The Fundamental Surplus

Lars Ljungqvist Thomas J. Sargent*

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Abstract

To generate big responses of unemployment to productivity changes, researchers have reconfigured matching models in various ways: by elevating the utility of leisure, by making wages sticky, by assuming alternating-offer wage bargaining, by introducing costly acquisition of credit, by assuming fixed matching costs, or by positing government mandated unemployment compensation and layoff costs. All of these redesigned matching models increase responses of unemployment to movements in productivity by diminishing the *fundamental surplus* fraction, an upper bound on the fraction of a job's output that the invisible hand can allocate to vacancy creation. Business cycles and welfare state dynamics of an entire class of reconfigured matching models all operate through this common channel.

Key words: Matching model, market tightness, fundamental surplus, unemployment, volatility, business cycle, welfare state.

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*Ljungqvist: Stockholm School of Economics and New York University (email: lars.ljungqvist@hhs.se); Sargent: New York University and Hoover Institution (email: thomas.sargent@nyu.edu). We are grateful for excellent research assistance by Isaac Baley, Cecilia Parlatore Siritto, and Jesse Perla. For their comments on earlier drafts we thank participants at the conference on Recursive Methods in Economic Dynamics, in Honor of 25 Years of [the treatise by] Stokey, Lucas, and Prescott, hosted by the Federal Reserve Bank of Minneapolis, participants at the European Summer Symposium in International Macroeconomics (ESSIM) 2015, and also Ross Doppelt, John Leahy, and Eran Yashiv. We thank Nicolas Petrosky-Nadeau and Etienne Wasmer for inspiring us to add section 5.4, and Lawrence Christiano, Martin Eichenbaum and Mathias Trabandt for conducting the experiment in section 8.2. Ljungqvist's research was supported by a grant from the Jan Wallander and Tom Hedelius Foundation.

1 Introduction

The matching framework is a workhorse for macro labor research about both business cycle and welfare state dynamics. Because it summarizes outcomes of labor market frictions without explicitly modeling them, Petrongolo and Pissarides (2001) call the matching function a black box. Under constant returns to scale in matching, a widely used assumption for which Petrongolo and Pissarides find ample empirical support, a ratio of vacancies to unemployment called market tightness drives unemployment dynamics. To get big responses of unemployment to movements in productivity, matching models require a high elasticity of market tightness with respect to productivity. We identify a channel through which economic forces that generate that high elasticity must operate. Understanding how disparate matching models must work through the same channel sheds light on features essential to produce big unemployment responses to movements in productivity.

With exogenous separation, a comparative steady state analysis decomposes the elasticity of market tightness into two multiplicative factors, both of which are bounded from below by unity. In a matching model of variety j, let $\eta^j_{\theta,y}$ be the elasticity of market tightness θ with respect to productivity y:

$$\eta^j_{\theta,y} \; = \; \Upsilon^j \, \frac{y}{y - x^j}. \tag{1}$$

The first factor Υ^j has an upper bound coming from a consensus about values of the elasticity of matching with respect to unemployment. The second factor $y/(y-x^j)$ is the inverse of what we define as the 'fundamental surplus fraction'. The fundamental surplus $y-x^j$ equals a quantity that deducts from productivity y a value x^j that the 'invisible hand' cannot allocate to vacancy creation, a quantity whose economic interpretation differs across models. Unlike Υ^j, the fraction $y/(y-x^j)$ has no widely agreed upon upper bound. To get a high elasticity of market tightness requires that $y/(y-x^j)$ must be large, i.e., that what we call the fundamental surplus fraction must be small.[1] Across reconfigured matching models, many details differ, but what ultimately matters is the fundamental surplus.

In the standard matching model with Nash bargaining, the fundamental surplus is simply what remains after deducting the worker's value of leisure from productivity, $x = z$. To induce them to work, workers have to receive at least the value of leisure, so the invisible hand cannot allocate that value to vacancy creation.

In other specifications of matching models appearing in Table 1, the fundamental surplus

[1]We call $y - x$ the fundamental surplus and $\frac{y-x}{y}$ the fundamental surplus fraction.

Table 1: Elasticities of market tightness and fundamental surpluses

Business cycle context	Elasticity	Key variables
Nash bargaining (Hagedorn and Manovskii 2008)	$\Upsilon^{\text{Nash}} \dfrac{y}{y-z}$	z, value of leisure
Sticky wage (Hall 2005)	$\Upsilon^{\text{sticky}} \dfrac{y}{y-\hat{w}}$	$\hat{w}$, sticky wage
... and a financial accelerator (Wasmer and Weil 2004)	$\Upsilon^{\text{sticky}} \dfrac{y}{y-\hat{w}-k}$	k, annuitized value of credit search costs
Alternating-offer bargaining (Hall and Milgrom 2008)	$\Upsilon^{\text{sticky}} \dfrac{y}{y-z-\beta(1-s)\gamma}$	γ, firm's cost of delay in bargaining*
Fixed matching cost (Pissarides 2009)	$\Upsilon^{\text{Nash}} \dfrac{y}{y-z-\beta(r+s)H}$	H, fixed matching cost*
Welfare state context†		
Unemployment insurance	$\Upsilon^{\text{Nash}} \dfrac{y}{y-z-b}$	b, unemployment benefit
Layoff costs	$\Upsilon^{\text{Nash}} \dfrac{y}{y-z-\beta s\tau}$	τ, layoff tax*

* Other parameters are the discount factor $\beta = (1+r)^{-1}$, and the separation rate s.

† Theories that attribute high European unemployment to productivity changes include a widened earnings distribution in Mortensen and Pissarides (1999), higher capital-embodied technological change in Hornstein et al. (2007), and shocks to human capital in Ljungqvist and Sargent (2007).

emerges after making other deductions from productivity. In a model with a sticky wage $\hat{w}$, the deduction is simply the wage itself, $x = \hat{w}$, since the invisible hand cannot allocate so much to vacancy creation that there remains too little to pay the wage. If there is also costly acquisition of credit, as in Wasmer and Weil's (2004) model of a financial accelerator, an additional deduction needed to arrive at the fundamental surplus is the annuitized value k of the average search costs for the formation of a unit that can post a vacancy, so here $x = \hat{w}+k$. Similarly, in the case of a layoff tax τ for which liability arises after the formation of an employment relationship, the fundamental surplus under Nash bargaining is obtained by deducting the value of leisure and also a value reflecting the eventual payment of the layoff tax, $x = z + \beta s\tau$, where the product of the discount factor β, match destruction probability s, and the layoff tax τ is an annuity payment that has the same expected present value as the layoff tax. In Hall and Milgrom's (2008) model of alternating-offer wage bargaining, we must deduct both the value of leisure and a quantity measuring a worker's ability to impose a cost of delay γ on the firm. When firms make the first wage offer, the fundamental surplus is obtained by making deduction $x = z + \beta(1 - s)\gamma$ (when it is assumed that a bargaining firm-worker pair faces the same separation rate s before and after an agreement is reached).

This paper finds (1) that as a fraction of productivity, the fundamental surplus must be small to produce high unemployment volatility during business cycles, and (2) that a small fundamental surplus fraction also explains high unemployment coming from adverse welfare state incentives. Within several matching models, we dissect the forces at work by presenting closed-form solutions for steady-state comparative statics, and also by reporting numerical simulations that confirm how those same forces also shape outcomes in stochastic models.

The following mechanical intuition underlies our findings. The fundamental surplus is an upper bound on what the invisible hand can allocate to vacancy creation. A given change in productivity translates into a larger percentage change in the fundamental surplus when the fundamental surplus fraction is small. That induces the invisible hand to make resources allocated to vacancy costs comove strongly with changes in productivity. The relationship is immediate in a matching model with a sticky wage because a free-entry condition in vacancy creation equates the expected cost of filling a vacancy to the expected present value of the difference between a job's productivity and the sticky wage. Consequently, a change in productivity has a direct impact on resources devoted to vacancy creation. If those resources are small relative to output, a given percentage change in productivity translates into a much larger percentage change in resources used for vacancy creation and hence big responses of unemployment to movements in productivity. The relationship is subtler but similar in other matching models in which changes in productivity that have large effects on the fundamental surplus must also affect the equilibrium amount of resources devoted to vacancy creation.

After setting forth a standard matching model in section 2, section 3 explains the role of the fundamental surplus as an object uniting seemingly disparate matching models. We derive steady-state comparative-statics expressions for the elasticity of market tightness with respect to productivity in models of welfare states and business cycles in sections 4 and 5, respectively. Section 6 discusses how the effects of the fundamental surplus are mediated through the dynamics of wages and profits. Stochastic versions of business cycle models are simulated in section 7. Section 8 demonstrates the usefulness of the fundamental surplus in richer environments that embed matching models. Section 9 offers concluding remarks. (Some computational details are relegated to an online appendix.)

We tell where important earlier accounts of the forces at work were incomplete or misleading. Rogerson and Shimer's (2011) attribution of a low (high) elasticity of market tightness to a high (low) elasticity of the wage with respect to productivity in Nash bargaining models identifies neither a necessary nor a sufficient condition. Hall's (2005) sticky wage and Hall and Milgrom's (2008) use of special bargaining protocols to suppress a worker's outside value fail to imply a high elasticity of market tightness because they need not imply a small fundamental surplus fraction. As astutely noted by Hagedorn and Manovskii (2008), a high elasticity of market tightness requires that profits are small and elastic. We show that for that to happen the fundamental surplus fraction must be small. Instead of stopping with proximate causes cast in terms of endogenous outcomes (e.g., small and elastic profits), we advocate focusing on primitives that determine the fundamental surplus. Failure to do in earlier work has occasionally obscured ultimate causes. For example, a decomposition of Petrosky-Nadeau and Wasmer (2013) assigns a multiplicative role to a financial accelerator in the model of Wasmer and Weil (2004). We show that eliminating endogenous quantities in favor of exogenous ones reveals how the fundamental surplus fraction in Table 1 is the essential determinant. On a positive note, Mortensen and Nagypál (2007) combined several forces in ways that are consistent with our argument that all components of productivity that the 'invisible hand' cannot allocate to vacancy creation contribute to increasing the elasticity of market tightness by diminishing the fundamental surplus fraction.

2 Preliminaries

To set the stage, we review key equations and equilibrium relationships for a basic discrete time matching model.[2] There is a continuum of identical workers with measure 1. Workers are infinitely lived and risk neutral with discount factor $\beta = (1 + r)^{-1}$. A worker wants to maximize the expected discounted sum of labor income plus the value of leisure. An employed worker gets labor income equal to the wage w and no leisure. An unemployed worker receives value of leisure $z > 0$ and no labor income.

The production technology has constant returns to scale with labor as the sole input. Each employed worker produces y units of output. Each firm employs at most one worker. A firm incurs a vacancy cost c each period it waits to find a worker. While matched with a worker, a firm's per-period earnings are $y - w$. All matches are exogenously destroyed with per-period probability s. Free entry implies that a new firm's expected discounted stream of vacancy costs plus earnings equals zero.

A matching function $M(u, v)$ determines the measure of successful matches in a period,

[2]See Ljungqvist and Sargent (2012, section 28.3), or for a continuous time version, see Pissarides (2000).

where u and v are aggregate measures of unemployed workers and vacancies. The matching function M is increasing in both arguments, concave, and homogeneous of degree 1. Homogeneity implies that the probability of filling a vacancy is $q(v/u) \equiv M(u, v)/v$. The ratio $\theta \equiv v/u$ of vacancies to unemployed workers is called *market tightness*. The probability that an unemployed worker will be matched in a period is $\theta q(\theta)$.

A firm's values J of a filled job and V of a vacancy satisfy the Bellman equations

$$J = y - w + \beta \left[sV + (1-s)J \right], \tag{2}$$

$$V = -c + \beta \big\{ q(\theta)J + [1 - q(\theta)]V \big\}. \tag{3}$$

After imposing the zero profit condition $V = 0$, equation (3) implies

$$J = \frac{c}{\beta q(\theta)}, \tag{4}$$

which we can substitute into equation (2) to arrive at

$$w = y - \frac{r+s}{q(\theta)} c. \tag{5}$$

A worker's values as employed E and as unemployed U satisfy the Bellman equations

$$E = w + \beta \big[sU + (1-s)E \big], \tag{6}$$

$$U = z + \beta \big\{ \theta q(\theta)E + [1 - \theta q(\theta)]U \big\}. \tag{7}$$

In the basic matching model, the match surplus $S \equiv J + E - U$ is split between a matched firm and worker according to Nash bargaining. The maximizers of Nash product $(E-U)^{\phi} J^{1-\phi}$ satisfy

$$E - U = \phi S \quad \text{and} \quad J = (1-\phi)S, \tag{8}$$

where $\phi \in [0, 1)$ measures the worker's 'bargaining power'. After solving equations (2) and (6) for J and E, respectively, then substituting them into equations (8), we find that the wage rate satisfies

$$w = \frac{r}{1+r} U + \phi \left(y - \frac{r}{1+r} U \right). \tag{9}$$

The annuity value of being unemployed, $rU/(1+r)$, can be obtained by solving equation (7) for $E - U$ and substituting this expression and equation (4) into equations (8):

$$\frac{r}{1+r} U \;=\; z + \frac{\phi\, \theta\, c}{1-\phi}\,. \tag{10}$$

Substituting equation (10) into equation (9) yields

$$w \;=\; z + \phi(y - z + \theta c)\,. \tag{11}$$

The two expressions (5) and (11) for the wage rate jointly determine the equilibrium value of θ:

$$y - z \;=\; \frac{r + s + \phi\, \theta\, q(\theta)}{(1-\phi) q(\theta)}\, c\,. \tag{12}$$

The assumptions of identical workers/vacancies, risk neutrality, and constant returns to scale in both production and matching imply that equilibrium market tightness θ does not depend on composition of workers between those employed and those unemployed. In a steady state, θ determines unemployment by the condition that the measure of workers laid off in a period, $s(1-u)$, must equal the measure of unemployed workers gaining employment, $\theta q(\theta) u$, which implies

$$u \;=\; \frac{s}{s + \theta q(\theta)}\,. \tag{13}$$

We proceed under the assumption that the matching function has the Cobb-Douglas form, $M(u,v) = A u^{\alpha} v^{1-\alpha}$, where $A > 0$, and $\alpha \in (0,1)$ is the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\, \theta / q(\theta)$.

