Harrod 1939

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Abstract

Harrod's 1939 "Essay in Dynamic Theory" is celebrated as one of the foundational papers in the modern theory of economic growth. Linked eternally to Evsey Domar, he appears in the undergraduate and graduate macroeconomics curricula, and his "fundamental equation" appears as the central result of the AK model in modern textbooks. Reading his *Essay* today, however, the reasons for his centrality are less clear. Looking forward from 1939, we see that the main stream of economic growth theory is built on neoclassical distribution theory rather than on the Keynesian principles Harrod deployed. Looking back, we see that there were many antecedent developments in growth economics, some much closer than Harrod's to contemporary developments. So what, then, did Harrod accomplish?

1 Harrod's Analysis

Harrod presented what he described as a 'tentative and preliminary attempt to give an outline of a "dynamic" theory.' His purpose, he concluded, was '. . . to present a tool of analysis, not to diagnose the current condition.'[1] Harrod's equilibrium analysis was

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[1]Harrod, p. 33

based on three assumptions: (1) saving is proportional to national income, $S_t = sY_t$; (2) investment, the demand for saving, is proportional to the growth of national income, $I_t = g(Y_{t+1} - Y_t)$; and (3) saving equals investment, the demand for saving equals the supply of saving, $S_t = I_t$. From this one derives the "fundamental equation",

$$\frac{Y_{t+1} - Y_t}{Y_t} \equiv \rho_w = \frac{s}{g},$$

in which ρ_w is the "warranted" rate of growth. Put differently, national income follows the first-order difference equation $Y_t = \frac{g}{g-s} Y_{t-1}$, with $1 > g > s > 0$. Domar (1946) attained the same result in a different model, now often conflated with Harrod's, and the same equation has recently reemerged as Piketty's Second Fundamental Law of Capitalism.[2]

Harrod supplemented his formal analysis with speculations about the consequences of deviations of actual aggregate income from warranted aggregate income. Harrod said that such deviations were bound to occur because the warranted rate of growth usually differs from a "natural" rate of growth that is determined by changes in productivity and the labor force. Harrod also discussed instabilities and, in passing, disappointments of investors' expectations, that gaps between warranted and natural growth rates would ignite.

Harrod's paper was widely read and cited for 20 or 30 years after 1939 because it raised interesting issues:

1. The dynamic properties of a fixed coefficient model.
2. The implications of the qualification that fixed coefficients like saving rate are not fixed exogenously, but instead are determined by economic forces.
3. Alternative senses and sources of instability.
4. Some possible interactions between a multiplier (reflecting consumption decisions) and an accelerator (reflecting real investment decisions)
5. Economic consequences of discrepancies between *ex ante* (or expected) and *ex post* (or realized) objects.

[2]Piketty (2014), p. 166.

Harrod discussed these things in ways that readers today will find difficult to comprehend and appreciate, partly because of progress that the study of economic dynamics has made since 1939, partly because Harrod chose not to use or extend some lines of work preceding 1939 that would be more familiar to today's reader, and partly because his analysis, done without benefit of a formal model, is hard to follow, and the analytic categories differ from those we use today. The explicit parts of the paper leading to the difference equation for warranted aggregate income are clear enough, but the paper's informal and speculative parts are obscure. The modern reader cannot be blamed for sometimes being puzzled what the paper is really about and how to relate Harrod's ideas to those prevailing today. Some of the difficulties the *Essay* presents are these:

1. What is capital? Today we think of capital as a factor of production; consequently the marginal product of capital is crucial to determining interest and wage rates, the distribution of output between capital owners and workers and so forth. For Harrod, as for Keynes, capital is something quite different. '. . . no distinction is drawn in this theory between capital goods and consumption goods. In measuring the increment of capital, the two are taken together; the increment consists of total production less total consumption.'[3]

2. What determines s and g? The parameter g describes for Harrod the demand for saving, not as we read it today, the inverse marginal or average product of capital. 'It may be expected,' Harrod writes, 'to vary as income grows and in different phases of the trade cycle; it may be somewhat dependent on the rate of interest.'[4] Similarly, 's is regarded as likely to vary with a change in the size of income. . . .'[5]

3. What is equilibrium? Harrod is clear that the "warranted" rate of growth is in fact the equilibrium growth rate of a model. If the key parameters s and g in fact vary with endogenous variables, then equilibrium is not yet determined until these additional relations are appended to the model. The easiest way to fill in the gaps, or course, is to read the *Essay* as a fixed-coefficients model, and this has become the tradition.

