (1981) optimal auction assumes that the seller and every bidder know the distribution from which bidders’ values are drawn.

Industrial organization. In a Markov perfect equilibrium (Maskin and Tirole, 2001), each firm’s beliefs about its rivals’ state-contingent policies equal those policies. Ericson and Pakes (1995) built a framework for empirical work on industry dynamics on that equilibrium concept. Structural estimation in industrial organization implements the communism of models as an estimator. In Rust (1987), a decision maker who knows the transition law of bus mileage chooses when to replace bus engines. The econometrician estimates that transition law and attributes it to the decision maker. The two-step estimators of Hotz and Miller (1993) and Bajari et al. (2007) first estimate transition laws and policy functions from data and then assume that agents use those estimated objects to form their expectations.

Labor economics. A worker in McCall’s (1970) search model knows the distribution of wage offers, an instance of rational expectations about an exogenous process. In Mortensen and Pissarides (1994), the job-finding rate is an equilibrium object that firms’ free entry determines, and workers and firms know it; that requires a fixed point. With aggregate shocks and heterogeneous workers and firms, the distribution of matches determines whom a worker or firm will meet, so the fixed point includes a law of motion for that distribution. Payne, Rebei, and Yang (2026) compute such equilibria globally with deep neural networks. After a shock calibrated to COVID-19, firms in their model foresee a pool of job seekers tilted toward the less skilled and offer jobs to a wider range of workers. Unemployment then falls about 25 percent faster during the recovery than when workers and firms act as if the distribution stays at its long-run average. Keane and Wolpin (1997) estimated a model of career choice in which young men know the distributions of the shocks to their future rewards, the same distributions that the econometricians estimate.

Public finance and policy. Ramsey problems (Lucas and Stokey, 1983), Mirrlees problems (Mirrlees, 1971), time-consistency problems (Kydland and Prescott, 1977; Calvo, 1978), and models of reputation (Chari and Kehoe, 1990; Stokey, 1991) all assume that private agents understand the government’s policy rule. That is what makes a policy rule a private sector forecasting rule (section 8.3).

International economics. In Dornbusch (1976), asset markets clear quickly under rational expectations and goods prices adjust slowly, so the exchange rate overshoots its long-run response to a monetary expansion. In sovereign debt models that descend from Eaton and Gersovitz (1981), such as Arellano (2008), lenders price a government’s bonds using default probabilities implied by the government’s own optimal default policy.

Information and higher-order beliefs. In Townsend’s (1983) model of “forecasting the forecasts of others,” in the beauty contests of Morris and Shin (2002), and in Huo and Pedroni (2020) and Huo and Takayama (2025), agents have different information but share a joint probability distribution (section 2). In models of rational inattention (Sims, 2003, 2006; Turmuhambetova, 2005), variables that interest a decision maker are polluted with measurement errors that the decision maker can shape at a cost. Sims (2003, 2006) constrains or penalizes the Shannon mutual information between the variables that a decision maker cares about and the signals that he or she observes. Turmuhambetova (2005) lets a decision maker choose an additional signal process and penalizes that choice according to the signal’s information content. In a rational expectations equilibrium of such a model, decision makers have different information sets because each shapes his or her information according to his or her own objectives. In Turmuhambetova’s (2005) portfolio choice model, for example, endogenous information induces heterogeneity in consumption and portfolio allocations across agents with different amounts of wealth.

Experimental economics. Rational expectations equilibria supply predictions that laboratory markets can test. Plott and Sunder (1982) found that in experimental security markets with insiders, prices came, with experience, to reveal the insiders’ information, as a rational expectations equilibrium predicts. Plott and Sunder (1988) found that markets aggregated traders’ diverse information and attained rational expectations equilibria when traders could trade a complete set of Arrow–Debreu securities or had identical preferences, but not in a third design. Whether a market attains a rational expectations equilibrium can depend on its institutions and trading instruments.

Two patterns stand out. First, in most of these fields the restriction is imposed without the words “rational expectations.” It enters as part of the definition of equilibrium. Second, empirical work in these fields estimates an objective model and hands it to agents as their subjective model: Hansen and Sargent (1980) in macroeconomics, Rust (1987) and Hotz and Miller (1993) in industrial organization, and Keane and Wolpin (1997) in labor economics. The restriction is a principal device through which economics connects theories of behavior to data.

7.3 Rational expectations in modern macroeconomic models

Several roles of the restriction are on display in current macroeconomic practice.

Expectations management. In New Keynesian models, households and firms respond today to the entire expected future path of interest rates. Rational expectations makes announcements about that path powerful; managing expectations and choosing policies are the same activity (section 8.3). Under rational expectations, a promise to hold the nominal interest rate low for a long time has large effects on current output and inflation, effects that do not shrink as the promised policy recedes into the future. Del Negro et al. (2023) call those implausibly large effects the “forward guidance puzzle.” Proposed resolutions alter other ingredients, such as the completeness of markets (McKay et al., 2016), or alter beliefs, for example by making agents myopic about distant events (Gabaix, 2020) or by limiting their depth of reasoning (Farhi and Werning, 2019).

Determinacy. Determinacy of rational expectations equilibrium (section 7.1) organizes debates about monetary and fiscal policy. Leeper (1991) showed that a unique equilibrium requires one policy to be “active” and the other “passive.” Monetary policy is active when it obeys the Taylor principle; fiscal policy is passive when it adjusts primary surpluses to stabilize government debt. The fiscal theory of the price level (Sims, 1994; Cochrane, 2023) studies equilibria in which fiscal policy is active and the price level adjusts so that the real value of government debt equals the present value of expected primary surpluses. Both versions of the determinacy argument rely on rational expectations of future policies. Changing beliefs changes determinacy conditions: in Gabaix’s (2020) model, equilibrium can be determinate even when monetary policy violates the Taylor principle.

Transitions after unanticipated shocks. The “MIT shock” of section 8.4 has become a computational device. Heterogeneous agent models are often solved by computing perfect foresight paths of prices after an unanticipated aggregate shock. Agents assign probability zero to the shock beforehand and know the entire subsequent path of prices afterward. To first order in aggregate shocks, the resulting impulse responses coincide with those of a rational expectations equilibrium with aggregate risk, so they deliver its linearized law of motion (Boppart et al., 2018; Auclert et al., 2021). Reiter (2009) instead linearized around the stationary cross-section distribution. These methods make rational expectations equilibria of heterogeneous agent models computable for small aggregate shocks; section 12.3 describes global methods for large ones.

Estimation. Bayesian estimation of rational expectations models such as Smets and Wouters (2007) is routine at central banks (Herbst and Schorfheide, 2016; Sargent, 2025). It imposes cross-equation restrictions like those of section 5.1. Auclert et al. (2021) use

sequence-space methods to estimate heterogeneous agent models. Payne et al. (2026) solve the direct problem of section 6 for a whole region of structural parameters at once, by treating the parameters as additional state variables of the master equation (section 12.3), and then estimate their search and matching models by the simulated method of moments. Solving and estimating one of their models takes about three hours.

8 Policy under rational expectations

Counterfactual policies produce outcomes that historical data have never displayed. A rational expectations model of policy must take a stand on what people inside the model know about situations they have not experienced.