3 Fundamental surplus as essential object

In equation (13), the derivative of steady-state unemployment with respect to market tightness is

$$\frac{d\,u}{d\,\theta} \;=\; -\frac{s\,[q(\theta) \,+\, \theta\, q'(\theta)]}{[s \,+\, \theta\, q(\theta)]^2} \;=\; -\left[1 + \frac{\theta\, q'(\theta)}{q(\theta)}\right] \frac{u\, q(\theta)}{s \,+\, \theta\, q(\theta)} \;=\; -(1-\alpha)\, \frac{u\, q(\theta)}{s \,+\, \theta\, q(\theta)}\,,$$

where the second equality uses equation (13) and factors $q(\theta)$ from the expression in square brackets of the numerator, and the third equality is obtained after invoking the constant elasticity of matching with respect to unemployment. So the elasticity of unemployment with respect to market tightness is

$$\eta_{u,\theta} \;=\; -(1-\alpha)\,\frac{\theta\, q(\theta)}{s\,+\,\theta\, q(\theta)} \;=\; -(1-\alpha)\,\left(1-\frac{s}{s\,+\,\theta\, q(\theta)}\right) \;=\; -(1-\alpha)\,(1-u)\,, \tag{14}$$

where the second equality is obtained after adding and subtracting s to the numerator, and the last third equality invokes expression (13).

To shed light on what contributes to significant volatility in unemployment, we seek forces that can make market tightness θ highly elastic with respect to productivity.

3.1 A decomposition of the elasticity of market tightness

After implicit differentiation of expression (12), we can compute the elasticity of market tightness with respect to productivity as

$$\eta_{\theta,y} \;=\; \frac{(r+s)\,+\,\phi\,\theta\, q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\, q(\theta)}\;\frac{y}{y-z} \;\equiv\; \Upsilon^{\text{Nash}}\,\frac{y}{y-z}\,. \tag{15}$$

(See online appendix A.1.) This multiplicative decomposition of the elasticity of market tightness is central to our analysis. Similar decompositions prevail in all of the reconfigured matching models to be described below. The first factor Υ^{Nash} in expression (15), has counterparts in other setups. A consensus about reasonable calibrations bounds its contribution to the elasticity of market tightness. Hence, the magnitude of the elasticity of market tightness depends mostly on the second factor in expression (15), i.e., the inverse of what in section 1 we defined to be the fundamental surplus fraction.

Shimer's (2005) critique is that for common calibrations of the standard matching model, the elasticity of market tightness is too low to explain business cycle fluctuations. Shimer noted that the average job finding rate $\theta\, q(\theta)$ is large relative to the observed value of the sum of the net interest rate and the separation rate $(r+s)$. When combined with reasonable parameter values for a worker's bargaining power ϕ and the elasticity of matching with respect to unemployment α, this implies that the first factor Υ^{Nash} in expression (15), is close to its lower bound of unity. More generally, the first factor in (15) is bounded from above by $1/\alpha$. Since reasonable values of the elasticity α confine the first factor, it is the second factor $y/(y-z)$ in expression (15) that is critical in generating movements in market tightness. For values of leisure within a commonly assumed range well below productivity, the second factor is not large enough to generate the observed high volatility of market tightness. This is Shimer's critique.

Shimer (2005, pp. 39-40) documented that comparisons of steady states described by expression (15) provide a good approximation to average outcomes from simulations of an economy subject to aggregate productivity shocks. We will derive some closed-form solutions for steady states in other setups. These will shed light on the findings from stochastic simulations to be reported in section 7.

3.2 Relationship to match surplus and outside values

The match surplus is the capitalized surplus accruing to a firm and a worker in the current match. It is the difference between the present value of the match and the sum of the worker's outside value and the firm's outside value. By rearranging equation (7) and imposing the first Nash-bargaining outcome of equations (8), $E - U = \phi S$, the worker's outside value can be expressed as

$$U = \frac{z}{1-\beta} + \frac{\beta}{1-\beta}\theta q(\theta)\,\phi S = \frac{z}{1-\beta} + \Psi_u^{\text{m.surplus}} + \Psi_u^{\text{extra}}, \tag{16}$$

where the second equality decomposes U into three nonnegative parts: (1) the capitalized value of choosing leisure in all future periods, $z(1-\beta)^{-1}$; (2) the sum of the discounted values of the worker's share of match surpluses in his or her as yet unformed future matches[3]

$$\Psi_u^{\text{m.surplus}} = \frac{r+s}{r}\,\frac{\theta\, q(\theta)}{r+s+\theta\, q(\theta)}\,\phi S\,; \tag{17}$$

[3]Let $\Psi_n^{\text{m.surplus}}$ be the analogous capital value of an employed worker's share of all match surpluses over lifetime, including current employment. The capital values $\Psi_u^{\text{m.surplus}}$ and $\Psi_n^{\text{m.surplus}}$ solve the Bellman equations

$$\begin{aligned}
\Psi_u^{\text{m.surplus}} &= 0 + \beta\Big\{\theta q(\theta)\Psi_n^{\text{m.surplus}} + [1-\theta q(\theta)]\,\Psi_u^{\text{m.surplus}}\Big\},\\
\Psi_n^{\text{m.surplus}} &= \psi + \beta\Big\{(1-s)\Psi_n^{\text{m.surplus}} + s\Psi_u^{\text{m.surplus}}\Big\},
\end{aligned}$$

where ψ is an annuity that, when paid for the duration of a match, has the same expected present value as a worker's share of the match surplus, $E - U = \phi S$:

$$\sum_{t=0}^{\infty}\beta^t(1-s)^t\psi = \phi S \qquad \Longrightarrow \qquad \psi = (r+s)\beta\phi S\,.$$

and, key to our new perspective, (3) the parts of fundamental surpluses from future employment matches that are not allocated to match surpluses

$$\Psi_u^{\text{extra}} = \frac{\theta q(\theta)}{r+s}\,\Psi_u^{\text{m.surplus}}, \tag{18}$$

which can be deduced from equation (16) after replacing $\Psi_u^{\text{m.surplus}}$ with expression (17).

We can use decomposition (16) of a worker's outside value U to shed light on the activities of the 'invisible hand' that make the elasticity of market tightness with respect to productivity be low for common calibrations of matching models. As noted above, those parameterizations entail a value of leisure z well below productivity and a significant share ϕ of match surpluses being awarded to workers, which together with a high job finding probability $\theta q(\theta)$ imply that the sum $\Psi_u^{\text{m.surplus}} + \Psi_u^{\text{extra}}$ in equation (16) forms a substantial part of a worker's outside value. Furthermore, Ψ_u^{extra} is the much larger term in that sum, which follows from expression (18) and that $\theta q(\theta)$ is large relative to $r+s$. That big term Ψ_u^{extra} makes it easy for the invisible hand to realign a worker's outside value in a way that leaves the match surplus almost unchanged when productivity changes. Offsetting changes in Ψ_u^{extra} can absorb the impact of productivity shocks so that resources devoted to vacancy creation can remain almost unchanged, which in turn explains why unemployment does not respond sensitively to productivity.

But in other calibrations with a high value of leisure (Hagedorn and Manovskii 2008), the fundamental-surplus components of a worker's outside value are so small that there is little room for the invisible hand to realign things as we have described, making the equilibrium amount of resources allocated to vacancy creation respond sensitively to variations in productivity. That results in a high elasticity of market tightness with respect to productivity. Put differently, since the fundamental surplus is a part of productivity, it follows that a given change in productivity translates into a greater percentage change in the fundamental surplus by a factor of $y/(y-z)$, i.e., the inverse of the fundamental surplus fraction. Thus, the small fundamental surplus fraction in those alternative calibrations having high values of leisure imply large percentage changes in the fundamental surplus. Such large changes in the amount of resources that could potentially be used for vacancy creation cannot be offset by the invisible hand and hence variations in productivity lead to large variations in vacancy creation, resulting in a high elasticity of market tightness with respect to productivity.[4]

[4]It is instructive to consider a single perturbation, $\phi = 0$, to common calibrations of the standard matching model, for which a worker's outside value in expression (16) solely equals the capitalized value of leisure and the worker receives no part of fundamental surpluses, $\Psi_u^{\text{m.surplus}} + \Psi_u^{\text{extra}} = 0$. How come that the elasticity of

Having described how the fundamental surplus supplements the concept of a worker's outside value, we now tell how the fundamental surplus relates to the match surplus. The fundamental surplus is an upper bound on resources that the invisible hand can allocate to vacancy creation. Its magnitude as a fraction of output is the prime determinant of the elasticity of market tightness with respect to productivity.[5] In contrast, although it is directly connected to resources that are devoted to vacancy creation, a small size of the match surplus relative to output has no direct bearing on the elasticity of market tightness. Recall that in the standard matching model, the zero-profit condition for vacancy creation implies that the expected present value of a firm's share of match surpluses equals the average cost of filling a vacancy. Since common calibrations award firms a significant share of match surpluses, and since vacancy cost expenditures are calibrated to be relatively small, it follows that equilibrium match surpluses must form small parts of output across various matching models, regardless of the elasticity of market tightness in any particular model.

From an accounting perspective, match surpluses and firms' profits emerge from fundamental surpluses. Hence, a small fundamental surplus fraction necessarily implies small match surpluses and small firms' profits. But, as we will show, the converse does not hold. Small match surpluses and small firms' profits need not imply small fundamental surpluses. Therefore, the size of the fundamental surplus fraction is the only reliable indicator of the magnitude of the elasticity of market tightness with respect to productivity, as conveyed by expression (15).

Dynamics that are intermediated through the fundamental surplus occur in other popular setups, including those with sticky wages, alternative bargaining protocols and costly acquisition of credit. For example, it matters little if the source of a diminished fundamental surplus fraction is Hagedorn and Manovskii's (2008) high value of leisure for workers, Hall's (2005) sticky wage, Hall and Milgrom's (2008) cost of delay for firms that participate in alternating-offer bargaining, or Wasmer and Weil's (2004) upfront cost for firms to secure credit. A small fundamental surplus fraction causes variations in productivity to have large effects on resources devoted to vacancy creation either because workers insist on being com-

market tightness with respect to productivity remains low for such perturbed parameterizations where large fundamental surpluses only end up as firms' profits which in an equilibrium are all used for vacancy creation? The answer lies precisely in the outcome that firms' profits would then be truly large; therefore, even though variations in productivity then affect firms' profits directly, the percentage wise impact of productivity shocks on such huge profits is negligible so market tightness and unemployment will hardly change. For further discussion of profits and fundamental surpluses, see section 6.

[5]We express the fundamental surplus as a flow value while the match surplus is typically a capitalized value.

pensated for their losses of leisure, or because firms have to pay the sticky wage, or because workers strategically exploit the firm's cost of delay under an alternating-offer bargaining protocol, or because firms must bear the cost of acquiring credit. Likewise, welfare state policies such as unemployment compensation and government-imposed layoff costs can reduce the fundamental surplus fraction. We turn to welfare state policies next.

4 Fundamental surplus and welfare states

4.1 Unemployment compensation

We begin by considering a simplified version of Mortensen and Pissarides' (1999) model of workers who are heterogeneous in their skills and enjoy a welfare state safety net. Mortensen and Pissarides make technological assumptions that imply that unemployed workers enter skill-specific matching functions to match with vacancies at their skill levels. An important assumption for Mortensen and Pissarides is that the value of leisure including unemployment compensation does not vary proportionately with workers' skills. We incorporate these features in our standard matching model by assuming that workers have different productivities y but a common value of leisure z, defined as a value that includes unemployment compensation.

We set a value of leisure $z = 0.6$ and let workers' productivities reside in the domain $[0.6, 1]$. At the high-end of these productivities, $z = .6$ is a value of leisure within a range typical of common calibrations of matching models. Following Mortensen and Pissarides (1999), we assume that $\phi = \alpha = 0.5$, which is also a common calibration: workers' bargaining weight ϕ falls mid-range and equals the elasticity α of matching with respect to unemployment, so that the Hosios efficiency condition is satisfied.

The model period is a day and the discount factor is $\beta = 0.95^{1/365}$, i.e., an annual interest rate of 5 percent.[6] A daily separation rate of $s = 0.001$ means that jobs last on average 2.8 years. For the highest productivity level $y = 1$, we target an unemployment rate of 5 percent, which by equation (13) implies that unemployed workers face a daily job finding probability equal to $\theta\, q(\theta) = 0.019$, which means that the probability of finding a job within a month is 44 percent. According to equilibrium expression (12), two parameters

[6] A short model period helps vacancy creation take place even at low values of the fundamental surplus fraction. Note that the lower bound on the average recruitment cost is attained when market tightness has dropped so low that a vacancy encounters an unemployed worker with probability one, i.e., the recruitment cost becomes certain and equals the one-period vacancy cost.