4. Why is equilibrium unstable? The informal and speculative stability analysis is obscure in part to the modern reader. The source of the difficulty is the same as that

[3]Harrod, p. 18.
[4]Harrod, p. 17.
[5]Harrod, pp. 24-5.

of our question (3). Harrod's model is not closed. A decade later, Baumol made a first attempt to close the model so that these questions can be answered.[6]

5. Is this a growth model? Harrod himself did not emphasize the contribution to economic growth. The *Essay* was intended to develop arguments first put forth in his 1936 book *The Trade Cycle*. Although the importance of lags had been recognized since Böhm Bawerk, Harrod is concerned to distinguish his analysis from 'time lag theories', especially those of Robertson, and from the econometric approach to dynamics being developed by Frisch and Tinbergen.[7]. Time lags lead to second-order difference equations, and the possibility of oscillatory behavior. Harrod's one concession to a theory of growth is this: 'Supposing damping measures could be introduced, to counteract the oscillation caused by the lag, would the system be stationary or advancing? And at what rate? Dynamic theory in my sense may throw some light upon this.'[8] The interaction of growth and business cycles is an intriguing possibility, but in that day it would have been impossible to address the "cross-frequency restrictions" that Harrod's model might impose on aggregate time series.

6. What about the role of prices; distributions of income, wealth, and beliefs; labor demand and supply, and so forth. He acknowledges, as noted earlier, a dependence of g on the price of capital, but so much is missing ....

Although Harrod's paper seems partly about economic growth, most economists today have been convinced by Solow's and Swan's claim that labor-capital substitutability was the pertinent assumption for a long-run analysis and their demonstration that using that assumption in place of Harrod's fixed coefficient model of investment pulls the rug out from under Harrod's distinction between a natural and warranted growth rate.

The business cycle analysis Harrod points to in the *Essay*, the instability of the equilibrium growth path, could not go far without an analysis of how the economy behaves off the warranted path. In any event, the lag-theorists won out. Milton Friedman's distributed lag implementation of his permanent income model of saving as an important source of business cycle dynamics displaced Harrod's analysis. Like Harrod's, at least in its time series distributed lag econometric implementation, Friedman's (1957) representation

[6]Baumol (1948, 1949).
[7]See Besomi (1998).
[8]Harrod, p. 14-15.

of adaptive expectations as a geometric distributed lag in disposable is a fixed coefficient model. But it wouldn't be a fixed coefficient model for long. Friedman informally motivated his distributed lag in income as being an operational version of Irving Fisher's theory of intertemporal consumption choice. That set the stage for John F. Muth (1960) to interpret the fixed coefficient, geometric distributed lag posited by Friedman as the outcome of a statistical prediction problem, thereby providing the first concrete illustration of the cross-equation restrictions that Muth's hypothesis of rational expectations imposes. After Muth, most of us sooner or later came to realize that for both investment and consumption functions, what we had earlier treated as fixed coefficients were not fixed with respect to the very things whose effects we want to study, namely, government tax and expenditure policies.

2 Harrod's Antecedents

Reading the journals of the interwar years, one is struck by the paucity of footnotes. By today's publishing standards, Harrod didn't provide a reader much help in relating his analysis to earlier and contemporary work. This is a shame because the 1920s and 30s were decades when some of the most important ideas underlying modern dynamic economic analysis were foreshadowed or created. From reading the paper it is impossible to tell whether Harrod was unaware of some of this work or knew about it and chose not to use it.

