What Nonconvexities Really Say about Labor Supply Elasticities

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Abstract

Rogerson and Wallenius (2013) draw an incorrect inference about a labor supply elasticity at an *intensive* margin from premises about an option to work part time that retiring workers decline. We explain how their false inference rests on overgeneralizing outcomes from a particular example and how Rogerson and Wallenius haven't identified an economic force beyond the two – indivisible labor and time separable preferences – that drive a high labor supply elasticity at an interior solution at an *extensive* margin.

KEY WORDS: Labor supply elasticity, nonconvexities, extensive margin, intensive margin.

1 Introduction

An example doesn't establish a general proposition, but a counterexample eradicates it. This paper provides a counterexample to Rogerson and Wallenius's (2013, hereafter RW) claim to infer a labor supply elasticity at an *intensive* margin from workers declining to work part time.

RW extended a model of Ljungqvist and Sargent (2007, hereafter LS), who constructed a nonstochastic, continuous-time lifecycle model with a constant wage, a risk-free asset, and a restriction in the spirit of Rogerson (1988) that in each "period" a worker can work either 0 hours or full-time hours $h_f > 0$. LS showed that when the market interest rate equals the subjective discount rate, their model yields the same aggregate outcomes, expected utilities, and aggregate labor supply elasticity as those from a model with employment lotteries like Rogerson's. In the LS model, rather than an economy-wide representative family that like Rogerson's assigns a fraction of people to work each period, an individual worker simply chooses the fraction of her lifetime to work. Those two conceptually different fractions, Rogerson's across a continuum of workers at a point in time, LS's across time for a single worker, turn out to be equal.[1]

The high labor supply elasticity at the extensive margin (the number of "periods" in a lifetime devoted to work) that emerges from the LS model depends on two key ingredients: (i) time-separable preferences, and (ii) an exogenously imposed labor supply indivisibility. The indivisibility causes individuals to partition their lives into spells working and spells not working. Time-separable preferences imply that the line between those two parts occurs at a constant per-period disutility of work. An interior solution for career length implies a high lifetime labor supply elasticity.

RW modify the basic LS model by assuming that each "period" the worker can choose to supply labor at one of *three* possible levels $\{0 < h_p < h_f\}$, where the addition of h_p

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[1]Larry Jones and Casey Mulligan anticipated aspects of this equivalence result. In the context of indivisible consumption goods, in the original 1988 version of his paper, Jones (2008) showed how timing could replace lotteries when there is no discounting. In the 2008 published version of his paper, he extended the analysis to cover the case of discounting. After showing that an indivisible-labor model with complete markets, lotteries and insurable preference heterogeneity is isomorphic with a divisible-labor, representative-agent model, Mulligan (2001, Appendix II) used numerical examples to illustrate that the elimination of insurance and lotteries from the former model might not make much of a quantitative difference.

Responding to Mulligan's (2001) contention that the aggregate implications of indivisible labor are few, Ljungqvist and Sargent (2011, pp. 488–489) explain why reasonable parameterizations of an indivisible-labor, heterogeneous-agent, time averaging model suggest a high aggregate labor supply elasticity at the extensive margin.

represents an opportunity to work part time. Allowing an employed worker to choose either h_p or h_f hours opens an "intensive margin" (or number of hours "within a period") to supplement the LS model's "extensive margin." RW claim that their model implies that the intertemporal elasticity of substitution (IES) for work at the intensive margin is high for any worker who eventually retires and also chooses *not* to exercise the option to work part time. RW establish their claim by constructing examples with a particular parametric utility function. This paper constructs a counterexample.

Our counterexample calls for altering RW's (2013, p. 1461) main conclusion "that based on existing estimates of the size of nonconvexities and measures of full-time work prior to retirement, it is hard to rationalize values of the IES that are less than 0.75." To the contrary, it is not hard. We show this by simply blending preferences that were actually used by RW themselves in closely related contexts.

The source of RW's erroneous inference is this. That a worker chooses to work full time and *not* part time restricts relative disutilities of work at the full-time and part-time work options. But those relative disutilities do not in general restrict the *curvature* of the disutility of work locally at the full-time work option that the worker chooses, which is the object that RW claim to have restricted. To complete their claim, RW would have had to discover a missing third economic force to supplement the aforementioned ingredients (i) and (ii) that drive the high labor supply elasticity at the extensive margin inherent in the LS model. This they have not done.