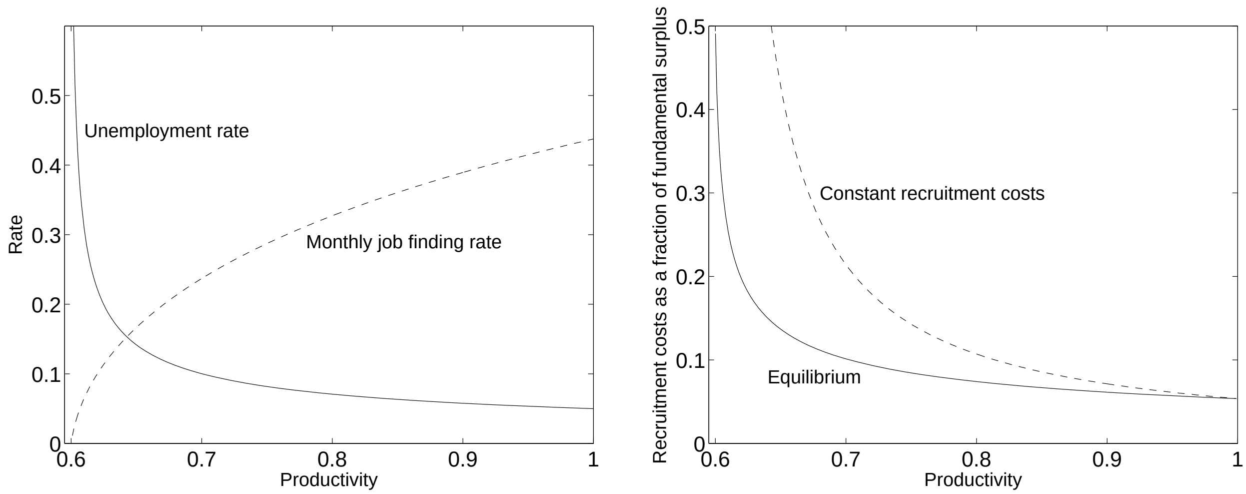


Figure 1: Outcomes for matching functions indexed by productivity. Panel A displays the unemployment rate and monthly job finding rate. Panel B displays annuitized recruitment costs as a fraction of the fundamental surplus, where the solid curve depicts equilibrium outcomes and the dashed curve shows what the fraction would be if recruitment costs were constant and equal to those prevailing at the highest productivity, $y = 1$.

remain to be set to attain this equilibrium outcome: the vacancy cost c and the efficiency parameter A of the matching function. Without any targets for vacancy statistics, this is a choice of normalization, as noted by Shimer (2005).[7] Therefore, we set $c = 0.1$ and adjust the parameter A to attain the unemployment target of 5 percent.

The solid line in Figure 1(a) depicts the equilibrium unemployment rate at different productivity levels. Unemployment spikes at low productivities as the fundamental surplus fraction $(y - z)/y$ approaches zero, in line with formula (15) for the elasticity of market tightness with respect to productivity. Specifically, as the fundamental surplus fraction diminishes, the elasticity of market tightness with respect to additional decreases in productivity increases and hence the increments in unemployment per unit fall in productivity become successively larger as we move to ever lower productivity levels.

The solid line in Figure 1(b) shows how the annuitized value of average recruitment costs becomes an ever larger fraction of the fundamental surplus. The annuitized value is just the difference between productivity and the wage rate, $y - w$, since vacancy creation breaks even when the expected present value of firms' share of match surpluses just cover average recruitment costs. Despite being an increasing fraction of the fundamental surplus, recruitment costs fall in absolute terms when productivity declines, an outcome reflected in

[7] For an account of the normalization, see footnote 31 in online appendix C.

the falling job finding probabilities in Figure 1(a). The dashed line in Figure 1(b) shows the ratio of recruitment costs to the fundamental surplus required if these costs were to have remained constant in absolute terms, and equal to those prevailing at the highest productivity, $y = 1$.

Given our $\phi = 0.5$ calibration (equal bargaining weights for workers and firms), the annuitized match surplus as a fraction of the fundamental surplus is twice that of the solid line in Figure 1(b), so it ranges from a mere 10 percent at high productivities to 100 percent in the limit when productivity approaches the value of leisure. Hence, at the lower range of productivities, recruitment costs are bound to fall with a drop in productivity. At low productivity levels, the annuitized match surplus comprises almost the entire fundamental surplus. Being unable to appropriate the value of a worker's leisure, the "invisible hand" has to let resources allocated to vacancy creation move with productivity. Together with the fact that a given percentage change in productivity becomes so much larger as a percentage change of a small fundamental surplus (of which almost all is now allocated to the match surplus), it follows that the elasticity of market tightness with respect to productivity explodes as productivity approaches the value of leisure.

Mortensen and Pissarides (1999, p. 258) compare the unemployment schedule in Figure 1(a) to that for an economy with lower unemployment compensation parameterized as a lower value of z – 'Europe' versus the 'US' – and conclude that "the relationship between the unemployment rate and worker productivity is *much more convex* in the 'European' case than in the 'US'." As we have shown, this outcome is a necessary consequence of a much smaller fundamental surplus fraction in the economy with a higher value of z.

Next, Mortensen and Pissarides hypothesize that the widening unemployment difference between Europe and the US after the late 1970s can be explained by 'skill-biased' technology shocks, modeled as a mean preserving spread of the distribution of productivities across workers. Figure 1(a) shows that moving workers to a lower range of productivities causes a larger increase in unemployment than the decrease caused by moving workers to higher productivities. Mortensen and Pissarides (1999, p. 259) add that such skill-biased shocks "induce reductions in the participation rate like those observed in the major European economies." In our language, this occurs because the fundamental surplus becomes too small or perhaps even negative, making vacancy creation shut down and market tightness become zero in the matching functions for workers with low productivities.[8]

[8]Instead of assuming that individual workers are permanently attached to their productivity levels, Ljungqvist and Sargent (2007) formulate a matching model with skills that accumulate through work ex-

4.2 Layoff taxes

We assume that the government imposes a layoff tax τ on each layoff. Tax revenues are returned as lump sum transfers to workers, but because they do not affect behavior, these transfers do not appear in the expressions below. To promote transparency, we retain exogenous job destruction and focus on how layoff taxes affect the elasticity of market tightness with respect to productivity. We again find that the elasticity of market tightness with respect to productivity depends on the size of the fundamental surplus fraction.

When liability for the layoff tax arises after the formation of an employment relationship, Nash bargaining solution (8) continues to hold, i.e., a worker and a firm split match surpluses, including a negative match surplus at a separation, $S = -\tau$, according to their respective bargaining powers

$$(1-\phi)(E-U) \;=\; \phi J\,. \tag{19}$$

Hence, in the presence of a layoff tax, Bellman equations for the value J to a firm of a filled job and the value E of an employed worker in expressions (2) and (6) become

$$J \;=\; y - w + \beta\big[-s(1-\phi)\tau + (1-s)J\big] \tag{20}$$

$$E \;=\; w + \beta\big[s(U-\phi\tau) + (1-s)E\big]\,, \tag{21}$$

where we have imposed $V = 0$ so that vacancies break even in an equilibrium. The no-profit condition for vacancies from expression (4) and the value of an unemployed worker from expression (7) remain the same.

Paralleling the steps in section 2, we can derive two expressions that the equilibrium wage must satisfy. (See online appendix A.2.) When equating those two expressions, we obtain the following equation for equilibrium market tightness θ:

$$y - z - \beta s\tau \;=\; \frac{r + s + \phi\,\theta\,q(\theta)}{(1-\phi)q(\theta)}\,c\,. \tag{22}$$

perience. They attribute high European unemployment to an increase in economic turbulence, modeled as skill loss at job separations, and to generous unemployment insurance that is paid as a fixed replacement of a worker's past earnings. If there are separate matching functions for all combinations of skill and benefit levels, the counterpart in Ljungqvist and Sargent's model to workers farthest to the left in Figure 1(a) would be low-skilled, high-benefit unemployed workers, i.e., the lowest fundamental surplus fractions are associated with workers who have suffered skill loss and are now entitled to high benefits based on their past high earnings. Ljungqvist and Sargent also show how the adverse consequences of the presence of a set of low-skilled, high-benefit unemployed workers are diluted when these workers are assigned to matching functions with other workers associated with a larger fundamental surplus, e.g., if there is a single matching function for all unemployed workers.

After implicit differentiation, we can compute the elasticity of market tightness as

$$\eta_{\theta,y} \;=\; \Upsilon^{\text{Nash}} \, \frac{y}{y - z - \beta s\tau} \,. \tag{23}$$

The only difference between the elasticity of market tightness with a layoff tax (23) and the earlier expression (15) without layoff taxes is that the fundamental surplus has an additional deduction of $\beta s\tau$. So long as the firm continues to operate, this is an annuity payment a having the same expected present value as the layoff tax:

$$\sum_{t=0}^{\infty} \beta^t (1-s)^t a \;=\; \sum_{t=1}^{\infty} \beta^t (1-s)^{t-1} s\,\tau \qquad \Longrightarrow \qquad a \;=\; \beta s\tau \,, \tag{24}$$

where the flow of annuity payments on the left side of the first equation starts in the first period of operating and ceases when the job is destroyed, while the future layoff tax on the right side occurs first after the initial period of operation. Since the "invisible hand" can never allocate those resources to vacancy creation, it is appropriate to subtract this annuity value when computing the fundamental surplus.

Some modelers have used the alternative assumption that firms are liable for the layoff tax immediately upon being matched with unemployed workers regardless of whether employment relationships are eventually formed, e.g. see Millard and Mortensen (1997). Under this assumption, online appendix A.3 derives the elasticity of market tightness as a two-factor decomposition similar to expression (23). While details differ, the message remains the same. Since the first factor of the decomposition is confined by a generally accepted upper bound, a high elasticity of market tightness requires that the second factor be large, i.e., that a properly defined fundamental surplus fraction be small.[9]

4.3 Fixed matching cost

In addition to a vacancy posting cost c per period, we now assume that a firm incurs a fixed cost H when matching with a worker. It is instructive to compare outcomes to those from

[9]The first factor is bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$. The second factor involves the Nash bargaining shares, $1-\phi$ and ϕ, for the following two reasons: (i) since the layoff tax must ex ante be financed out of the firm's match surplus, the deduction from the fundamental surplus associated with the layoff tax is amplified by a smaller share $1-\phi$ of the match surplus going to the firm; and (ii) because firms are liable for the layoff tax after merely meeting unemployed workers, workers exploit that fact in bargaining. The latter item (ii) brings an implicit interest cost, weighted by the worker's bargaining power ϕ, to be deducted from the fundamental surplus as if the layoff tax had been incurred already at the beginning of the employment relationship. This resembles the upfront fixed matching cost in section 4.3.

layoff taxes.

If the firm incurs the cost H after bargaining with the worker (e.g., a training cost before work commences), online appendix A.4 derives the elasticity of market tightness to be

$$\eta_{\theta,y} = \Upsilon^{\text{Nash}} \frac{y}{y - z - \beta(r+s)H}, \tag{25}$$

which is almost identical to expression (23), which we derived for the case in which liability for layoff taxes arises after the formation of employment relationships. The only difference is that the fixed matching cost H is incurred at the start and the layoff tax τ at the end of a match. Hence, the fundamental surplus under a fixed matching cost is further reduced by an additional interest cost for the upfront expenditure, βrH.[10]

Beyond reaffirming Pissarides's (2009) insight that the addition of a fixed matching cost increases the elasticity of market tightness, our analysis thus adds the important refinement that the quantitative importance of that fixed cost is inversely related to the ultimate size of the fundamental surplus fraction.

5 Fundamental surplus and business cycles

5.1 Alternative calibration

To confront the Shimer critique, Hagedorn and Manovskii (2008) propose an alternative calibration of a standard matching model that effectively places the economy at the left end of our Figure 1(a). To illustrate, let the productivities 0.61, 0.63, and 0.65 in Figure 1(a) represent three different economies with homogeneous workers. For each economy, we renormalize the efficiency parameter A in the matching function to make the unemployment rate be 5 percent at the economy's postulated productivity level. Figure 2 shows how the steady-state unemployment rate would change if we were to perturb productivity around each economy's productivity level. Specifically, the elasticity of market tightness with respect to productivity would be 65, 22, and 14 in an economy with productivity 0.61, 0.63 and 0.65, respectively. According to formula (14) for the elasticity of unemployment with respect to market tightness (evaluated at our calibration with $\alpha = 0.5$), the corresponding elasticities

[10] Under the alternative assumption that the firm incurs the fixed matching cost before bargaining with the worker, we derive a two-factor decomposition of the elasticity of market tightness in online appendix A.5. Two similarities emerge in comparison to the case of firms being liable for a layoff tax after merely meeting unemployed workers. The first factor of the decomposition is bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$. The second factor involves the firm's share $1-\phi$ of the match surplus as in item (i) in footnote 9.

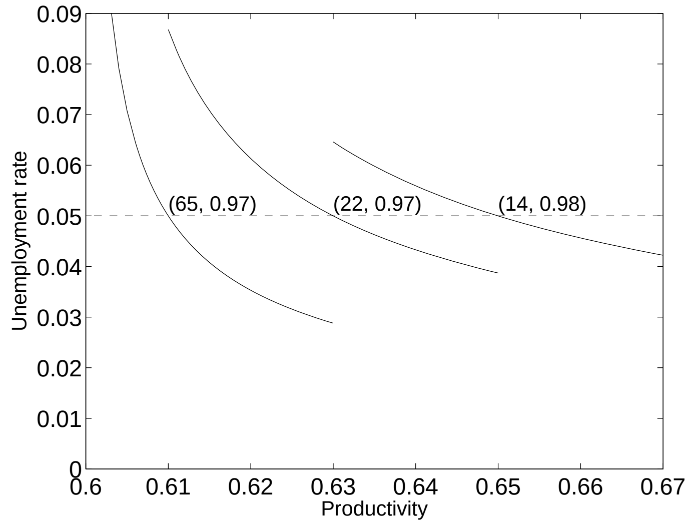


Figure 2: Unemployment for three segments of the schedule in Figure 1(a), with A adjusted to generate 5 percent unemployment at productivities 0.61, 0.63 and 0.65, respectively, with the elasticities of market tightness and of the wage rate, respectively, within parentheses $(\eta_{\theta,y},\, \eta_{w,y})$.

of unemployment with respect to productivity are roughly negative half those numbers for the elasticity of market tightness with respect to productivity.