A list of significant available contributions that Harrod neglected would include: (1) Earlier work on the properties of fixed coefficients models, for example, the Ricardian trade model (a timeless analysis), Leontief's input-output analysis, and the von Neumann fixed-coefficient equilibrium model of economic growth, prices, and interest rates. (2) Edgeworth's (1881) analysis of "out of equilibrium trades" and their potential effects on equilibrium quantities and prices. (3) Pigou's analysis of the interaction of time-to-build and gaps between anticipated and realized prices and quantities as sources of equilibrium dynamics (*Industrial Fluctuations*). (4) What turned out to be a workhorse framework for equilibrium dynamics that J.R. Hicks (*Value and Capital*) created by including dated commodities within a Walrasian model of general equilibrium. Hicks's "temporary equilibrium" allowed discrepancies between anticipated and realized prices. Irving Fisher's *Theory of Interest*

contained key ingredients that set the stage for Hicks. In effect, Hicks showed how "equilibrium dynamics are a special case of static equilibrium". Hicks's work directly set the stage for Arrow's and Debreu's contingent commodities: if it is fruitful to index commodities by dates, why not also index them by random events? Doing that provides a sense in which "an equilibrium with risky commodities is a special case of an equilibrium without them". (5) Frank Ramsey's (1928) intertemporal model of the equilibrium determination of investment and consumption. When combined with the ideas of Hicks, Arrow, Debreu, Muth, and Lucas, Ramsey's framework became the foundation of the dominant approach to equilibrium growth and business cycles that continues to be refined today; (6) Work by Slutsky, Yule, and Frisch that showed how random shocks impinging on simple low order linear difference equations creates a rich structure for distinguishing random impulses from propagation mechanisms embedded in the difference equations. (7) Samuelson's (1939) contemporary work on interactions between a multiplier and accelerator. (8) The analysis of stability of an equilibrium in terms of what we would now call fast and slow dynamics. Presumably, the "fast dynamics" correspond to Samuelson's "out of equilibrium" dynamics that related rates of change of prices to excess demand, while the "slow dynamics" would be those driven by the exogenous forces pushing supply and demand curves to change over time. These were organized by Samuelson into his correspondence principle as a device for disciplining comparative static analysis.

3 After Harrod

It is worthwhile to list some of the most important subsequent contributions that made progress in addressing issues that Harrod raised.

(1) Theories based on consumer optimization aimed at providing a theory of Harrod's saving coefficient. We have already mentioned the permanent income theory of consumption and saving and how it developed in a way that featured a peculiar division of labor: Friedman used Irving Fisher's theory of optimal consumption choice conditional on exogenous forecasts of a stream of future disposal incomes (i.e., Friedman studied 'control'); while Muth studied optimal prediction. Muth and his coauthors exploited the Theil-Simon separation of prediction and control (legitimate for linear, quadratic, Gaussian problems) extensively in their 1960 book.

(2) Theories based on firm optimization aimed at providing a theory of Harrod's investment coefficient. Like Harrod, Domar (1946) also wrote in a Keynesian framework. Unlike Harrod, he was concerned directly with economic growth. For him, g was an inverse marginal product of capital. Abba Lerner (*Economics of Control*) and Trygve Haavelmo (*A Study on the Theory of Investment*) characterized the difficulties in linking marginal product of capital to a Keynesian investment function: a neoclassical theory of the firm gives rise to a demand function for a stock of capital, not an investment function like the one Harrod and earlier Keynesian econometric models assumed. Subsequent partially effective attempts to solve the conceptual problem raised by Haavelmo were pursued by Jorgenson and Eisner; success occurred only in the mid to late 60s with the development of the adjustment cost model of Lucas (1967), Gould (1968), and Treadway (1969), culminating in Hayashi (1982).

(3) Malinvaud (1953) contributed what can be regarded as a multi-sector Ramsey-Cass-Koopmans model of optimal economic growth. This is the first "modern" growth model. Malinvaud raised the question of when equilibrium prices would signal an efficient allocation of resources. This led to a large and productive literature that opened many doors even before a practical answer was finally provided by the so-called Cass conditions. The same issue arises in distinguishing those prices that support weak optima from those that support full optima in overlapping generations models (themselves a development of dynamic analysis that goes back to Allais and Samuelson).

(4) The complete markets general equilibrium models of Arrow and Debreu with all trading occurring at time zero, so that traders have no need to forecast prices. In this setting, Pigou's (and subsequently, Lucas's) gap between anticipated and realized outcomes does not appear.

(5) Radner's sequential equilibrium trading model put traders in situation in which they have to forecast prices. This model also highlighted the importance of the distinction between complete and incomplete markets that, although implicitly present in Arrow, had not heretofore been adequately appreciated. These structures open a roll for gaps between anticipated and realized prices, a gap that played a central role in Lucas's 1972 equilibrium model of the Phillips curve.