2   Setup

In the Ljungqvist and Sargent (2007) time-averaging framework, a worker chooses $c(t) \geq 0$, $h(t) \in \{0, h_f\}$, for $t \in [0, 1]$, to maximize

$$\int_0^1 e^{-\delta t}[u(c(t)) - \alpha v(h(t))]dt \tag{1}$$

subject to

$$\int_0^1 e^{-rt}wh(t)dt \geq \int_0^1 e^{-rt}c(t)dt \tag{2}$$

where $\delta \geq 0$ and $r \geq 0$. Following RW, we assume a zero interest rate and no subjective discounting, $r = \delta = 0$.

Prescott (2006) and Prescott et al. (2009) extended the LS model to include an intensive margin, $h(t) \in [0, 1]$, i.e., a continuous choice of hours worked in periods of working.[2] To generate outcomes with periods of working and not working, they replace the expression for labor income on the left side of budget constraint (2) with a nonlinear mapping from hours worked to labor income. RW let that mapping take the form of a wage schedule that is increasing in hours worked,

$$w(h) = w_0 h^\theta, \tag{3}$$

where $\theta \geq 0$. RW report empirical evidence for choosing $\theta = 0.4$, a value that we adopt throughout our analysis.

RW specialize things by requiring that $h(t) \in \{0, h_p, h_f\}$, with corresponding hourly wages w_p and w_f for part-time and full-time work, respectively, where according to (3) the relative wage satisfies

$$\frac{w_p}{w_f} = \frac{w(h_p)}{w(h_f)} = \left[\frac{h_p}{h_f}\right]^\theta. \tag{4}$$

Prescott et al. (2009) assumed a functional form for the disutility of work with a constant IES ϕ for *work*:

$$v^{\mathrm{P}}(h) = h^{1+\frac{1}{\phi}}. \tag{5}$$

RW depart from Prescott et al. (2009) by instead specifying a utility function $x(1-h) = \frac{1}{1-\frac{1}{\gamma}}(1-h)^{1-\frac{1}{\gamma}}$ of leisure $(1-h)$ that has a constant IES γ for *leisure*. As a consequence, for RW the disutility of work becomes

$$v^{\mathrm{RW}}(h) = x(1) - x(1-h) = \frac{1}{1-\frac{1}{\gamma}}\left[1 - (1-h)^{1-\frac{1}{\gamma}}\right], \tag{6}$$

so with RW's preference specification the IES for *work* is $\gamma(1-h)/h$, an object that varies with h.[3]

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[2]In his original Nobel prize lecture in 2004, Prescott (2005) praised and exclusively relied on the employment lotteries model as the aggregation theory of households' labor supply. But then as a discussant of Ljungqvist and Sargent's (2007) time averaging model at the 2006 NBER Macro Annual conference, Prescott (2007, pp. 233) exclaimed that the "important contribution of their paper is the initiation of an important research program, a program that already has begun to bear fruit." As an example of the fruit that he had in mind, Prescott (2006, p. 223–225) added a section entitled "The Life Cycle and Labor Indivisibility" to the version of his Nobel lecture published in the *JPE*. There he extended the Ljungqvist-Sargent model to include an intensive margin.

[3]In the Appendix, we show that RW in some instances fail to properly account for how the IES for work varies with h, and therefore make misleading inferences.

3 Example and counterexample

This section describes the logical structure of RW's claim that the IES for work at the intensive margin, must be high if a worker is to choose to retire by transiting directly from full-time work to no work. Their argument relies on the shape of the RW utility function $v^{\text{RW}}(h)$ and setting parameter value $h_f = 0.385$ (with the motivation that full-time work on an annual basis constitutes 2,000 hours out of an assumed total discretionary time of 5,200 hours).

Let e_f (e_p) denote the fraction of a worker's lifetime devoted to full-time (part-time) work, with any remaining part, $1 - e_f - e_p$, being spent in retirement. The worker's optimization problem becomes

$$\max_{e_f, e_p} \left\{ u(e_f w_f h_f + e_p w_p h_p) - e_f \alpha v(h_f) - e_p \alpha v(h_p) \right\} \tag{7}$$

$$\text{subject to} \qquad e_f,\, e_p \geq 0, \tag{8a}$$

$$e_f + e_p \leq 1. \tag{8b}$$

When none of constraints (8) binds, first-order necessary conditions for an optimum imply

$$\frac{v(h_p)}{w_p h_p} = \frac{v(h_f)}{w_f h_f}, \tag{9}$$

so that the disutility of work per unit of labor income is identical across part-time and full-time work.