8.1 Counterfactuals

A counterfactual policy is one that has not been in place historically. Marschak described the challenge:

The economist’s objectives are similar to those of an engineer but his data are like those of a meteorologist. The economist is often required to estimate the effects of a given (intended or expected) change in the “economic structure”, i.e., in the very mechanism that produced his data. None of these changes can he produce beforehand, as in a laboratory experiment; and since some of the changes envisaged have never happened before, the economist often has to estimate the results of changes he has never observed. (Marschak, 1950, p. 2)

A counterfactual question asks what a *stochastic process* would have been had government policy been different. A rational expectations model tells how agents’ predictive densities depend on government policies; equation (9) is an example. Policy parameters, such as (A, G) in the Cagan model, are components of θ_s , so $h(\theta_s)$ shows how expectations respond to policy. More generally, let v be a policy parameter, a component of θ_s , and write fixed point (4) as $\theta = \bar{T}(\theta; v)$. On a differentiable branch of equilibria,

$$\frac{\partial \theta}{\partial v} = [I - D_\theta \bar{T}]^{-1} D_v \bar{T}, \quad (19)$$

provided that the inverse exists. The term $D_v \bar{T}$ measures how a policy change affects outcomes at fixed beliefs; the inverse adds the feedback from outcomes to beliefs and back. A model with “behaviorally biased” agents must explain how those biases depend on government policies. Models that contain robust decision makers do that (section 10.1).

A model’s responses to new policies come from its controlled laws. Two controlled laws, $y_{t+1} = u_t + \varepsilon_{t+1}$ and $y_{t+1} = 2u_t + \varepsilon_{t+1}$, illustrate the point. If policy has always set $u_t = 0$, the two laws generate the same data and the same correct forecasts. They disagree about setting $u_t = 1$. A rational expectations model resolves the disagreement by assuming a controlled law. Evidence from other environments or from experiments must support that assumption.

“Different” has at least two meanings. A different policy can be *possible*: historically imagined, but never tried. A different policy can be *unimagined*: historically not even contemplated, an “innovation.” Ramsey, Mirrlees, and other optimal policy design problems take a heroic stand: they treat every policy in an admissible set as possible, and they assume that agents understand the consequences of each.

A Ramsey or Mirrlees planner conducts a thorough analysis of all possible counterfactuals. The planner understands agents’ responses to each possible plan. That is expectations management on steroids. The model builder also decides which equilibria to admit. In the Cagan model (6), the model builder rules out inflation paths that grow at the explosive rate $\left(\frac{1+\alpha}{\alpha}\right)^t$. That eliminates bubbles $c\left(\frac{1+\alpha}{\alpha}\right)^t$ that also satisfy the equilibrium difference equation (Sargent, 2024). A Ramsey planner can instead design policies that rule out such bubbles.

8.2 Time inconsistency and credibility

Rational expectations brought “dynamic programming squared” to problems of time inconsistency and credibility. Agents can solve Bellman equations under any subjective model. Rational expectations supplied the equilibrium restriction on the probabilities that enter nested Bellman equations. Examples are Ramsey plans for monetary-fiscal policies (Lucas and Stokey, 1983), repeated moral hazard and enforcement problems, and Mirrlees problems (Mirrlees, 1971) and their dynamic extensions. In these problems a value that emerges from one Bellman equation appears as an argument in another Bellman equation. An agent’s value function, a backward-forward looking object, appears as an argument of a principal’s Bellman equation. Kydland and Prescott (1977) interpreted time inconsistency as rendering dynamic programming inapplicable. Kydland and Prescott (1980) rescinded that claim by redefining the state that confronts a planner (Sargent, 2024). Chari et al. (2020) used a redefined state to reinterpret time inconsistency. Their Ramsey planner in a growth model must deliver households an exogenous value W of initial wealth measured in utility. The Ramsey problem then has a recursive representation with state (k, W) , where k is capital and W is promised wealth in utility terms. Suppose that each period’s government chooses current policies and the value of wealth that its successor must respect, and nothing else. The Ramsey outcome is a Markov equilibrium of that partial commitment environment. Partial commitment to the

value of wealth delivers the full commitment outcome. Partial commitment to next period's tax rates typically does not. The state variable, a promised value, carries the commitment.

8.3 Actions, forecasts, and promises

A communism of models sometimes requires one variable to play multiple roles. In the Calvo–Chang model studied by Sargent (2024), the inflation rate π_t plays three roles: it is the actual rate of inflation; it is the public's expected rate of inflation; and it is a promised rate of inflation chosen by a Ramsey planner at time 0. That model is deterministic. In a stochastic model, the public forecasts a policy action by its conditional expectation given the policy rule, so the policy rule still determines the forecasting rule. In a model of a credible plan, a government's sequence of money creation rates $\bar{\mu}$ is both a sequence of actions chosen by the government and a sequence of private agents' forecasts of those actions. In a rational expectations model in which the government's policy rule is part of the model, a government policy rule is a private sector forecasting rule. Managing expectations and choosing policies are the same activity.

8.4 Regimes

One response to the challenge posed by counterfactuals is to put prospective changes in policy inside the model, so that people inside the model have probabilities over them. That brings us to the notion of a policy regime.

A *regime* is a fixed government policy, i.e., a strategy, a sequence of possibly history-dependent functions. A regime can be represented recursively in terms of some state, and different theories of government policy bring different states: an exogenous Markov state; a promised inflation rate; or a promised inflation rate together with continuation values (Sargent, 2024). Finding the state is an art. A *regime-switching model* is a collection of regimes, each indexed by a Markov-switching state j_t . Parameters describe regimes. Hyper-parameters describe switches.

Sims (1982) reinterpreted Lucas's examples as neglected nonlinearities. A vector autoregression fit to data from a hidden Markov regime-switching model is an information projection of that model onto a manifold of linear models (see section 14.2).

Lucas's parallel universes. Lucas (1976) compared parallel universes. Each of two rational expectations equilibria describes outcomes under a *permanently operating* policy rule. The comparison says nothing about transitions between regimes. It studies stationary ergodic implications of each permanently operating regime. Policy analysis becomes a comparison of

stationary distributions. Policy advice emerges as a recommendation to engineer a complete surprise, i.e., an “MIT shock.” That is the “innovation” of section 8.1. Section 7.3 describes how MIT shocks have since become a computational device. Sargent (1982) studied the ends of four big inflations after World War I as laboratories for the study of regime changes. In Austria, Hungary, Poland, and Germany, inflation stopped abruptly rather than gradually once governments created independent central banks and simultaneously altered their fiscal policy regimes, experiences that are difficult to reconcile with views that attribute momentum to inflation. In a footnote, Sargent (1982, n. 39) recorded a deep objection to that interpretation: in a rational expectations model of a single economy, what look like changes in regime are realizations of events under a single, more complex regime. The next paragraph describes models built on that objection.

Complete rational expectations models. Sargent and Wallace (1976), Sims (1982), and Sargent (1984) refined the Lucas critique when they posited that prospective switches of government policy belong *inside* a historically realistic rational expectations equilibrium. Proposals for new policies do not emerge in a vacuum. Svensson and Williams (2008, 2009) provide a way to implement this idea. Their Markov-jump linear-quadratic dynamic programming problems have N regimes, one for each Markov-jump state, and N coupled Riccati equations. Because the Riccati equations are coupled through the probabilities of switching among regimes, agents’ beliefs about prospective switches are part of the equilibrium. A change of policy regime then alters equilibrium allocations through those beliefs, which update as regimes switch, rather than through an unanticipated break. Rational expectations econometrics can infer agents’ beliefs about prospective switches, e.g., with a hidden Markov model.