(6) Theoretical work on rational expectations and recursive competitive equilibria. In recursive competitive equilibria, private decision makers must forecast equilibrium prices.

Nevertheless, in some important settings recursive competitive equilibria and Arrow-Debreu equilibria with all trades at time 0 have equivalent allocations. In these settings, the two equilibrium concepts they are just different ways of implementing the same equilibrium allocations. This work has helped identify and clarify the role of assumptions about market completeness in making room for gaps between anticipated and realized prices to affect equilibrium allocations.

(7) Work of Chow, Howrey, and others building souped up fixed coefficients models capable of matching aggregate economic times series. Those models combined some kind of permanent income consumption function with a distributed lag accelerator. Something like these two components seem to be what are needed to match basic dynamics of C, I, Y data (e.g., Sargent's 1989 JPE paper.) Refinements in the 1980s and 1990s of real business cycle theories, especially their log-linear manifestations, extended this tradition by offering theories of the determinants of what earlier work had treated as fixed coefficients.

4 Concluding Remarks

Harrod's paper is in a Keynesian tradition of wanting something *not* having what were ordinarily thought of as microfoundations based on theories of choice and equilibria available in the 1930s, or at least theories of choice available to Keynes and his colleagues.[9] The purpose was to create a *New Economics* (the title of Seymour Harris's 1947 edited collection of essays) capable of explaining observations that Keynes and has circle were convinced equilibrium classical economics could not. Most of these things (Thomas Kuhn and Edward C. Prescott would call them "puzzles") involved dynamics, frictions, shocks, disappointed anticipations, and apparent violations of static first-order conditions (see chapter 2 of Keynes's *General Theory*). The sense in which Keynes's *General Theory* was to be "more general" than classical equilibrium theory was that by relaxing the restrictions imposed by those (static) first order conditions, you could reconcile more outcomes to the theory. This approach missed the mark in two ways. First, a theory without restrictions is useless. A theory that predicts everything predicts nothing, and could provide not guide for policy analysis. If the restrictions were to be empirical, how would we be able to distinguish those relations that are stable when policies change from those that

[9]See Tobin's interpretation and celebration of Keynes's along theses lines in the 1947 Harris volume.

are not (and are therefore useless for policy analysis)? This, of course, is the essence of Marschak's analysis of the meaning of structural equations and, in the context of models in which private agents optimally solve dynamic choice problems, the Lucas critique. Second, the content of the neoclassical restrictions was not well-understood. Indeed this question was not even seriously investigated until Sonnenschein took it up in the early 1970s, leading to the Debreu-Mantel-Mas Colell-Sonnenschein theorem on the lack of restrictions for equilibrium prices, and the subsequent Brown and Matzkin (1996) work on restrictions for comparative statics. The problem is not, as Keynes and his circle thought, that the neoclassical restrictions were too severe.[10] The framework of equilibrium theory imposes very few restrictions *per se*, and the art of applied general equilibrium analysis is to find the assumptions about the primitives, preferences and technologies, that lead to the most useful restrictions on equilibrium outcomes.

Attempts to pursue that Keynesian vision of a more general theory attracted some enthusiasm in the late 1960s and early 1970s but ran out of steam soon after the publication of Barro and Grossman's *Money, Employment, and Inflation*, which built on Clower's earlier efforts to create a disequilibrium analysis. Along with the equilibrium dynamics of Hicks and Arrow and Debreu, two broad forces eventually subverted Keynesian economics. One was a mostly American project to put microfoundations underneath Keynesian economics, such as the 1970 Phelps *et al.* volume. The other was the macroeconometric project begun by Tinbergen and carried forward by Klein, Goldberger, Modigliani, and others, a project that, building on possibilities that Slutsky and Frisch had demonstrated, aimed to put a quantitative discipline on the key objects of business cycle and growth theory.

To answer the question with which we began, Harrod's accomplishment in the *Essay* was not the establishment of a theory that provided a foundation for future developments. His power was in asking the right questions. His struggles in the *Essay* raised questions that the profession spent the next half-century addressing. The *Essay* is important as a signpost on the road, not announcing a destination, but pointing in the direction that modern economic analysis subsequently took.

[10] Essentially none on prices, and only those of revealed preference on comparative statics.

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