Using RW's specification (6) for the disutility of work, and invoking wage schedule (3), condition (9) can be solved to obtain[4]

$$\theta = \left[\log \left(\frac{1 - (1 - h_p)^{1 - \frac{1}{\gamma}}}{1 - (1 - h_f)^{1 - \frac{1}{\gamma}}} \right) \Big/ \log\left(h_p / h_f \right) \right] - 1. \tag{10}$$

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[4]RW (2013, online Appendix) arrive at expression (10) in a different way. They assume logarithmic utility over consumption, and compare two individuals who work either full time only or part time only over their entire careers. Under the assumption that both workers are at an interior solution for career length, they solve for parameter configurations that equalize the lifetime utilities of the two individuals. However, condition (10) characterizes yet more instances of indifference between the two work options, including parameterizations where the optimum cannot be attained by only using the part-time work option, and where as in our derivation, the specification of the utility function for consumption does not matter.
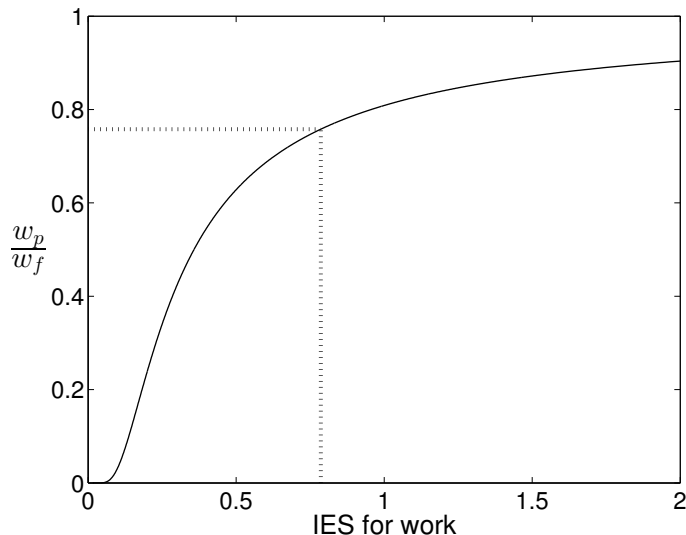
Figure 1: Part-time wage as a fraction of the full-time wage, at which indifference prevails between the two work options at an interior solution to career length, for different calibrations of the RW preferences as given by the associated IES at full-time work. For a part-time wage above (below) the solid line, the worker would prefer to work part time (full time).

Given $h_f = 0.385$ and $h_p = h_f/2$, by varying (θ, γ) pairs, condition (10) can be used to sweep out the relationship between the relative wage w_p/w_f and the IES for work at the full time work option shown in figure 1. For each (θ, γ) pair that satisifies condition (10), the figure maps θ into the relative wage w_p/w_f given by (4), and γ into the implied IES for work at the full-time work option, IES $= \gamma(1 - h_f)/h_f$. Thus, for a given IES at the full-time work option, the solid line depicts the razor's-edge value of the relative wage w_p/w_f at which the worker would be indifferent among all combinations of part-time and full-time work that result in the same optimally chosen lifetime labor income. For any part-time wage that lies above (below) the solid line, the indifference breaks in favor of the part-time (full-time) work option. At low values of the IES, a worker doesn't prefer the full-time work option unless the part-time wage is very low. For example, under RW's assumption that $\theta = 0.4$, the part-time wage is approximately 75 percent of the full-time wage, and the dotted lines in figure 1 confirm RW's (2013, p. 1460) remark that "[e]ven with only a half-time option $[h_p = h_f/2]$ it is difficult to rationalize a value of the IES below 0.75."