9 Puzzles and the hard core

Rational expectations models are coherent, but rarely comprehensive. Users of these models accumulate “puzzles,” i.e., patterns that their models miss. Examples are the equity premium puzzle, the risk-free rate puzzle, and other asset pricing anomalies, and, in growth theory, the failure of incomes in poor countries to converge to those in rich countries. Diagnostics point to features that call for modification. How economists respond to puzzles reveals how thoroughly they have embraced the rational expectations restriction.

A standard response is to insist on rational expectations and to modify specifications of

- preferences: Kreps and Porteus (1978) recursive preferences; hyperbolic discounting (Laibson, 1997); Gul and Pesendorfer (2001) temptation preferences; habits; reference

dependence (Barberis et al., 2001);

- technologies: Romer (1986) externalities; adjustment costs;
- shock processes: stochastic volatility; long-run risks (Bansal and Yaron, 2004; Hansen et al., 2008).

In their sophisticated versions, each of these responses keeps agents' beliefs at the rational expectations benchmark. Naive hyperbolic discounters, by contrast, misforecast their own future behavior. In the language of Lakatos (1970), the rational expectations restriction functions as the hard core of a research program, and specifications of preferences, technologies, and shock processes form its protective belt. That rational expectations functions as the hard core is not merely an observation about the sociology of science. Preferences, technologies, and shock processes can be disciplined independently: preferences by introspection and experiments, technologies by engineering evidence, and shock processes by financial and other time series. The beliefs that rational expectations eliminates cannot be disciplined from outside the model, for example with survey data (section 12.2), without reintroducing the free parameters that rational expectations was designed to remove. Another response modifies beliefs while keeping a shared baseline model. Section 10, which follows, describes how agents' concerns about ambiguity and misspecification add uncertainty components to market prices of risk. Section 14 describes other disciplined departures.

Part III. Defending rational expectations and taking stock

10 Rational expectations with uncertain agents

Section 10.1 describes equilibria in which agents share an approximating model that they distrust. Section 10.2 describes how two kinds of distrust, ambiguity and misspecification, show up in equilibrium prices.

10.1 Equilibria with a shared approximating model

In a rational expectations equilibrium with uncertain agents, the communism of models applies to an *approximating* model, also called a *baseline* model. All agents share an approximating model, which is the objective model that the equilibrium generates, but no agent fully trusts it. Each agent surrounds the approximating model with a set of nearby models whose divergences from it are measured by relative entropy, and chooses a decision rule that performs well across

that set. Because agents’ objectives differ, the worst-case models that shape their decisions differ too, even though all agents share one approximating model. An equilibrium is a fixed point of the form (20) below. The communism of models holds for the approximating model. Agents’ worst-case models differ from it, which is why section 14 also lists robustness among departures anchored to the restriction.

Three examples illustrate such equilibria. Hansen and Sargent (2008, ch. 12) construct competitive equilibria with robustness in a pure endowment economy. They formulate a robust planning problem, decentralize its allocation with robust consumers who trade Arrow securities, and give the robust planning problem an *ex post* Bayesian interpretation. The lecture “Robust Markov Perfect Equilibrium” in Sargent and Stachurski (2026a), at https://python-advanced.quantecon.org/rob_markov_perf.html, provides another example. Two duopolists share a baseline model of the state transition law but doubt it. Each firm chooses a decision rule while a fictitious adversary distorts the transition law, subject to a relative entropy penalty. The firms’ worst-case transition laws differ, so the firms hold different beliefs about future output and prices even though they share a common baseline model. Hansen et al. (2026) provide an example in a heterogeneous agent economy. Consumers in their Bewley economy share a baseline model of their endowments but differ in their discount factors and in how much they distrust that model. Along a locus of those two parameters, any distribution of consumers supports the same interest rate and the same joint distribution of consumption, debt, and income as an economy of consumers who trust the baseline model, while consumers’ worst-case forecasts and welfare evaluations differ.

With a multiplier penalty ξ_i on KL divergence, agent i ’s worst-case density twists the approximating model:

$$\tilde{f}_i(x_{t+1} \mid x_t) \propto \exp\left(-\frac{V_i(x_{t+1})}{\xi_i}\right) f_\theta(x_{t+1} \mid x_t),$$

where $V_i(x_{t+1})$ is agent i ’s continuation value at next period’s state. The penalty that produces this twist is $\xi_i KL(\tilde{f}_i(\cdot \mid x_t), f_\theta(\cdot \mid x_t))$, a divergence between one-step-ahead conditional densities at the current state. It measures the divergence of the subjective model relative to the approximating model, the reverse of the order in (3). The two orders answer different questions. Maximum likelihood (section 6) and the information projections of section 14.2 average log likelihood ratios under the model that generates the data. A robust agent averages them under the worst-case model, the model under which the agent evaluates decisions. That order delivers the exponential twist above and links robustness to the risk-sensitive control of

Jacobson (1973). Writing agent i 's twisted model as $S_i(\theta)$, an equilibrium is a fixed point

$$\theta = \mathsf{T}(S_1(\theta), S_2(\theta), \dots, S_I(\theta)). \quad (20)$$

In the formulations of Hansen (2014) and Hansen and Sargent (2022), robustness adds two parameters per agent, one that measures doubts about a prior over θ_s and one, related to ξ_i , that measures doubts about the likelihood. Section 10.2 shows how those two parameters put two distinct components into equilibrium prices. Calibrating them with detection error probabilities keeps worst-case models statistically close to the approximating model. Because worst-case probabilities have an *ex post* Bayesian interpretation, robustness provides a structural model of “behavioral biases” and of belief heterogeneity. It moves agents toward the econometrician’s footing,¹³ and estimation proceeds essentially as in rational expectations econometrics (Szöke, 2022). For applications to asset pricing, including the equity premium puzzle of section 9, see Hansen et al. (1999), Tallarini (2000), Barillas et al. (2009), and Cogley and Sargent (2008). For fiscal policy, see Karantounias (2013) and Ferriere and Karantounias (2019). For a textbook treatment, see Hansen and Sargent (2008); for code, see the lecture “Robust Permanent Income and Pricing” in Sargent and Stachurski (2026b).

10.2 Ambiguity, misspecification, and uncertainty prices

Hansen and Sargent (2024) separate two kinds of doubt by pulling apart the two ingredients of the Bayesian econometrician’s predictive density in section 6.2: a family of likelihoods $\{f(Y \mid \theta(\theta_s))\}_{\theta_s \in \Theta_s}$ and a prior over θ_s . A rational expectations agent knows θ_s . A Bayesian agent knows the family and a prior, which is rational expectations again in a bigger model (section 14.3). An agent who knows the family but does not know or trust a prior over its members faces *ambiguity*. An agent who distrusts the likelihoods themselves, suspecting that every member of the family is misspecified, faces *misspecification*. Hansen and Sargent (2021) call members of the family *structured* models,¹⁴ and they use *uncertainty* for ambiguity and misspecification together, ignorance about the probability distributions of the risks that the models describe.

Hansen and Sargent (2021) formalize these ideas in continuous time. Each alternative to a baseline diffusion driven by a Brownian motion B is represented by a positive martingale

¹³After I presented a joint paper with Lars Hansen about robustness at the Federal Reserve Bank of Minneapolis, Bob Lucas asked me “why should the people in our models be like us?” (Sargent, 2025).

¹⁴Structured models are “more or less tightly parameterized statistical models” that “are not what econometricians working in the tradition either of the Cowles commission or of rational expectations econometrics would call ‘structural’ models” (Hansen and Sargent, 2021, p. 222, n. 1). In the language of section 3, they are statistical models whose parameters need not be structural.