The relationship between unemployment and productivity in Figure 2 foretells our numerical simulations of economies with aggregate productivity shocks in section 7.2. There we replicate earlier findings that the empirical volatility of unemployment can be reproduced under the Hagedorn-Manovskii calibration but not under the 'common' calibration of the matching model underlying the Shimer critique as presented in section 3.1. As we have demonstrated, what accounts for these different outcomes are the sizes of the fundamental surplus fractions. However, as reported and challenged by Hagedorn and Manovskii (2008, p. 1695), another "prominent explanation of the findings in Shimer (2005) is that the elasticity of wages is too high in his model (0.964). The argument is then that an increase in productivity is largely absorbed by an increase in wages, leaving profits (and, thus, the incentives to post vacancies) little changed over the business cycle." Adhering to that line of reasoning, Rogerson and Shimer (2011, p. 660) emphasize that wages are rigid under the calibration of "Hagedorn and Manovskii (2008), although it is worth noting that the authors do not interpret their paper as one with wage rigidities. They calibrate ... a small value for the workers' bargaining power [$\phi = 0.052$]. This significantly amplifies productivity shocks

..." Evidently, Figure 2 contradicts Rogerson and Shimer's emphasis on wage rigidity because the elasticity of the wage with respect to productivity approximates 0.97 for all three of our economies in Figure 2, where we assume a much higher worker bargaining weight $\phi = 0.5$, yet still obtain Hagedorn and Manovskii's high elasticity of market tightness and unemployment with respect to productivity.

Hagedorn and Manovskii (2008) make this point by taking their calibration and *raising* a worker's bargaining weight in a way that generates the same high wage elasticity as in a common calibration of the matching model, while leaving unemployment very sensitive to changes in productivity. The small fundamental surplus fraction in the Hagedorn-Manovskii calibration governs these results. Hagedorn and Manovskii (2008) also perturb a common calibration of the matching model by *lowering* a worker's bargaining weight in a way that yields the same low wage elasticity as in the Hagedorn-Manovskii calibration, while now retaining the outcome that unemployment is insensitive to changes in productivity. That happens because the fundamental surplus fraction remains high in a common calibration of the matching model. We report outcomes from a similar exercise in Figures 3(a) and 3(b) where we recompute our relationship between unemployment and productivity for different values of a worker's bargaining weight $\phi \in \{0.05, 0.1, 0.2, 0.5, 0.8\}$. The solid line in Figure 3(a) with $\phi = 0.5$ depicts the same curve that appeared in Figure 1(a). The corresponding elasticities of wages with respect to productivity are depicted in Figure 3(b). We conclude from Figure 3(a) that high responses of unemployment to changes in productivity occur only at low fundamental surplus fractions. Figure 3(b) confirms that the Shimer critique of common calibrations does not hinge on a high wage elasticity and that the Hagedorn-Manovskii result is not predicated on a low wage elasticity. We provide yet another perspective of these issues in section 6.2.

In conclusion, we do not deny that common calibrations of the matching model and the specific alternative calibration of Hagedorn and Manovskii (2008) are characterized by high and low wage elasticities, respectively; but we do assert that the size of the calibrated fundamental surplus fraction and not these wage elasticities is the important thing.[11]

[11]For a further discussion of the determinants of the wage elasticity, see online appendix B. Also, at the end of that appendix, we provide an explanation of why the wage elasticities in Figure 2 are higher than those for the solid line ($\phi = 0.5$) in Figure 3(b).

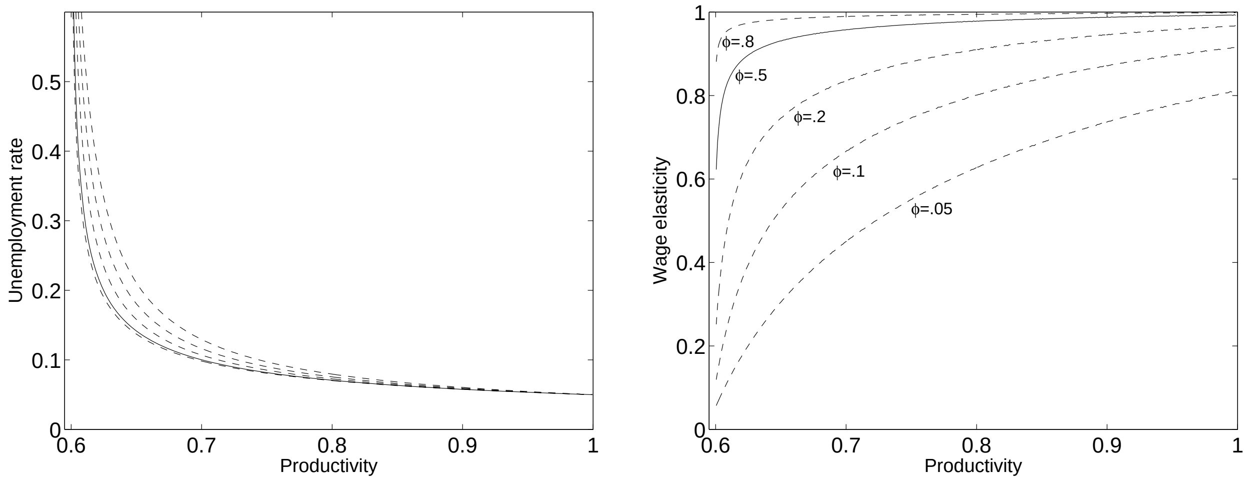


Figure 3: Outcomes for matching functions indexed by productivity. Panel A displays the unemployment rate for different worker's bargaining weights, $\phi \in \{0.05, 0.1, 0.2, 0.5, 0.8\}$ (from top to bottom curve). Panel B displays the wage elasticity with respect to productivity (where the curves now appear in the opposite vertical order).

5.2 Sticky wages

Another response to the Shimer critique is Hall's (2005) analysis of sticky wages, actually a constant wage in his main analysis. Hall notes that a constant wage in a matching model can be consistent with no private inefficiencies in contractual arrangements. Specifically, matching frictions imply a range of wages that the firm and worker both prefer to breaking a match. Hence, the standard assumption of Nash bargaining in matching models is just one way to determine a wage. As an alternative to bargaining, Hall posits a 'wage norm' $\hat{w}$ inside the Nash bargaining set that must be paid to workers. How does such a constant wage change the elasticity of market tightness with respect to productivity? The answer again hinges on the size of an appropriately defined fundamental surplus fraction.

Given a constant wage $w = \hat{w}$, an equilibrium is again characterized by the zero-profit condition for vacancy creation in expression (5) of the standard matching model,

$$\hat{w} \;=\; y - \frac{r+s}{q(\theta)}c\,. \tag{26}$$

There exists an equilibrium for any constant wage $\hat{w} \in [z, y-(r+s)c]$, where the lower bound is a worker's utility of leisure and the upper bound is determined by the zero-profit condition for vacancy creation when the probability of a firm filling a vacancy is at its maximum value of one, $q(\theta) = 1$. After implicitly differentiating (26), we can compute the elasticity of market tightness as

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\frac{y}{y-\hat{w}} \;\equiv\; \Upsilon^{\text{sticky}}\;\frac{y}{y-\hat{w}}\,. \tag{27}$$

This equation for $\eta_{\theta,y}$ resembles the earlier one in (15). Not surprisingly, if the constant wage equals the value of leisure, $\hat{w} = z$, then the elasticity (27) is equal to that earlier elasticity of market tightness in the standard matching model with Nash bargaining when the worker has a zero bargaining weight, $\phi = 0$. With such lopsided bargaining power, the equilibrium wage would indeed be the constant value z of leisure.

From this similarity, we are reminded that the first factor in expression (15) can play a limited role in magnifying the elasticity $\eta_{\theta,y}$ because it is bounded from above by the inverse of the elasticity of matching with respect to unemployment, α. In (27) the bound is attained. So again it is the second factor, the inverse of the fundamental surplus fraction, that tells whether the elasticity of market tightness is high or low. The proper definition of the fundamental surplus is now the difference between productivity and the stipulated constant wage.

In Hall's (2005) model, all of the fundamental surplus goes to vacancy creation (as also occurs in the standard matching model with Nash bargaining when the worker's bargaining weight is zero). A given percentage change in productivity is multiplied by a factor $y/(y - \hat{w})$ to become a larger percentage change in the fundamental surplus. Because all of the fundamental surplus now goes to vacancy creation, there is a correspondingly magnified impact on unemployment. Our interpretation is born out in numerical simulations of economies with aggregate productivity shocks in section 7.1.

5.3 Alternating-offer wage bargaining

Hall and Milgrom (2008) proposed another response to the Shimer critique. They replaced standard Nash bargaining with alternating-offer bargaining. A firm and a worker take turns making wage offers. The threat is not to break up and receive outside values, but instead to continue to bargain because that choice has a strictly higher payoff than accepting the outside option. After each unsuccessful bargaining round, the firm incurs a cost of delay $\gamma > 0$ while the worker enjoys the value of leisure z. There is a probability δ that the job opportunity is exogenously destroyed between bargaining rounds, sending the worker to the unemployment pool.

It is optimal for both bargaining parties to make barely acceptable offers. The firm always offers w^f and the worker always offers w^w. Consequently, in an equilibrium, the first wage offer is accepted. Hall and Milgrom assume that firms make the first wage offer.

In their concluding remarks, Hall and Milgrom (2008, p. 1673) choose to emphasize that "the limited influence of unemployment [the outside value of workers] on the wage results in large fluctuations in unemployment under plausible movements in [productivity]." But it is more enlightening to emphasize that once again the key force is that an appropriately defined fundamental surplus fraction is calibrated to be small. Without a small fundamental surplus fraction, it matters little that the outside value has been prevented from influencing bargaining. We illustrate this idea by computing the elasticity of market tightness with respect to productivity.

After a wage agreement, a firm's value of a filled job, J, and the value of an employed worker, E, remain given by expressions (2) and (6) in the standard matching model. These can be rearranged to become

$$E = \frac{w + \beta\, s\, U}{1 - \beta(1-s)}, \tag{28}$$

$$J = \frac{y - w}{1 - \beta(1-s)}, \tag{29}$$

where we have imposed a zero-profit condition on vacancy creation, $V = 0$, in the second expression. Thus, using expression (28), the indifference condition for a worker who has just received a wage offer w^f from the firm and is choosing whether to decline the offer and wait until the next period to make a counteroffer w^w is

$$\frac{w^f + \beta\, s\, U}{1 - \beta(1-s)} = z + \beta \left[(1-\delta) \frac{w^w + \beta\, s\, U}{1 - \beta(1-s)} + \delta\, U \right]. \tag{30}$$

Using expression (29), the analogous condition for a firm contemplating a counteroffer from the worker is

$$\frac{y - w^w}{1 - \beta(1-s)} = -\gamma + \beta(1-\delta)\, \frac{y - w^f}{1 - \beta(1-s)}. \tag{31}$$

After collecting and simplifying the terms that involve the worker's outside value U, expression (30) becomes

$$\frac{w^f}{1 - \beta(1-s)} = z + \beta(1-\delta)\, \frac{w^w}{1 - \beta(1-s)} + \beta\, \frac{1-\beta}{1 - \beta(1-s)}\, (\delta - s)\, U. \tag{32}$$

As emphasized by Hall and Milgrom, the worker's outside value U has a small influence on bargaining; when $\delta = s$, the outside value disappears from expression (32). That is, under continuing bargaining that ends only either with an agreement or with destruction of the job, the outside value will matter only if the job destruction probability differs before and after reaching an agreement. To strengthen Hall and Milgrom's (2008) observation that the outside value has at most a small influence under their bargaining protocol, we proceed under the assumption that $\delta = s$, so the two indifference conditions (32) and (31) become

$$w^f = (1-\tilde{\beta})\, z + \tilde{\beta}\, w^w, \tag{33}$$

$$y - w^w = -(1-\tilde{\beta})\, \gamma + \tilde{\beta}\, (y - w^f)\,, \tag{34}$$

where $\tilde{\beta} \equiv \beta(1-s)$. Solve for w^w from (34) and substitute into (33) to get

$$w^f = \frac{(1-\tilde{\beta})\left[z + \tilde{\beta}(y+\gamma)\right]}{1-\tilde{\beta}^2} = \frac{z + \tilde{\beta}(y+\gamma)}{1+\tilde{\beta}}\,. \tag{35}$$

This is the wage that a firm would immediately offer a worker when first matched; the offer would be accepted.[12] In an equilibrium, this wage must also be consistent with the no-profit condition in vacancy creation. Substitution of $w = w^f$ from expression (35) into the no-profit condition (5) of the standard matching model results in the following expression for equilibrium market tightness:

$$\frac{z + \tilde{\beta}(y+\gamma)}{1+\tilde{\beta}} = y - \frac{r+s}{q(\theta)}\, c. \tag{36}$$

After implicit differentiation, we can compute the elasticity of market tightness as

$$\eta_{\theta,y} = \frac{1}{\alpha}\; \frac{y}{y - z - \tilde{\beta}\,\gamma}\,, \tag{37}$$

where the fundamental surplus is the productivity that remains after making deductions for the value of leisure z and a firm's discounted cost of delay $\tilde{\beta}\gamma$. The latter item captures the worker's prospective gains from his ability to exploit the cost that delay imposes on the firm. What remains of productivity is the fundamental surplus that could potentially be extracted

[12] When firms make the first wage offer, a necessary condition for an equilibrium is that w^f in expression (35) is less than productivity y, i.e., the parameters must satisfy $z + \tilde{\beta}\gamma < y$.

by the ‘invisible hand’ and devoted to sustaining vacancy creation in an equilibrium.