RW's finding that the IES for work must be high in order for workers to refrain from part-time work depends sensitively on their decision to assume utility function (6). To show this, we take an alternative utility function created by mixing RW's (6) with the earlier

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Prescott et al. (2009) utility function (5):

$$v^{\text{alt}}(h) = \mu_1 v^{\text{RW}}(h) + \mu_2 v^{\text{P}}(\max\{0,\, h - h_p\}). \tag{11}$$

The utility function (11) augments the RW disutility of work, $v^{\text{RW}}(h)$, with extra disutility for hours of work above h_p measured by the Prescott et al. (2009) disutility $v^{\text{P}}(\max\{0,\, h - h_p\})$. We set the preference parameter γ in $v^{\text{RW}}(\cdot)$ to be the favorite value of RW, in the sense of implying hours worked $h_f = 0.385$,[5] while we set the preference parameter ϕ in $v^{\text{P}}(\cdot)$ to be 0.25. Next, we set the weights μ_1 and μ_2 so that $v^{\text{alt}}(h_f) = 1$ and $v^{\text{alt}}(h_p) = 0.5 \cdot 0.75$.[6] This makes the disutilities of full-time and part-time work take values that make the worker indifferent between h_p and h_f at an interior solution to career length given a part-time wage that equals 75 percent of the full-time wage.

The solid line in figure 2 depicts our alternative specification of preferences in the upper panel together with the associated IES for work in the lower panel. The circle in the upper panel marks what the disutility of part-time work ($h_p = h_f/2$) would have to be in order for the worker to be indifferent between h_p and h_f at an interior solution for career length, given a part-time wage that is equal to 75 percent of the full-time wage. By construction, the solid line goes through the circle, i.e., the alternative preference specification satisfies condition (9). For comparison, the dashed line in figure 2 represents the RW preferences that go into constructing the alternative utility function (11), but where that RW disutility of work is now also normalized to be one at the full-time work option. Given this normalization, the fact that the RW disutility of work lies above the circle in the upper panel of figure 2 confirms that the worker strictly prefers the full-time work option.

Since a worker with either of the two preference orderings summarized in figure 2 could be seen choosing only full-time work and then transiting directly to no work in retirement, our counterexample illustrates that a worker's not taking the part-time work option is not enough to infer a high IES for work at the work level h_f that he chooses.

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[5]This is the value targetted by RW in a setting in which individuals face a continuous choice of hours. See our discussion of figure 3 in the Appendix.

[6]The weights μ_1 and μ_2 then satisfy

$$\mu_1 = \frac{0.5 \cdot 0.75}{v^{\text{RW}}(h_p)} \qquad \text{and} \qquad \mu_2 = \frac{1 - \mu_1 v^{\text{RW}}(h_f)}{v^{\text{P}}(h_f - h_p)}.$$
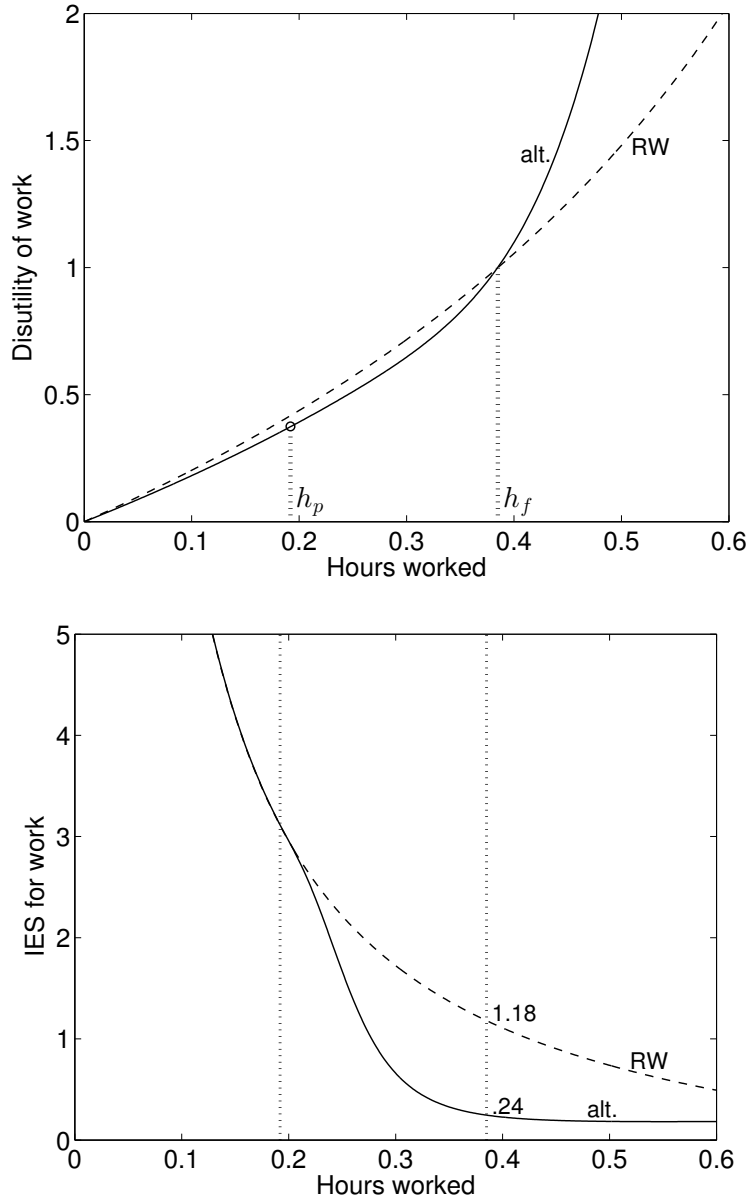
Figure 2: Preferences over work according to RW and an alternative utility specification, as represented by the dashed and solid lines, respectively. In the upper panel, the disutility of work is normalized to one at full-time work ($h_f = 0.385$), and the circle marks what the disutility of part-time work ($h_p = h_f/2$) would have to be in order for the worker to be indifferent between full and part time at an interior solution to career length, given a part-time wage that is equal to 75 percent of the full-time wage. The IES at full-time work is 1.18 and 0.24 for the RW and the alternative specification, respectively, as depicted in the lower panel.