M^v , a likelihood ratio with $dM_t^v = M_t^v v_t \cdot dB_t$. Under the alternative model, the increment dB_t acquires the drift $v_t dt$. Structured models have drift distortions s_t that lie in sets Ξ_t generated by alternative parameter values. A representative investor ranks consumption processes with continuation values

$$V_t = \min_{\{v_{t+\tau}\}} E_t \left[\int_0^\infty \beta^\tau \frac{M_{t+\tau}^v}{M_t^v} \left(\log c_{t+\tau} + \frac{\xi}{2} \min_{s \in \Xi_{t+\tau}} |v_{t+\tau} - s|^2 \right) d\tau \right]. \quad (21)$$

Minimization over structured models expresses ambiguity. The relative entropy penalty on the distance between v and the set of structured models expresses misspecification.¹⁵ The size of that set and the penalty ξ are the two parameters of section 10.1. As $\xi \rightarrow \infty$, the investor has max–min preferences over structured models. When the set of structured models shrinks to the baseline model, only misspecification concerns remain, as in Hansen and Sargent (2001).

Equilibrium prices equal shadow prices of a robust planning problem. The vector of local prices of exposures to dB_t is a risk price, which comes from the curvature of utility, plus an uncertainty price $-v_t^*$, where v_t^* is the worst-case drift distortion. With s_t^* the drift distortion of the worst-case structured model, Hansen and Sargent (2021, pp. 239–40) decompose the uncertainty price as

$$\underbrace{-v_t^*}_{\text{uncertainty price}} = \underbrace{-s_t^*}_{\text{ambiguity price}} \underbrace{-(v_t^* - s_t^*)}_{\text{misspecification price}} \quad (22)$$

The first component comes from not knowing which structured model is best, the second from fearing that all of them are misspecified.¹⁶ Uncertainty prices enter the stochastic discount factor through the worst-case likelihood ratio. In the Euler equation (17), a robust investor’s E_t^Q is an expectation under the worst-case model. Measured with objective expectations, the stochastic discount factor is therefore a marginal rate of substitution times a likelihood ratio. An econometrician who imposes rational expectations sets that likelihood ratio to one and must attribute uncertainty prices to risk aversion. Earlier quantitative work had found that “investors’ responses to modest amounts of model ambiguity can replace the implausibly large risk aversions during economic downturns that are required to explain observed market prices of risk” (Hansen and Sargent, 2021, p. 223).

Four features connect this work to the roles of rational expectations. First, agents, the

¹⁵I write v , s , B , and c for their U , S , W , and C , β^τ for their $\exp(-\delta\tau)$, and ξ for their $\theta\delta$.

¹⁶Earlier work surrounded a single baseline model with a relative entropy neighborhood. In the terms used here, the “market price of uncertainty” that Anderson et al. (2003, p. 68) linked to “a bound on the error in statistically discriminating between an approximating and a worst case model” is a misspecification price when the set of structured models contains only the baseline model (Hansen and Sargent, 2021, p. 226, n. 9).

model builder, and the econometrician share the baseline model and the family of structured models, and uncertainty prices are measured relative to the baseline model. Hansen and Sargent estimated the baseline model from time series of macroeconomic quantities alone. They deliberately ignored asset prices and so did not impose the cross-equation restrictions that their asset pricing theory implies (Hansen and Sargent, 2021, p. 243). Second, detection error probabilities discipline the two new parameters. Statistical tests would take a very long time to tell the worst-case models apart from the baseline model or from each other: half-lives of mistake probabilities range from 60 to 668 quarters. Misspecification contributes roughly a third to a half of long-horizon uncertainty prices (Hansen and Sargent, 2021, Tables 1 and 2). Third, uncertainty prices vary over time without stochastic volatility, because the investor fears high persistence of low growth rates and low persistence of high growth rates. Fourth, admitting only models that histories of infinite length cannot distinguish from a structured model is what self-confirming equilibria do (section 14.2). Hansen and Sargent admit models with small positive relative entropy (Hansen and Sargent, 2021, p. 234, n. 16).¹⁷

11 What rational expectations asks of people

This section describes what the restriction asks of the people inside a model and four defenses of asking it: the “as if” stance of Lucas and Prescott (1971); Sims’s histograms, which rest on stationarity; steady states of adaptive processes; and market selection.

A rational expectations model is a coherent model of *what* future outcomes people inside the model forecast. It is not a model of *how* people think about forecasting. It does not describe the machinery or thought process that leads them to forecast as they do. Thus, Lucas and Prescott (1971, p. 664) wrote

It remains for us to link the anticipated price sequence to the actual price sequence—also a sequence of functions on E^t . Typically, this is done by postulating the process by which firms actually develop forecasts from actual past values and other information. In this paper, we shall not take this route but rather postulate a property of the outcome of this (unspecified) process. Specifically, we assume that expectations of firms are rational, or that the anticipated price at time t is the same function of $(u_1, \dots, u_t)$ as is the actual price. That is, we assume that firms know the true distribution of prices for all future periods.

¹⁷Hansen and Sargent (2010, p. 130) study a discrete-time precursor in which a representative consumer learns by Bayes’ law about two models of consumption growth and their parameters, and “doubts the Bayesian model-mixing probabilities as well as the specification of each model.” Those doubts put countercyclical uncertainty premia into prices of risk.

That stance resembles Newton’s refusal to explain the cause of gravity (“hypotheses non fingo”), and it is a version of the “as if” reasoning that physicists use in the principle of least action and that Friedman and Savage (1948, p. 298) and Friedman (1953, p. 21) used to predict the shots taken by an expert billiards player. No particle solves the variational problem in the principle of least action. No billiards player solves the equations of mechanics. Nobody inside a rational expectations model computes its equilibrium. That is the model builder’s and the econometrician’s task.

What must people inside a rational expectations model know? Rational expectations econometrics attributes knowledge of θ_s to them (section 6). Their decisions require less. An agent’s decisions depend on the objective model only through the conditional distributions of variables that enter his or her own decision problem, typically exogenous shocks and prices. Restriction (5) requires those distributions to be correct. An agent need not know other agents’ preferences, how goods are produced, or the state variables that a model builder uses to compute an equilibrium.¹⁸ Prescott and Mehra wrote that knowing H , p , and $F(\zeta' | \zeta)$ “is sufficient for forecasting future prices (actually forming predictive probability distributions of future prices)” (section 4). Those predictive distributions are what a household needs. The functions H and p are one way to represent them. Hayek (1945, pp. 526–27) admired the same economy in a price system: “The most significant fact about this system is the economy of knowledge with which it operates, or how little the individual participants need to know in order to be able to take the right action.” The mean field games of section 7.2 share that economy. Lasry and Lions (2007, p. 229) describe “a very large number of ‘rational players’ with a limited information (or visibility) on the ‘game,’ ” each of whom “chooses his optimal strategy in view of the global (or macroscopic) informations that are available to him and that result from the actions of all players.”