While Hall and Milgrom (2008, p. 1670) notice that their “sum of z and γ is … not very different from the value of z by itself in … Hagedorn and Manovskii’s calibration” (as studied in our section 5.1), they downplay this similarity and choose to emphasize differences in mechanisms across Hagedorn and Manovskii’s model and theirs. But properly focusing on the fundamental surplus tells us that it is their similarity that should be stressed – the two models are united in requiring a small fundamental surplus fraction to generate high unemployment volatility over the business cycle.

To summarize, we do not doubt that the alternative bargaining protocol of Hall and Milgrom (2008) suppresses the influence of the worker’s outside value during bargaining. But this outcome would be irrelevant had Hall and Milgrom not calibrated a small fundamental surplus fraction.

5.4 A financial accelerator

Wasmer and Weil (2004) explore how a financial accelerator affects the elasticity of labor market tightness with respect to productivity. They assume that matching in a credit market precedes matching in the labor market. Credit matching determines equilibrium measures e and f of entrepreneurs and financiers, respectively, the two inputs into a matching function for the credit market. Matched entrepreneur-financier pairs then post vacancies in a matching function for labor. As before, the labor market matching function matches vacancies with workers. Filled jobs and the entrepreneur-financier matches that helped to create them are exogenously destroyed with per-period probability s.

The credit market matching function has constant returns to scale. Credit market tightness $\sigma \equiv e/f$ determines the probability $p(\sigma)$ $(\sigma p(\sigma))$ that an entrepreneur (a financier) finds a counterparty. Per-period credit market search costs of an entrepreneur and a financier are denoted $\epsilon > 0$ and $\kappa > 0$, respectively. A successfully matched entrepreneur-financier pair immediately posts one vacancy in the matching function for the labor market. An entrepreneur-financier pair shares the value of a vacancy according to Nash bargaining, ξ and $1-\xi$ being the bargaining power of the entrepreneur and the financier, respectively. In their main setup, Wasmer and Weil (2004) assume a sticky wage $\hat{w}$ in the labor market. The key question ultimately to be studied is how the elasticity of labor market tightness differs from that of Hall’s (2005) sticky wage model described above in section 5.2.

In an equilibrium, costly search in the credit market assumes a strictly positive value of a vacancy in the labor market, $V > 0$. Nash bargaining awards the entrepreneur and the financier ξV and $(1-\xi)V$, respectively. Free entry on both sides of the credit market ensures that the two parties expect to break even, so that the per-period search costs of an entrepreneur and a financier equal their respective expected payoffs:

$$\epsilon \;=\; p(\sigma)\xi V \quad \text{and} \quad \kappa \;=\; \sigma p(\sigma)(1-\xi)V. \tag{38}$$

The zero expected profits conditions (38) imply that the equilibrium value of a vacancy in the labor market equals total search costs in the credit market divided by the number of entrepreneur-financier pairs formed, which is the average search cost incurred for the formation of an entrepreneur-financier pair in the credit market:

$$V \;=\; \frac{\epsilon}{p(\sigma)} + \frac{\kappa}{\sigma p(\sigma)} \;\equiv\; K(\sigma). \tag{39}$$

The zero expected profits conditions (38) also imply that equilibrium credit market tightness σ is a function solely of relative bargaining powers and relative per-period search costs,

$$\sigma \;=\; \frac{1-\xi}{\xi}\frac{\kappa}{\epsilon} \;\equiv\; \sigma^{\star}. \tag{40}$$

The value of a vacancy continues to be given by equation (3), which can be solved for the value of a filled job

$$J \;=\; \frac{1}{\beta q(\theta)}\left[c + \Big(1-\beta[1-q(\theta)]\Big)K(\sigma^{\star})\right], \tag{41}$$

where we have invoked equilibrium outcomes (39) and (40), i.e., $V = K(\sigma^{\star})$. Another expression for the value J of a filled job is obtained by solving a pertinent version of Bellman equation (2), namely,[13]

$$J \;=\; y - \hat{w} + \beta(1-s)J\,,$$

"forward" to obtain

$$J \;=\; \frac{y - \hat{w}}{\beta(r+s)}\,. \tag{42}$$

The equilibrium value of labor market tightness θ adjusts to equate expressions (41) and

[13]Note that upon the destruction of a job in the Wasmer and Weil setup, the entrepreneur-financier pair also breaks up so that the value of a vacancy V vanishes in the Bellman equation (2).

(42) for the value J of a filled job

$$q(\theta)\Big[y - \hat{w} - (r+s)\beta K(\sigma^\star)\Big] \;=\; (r+s)\Big[c \,+\, (1-\beta)K(\sigma^\star)\Big]\,. \tag{43}$$

After implicit differentiation, we can compute the elasticity of market tightness as

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\frac{y}{y-\hat{w}-k}\,. \tag{44}$$

where $k \equiv (r+s)\beta K(\sigma^\star)$. Comparing (44) to the elasticity (27) that emerges from Hall's (2005) sticky wage model, we observe that (44) deducts another term k from the fundamental surplus, namely, the annuitized value of the average search costs incurred to form an entrepreneur-financier pair.[14] In the Wasmer and Weil setup, the invisible hand cannot use these resources to pay for vacancy costs in the labor market, since they are required to assure that entrepreneur-financier pairs on average earn zero expected profits from investing in credit market search costs.[15] [16]

[14]To have the same expected present value as the average search costs in the credit market, $K(\sigma^\star)$, we compute an annuity k with a stream that starts when an entrepreneur-financier pair matches with a worker, and ends at the job's stochastic destruction:

$$\sum_{t=0}^{\infty}\beta^t(1-s)^t k \;=\; K(\sigma^\star) \qquad \Longrightarrow \qquad k \;=\; (r+s)\beta K(\sigma^\star)\,.$$

[15]Petrosky-Nadeau and Wasmer (2013, first equation on page 201) construe a multiplicative role of the financial accelerator in Wasmer and Weil's model, as said to be represented by the second factor $J/(J-K(\sigma^\star))$ in the following steady-state version of their expression for the elasticity of market tightness,

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;\frac{J}{J-K(\sigma^\star)}\;\frac{y}{y-\hat{w}}\,.$$

Though, after invoking equilibrium expression (42) for the value J of a filled job, our expression (44) for the elasticity reemerges. Hence, we conclude that there is no multiplicative role of the financial accelerator but rather, the annuitized value of the average search costs in the credit market simply diminishes the fundamental surplus.

[16]For the elasticity of market tightness under Nash-bargained wages in the Wasmer and Weil setup, see online appendix A.6 where we adopt the block-bargaining assumption of Petrosky-Nadeau and Wasmer's (2013) extension, i.e., the entrepreneur and the financier form a block when bargaining with the worker. The resulting decomposition of the elasticity of market tightness for this setting resembles the case of a fixed matching cost incurred by the firm before bargaining with the worker. (See footnote 10.)

6 Fundamental surpluses and profits

To shed further light on why a small fundamental surplus fraction is necessary for small changes in productivity to have large effects on unemployment, we examine the chain of causality that operates through the relative impact of productivity on firms' profits. We use the fact that, in an equilibrium, free entry into job creation implies that the expected present value of a firm's profits is related to the average vacancy expenditures for filling a job. Except for models of layoff taxes, fixed matching costs, and a financial accelerator, all our preceding derivations of analytical expressions for the elasticity of market tightness with respect to productivity involve the equilibrium condition:[17]

$$c \;=\; \beta\, q(\theta)\, \frac{y - w(y,\theta)}{1-\beta(1-s)}\,, \tag{45}$$

where we have substituted expression (29), the value of a job expressed as the expected present value of its profits, into break-even condition (4) for vacancy creation. For each particular model, the equilibrium wage $w(y,\theta)$ is a nondecreasing function in productivity y and market tightness θ.

Differentiating expression (45) with respect to y and θ yields

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\,\left[1 - w_y(y,\theta) - w_\theta(y,\theta)\,\frac{d\,\theta}{d\,y}\right]\,\frac{y}{y - w(y,\theta)}\,, \tag{46}$$

which is an alternative to our earlier formulas (15), (27) and (37) for the elasticity of matching with respect to productivity. It is a decomposition into three multiplicative components. As discussed above, the magnitude of the first component $1/\alpha$ is restricted by a consensus about what are reasonable estimates of the elasticity α. Delineated by square brackets, the second component involves nonnegative derivatives of the wage and the positive derivative of market tightness with respect to productivity. An upper bound of unity for the second component would be attained if $w_y = w_\theta = 0$. Thus, we conclude that a necessary condition for expression (46) to be really large is that its third component is large, i.e., for any possibility of market tightness to respond sensitively to small changes in productivity, a firm's profit relative to productivity, $(y-w)/y$, must be small.

Interpretations of the three multiplicative components in expression (46) are straight-

[17] The argument below can easily be modified so that the conclusions extend to the models of layoff taxes, fixed matching costs, and a financial accelerator.

forward. The second component is the fraction of an increment in productivity that is not siphoned off to the wage but passed on to a firm's profit. In relative terms, any change in productivity that is passed on to profits translates into a greater percentage change in profits by a factor of $y/(y-w)$, i.e., the third component. The first component $1/\alpha$ determines then how these dynamics map into an elasticity of market tightness with respect to productivity, where the free-entry condition in vacancy creation ensures that the relative change in a firm's profit is also the relative change in average vacancy expenditures for filling a job. The larger such a relative change in vacancy expenditures caused by a change in productivity, the larger is the elasticity of market tightness with respect to productivity.

To understand how these observations square with the all-important concept of the fundamental surplus fraction, we start by considering instances when the wage is invariant to market tightness, and then proceed to studying outcomes under Nash bargaining.

6.1 Wage invariant to market tightness

If the wage $w(y)$ is a function solely of productivity, expression (46) simplifies to become

$$\eta_{\theta,y} \;=\; \frac{1}{\alpha}\;[1-w'(y)]\;\frac{y}{y-w(y)}\,. \tag{47}$$

In the case of a constant wage in section 5.2, $w(y)=\hat{w}$ and $w'(y)=0$, this is immediately seen to be our earlier expression (27). As already discussed, the firm's profit then coincides with the fundamental surplus.

Another example of a wage invariant to market tightness is the outcome under alternating-offer bargaining in section 5.3 when the parameterization is such that outside values have no influence on bargaining ($\delta = s$). Under the assumption that firms formulate the first wage offer, we can use equilibrium wage expression (35) to compute

$$w'(y) \;=\; \frac{\tilde{\beta}}{1+\tilde{\beta}} \qquad \text{and} \qquad y-w(y) \;=\; \frac{y-z-\tilde{\beta}\gamma}{1+\tilde{\beta}}\,. \tag{48}$$

Given the constancy of derivative $w'(y)$ in (48), the second component of expression (47) is also a constant (less than one). Hence, attaining a high elasticity of market tightness can focus on making the third component large, i.e., making profit small. As compared to the fundamental surplus identified in expression (37), we see that a firm's profit $y-w(y)$ in (47) constitutes a constant fraction $(1+\tilde{\beta})^{-1}$ of the fundamental surplus.

Thus, two common calibration targets are well aligned in these frameworks: (1) a high elasticity of market tightness with respect to productivity chosen to generate business cycle fluctuations, and (2) small profits chosen to be consistent with data on small vacancy expenditures relative to output. Hitting both targets is helped by setting a sufficiently high constant wage in the sticky-wage model, or a sufficiently high firm's cost of delay in bargaining in the alternating-offer bargaining model (for a given value of leisure).

Things get more complicated under Nash bargaining, to which we turn next. While a small fundamental surplus fraction trivially implies small profits (because the former is an upper bound on what the 'invisible hand' could ever allocate to vacancy creation and the latter reflects what is actually allocated in an equilibrium), small profits are no longer a sure indication of the size of the fundamental surplus fraction.

6.2 Wage under Nash bargaining

As discussed and refuted in section 5.1, a prominent explanation for the low elasticity of market tightness with respect to productivity in common calibrations of the standard matching model is that the elasticity of wages with respect to productivity is too high. Rogerson and Shimer (2011) misattribute the success of Hagedorn and Manovskii (2008) in raising the elasticity of market tightness to their calibration of a rigid wage. To supplement our earlier numerical counterexamples and discussion of the steady-state comparative-statics expression (15) for the elasticity of market tightness (as well as for the wage elasticity in online appendix B), our expression (46) for the elasticity of market tightness can be used to provide yet another perspective on these issues.

For a given value of leisure z, recall that we generated different wage elasticities by varying a worker's bargaining weight ϕ. The striking finding is that the elasticity of market tightness is almost invariant to such perturbations. In terms of expression (46), the explanation is that varying a worker's bargaining weight ϕ has countervailing effects on the second and third components. Hagedorn and Manovskii (2008, p. 1696) summarize their explorations: "What matters for the incentives to post vacancies is the size of the percentage changes of profits in response to changes in productivity. These percentage changes are large if the size of profits is small and the increase in productivity is not fully absorbed by an increase in wages. In the standard [matching] model, conditional on the choice of z, the bargaining parameter [ϕ] determines both the level and the volatility of wages. Thus, if we fix z and raise [ϕ], wages rise and become more cyclical, meaning that profits become smaller but less cyclical. These two opposing effects almost exactly cancel each other out. Thus, the volatility of labor market tightness is almost independent of [ϕ] and is determined only by the level of z."