4   Concluding remarks

As emphasized in the introduction, the high labor supply elasticity inherent in the LS model at an interior solution for career length rests on two pillars – indivisibilities in labor supply and time separable preferences. Labor indivisibilities cause workers to divide their lifetimes into parts working and not working. Time-separable preferences imply that the marginal choice between those two parts occurs at a constant per-period disutility of work, giving that high elasticity of labor supply at an interior solution for career length.

The labor-supply-indivisibility pillar is typically justified by the observation that workers' hours of work are mostly bunched at a few common values with the 'full-time' value being predominant. Alternative assumptions about technologies and preferences can generate that observation. Simple examples would include a setup cost at work and a fixed disutility of work. The time-separable-preferences pillar is typically justified as doing a good job of approximating workers' wishes to rest and refresh between periods.[7]

RW want but don't supply a third pillar on which to rest an implication about a labor supply elasticity at an intensive margin. What underlying economic force do RW have in mind that would fortify their example and exclude our counterexample? Without adding to their mix additional postulates and forces beyond what they have, generality cannot be credited to RW's claim that the intertemporal elasticity of substitution for work at the intensive margin must also be high under their two premises that (1) most workers transit directly from full-time work to no work in retirement, and (2) there exists at least one intermediate part-time work option that workers choose not to exercise.

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[7]We digress to note that while RW choose the year as a 'period' that could match their access to data on hours worked at an annual frequency, a better delineation based on real-world observations might be the week as a 'period'. Not only does the work week, including one or two days off, seem to be a common form for organizing work, it would also explain why European welfare states have chosen to extend additional leisure to the working-age population by legislating number of weeks of vacation rather than shortening work weeks. Notwithstanding France and its introduction of a 35-hour work week, since that reform seems to have caused economic problems that prompted legislators partially to reverse it.

A   Further corrections of RW's inferences

Along with Prescott (2006) and Prescott et al. (2009), RW also consider a continuous choice set at the intensive margin, $h(t) \in [0, 1]$, but using their utility function (6) that exhibits a constant IES for leisure, γ, and an implied IES for work, $\gamma(1 - h)/h$. Since the IES for work varies with hours worked, RW (2013, p. 1453, footnote 13) decide to index each calibration of the preferences by "the value of the IES at annual work hours of 2,000 [$h_f = 0.385$], even if retirement occurs from a lower level" (they should also have said "or from a higher level," as we will see below). Thus, the relationship between the IES at some particular hours worked $h^\star$ and the 'RW indexation' is

$$\text{IES}\Big|_{h=h^\star} = \frac{h_f}{h^\star} \frac{1 - h^\star}{1 - h_f} \cdot (\text{RW indexation}). \tag{12}$$

Unfortunately, RW use the RW indexation to draw misleading inferences about the characteristics of preference specification (6). For example, by comparing the RW indexation of alternative calibrations that are targeted to explain different outcomes at the intensive margin, RW suggest that a lower value of the IES for work is required to rationalize outcomes with fewer hours of work at the intensive margin, when in fact the opposite is true.