Hellwig and Venkateswaran (2025) formalize Hayek’s point in a rational expectations model. Monopolistically competitive firms observe only their own sales and wages. Those signals reveal each firm’s current marginal cost and revenue. They can be poor indicators of aggregate shocks. With flexible prices, equilibrium prices and quantities equal those of an economy in which firms observe all aggregate and firm-specific shocks. With sticky prices, a firm must forecast its future marginal costs. The equivalence then requires the firm’s current marginal cost to be its best forecast of future marginal costs. That happens only when firm-specific and aggregate shocks are equally persistent and pricing decisions are not strategic complements. Firms then “do not need to know whether their marginal costs are

¹⁸Fernando Alvarez tells first-year macroeconomics students that a competitive equilibrium is like living through COVID. Isolated from other people, you buy and sell things on Amazon. You need to know prices. You need not know how things are produced or who produces them.

driven by firm-specific shocks, aggregate shocks, or the other firms' pricing decisions" (Hellwig and Venkateswaran, 2025, p. 2). Otherwise, dispersed information slows the response of prices to nominal shocks.

It is helpful to recall Christopher Sims's response to widespread early-1970s statements like "I don't believe that real people know enough to have rational expectations."¹⁹ Sims presented what I continue to find to be a telling answer, namely, that people who live inside a rational expectations equilibrium need only form joint histograms. Under stationarity and ergodicity, a law of large numbers makes those histograms converge to the joint probability distribution that the equilibrium generates. That joint distribution is an optimal forecasting model. Sims's argument is nonparametric (except that it does require choosing a bin-mesh size). It requires people inside the model only to tabulate histograms.

Sims's argument rests on foundations of time series statistics required to justify applying a law of large numbers (an ergodic theorem), such as stationarity; ergodicity; and transformations that attain stationarity and ergodicity in the midst of growth, e.g., cointegration and scalings by additive and multiplicative functionals (Hansen and Scheinkman, 2009). Rational expectations econometrics also requires those foundations.

Lucas and Prescott (1971, p. 664, n. 9) had rested their defense of rational expectations on the same foundations:

... we can think of no objection to this assumption which is not better phrased as an objection to our hypothesis that the stochastic component of demand has a regular, stationary structure. If the demand shift assumption is reasonable, then expectations rational in our sense are surely more plausible than any simple, adaptive scheme. If the demand assumption is unreasonable, then adopting an alternative expectations hypothesis will certainly not improve matters.

Sims's histograms and Lucas and Prescott's footnote both locate the case for rational expectations in the stationarity of the environment. In Lucas and Prescott's words, an objection to rational expectations is better phrased as an objection to stationarity. Rational expectations itself does not require stationarity: perfect foresight models and models with known time-varying laws of motion impose it without a stationary distribution. Stationarity is what the two defenses require.

Sims's histograms are an account of how people come to know θ , the parameters of the objective law of motion. They need not reveal the structural parameters θ_s . Section 6.2 said that agents inside a rational expectations model already know θ_s , and hence θ ; they do not

¹⁹Chris Sims told me this in the early 1970s in the hallway at the Department of Economics of the University of Minnesota. I had just returned from presenting Sargent (1973) at the Brookings Institution and had told Chris about some of the skepticism and criticisms the paper had provoked.

learn. Sims’s account has them learn θ from histograms. The two accounts are consistent only in the limit. Sims’s histograms tabulate data generated by an equilibrium in which people already know θ . Whether people who are still learning drive the data toward that equilibrium is a separate question. Bray and Kreps (1987) and Marcet and Sargent (1989) studied it. Section 14.3 returns to it.

Knowing θ suffices for forecasting within the stationary process that generated the histograms. Would agents also need θ_s to forecast after a change in government policy? Sims had in mind two interconnected things. First, people have access to a very long sample. Second, during that sample the economy has moved among all possible policy regimes, which have switched often. Switches of regime are then themselves part of the stationary joint distribution that the histograms tabulate, as in the complete rational expectations models of section 8.4. A joint histogram over a long enough sample approximates that joint distribution well, including the probabilities of regime switches and the behavior of outcomes after them. Ljungqvist and Sargent (2027, ch. 5) describe the technicalities that underlie Sims’s argument. The argument does not cover policies that have never been tried, the “innovations” of section 8.1, and it requires regime switches to be frequent enough that histograms fill in samples of realistic length (section 12.1).

Two other defenses also appeal to long experience. Lucas (1986) described economics as studying decision rules that are steady states of adaptive processes, and explored what happens when decision makers recursively update least squares estimates of the parameters of a possibly misspecified statistical model. Under suitable conditions, stochastic approximation results make their estimates and decision rules converge to parts of what section 14.2 calls a restricted perceptions equilibrium, which is a rational expectations equilibrium when their model is correctly specified (Sargent, 2025). Market selection, an argument that Alchian (1950) and Friedman (1953) made for profit maximization, offers another defense: traders whose beliefs are less accurate lose wealth to traders whose beliefs are more accurate. Blume and Easley (2006) show that when allocations are Pareto optimal, survival depends only on discount factors and beliefs, so that among traders with equal discount factors those whose beliefs have the smallest KL divergence (3) survive, but that with incomplete markets the market selection hypothesis can fail. Section 12 describes environments in which stationarity and a law of large numbers do not suffice.

12 Where the restriction strains

The roles described so far rest on assumptions that do not always hold. This section describes three places where the restriction strains.

12.1 Rare and off-path events

Models in which decisions hinge on probabilities of rare but consequential events work the restriction hard. The equilibrium joint probability distribution is shaped by influential decision makers' understandings of conditional probabilities of events that rarely occur in equilibrium. Examples are peso problems, rare disasters (Rietz, 1988; Barro, 2006), and run probabilities in Diamond and Dybvig (1983). A rational expectations model asserts that agents know the consequences of such events. Those consequences are learned slowly, if at all, from observations generated by the model. Induction from experience is limited. A rational expectations model therefore expresses faith in an extraneous source of deduction. How much can people deduce, possibly aided by instruction or by experience in related environments? Two kinds of events must be distinguished. *Rare* events occur along equilibrium paths, but infrequently. Sims's histograms eventually learn their probabilities, but slowly. *Off-path* events never occur in equilibrium. No amount of data reveals their consequences. Runs in Diamond and Dybvig (1983) can be off-path events: in an equilibrium without runs, agents' beliefs about what a run would bring are never tested. Self-confirming equilibria (section 14.2) allow beliefs about off-path events to be wrong.

12.2 Survey evidence

Surveys of expectations record disagreement among forecasters and forecast errors that can be predicted. Such evidence is not new. Tests of rational expectations with survey data began in the 1970s (Pesando, 1975), and Lovell (1986) reviewed many of them. Some of that evidence speaks against models in which all agents share the same information, not against rational expectations. Coibion and Gorodnichenko (2015) interpret predictable errors in average forecasts as evidence of information rigidities, which rational expectations models with dispersed or costly information, such as those of sections 2 and 7.2, generate. Bordalo et al. (2020) find that individual forecasters overreact to their own news while average forecasts underreact. Individual overreaction is harder to reconcile with rational expectations. Let $\hat{y}_{i,t}$ be forecaster i 's forecast at t of a variable y_T . If the forecast is a conditional expectation

given information that grows over time, the law of iterated expectations implies

$$E[(y_T - \hat{y}_{i,t})(\hat{y}_{i,t} - \hat{y}_{i,t-1})] = 0,$$

so a rational forecaster’s error cannot be predicted from his or her own forecast revision. A test with survey data tests this implication jointly with assumptions about what respondents report. Hansen et al. (2026) show that optimizing consumers who doubt their model can generate both patterns. Their worst-case forecasts overreact to news in samples in which distances from bliss are concentrated and underreact in samples in which those distances are dispersed. Those are forecasts of the consumers’ own consumption, and reading a worst-case forecast as a reported belief is one of the assumptions about what respondents report. Survey questions often ask about aggregate conditions. In the flexible-price economy of section 11, firms’ beliefs about aggregate conditions are widely dispersed, but “the extent of this dispersion in beliefs has absolutely no effects on allocations or aggregate dynamics” (Hellwig and Venkateswaran, 2025, p. 7).