While our formula (46) cast in terms of endogenous quantities symptomizes these countervailing forces, it cannot tell us whether that invariant elasticity of market tightness is low or high. But after evaluating formula (46), by invoking wage expression (11) and using equilibrium condition (12) to eliminate intermediate variables, our earlier expression (15) reemerges. Once again, it is abundantly clear that the elasticity of market tightness "is determined only by the level of z," or more properly, by the fundamental surplus fraction $(y - z)/y$.

6.3 Profits under Nash bargaining

It is useful to compute the relationship of the fundamental surplus to a firm's portion of the match surplus, namely, a firm's profits. After substituting Nash bargaining outcome (8) into break-even condition (4) for vacancy creation, the equilibrium condition $c = \beta q(\theta)(1 - \phi)S$ emerges, which when substituted into equilibrium condition (12) yields

$$S = \frac{1 + r}{r + s + \phi\, \theta\, q(\theta)}\, (y - z). \tag{49}$$

In section 7, to study effects of alternative parameter values, we will draw isoquants for the match surplus. So suppose that a parameterization in expression (49) has the same match surplus as an alternative parameterization indexed by the superscript star, i.e., $S = S^\star$, so that

$$\frac{1 + r}{r + s + \phi\, \theta\, q(\theta)}\, (y - z) \;=\; \frac{1 + r^\star}{r^\star + s^\star + \phi^\star\, \theta^\star\, q^\star(\theta^\star)}\, (y - z^\star). \tag{50}$$

Further, let the two parameterizations share the same interest rate, $r = r^\star$, and separation rate, $s = s^\star$, and also let each matching function be calibrated to target the same unemployment rate, i.e., by expression (13), the job finding rates will be the same, $\theta q(\theta) = \theta^\star q^\star(\theta^\star)$, so that expression (50) can be rearranged to read,

$$\frac{y - z}{y - z^\star} = \frac{r + s + \phi\, \theta\, q(\theta)}{r + s + \phi^\star\, \theta\, q(\theta)}\,. \tag{51}$$

To arrive at an analogous expression for isoprofit curves, we simply multiply the left and right sides of expression (49) by $(1 - \phi)$. Then, we follow the same steps as above, but now for two parameterizations with the same value for a firm's profits, $(1-\phi)S = (1-\phi^\star)S^\star$, which yields

$$\frac{y-z}{y-z^\star} = \frac{1-\phi^\star}{1-\phi}\,\frac{r+s+\phi\,\theta\,q(\theta)}{r+s+\phi^\star\,\theta\,q(\theta)}\,. \tag{52}$$

According to expressions (51) and (52), along an isoquant for the match surplus or for a firm's profits, a smaller fundamental surplus is associated with a lower worker's bargaining power. Could this be the reasoning underlying the common attributions of high elasticities of market tightness to low elasticities of the wage with respect to productivity? (See section 5.1.) Although we are unaware of earlier such rationalizations, it is evidently true that holding the match surplus or a firm's profits constant, the fundamental surplus (fraction) and a worker's bargaining power move in opposite directions. Hence, a higher fundamental surplus fraction increases the elasticity of market tightness and the associated lower bargaining power of a worker decreases the elasticity of the wage with respect to productivity.

But as the numerical analysis of the next section will illustrate, when we allow for the match surplus to change across parameterizations, relationships between the elasticities of market tightness and the wage vanish. Instead, expression (15) is back in full force, making the fundamental surplus fraction the sole determinant of the elasticity of market tightness. The quantitative analysis in section 7.2 demonstrates how small match surpluses or small firms' profits are unreliable indicators of the elasticity of market tightness in Nash bargaining models.

7 Business cycles

To illustrate how a small fundamental surplus fraction is essential for generating ample unemployment volatility over the business cycle in matching models, we turn to Hall's (2005) specification with discrete time and a random productivity process. The monthly discount factor β corresponds to a 5-percent annual rate and the value of leisure is $z = 0.40$. The elasticity of matching with respect to unemployment is $\alpha = 0.235$, and the exogenous monthly separation rate is $s = 0.034$. Aggregate productivity takes on five values y_s uniformly spaced around a mean of one on the interval $[0.9935, 1.00565]$, and is governed by a monthly transition probability matrix Π with probabilities that are zero except as follows: $\pi_{1,2} = \pi_{4,5} = 2(1-\rho)$, $\pi_{2,3} = \pi_{3,4} = 3(1-\rho)$, with the upper triangle of the transition matrix symmetrical to the lower triangle and the diagonal elements equal to one minus the sums of the nondiagonal elements. The resulting serial correlation of y is ρ, which is parameterized to be $\rho = 0.9899$.

However, to facilitate our sensitivity analysis, we alter Hall's model period from one month to one day. (See online appendix C.) As mentioned in footnote 6, a shorter model period fosters the existence of equilibria with vacancy creation.

7.1 Hall's (2005) sticky wage model

As in Hall's (2005) analysis of a constant wage, we postulate a fixed wage $\hat{w} = 0.9657$, which equals the flexible wage that would prevail at the median productivity level under standard Nash bargaining (with equal bargaining weights, $\phi = 0.5$). Figure 4(a) reproduces Hall's figures 2 and 4 for those two models. The solid line and the upper dotted line depict unemployment rates at different productivities for the sticky-wage model and the standard Nash-bargaining model, respectively.[18] Unemployment is almost invariant to productivity under Nash bargaining but responds sensitively under the sticky wage. The outcomes are explained by differences in job-finding rates as shown by the dashed line and the lower dotted line for the sticky-wage model and the standard Nash-bargaining model, respectively, expressed at our daily frequency.[19] Under the sticky wage, high productivity causes firms to post many vacancies, making it easy for unemployed workers to find jobs, while the opposite is true when productivity is low.

Starting from this verification of our conversion of Hall's (2005) model into a daily frequency, we conduct a sensitivity analysis of the choice of the fixed wage. The solid line in Figure 4(b) shows how the average unemployment rate varies with the fixed wage $\hat{w}$. Note how a small range of wages spans outcomes from a very low to a very high average unemployment rate. Small variations in a fixed wage close to productivity generate large changes in the fundamental surplus fraction, $(y - \hat{w})/y$, which by free entry of firms map directly into the amount of resources devoted to vacancy creation. The dashed line in Figure 4(b) traces out the implications for the volatility of unemployment. The standard deviation of unemployment is nearly zero at the left end of the graph when the job-finding probability is almost one for all productivity levels. Unemployment volatility then increases for higher constant wages until, outside of the graph at the right end, vacancy creation becomes so

[18]Unemployment is a state variable that is not just a function of the current productivity, as are all of the other variables, but depends on the history of the economy. But high persistence of productivity and the high job-finding rates make the unemployment rate that is observed at a given productivity level be well approximated by expression (13) evaluated at the market tightness θ prevailing at that productivity (see Hall (2005, p. 59)).

[19]Our daily job-finding rates are roughly 1/30 of the monthly rates in Hall (2005, figures 2 and 4), confirming our conversion from a monthly to a daily frequency.

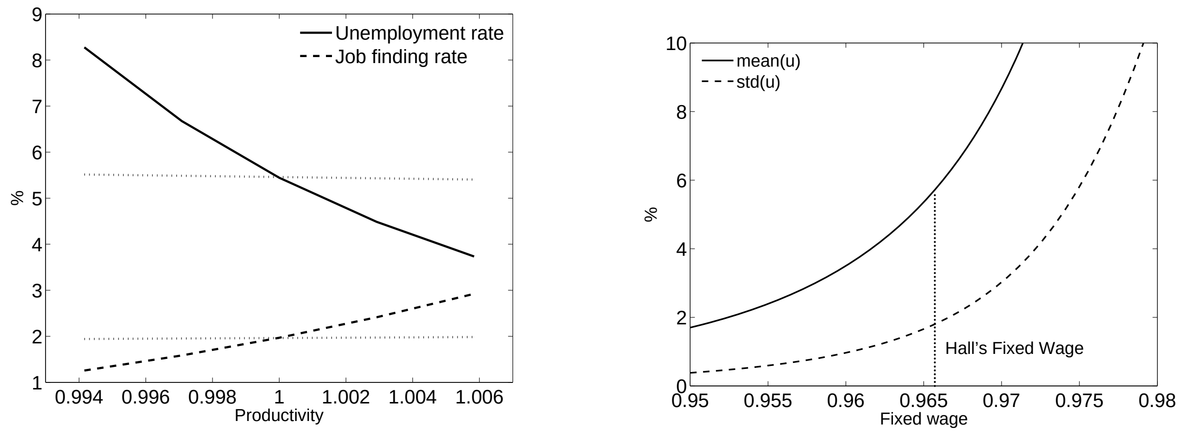


Figure 4: Outcomes in the sticky-wage model. Panel A displays unemployment rates and daily job-finding rates at different productivities (given a fixed wage $\hat{w} = 0.9657$), where the dotted lines with almost no slopes are counterparts from a standard Nash-bargaining model. Panel B displays the average unemployment rate and the standard deviation of unemployment for different postulated values of the fixed wage.

unprofitable that average unemployment converges to its maximum of 100 percent and there are no more fluctuations.

At Hall's fixed wage $\hat{w} = 0.9657$, Figure 4(b) shows a standard deviation of unemployment equal to 1.80 percentage points, which is close to the target of 1.54 to which Hall (2005) calibrated his model. Next, we will see how the standard Nash-bargaining model can attain the same volatility of unemployment by elevating the value of leisure.

7.2 Hagedorn and Manovskii's (2008) high value of leisure

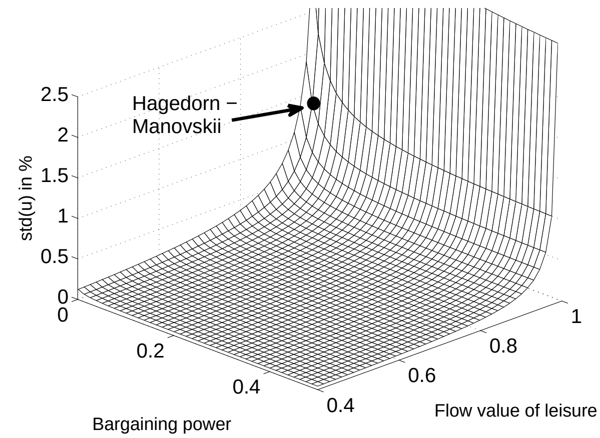
Using Hall's (2005) parameters but assuming standard Nash wage bargaining, we revisit the analysis of Hagedorn and Manovskii (2008) by postulating a high value of leisure, $z = 0.960$, and a low bargaining power of workers, $\phi = 0.0135$. For those parameter values, we do indeed obtain a high standard deviation of 1.4 percentage points for unemployment. To shed light on the sensitivity of outcomes to the choice of parameters, Figure 5(a) depicts outcomes for different constellations of $z \in [0.4, .99]$ and $\phi \in [0.001, 0.5]$. For each pair (z, ϕ), we adjust the efficiency parameter A of the matching function to make the average unemployment rate be 5.5 percent. As we would expect, a high value of leisure, i.e., a small fundamental surplus fraction, is essential for obtaining large variations in market tightness and a high volatility of unemployment.

As detailed in section 5.1 (and elaborated upon in online appendix B), Hagedorn and Manovskii (2008) require a low bargaining power of workers to match the elasticity of wages with respect to productivity. When we revisit their argument based on $(z, \phi) = (0.960, 0.0135)$ in the otherwise same parametrized environment of Hall (2005), the resulting wage elasticity of 0.44 is approximately the value to which Hagedorn and Manovskii calibrated their original analysis. Following the computational approach in Figure 5(a), Figure 5(b) provides a sensitivity analysis for the choice of parameter values (z, ϕ). The figure verifies the necessity for a low ϕ to obtain a low wage elasticity.

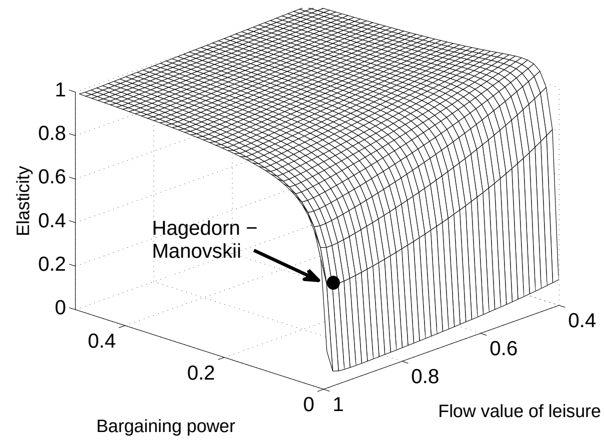
Taken together, Figures 5(a) and 5(b) show how the low wage elasticity of Hagedorn and Manovskii (2008) is incidental to and neither necessary nor sufficient to obtain a high volatility of unemployment. Hence, earlier misinterpretations stressing the importance of a rigid wage (Rogerson and Shimer 2011) should be set aside and replaced by the correct statement that the fundamental surplus fraction must be small.

A deceptive first impression of Figure 5(c) is that the match surplus might also be closely related to the elasticity of market tightness, because small match surpluses at values of leisure close to one coincide with high standard deviations of unemployment in Figure 5(a). But this outcome follows merely from the fact that the fundamental surplus is an upper bound on resources that the invisible hand can allocate to vacancy creation and hence, when the fundamental surplus becomes tiny at values of leisure close to one, the match surplus must also become small. To appreciate the forces actually driving outcomes, we should check if isoquants for the match surplus line up with isoquants for the standard deviation of unemployment, as they clearly do for the fundamental surplus. The diagonal dashed line in Figure 5(d) is the isoquant for the match surplus of Hall's (2005) version of a Nash bargaining model with parameter values $z = 0.4$ and $\phi = 0.5$. Since Hall's parameterization exemplifies a common calibration of the standard matching model, it follows that the elasticity of market tightness and the standard deviation of unemployment are small. However, to the northwest along that same isoquant in Figure 5(d), there are other Hagedorn-Manovskii (2008) like calibrations with the exactly same match surplus but with much higher standard deviations of unemployment. This reasoning leads us to conclude that the size of the match surplus is not a good indicator of the elasticity of market tightness.