As shown by RW, under a parameterization that yields an interior solution for career length, the optimal value of hours worked, $h^\star$, satisfies

$$\frac{h^\star}{1 + \theta} = \frac{1}{1 - \frac{1}{\gamma}} \Big[1 - (1 - h^\star)^{1 - \frac{1}{\gamma}}\Big] (1 - h^\star)^{\frac{1}{\gamma}}. \tag{13}$$

At different calibration targets for hours worked, $h^\star \in (0, 1)$, the solid line in figure 3 depicts the implied IES for labor. The dashed line shows the associated RW indexation for each such calibration target $h^\star$. As can be seen, the 'distortion' in expression (12) between the actual IES and the RW indexation becomes large when actual hours drift away from $h_f = 0.385$. For example, at RW's repeated reference to $h = 0.212$, the IES is then actually 2.3 times larger than what the RW indexation suggests. Attempting to draw inferences about the IES by using the RW indexation would misrepresent the direction of the change in IES across calibration targets $h^\star$. In particular, while the correct inference along the solid line is that it becomes easier to infer low values of the IES as the hours worked get closer to the upper limit of the maximum at the intensive margin, the RW indexation falsely suggests the opposite. Hence, the second half of the following inference of RW (2013, p. 1454) is misleading:
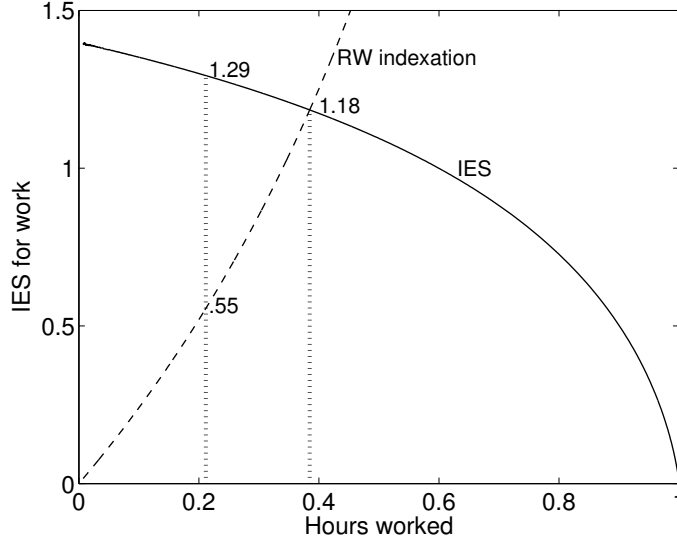
Figure 3: Calibration of RW preferences that generates a particular choice of hours worked, $h^\star \in (0,1)$, where the solid line depicts the associated IES at $h^\star$. The dashed line is the corresponding RW indexation of such a calibration, namely, the IES that would prevail for the calibrated preferences at a hypothetical value of hours worked equal to $h_f = 0.385$.

> If we consider $\theta = 0.4$ as a reasonable magnitude, values of the IES below 1.00 are not consistent with retirement if annual hours are 2,000 [h=0.385]. Note, however, that retirement from a level of annual hours equal to 1,100 [h=0.212] ... is consistent with an IES a bit above 0.50.