12.3 Large state spaces

Challenges to rational expectations are not confined to heterogeneous agent models. The puzzles of section 9 arose in representative agent models. Heterogeneous agent models with aggregate risk add a challenge of their own: their equilibrium prices depend on the cross-section distribution of wealth and income. Two systems of equations can characterize an equilibrium. One couples a backward Bellman equation for each agent’s value, in which the rest of the economy enters only through the stochastic process of prices, with a forward equation for the cross-section distribution. The other makes the distribution a state variable, as K is in section 4, and folds both equations into a single Bellman equation over an infinite-dimensional state. That equation, the “master equation” of mean field games, has been nicknamed the “Monster equation.” Under regularity conditions, the two systems deliver the same equilibrium (Cardaliaguet et al., 2019).

The master equation is a device that a model builder can use to compute an equilibrium. Agents inside the model need to forecast only the prices that enter their decision problems. With aggregate risk, each date’s price is a function of the history of aggregate shocks, given the initial distribution. Lucas and Prescott (1971, p. 664) assumed that firms know such functions (section 11), and sequence-space methods compute them to first order (Auclert et al., 2021). Model builders also compute such equilibria by perturbation to higher orders (Bilal, 2023) and globally, with deep neural networks that approximate solutions of master equations, both for incomplete-markets models (Gu et al., 2026) and for search and matching

models with heterogeneous workers and firms (Payne et al., 2026). Rational expectations restricts agents’ predictive distributions of prices. It does not restrict the state variables that a model builder uses to represent them. The difficulty of computing a rational expectations equilibrium is therefore a challenge for the model builder. Whether agents can learn the predictive distributions of prices that the equilibrium implies is a separate question, the one that Sims’s histograms answer in stationary environments (section 11).

Lucas’s student and coauthor Moll (2026) is not convinced by the “as if” metaphor of section 11. He contends that modern heterogeneous agent rational expectations models are troubled because they attribute implausible behavior to people inside them:

... forecasting behaviour implied by rational expectations is not just computationally challenging, it is conceptually implausible. If even our most advanced computational tools struggle with the Monster equation, how can we justify the assumption that real-world households and firms solve the associated decision problems? ... Instead of solving Monster equations we should replace the rational expectations assumption and solve the simpler equations corresponding to their actual price-forecasting behaviour. (Moll, 2026, p. 1174)

Section 3.5 of Moll (2026) implies a limit to Sims’s argument of section 11. Because equilibrium prices in these models are not Markov (their conditional distribution depends on the entire cross-section distribution), the histogram that would reveal θ must be taken over the infinite-dimensional state, or over long histories of prices. A law of large numbers still applies, but it fills such histograms very slowly. In the sticky-price economy of section 11, the same property determines how much firms need to know: their own market signals suffice when their marginal costs are Markov in those signals. Section 14.4 describes what alternatives to rational expectations about such prices would keep and lose.

13 The roles collected

Sections 5–10 described the roles that the rational expectations restriction plays, and sections 11 and 12 described what those roles ask of people and where they strain. Collected in one place, the roles are:

1. *Discipline on beliefs.* The restriction eliminates free parameters that would otherwise describe agents’ subjective models (sections 5.2 and 5.4).
2. *Identification and estimation.* Cross-equation restrictions link structural parameters to the parameters of descriptive models and help make that link invertible. Estimators

hand an estimated objective model to agents as their subjective model (sections 5.1, 6, 7.2, and 7.3).

3. *Policy evaluation.* Agents' beliefs respond to policy rules, so a model computes how outcomes respond to policies (sections 7.1, 7.3, and 8.1).
4. *Policy design and credibility.* Ramsey plans, time consistency, and reputations become problems that dynamic programming can solve (sections 8.2 and 8.3).
5. *Inference from equilibrium outcomes.* Agents know how prices and other agents' actions depend on private information, so they can extract information from them. That underlies theories of information aggregation, no-trade theorems, and higher-order beliefs (sections 2, 5.3, and 7.2).
6. *Welfare comparisons.* Agents' subjective expected utilities equal objective ones, which settles whose probabilities welfare comparisons should use (section 7.1).
7. *Determinacy.* Whether a policy rule delivers a unique equilibrium becomes a criterion for choosing it (sections 7.1, 7.3, and 8.1).
8. *Benchmark.* Departures from rational expectations, including "behavioral biases," are measured from it (sections 6.2, 7.1, 10.2, and 14).
9. *Computation.* Equilibria are fixed points that relaxation and stochastic approximation algorithms can find. Perturbation and deep learning methods compute them in models with large state spaces (sections 5.5, 7.3, and 12.3).

Each role also uses ingredients that the restriction does not supply. Identification requires rank conditions. Counterfactuals require primitives that stay fixed across policies and an account of what agents know about a new policy. Welfare comparisons require a criterion. Every role requires an equilibrium that exists and a rule for selecting one when there are several. Section 7.2 shows these roles at work throughout economics, often under other names.

The nine roles are also criteria.²⁰ A proposed alternative equilibrium concept can be judged by how many of them it keeps. Section 14.4 applies that test to three alternatives.

²⁰Fernando Alvarez regrets that there are only nine. Ten could have been commandments.

Part IV. Alternatives and an accounting

14 Alternatives anchored to the restriction

There is a wilderness of models that do not impose rational expectations.²¹ Most proposed replacements for rational expectations have long histories. Adaptive expectations, a forecasting heuristic, came first (section 5.4). Modern heuristics make agents myopic about distant events (Gabaix, 2020) or limit their depth of reasoning (Farhi and Werning, 2019). Temporary equilibrium theory (Grandmont, 1977) takes agents' forecasting rules as given. Lucas and Prescott (1971, p. 664) called that route typical and declined to take it (section 11). In the internal rationality of Adam and Marcet (2011), agents optimize under subjective beliefs about prices. Least squares learning (Bray, 1982; Marcet and Sargent, 1989), reinforcement learning by artificially intelligent agents (Marimon et al., 1990; Sargent, 1993), restricted perceptions and self-confirming equilibria (Fudenberg and Levine, 1993; Sargent, 1999; Evans and Honkapohja, 2001), and doubts about models (Hansen and Sargent, 2008) have been studied for decades. Those literatures have already worked out much of what each departure keeps and loses.

This section sketches the disciplined parts of the wilderness and points to fuller treatments elsewhere, and it closes with an accounting of what they keep and lose (section 14.4). Departures in a first class share a template. An *outcome map* T sends a list of agents' subjective models into the objective model that their decisions generate. A *belief map* B_i sends the objective model into agent i 's subjective model. An equilibrium concept is a choice of subjective families $\mathcal{F}_i$ together with belief maps B_i , and an equilibrium is a fixed point

$$\theta \in \mathsf{T}_{\mathcal{F}}(B_1(\theta), \dots, B_I(\theta)). \quad (23)$$

The outcome map has the same economic content across equilibrium concepts, but its domain depends on the families $\mathcal{F}_i$. The belief maps carry the differences, as Table 1 shows. Rational expectations is a special case of each of the first four rows of Table 1. Robustness reduces to it as $\xi_i \rightarrow \infty$ when there is no ambiguity, that is, when the only structured model is the approximating model (section 10.2). A restricted perceptions equilibrium is a rational expectations equilibrium when each Δ_i contains the objective model, because then $g_{\Pi_i(\theta)} = f_{\theta}$. Every rational expectations equilibrium is self-confirming when agent i 's family contains

²¹Sims (1980, p. 4) used the word for a different set of models, those in which prices adjust sluggishly: “If instead we take the view that prices themselves may adjust sluggishly, we enter the wilderness of ‘disequilibrium economics.’ This phrase must, it seems to me, denote a situation in which we cannot suppose that business behavior is invariant under changes in the public’s tastes.”