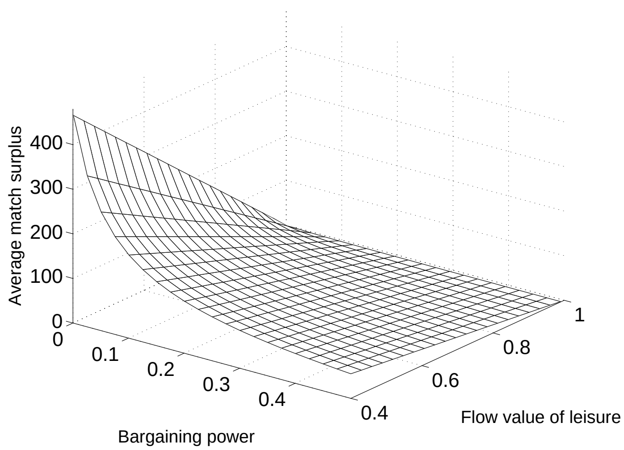
A similar message is conveyed by the isoprofit curves in Figure 5(d): the three solid curves with the middle one representing a firm's profits in Hall's (2005) Nash bargaining model. Recall our discussion in the introduction of Hagedorn and Manovskii's (2008) discerning observation that profits need to be small and elastic for unemployment to respond sensitively



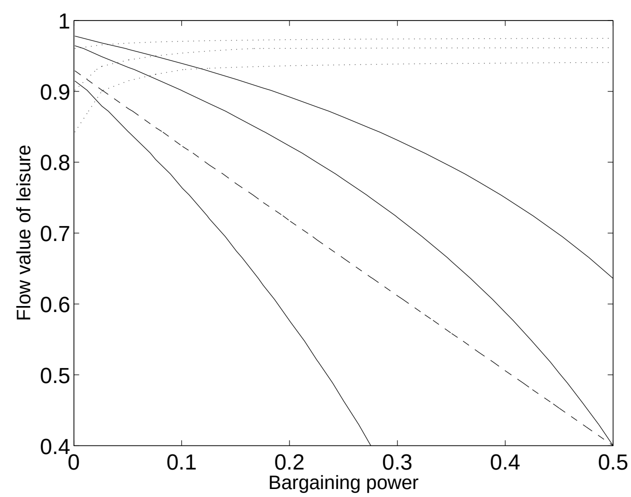
(a) Std of unemployment



(b) Wage elasticity



(c) Match surplus



(d) Isoprofit curves (solid)

Figure 5: Outcomes in the Nash-bargaining model for different constellations of the value of leisure z, and the bargaining power of workers ϕ. Panel (a) displays the standard deviation of unemployment in percentage points. Panel (b) shows the wage elasticity with respect to productivity. Panel (c) depicts the match surplus. In panel (d), the diagonal dashed line is an isoquant for the match surplus distilled from panel (c), the three solid lines are isoprofit curves, and the three dotted curves are isoquants for the standard deviation of unemployment in panel (a).

to movements in productivity. Along the top two isoprofit curves or any higher ones not shown in Figure 5(d), profits are held constant at relatively low levels. But for any such small profits to be elastic also with respect to changes in productivity, i.e., for small profits to be associated with high standard deviations of unemployment, the fundamental surplus fraction must be small in Figure 5(d). We are again reminded that the fundamental surplus fraction is the determinant of the elasticity of market tightness, and that small match surpluses or small firms' profits are unreliable indicators thereof.[20]

7.3 Hall and Milgrom's (2008) alternating-offer bargaining

Finally, we turn to Hall and Milgrom's (2008) model of alternating-offer wage bargaining as another way to increase unemployment volatility in a matching framework. Except for the wage formation process, their environment is the same as that of Hall (2005) but parameterized quite differently. It is important to stress one difference between Hall and Milgrom's parameterization and that of Hall (2005) because it plays an important role in setting the fundamental surplus: they raised the value of leisure to $z = 0.71$ from Hall's value of $z = 0.40$. Section 5.3 taught us that the value of leisure and the firm's cost of delay in bargaining (γ) are likely to be critical determinants of the elasticity of market tightness with respect to productivity and hence of the volatility of unemployment.

Hall and Milgrom (2008) choose instead to emphasize how much the outside value of unemployment is suppressed in alternating-offer wage bargaining since disagreement no longer leads to unemployment but instead to another round of bargaining. Hence, a key parameter from Hall and Milgrom's perspective is the exogenous rate δ at which parties break up between bargaining rounds. Figure 6 shows how different constellations of (γ, δ) affect the standard deviation of unemployment. For each pair (γ, δ), we adjust the efficiency parameter A of the matching function to make the average unemployment rate be 5.5 percent. For the record, Hall and Milgrom (2008) argue that productivity shocks are not the sole source of unemployment fluctuations and consequently they lower their target standard deviation of unemployment to 0.68 percentage points – a target attained with their parameterization $(\gamma, \delta) = (0.27, 0.0055)$ and reproduced in Figure 6.

Figure 6 supports our earlier finding that the cost of delay γ together with the value of leisure z are paramount for generating higher volatility of unemployment. Without a cost of

[20] The isoquant for the match surplus and the isoprofit curves in Figure 5(d) depict averages in our stochastic specification of the Nash bargaining model. The steady-state expressions (51) and (52) provide almost perfect approximations to those outcomes.

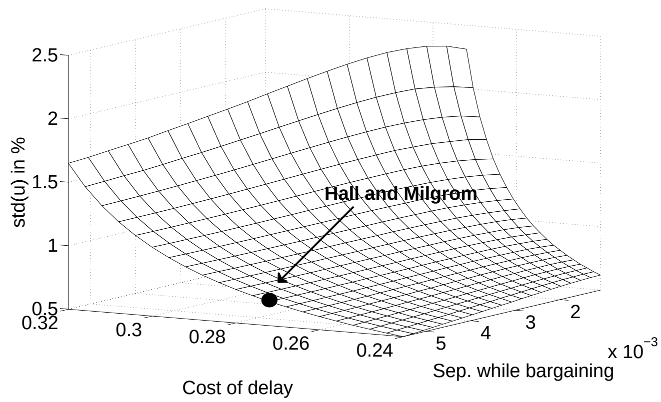


Figure 6: The standard deviation of unemployment in percentage points for different constellations of firms' cost of delay in bargaining (γ) and the exogenous separation rate while bargaining (δ) in the alternating-offer bargaining model.

delay sufficiently high to reduce the fundamental surplus fraction, the exogenous separation rate between bargaining rounds matters little.[21]

8 More complicated environments

To watch the fundamental surplus working its will in more complicated environments, we investigate the roles of matching models in Hornstein et al. (2007) and Christiano et al. (2015). For further details on our two explorations, see online appendix D.

8.1 A suite of welfare state policies

Hornstein et al. (2007) (henceforth HKV) put a matching function into a model with vintage capital and use it to argue a substantial increase in European unemployment to an acceleration in the rate of capital-embodied technological change $\triangle$. HKV suggest that pre-1970 and post-1990 periods are characterized by $\triangle = 0.04$ and $\triangle = 0.077$, respectively, which together with HKV's assumptions about government policies for them explains why the unemployment rate increased in post-1990 by over four percentage points in Europe but by

[21]To be specific, our formula (37) for the steady-state comparative statics is an approximation of the elasticity of market tightness at the rear end of Figure 6 where the exogenous rate δ at which parties break up between bargaining rounds is equal to Hall and Milgrom's (2008) assumed job destruction rate of 0.0014 per day.

just one percentage point in the U.S.

In HKV's framework, the formation of a job coincides with investing in a one-worker machine with capital-embodied productivity at the frontier of an exogenously moving technology frontier at the time of investment. The two inputs into their single matching function are homogeneous unemployed workers and vacant machines of different vintages. Matched worker-machine pairs engage in Nash bargaining to set vintage-specific wage rates. All worker-machine pairs are subject to exogenous separation shocks that return separated workers and machines to the matching function. When the capital-embodied productivity of a vintage falls too far behind the current technology frontier, the match surplus evaporates and machines are scrapped.

HKV calibrate government policies to be more active in Europe than in the U.S.: unemployment benefits $b^{\mathrm{EU}} = 0.33$ versus $b^{\mathrm{US}} = 0.05$, which correspond to replacement rates of 75% and 10% of average wages in pre-1970 (when $\triangle = 0.04$) in Europe and the U.S., respectively; a layoff tax $\tau^{\mathrm{EU}} = 0.45$ in Europe, which is equivalent to one year of average wages in pre-1970 (when $\triangle = 0.04$) versus no layoff tax in the U.S.; and a pair of European income and payroll taxes $\{24\%, 21\%\}$ versus a U.S. pair $\{17\%, 8\%\}$. To generate balanced growth paths, HKV assume that unemployment benefits b, layoff taxes τ, and the investment cost for machines change at the economy's growth rate.[22]

To understand how $\triangle$ affects unemployment outcomes, we compare with the forces at work earlier in Figure 1(a). A higher $\triangle$ acts as an obsolescence shock because it shortens the equilibrium lifespan of machines, leaving less time to recover the machine cost. Holding other things the same, a shorter machine life also implies that the annuity value of the layoff tax increases and that the fundamental surplus falls (in line with our discussion of expression (23)). But other things are not the same – the fundamental surplus now shrinks faster with the age of a machine since both the unemployment benefit and the layoff tax grow at the economy's growth rate, which has risen as a consequence of a higher $\triangle$ (while the vintage-dependent output evolves over a machine's lifespan in the same way as before). The implied reduction in the fundamental surplus fraction can be thought of as a leftward movement along the curve in a graph like Figure 1(a). Thus, HKV's finding of a larger increase in European unemployment is a manifestation of how welfare state policies had *already* diminished an initial fundamental surplus fraction. Consequently, Europe was positioned farther to the left along the curve in a graph like Figure 1(a) *before* the onset of higher capital-embodied

[22] To attain the same unemployment rate of 4% in both Europe and the U.S. in pre-1970, HKV assume a significantly lower exogenous separation rate in Europe than in the U.S. which remain fixed over time.

technological change, which then moved the economy even further to the left along the curve.

Viewing things through the lens of the fundamental surplus sharpens and clarifies the message of HKV's policy experiments. What matters in HKV's framework, is the size of the fundamental surplus fraction. It is determined by HKV's suite of welfare state policies. To make that point precise, online appendix D.1 suggests an alternative representation of HKV's analysis. Selecting one of HKV's possible government policies, we consider only unemployment benefits, and find that our alternative setting of $\hat{b}^{\mathrm{EU}} = 0.594$ and $\hat{b}^{\mathrm{US}} = 0.089$ can reproduce the outcomes of HKV's bundle of policies. Hence, if HKV were to ease any one policy, it would be tantamount to a reduction in our $\hat{b}$, which enlarges the fundamental surplus fraction and therefore makes unemployment respond less to the pace of capital-embodied technological change. This clarifies HKV's (2007, p. 1110) conclusions that, "first, it would be inaccurate to point to one particular institution as the culprit and second, looking ahead, reforming any one institution could reduce dramatically the elasticity of the unemployment rate to obsolescence shocks."[23]

It is instructive to clarify how one factor contributes to HKV's diminished post-1990 fundamental surplus fraction. HKV's assumption of constant values for b and τ, means that policies actually become more active when $\triangle$ increases. In the case of Europe, the unemployment benefit, $b^{\mathrm{EU}} = 0.33$, corresponds to a replacement rate of 83% (75%), and the layoff tax, $\tau^{\mathrm{EU}} = 0.45$, is equal to 1.14 (1.0) annual average wages in post-1990 (pre-1970). In terms of our alternative representation in online appendix D.1, $\hat{b}^{\mathrm{EU}} = 0.594$ corresponds to a replacement rate of 93% (85%) in post-1990 (pre-1970). If instead we let b^{EU} and τ^{EU} (or $\hat{b}^{\mathrm{EU}}$) vary with $\triangle$ in ways sufficient to keep them constant as fractions of the average wage across periods, the European unemployment rate would have increased by merely 1.4 percentage points post-1990. This calculation indicates that HKV's explanation of higher European unemployment is not only higher capital-embodied technological change but instead is driven in critical ways by their implicit (and apparently inadvertent) assumptions that make unemployment benefits become more generous and the layoff tax become more burdensome. Evidently, the calibrated fundamental surplus fraction for Europe in pre-1970 is simply not small enough by itself capable of generating a large unemployment response to

[23]Another conclusion of HKV is also immediate through the lens of the fundamental surplus. When comparing their high unemployment benefits to Hagedorn and Manovskii's (2008) calibration of a high value of leisure, HKV (2007, p. 1113) conclude that their "interpretation of the recent European experience builds on similar mechanisms as those discussed in the recent literature on short-run dynamics." See also Costain and Reiter (2008) for an analysis of how calibrations of Nash-bargaining matching models affect both the volatility of unemployment over the business cycle and the sensitivity of unemployment to changes in unemployment benefits.

the postulated increase in technological change.

8.2 Nash versus alternating-offer bargaining

Within a DSGE model with habit formation in preferences, adjustment costs in capital formation, capacity utilization costs, Calvo sticky price frictions, and a Taylor rule for monetary policy, Christiano et al. (2015) (henceforth CET) incorporate a matching model, either with Nash bargaining or with Hall and Milgrom's (2008) alternating-offer bargaining (AOB), to study how labor markets respond to neutral and investment-specific technology shocks and monetary policy shocks. CET show that their estimated Nash and AOB models equally well account for patterns in the data and they "generate impulse response functions that are virtually identical to each other."