Next, RW consider additional sources of nonconvexity in the form of fixed time and consumption costs associated with work. Here we focus on the fixed time cost $\bar{h}$ that is incurred in any period that the individual supplies labor. The fixed time $\bar{h}$ neither earns labor income nor contributes to the increasing wage schedule. As shown by RW, under parameter settings that yield an interior solution for career length, the optimal value of hours worked, $h^\star$ that includes the fixed time cost, satisfies

$$\frac{h^\star - \bar{h}}{1 + \theta} = \frac{1}{1 - \frac{1}{\gamma}}\left[1 - (1 - h^\star)^{1 - \frac{1}{\gamma}}\right](1 - h^\star)^{\frac{1}{\gamma}}. \tag{14}$$

In one comparison across targets for hours worked, RW vary the parameterization of the fixed time cost so that it is a constant fraction of the target for hours worked. But for a given calibration target $h^\star$, the associated fixed time cost $\bar{h}$ enters as a constant in the worker's

optimization and the optimal choice is characterized by (14). In figure 4, we adopt RW's parameterization that the fixed time cost $\bar{h}$ constitutes a fraction $\frac{335}{335+1100}$ of targeted hours worked $h^\star$. As before, the solid line shows the IES for labor but now as a function of the *observed* hours worked net of the fixed time cost, $h^\star - \bar{h}$, i.e., the IES for some calibration target $h^\star$ is registered in figure 4 at $h^\star - \bar{h}$. And as before, the dashed line depicts the RW indexation that coincides with the IES when $h^\star = 0.385$, i.e., the IES and the RW indexation are the same in figure 4 when $h^\star - \bar{h} = 0.385 - \bar{h} = \frac{1100}{335+1100} \cdot 0.385$. Since that point of equality occurs between RW's two focal points of $(h^\star - \bar{h}) \in \{0.212, .385\}$, inferences based on the RW indexation are bound to underestimate (overestimate) the IES at the lower (higher) value of observed hours of work.

So RW (2013, pp. 1455-1456, including footnote 14) make a couple of misleading inferences when they claim:

> We have already noted [above] that generating retirement from annual hours of 1,100 [h=0.212] is significantly easier than generating retirement from annual hours of 2,000 [h=0.385]. To pursue this further we consider the implications of our framework for the case of retirement from annual hours of 1,100 and assuming a fixed time cost of 335 hours. In this case we find that a value of $\theta = 0.40$ is consistent with an IES equal to 0.41. ... If we assume that annual hours at retirement are 2,000 and the fixed time cost is one quarter of total work plus fixed time costs, then the implied value of the IES given $\theta = 0.4$ is approximately 0.94.

Contrary to what might be inferred from the RW indexation, it remains true in figure 4 that it is more difficult to generate retirement from fewer hours of work at the intensive margin, in the sense that it requires a higher IES for work. In particular, at calibration target $h^\star - \bar{h} = 0.212$, the IES equals 0.67 which is almost two thirds higher than the RW indexation of 0.41. The model implies that the IES falls slightly to 0.58 when calibrating to $h^\star - \bar{h} = 0.385$ rather than shooting up close to unity as asserted by RW when they misleadingly read off the IES from the RW indexation.
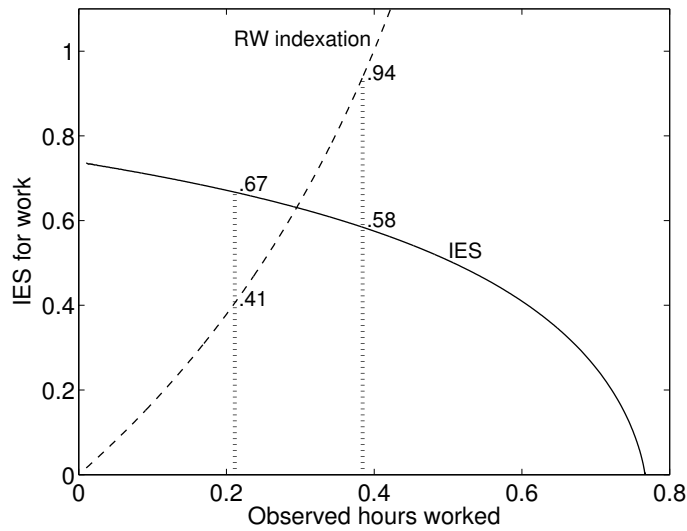
Figure 4: Calibration of RW preferences that generates a particular choice of observed hours worked, under the assumption of a fixed-time requirement that makes actual hours of work $h^\star$ larger by a factor of $\frac{335+1100}{1100}$. The solid line depicts the associated IES at the observed hours worked. The dashed line is the corresponding RW indexation of such a calibration, namely, the IES that would prevail for the calibrated preferences at a hypothetical value of actual hours of work equal to $h_f = 0.385$.

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