Table 1: Belief maps and equilibrium concepts

Concept	Agent i 's family	Belief map $B_i(\theta)$	Equilibrium
Rational expectations	$\{f(\cdot \theta)\}_{\theta \in \Theta}$	θ	$\theta = T(\theta, \dots, \theta)$
Robustness	approximating $f(\cdot \theta)$ or structured models; relative entropy neighborhood	worst-case twist $S_i(\theta)$	$\theta = T(S_1(\theta), \dots, S_I(\theta))$
Restricted perceptions	$\{g_\delta\}_{\delta \in \Delta_i}$, possibly misspecified	projection $\Pi_i(\theta)$, (25)	$\theta = T(\Pi_1(\theta), \dots, \Pi_I(\theta))$
Self-confirming	$\{f_i(\cdot \theta_i)\}_{\theta_i \in \Theta_i}$	set $\mathcal{C}_i(\theta)$: correct on path	$\theta \in T(\mathcal{C}_1(\theta), \dots, \mathcal{C}_I(\theta))$
Adaptive learning	$\{g_\delta\}_{\delta \in \Delta_i}$	recursion (15)	$\theta_t = T_t(\hat{\delta}_{1t}, \dots, \hat{\delta}_{It})$; no fixed point

the objective family, because then $\theta \in \mathcal{C}_i(\theta)$. Measured by the KL divergence (3), rational expectations sets the divergence to zero, a self-confirming equilibrium sets it to zero for outcomes on the equilibrium path, and a restricted perceptions equilibrium minimizes it over a possibly misspecified family. Robustness instead penalizes the divergence of a worst-case model relative to the approximating model, the reverse order. In that precise sense, rational expectations is the benchmark from which departures, including “behavioral biases,” are measured (section 7.1). Hansen et al. (2026) show how much freedom the choice of a benchmark leaves and what bounds it.²² Departures in a second class, such as adaptive learning along a transition path, have no fixed point. Agents’ beliefs are state variables, and along a transition they are inconsistent with the outcomes that they generate.

14.1 Robustness

Section 10.1 described the equilibria in the second row of Table 1. Because agents share an approximating model, a communism of approximating models survives, and in the formulations cited there the worst-case twists S_i add two parameters per agent. Section 10.2 describes how those parameters, one for ambiguity and one for misspecification, show up in equilibrium prices.

²²In the Bewley economy of section 10.1, quantity data identify neither a consumer’s type nor the reference model from which a bias is measured. The reference model that an analyst chooses determines whether a pattern in elicited forecasts is called a bias at all. Measured from a trusting benchmark, robust consumers’ worst-case forecasts look pessimistic and their discount factors look impatient; measured from a point near the boundary of admissible robustness, consumers look optimistic and patient. A bound on admissible discount factors, detection error probabilities, and variation in the environment limit that freedom.

14.2 Restricted perceptions and self-confirming equilibria

In a restricted perceptions equilibrium, agent i 's family of subjective models $\{g_\delta\}_{\delta \in \Delta_i}$ need not contain the objective model f_θ .

Definition 14.1. *A restricted perceptions equilibrium is a $(\theta, \delta_1, \dots, \delta_I)$ that satisfies*

$$\theta = \mathsf{T}(\delta_1, \dots, \delta_I), \tag{24}$$

$$\delta_i = \arg \min_{\delta \in \Delta_i} KL(f_\theta, g_\delta), \quad i = 1, \dots, I. \tag{25}$$

The minimizer in (25) is a population maximum likelihood estimator (section 6), the information projection of f_θ onto agent i 's family (Sargent, 2025).²³ Writing it as $\Pi_i(\theta)$, a restricted perceptions equilibrium is a fixed point $\theta = \mathsf{T}(\Pi_1(\theta), \dots, \Pi_I(\theta))$ of T composed with information projections. When agent i 's family consists of linear regressions with Gaussian disturbances, $\Pi_i(\theta)$ is a population least squares regression. That is the projection that the Krusell–Smith algorithm of section 7.2 and least squares learning (section 14.3) compute. The Berk–Nash equilibrium of Esponda and Pouzo (2016) generalizes this concept to games in which agents entertain sets of models. In a self-confirming equilibrium (Fudenberg and Levine, 1993, 2009), agent i 's parameter need only lie in the set $\mathcal{C}_i(\theta)$ of parameters θ_i for which $f_i(\cdot \mid \theta_i)$ and f_θ assign the same probabilities to events on the equilibrium path. Self-confirming equilibria typically come in sets. An agent's model can misrepresent outcomes after historically unprecedented policies, so a government that uses such a model can adopt inferior policies.

14.3 Adaptive learning

Least squares learning is the subject of a literature of the 1980s and 1990s that includes Marcet and Sargent (1989), Sargent (1993), and Sargent (1999). That literature asks how people might come to have rational expectations. Game theorists ask how players might come to play a Nash equilibrium (Fudenberg and Levine, 1998). The lectures in the section “Bounded Rationality in Macroeconomics” of Sargent and Stachurski (2026b), which begins at https://python.quantecon.org/bounded_rationality.html, describe that literature in detail, and lectures in the same series titled “Self-Confirming Equilibria,” “Adaptive Learning and Escape Dynamics,” and “Escaping Nash Inflation” implement models with escape dynamics.

²³Information theorists call this minimizer a reverse information projection; their information projection minimizes over the first argument of (3) (Csiszár and Matúš, 2003).

Models that replace rational expectations with forecasting rules describe a temporary equilibrium

$$\theta_t = \mathsf{T}_t(\hat{\delta}_{1t}, \dots, \hat{\delta}_{It}), \quad (26)$$

in which agents' beliefs $\hat{\delta}_{it}$ are taken as given and T_t maps them into the objective law of motion of outcomes at t . In a learning model, agent i updates $\hat{\delta}_{it}$ with a recursion like (15), a stochastic counterpart of the relaxation iterations of section 5.5. Marcet and Sargent (1989) showed that the limiting behavior of beliefs is governed by the mean dynamics

$$\frac{d\delta}{d\tau} = \Pi(\mathsf{T}(\delta)) - \delta, \quad (27)$$

where Π projects the actual law of motion onto agents' family of perceived laws of motion, as in (25). Rest points of (27) are restricted perceptions equilibria.²⁴ With a decreasing gain, least squares learning can converge to a rational expectations equilibrium when agents' families contain the actual law of motion and the equilibrium is stable under (27), a property that Evans and Honkapohja (2001) call E-stability. Results of that kind rely on what I call *prompting* by the model builder: a correctly specified family, suitable initial beliefs, and devices that keep beliefs in a domain of attraction, such as the projection facility of Marcet and Sargent (1989). With a constant gain, beliefs keep fluctuating, and in some models escapes from self-confirming equilibria recur (Cho et al., 2002).²⁵

Learning models add free parameters: families, gains, and initial beliefs (Sargent, 1993, ch. 7, p. 168). The family determines the possible limits, which are restricted perceptions equilibria. With a decreasing gain and a unique limit, gains and initial beliefs affect only transient dynamics, so the stationary distribution of the data contains no information about them. When there are several possible limits, initial beliefs select among them. With a constant gain and an ergodic belief process, initial beliefs wash out. The gain governs both the size of belief fluctuations around a self-confirming equilibrium and the frequency of escapes from it. The fluctuations carry information about the gain, but escapes are rare, so for them the rare-events problem of section 12.1 reappears. Bayesian learning within a coherent model, by contrast, is rational expectations again, in a bigger model with priors as additional parameters (Bray and Kreps, 1987).

In 1993 I described the adaptive learning program this way:

²⁴Sargent (1999) and Cho et al. (2002) call them self-confirming equilibria, a usage that differs from that of section 14.2. They are special cases of the Berk–Nash equilibria of Esponda and Pouzo (2016).

²⁵Applications include escapes from Nash inflation (Cho et al., 2002), recurrent currency crises (Kasa, 2004; Cho and Kasa, 2005), illusory algorithmic collusion (Cho and Williams, 2024), recurrent Latin American stabilizations (Marcet and Nicolini, 2003; Sargent et al., 2009), and U.S. inflation-unemployment dynamics (Sims, 1988; Chung, 1990; Sargent, 1999).

Bounded rationality is a movement to make model agents behave more like econometricians. Despite the compliment thereby paid to their kind, macroeconometricians have shown very little interest in applying models of bounded rationality to data. Within the economics profession, the impulse to build models populated by econometricians has come primarily from theorists with different things on their minds from most econometricians. (Sargent, 1993, ch. 7, pp. 167–68)

Exceptions include Chung (1990), who showed that such models can be estimated. Moll (2026) renews the theorists’ impulse, with a difference: he asks that models of beliefs be disciplined by survey evidence on expectations (section 12.2), data that are far richer now than in 1993.

14.4 What alternatives keep and lose

Abandoning rational expectations everywhere would put at risk the equilibrium concepts of game theory and mechanism design and the structural estimators of industrial organization and labor economics (section 7.2). Moll (2026) proposes something narrower: replacing rational expectations about equilibrium prices in heterogeneous agent models with aggregate risk, while keeping optimization, market clearing, and rational expectations about exogenous processes (Moll, 2026, p. 1189, n. 22). He proposes three criteria for a replacement: computational tractability, consistency with empirical evidence on expectations, and endogeneity of beliefs to “model reality,” including policy changes. The third criterion keeps two of the roles collected in section 13: discipline on beliefs, because in stationary environments price beliefs should be approximately consistent with equilibrium prices; and policy evaluation, because price beliefs should respond to policy changes (Moll, 2026, pp. 1192–93). The alternative he prefers is close to the Krusell and Smith (1998) algorithm already in use, whose fixed point is a restricted perceptions equilibrium. Moll would treat that fixed point as a description of how people forecast and would discipline it with survey evidence (Moll, 2026, pp. 1177 and 1188).

Where such proposals apply, Table 2 compares what three alternatives keep and lose of the roles collected in section 13. The phrase “discipline on beliefs” means different things in each column of Table 2. For restricted perceptions, parameters are pinned down by information projections, though the family of models remains free. For robustness, a communism of approximating models survives, with two additional parameters per agent. For adaptive learning, discipline is lost, because families, gains, and initial beliefs are all free.

Discipline on beliefs, identification, and welfare comparisons are at risk under both restricted perceptions and adaptive learning. Adaptive learning also puts policy evaluation and policy design at risk: optimal policy under learning can be computed, but the plans depend

Table 2: What three alternatives keep and lose

Role	Price beliefs as restricted perceptions	Robustness	Adaptive learning
Discipline on beliefs	parameters pinned down by projections; families free	kept; two parameters per agent	lost: families, gains, initial beliefs
Identification and estimation	families must be specified	essentially as with rational expectations; uncertainty prices are measured relative to a shared baseline model	identification of gains and initial beliefs needs its own analysis
Policy evaluation	beliefs respond through the projection	beliefs respond through the worst-case twist	beliefs respond only gradually
Policy design and credibility	requires modeling how agents read policy	kept	possible, but plans hinge on gains and initial beliefs
Inference from equilibrium outcomes	inference through a best-fitting, possibly misspecified price function	kept, with worst-case inference	agents must first learn how prices reflect information (Bray, 1982)
Welfare comparisons	whose beliefs count?	agents' own robust criteria	whose beliefs count?
Determinacy	a fixed point remains, so the criterion applies	a fixed point remains, so the criterion applies	replaced by stability under learning
Benchmark	kept: a special case when families contain f_θ	kept: the limit as both kinds of doubt vanish	kept: a possible limit of learning
Computation	a fixed point remains	at least as hard as rational expectations	no fixed point in beliefs; each period's temporary equilibrium must still be computed

on gains and initial beliefs (Gaspar et al., 2006; Molnár and Santoro, 2014). Robustness, which retains a communism of approximating models, keeps most roles, and every alternative keeps rational expectations as its benchmark. What a departure loses can also be measured in a model's predictions. In the search model of Payne et al. (2026), unemployment falls about 25 percent faster during the recovery from a shock calibrated to COVID-19 when workers and firms forecast the distribution of matches than when they act as if it stays at its long-run average (section 7.2). Two less drastic responses retain coherence. One reverse engineers environments in which forecasting rules that survey evidence suggests people use are optimal, as Muth (1960) did, a harder task in heterogeneous agent models with aggregate risk than in Muth's univariate example. The other imposes the restriction only partially, as generalized method of moments estimation of Euler equations does (section 6.3).

15 Concluding remarks

A rational expectations model is a fixed point $\theta = T(\theta, \dots, \theta)$ of a mapping from subjective to objective statistical models. That communism of models plays the roles collected in section 13: it disciplines beliefs, helps identify structural parameters, lets models evaluate and design policies, lets agents extract information from prices and other agents' actions, settles whose probabilities welfare comparisons should use, makes determinacy a criterion for choosing policy rules, provides a benchmark for departures, and connects economics to fixed-point computation. It appears, often unnamed, throughout economics: as equilibrium price expectations, Euler-equation orthogonality conditions, correct beliefs about strategies, common priors, and known transition laws in structural econometrics. In current macroeconomics, it gives forward guidance its power, organizes debates about monetary and fiscal policy through determinacy, and underlies the MIT-shock computations and global solution methods that make heterogeneous agent models tractable. When models that impose it fail, economists usually modify preferences, technologies, and shock processes, not the restriction.

The restriction strains where stationarity and a law of large numbers give it the least support: for rare events, off-path events, unprecedented policies, and, as Moll (2026) emphasizes, equilibrium prices that depend on infinite-dimensional cross-section distributions. Agents in such models need to forecast only the prices that enter their decision problems. Computing the equilibrium is the model builder's task. Disciplined alternatives remain anchored to the restriction. Equilibria in which agents share but distrust an approximating model keep a communism of approximating models, and they add ambiguity and misspecification components to prices measured relative to that model. The roles are also criteria for judging proposed alternative equilibrium concepts. The accounting of section 14.4 shows that abandoning the

restriction would cost most in discipline on beliefs, identification, and welfare comparisons, and that a leading proposed alternative is close to the Krusell–Smith algorithm already in use. I hope that collecting the roles and the accounting in one place helps students and researchers decide when to impose the restriction and when to depart from it. Rational expectations remains the benchmark from which departures are measured.

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