The principle difference in parameter estimates is the replacement rate from unemployment insurance: 0.88 in the Nash model versus 0.37 in the AOB model. CET find that restricting the replacement rate in the Nash model to be the same as that of the AOB model and then recalculating the impulse response functions "leads to a dramatic deterioration in the performance of the Nash model. All of the quantity variables like hours worked, real GDP as well as unemployment are now much less responsive to a monetary policy shock ... [there is] a similar pattern with respect to the technology shocks." CET conclude that "empirically plausible versions of the Nash model must assume a very high [replacement rate]." We suggest that CET could have enlightened readers further by noting that what underlies their finding is that a lower replacement rate enlarges the fundamental surplus fraction and therefore diminishes the sensitivity of unemployment to shocks.

In response to our request, CET generously conducted what can be regarded as an obverse perturbation of the AOB model, namely, cutting in half both its replacement rate 0.37 and a firm's cost of delay in bargaining, where the latter in CET's framework is a firm's cost to make a counteroffer that is calibrated to 0.6 of a firm's daily revenue per worker.[24] As expected, and in line with our analyses in sections 5.3 and 7.3, the perturbation of the AOB model leads to a similarly dramatic deterioration in performance as that of the perturbed Nash model; in fact, both models generate almost identical impulse response functions after these perturbations. (See online appendix D.2.) So corresponding to CET's finding about the Nash model, we conclude that empirically plausible versions of the AOB model must combine very high values of the replacement rate and of a firm's cost to make a counteroffer.

[24]CET assume that it takes one day for a wage offer to be extended, alternatingly, by a firm or a worker.

The role of these high values is to diminish the fundamental surplus fraction and thereby increase the sensitivity of unemployment to shocks.[25]

9 Concluding comments

The next time you see unemployment respond sensitively to small changes in productivity in a model that contains a matching function, we hope that you will look for forces that suppress the fundamental surplus, i.e., deductions from productivity before the 'invisible hand' can allocate resources to vacancy creation.

The fundamental surplus fraction is the single intermediate channel through which economic forces generating a high elasticity of market tightness with respect to productivity must operate. Differences in the fundamental surplus explain why unemployment responds sensitively to movements in productivity in some matching models but not in others. The role of the fundamental surplus in generating that response sensitivity transcends diverse matching models having very different outcomes along other dimensions that include the elasticity of wages with respect to productivity, and whether or not outside values affect bargaining outcomes.

For any model with a matching function, to arrive at the fundamental surplus take the output of a job, then deduct the sum of the value of leisure, the annuitized values of layoff costs and training costs and a worker's ability to exploit a firm's cost of delay under alternating-offer wage bargaining, and any other items that must be set aside. The fundamental surplus is an upper bound on what the "invisible hand" could allocate to vacancy creation. If that fundamental surplus constitutes a small fraction of a job's output, it means that a given change in productivity translates into a much larger percentage change in the fundamental surplus. Because such large movements in the amount of resources that could potentially be used for vacancy creation cannot be offset by the invisible hand, significant variations in market tightness ensue, causing large movements in unemployment.

In the standard matching model with common calibrations, unemployment responds weakly to productivity shocks because the fundamental surplus fraction is large. There is no way to glean this information from the match surplus. The fundamental surplus can be large even when the match surplus is small, as it is commonly in matching models calibrated to be

[25]As demonstrated in section 4.3, CET's assumption of a fixed matching cost also suppresses the fundamental surplus fraction, but as in CET's perturbation of the Nash model, we leave that parameter unchanged in the perturbed AOB model.

consistent with observations indicating that expenditures that can be construed as vacancy costs are small; hence, the implied calibrated match surpluses must be small because firms are also commonly calibrated to receive a significant share of match surpluses, a share that firms use to finance the small vacancy costs. Consequently, given a small match surplus in a worker's current job as well as the discounted values of match surpluses expected to accrue from future jobs, it follows that a worker's outside value must mostly be made up of something else. What matters is whether this something else also belongs to the fundamental surplus. When it does, as in the standard matching model under common calibrations, the invisible hand can, by diverting resources from workers' outside values, mitigate the impact of productivity changes on match surpluses and hence on resources allocated to vacancy creation. This explains why a large fundamental surplus fraction attenuates the effects of productivity changes on unemployment and why the usual manifestation of a small match surplus is uninformative about this issue.

The insights from our analytical expressions in bare bone matching models help us to understand unemployment responses in richer matching environments. As an example, we reexamined Hornstein et al.'s (2007) matching model with vintage capital, which attributes a large increase in European unemployment to an increase in capital-embodied technological change. To complement the authors' explanations based on shifts of and movements along two endogenous curves of job creation and job destruction, respectively, we instead focused on the exogenous determinants of the fundamental surplus. In doing so, we discovered that the outbreak of European unemployment in their model is not really attributable to a higher pace of technological change as Hornstein et al. had suggested but instead to their implicit assumptions that unemployment benefits and the layoff tax had become more onerous in post-1990. In another example, we reexamined how Christiano et al. (2015) embed matching models in a DSGE framework to generate unemployment volatility. In addition to the authors' emphasis on the importance of a high replacement rate in unemployment insurance in their Nash bargaining model, our exploration of the fundamental surplus fraction uncovered the importance of a high *combined* value of the replacement rate and a firm's cost to make a counteroffer in their alternating-offer bargaining (AOB) model. This finding challenges Christiano et al.'s interpretation of their estimation "result as indicating that the replacement ratio does not play a critical role in the AOB model's ability to account for data."

We conclude by emphasizing that the concept of the fundamental surplus unites analyses of welfare state dynamics such as Hornstein et al. (2007), who mix various welfare state policies to diminish the fundamental surplus fraction, with analyses of business cycle dynamics such as Mortensen and Nagypál (2007), who combine various approaches to diminish the fundamental surplus fraction. Generating large movements in unemployment in the presence of welfare state policies, or getting large unemployment fluctuations over the business cycle requires the same thing: that the fundamental surplus fraction must be small.[26]

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[26]Extending a challenge that Chodorow-Reich and Karabarbounis (2015) presented to the matching literature, suppose that movements in productivity are associated with offsetting comovements in primitives that affect deductions from productivity in the fundamental surplus. By arresting the fundamental surplus fraction, those offsetting changes would make unemployment unresponsive to productivity changes. While Chodorow-Reich and Karabarbounis investigated only the consequences of a procyclical value of leisure, similar consequences would flow from procyclicality in, e.g., layoff costs, fixed matching costs, a firm's cost of delay in alternating-offer wage bargaining, and costs of credit acquisition.

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For Online Publication

A Elasticities of market tightness

A.1 Standard matching model

The equilibrium expression (12) for market tightness can be rewritten as

$$\frac{1-\phi}{c}(y-z) \;=\; \frac{r+s}{q(\theta)} \,+\, \phi\,\theta\,. \tag{53}$$

Implicit differentiation yields

$$\begin{aligned}
\frac{d\,\theta}{d\,y} \;&=\; -\frac{\frac{1-\phi}{c}}{-\left(\frac{-q'(\theta)\,(r+s)}{q(\theta)^2} + \phi\right)} \;=\; -\frac{\left(\frac{r+s}{q(\theta)} + \phi\,\theta\right)\,\frac{1}{y-z}}{-\left(\frac{\alpha(r+s)}{\theta q(\theta)} + \phi\right)} \\
&=\; \frac{(r+s)\,+\,\phi\,\theta\,q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\,q(\theta)}\;\frac{\theta}{y-z} \;\equiv\; \Upsilon^{\mathrm{Nash}}\,\frac{\theta}{y-z}\,,
\end{aligned} \tag{54}$$

where the second equality is obtained after using equation (53) to rearrange the numerator, while in the denominator, we invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$; the third equality follows from multiplying and dividing by $\theta\,q(\theta)$. The elasticity of market tightness is then given by (15).

A.2 Layoff taxes under Nash product $(E-U)^{\phi}J^{1-\phi}$

Bellman equations (20) and (21) can be solved for J and E to get

$$J \;=\; \frac{y\,-\,w\,-\,\beta s(1-\phi)\tau}{1\,-\,\beta(1-s)}\,, \tag{55}$$

$$E \;=\; \frac{w\,+\,\beta s(U-\phi\tau)}{1\,-\,\beta(1-s)}\,. \tag{56}$$

The no-profit condition for vacancies from expression (4) and the value of an unemployed worker from expression (7) remain the same.

After equating the right sides of expressions (4) and (55) and then rearranging, we find that the equilibrium wage must satisfy

$$w \;=\; y \;-\; \frac{r+s}{q(\theta)}c \;-\; \beta s(1-\phi)\tau\,. \tag{57}$$

As compared to wage expression (5) in an economy without layoff taxes, the right side of expression (57) has an additional negative term involving the layoff tax, namely, $-\beta s(1-\phi)\tau$. However, the shared negative term $-(r+s)c/q(\theta)$ becomes less negative with a layoff tax because, as we will show, market tightness falls, increasing the probability $q(\theta)$ of filling a job. But as we shall also show, the former negative effect outweighs the latter positive one, so the equilibrium wage falls when there are layoff taxes.

To obtain another useful equation for the equilibrium wage, use expressions (55) and (56) to eliminate J and E from equation (19):

$$(1-\phi)\left\{\frac{w \,+\, \beta s(U-\phi\tau)}{1 \,-\, \beta(1-s)} \,-\, U\right\} \;=\; \phi\,\frac{y \,-\, w \,-\, \beta s(1-\phi)\tau}{1 \,-\, \beta(1-s)}\,. \tag{58}$$

After multiplying both sides by $(1-\beta(1-s))$ and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\big(y \,-\, (1-\beta)U\big)\,. \tag{59}$$

Regarding the annuity value of being unemployed, $(1-\beta)U$, we can follow the same steps as in section 2 to arrive at expression (10), i.e.,

$$(1-\beta)U \;=\; z \,+\, \frac{\phi\,\theta\,c}{1-\phi}\,. \tag{60}$$

After using expression (60) to eliminate $(1-\beta)U$ in expression (59), and simplifying, we obtain our second wage equation:

$$w \;=\; z \,+\, \phi(y - z \,+\, \theta c)\,. \tag{61}$$

While this expression for the wage is identical to the corresponding expression (11) for a model without layoff taxes, it is now evaluated at a lower market tightness θ, and hence, the equilibrium wage rate is lower with layoff taxes. To confirm this, we equate the right sides of (57) and (61), and after rearranging, obtain the equilibrium expression for market tightness in section 4.2:

$$y - z - \beta s\tau \;=\; \frac{r \,+\, s \,+\, \phi\,\theta\,q(\theta)}{(1-\phi)q(\theta)}\,c\,. \tag{22}$$

Since the left side of (22) is lower than the left side of (12), it follows that market tightness θ must be lower on the right side of the former expression than in the latter.

Paralleling the steps in appendix A.1, implicit differentiation of expression (22) yields

$$\frac{d\,\theta}{d\,y} \;=\; \Upsilon^{\text{Nash}}\,\frac{\theta}{y - z - \beta s\tau}\,. \tag{62}$$

The elasticity of market tightness is then given by (23).

A.3 Layoff taxes under Nash product $(E-U)^{\phi}(J+\tau)^{1-\phi}$

Under the alternative assumption that firms are liable for the layoff tax immediately upon being matched with unemployed workers regardless of whether employment relationships are eventually formed, the firm's threat point is $-\tau$ and the Nash product to be maximized is $(E-U)^{\phi}(J+\tau)^{1-\phi}$ so that the outcomes satisfy

$$E - U \;=\; \phi\bigl(S+\tau\bigr) \quad \text{and} \quad J \;=\; (1-\phi)S - \phi\tau\,, \tag{63}$$

i.e., a worker and a firm split match surpluses so that

$$(1-\phi)(E-U-\phi\tau) \;=\; \phi(J+\phi\tau)\,. \tag{64}$$

Hence, as compared to expression (2), the Bellman equation for a firm's value of a filled job is modified, but the Bellman equation for the value of an employed worker continues to be the same as expression (6), i.e.,

$$\begin{aligned} J &= y - w + \beta\bigl[-s\,\tau + (1-s)J\bigr] \\ E &= w + \beta\bigl[s\,U + (1-s)E\bigr]\,, \end{aligned}$$

where we have imposed that $V=0$ so that vacancies break even in an equilibrium. These Bellman equations imply

$$J \;=\; \frac{y - w - \beta s\tau}{1 - \beta(1-s)} \tag{65}$$

$$E \;=\; \frac{w + \beta sU}{1 - \beta(1-s)}\,. \tag{66}$$

The no-profit condition for vacancies and the value of an unemployed worker continue to be as in expressions (4) and (7).

After equating the right sides of expressions and (4) and (65) and rearranging, it follows that the equilibrium wage satisfies

$$w \;=\; y - \frac{r+s}{q(\theta)}c \;-\; \beta s\tau\,. \tag{67}$$

To obtain another equation for the equilibrium wage, use expressions (65) and (66) to eliminate J and E from equation (64),

$$(1-\phi)\left\{\frac{w \,+\, \beta sU}{1 \,-\, \beta(1-s)} \,-\, U \,-\, \phi\tau\right\} \;=\; \phi\,\frac{y \,-\, w \,-\, \beta s\tau}{1 \,-\, \beta(1-s)} \,+\, \phi\tau\,. \tag{68}$$

After multiplying both sides by $(1-\beta(1-s))$, and simplifying, we find that the equilibrium wage satisfies

$$w \;=\; (1-\beta)U \,+\, \phi\Big(y \,-\, (1-\beta)U \,+\, (1-\beta)\tau\Big)\,. \tag{69}$$

To obtain an expression for $(1-\beta)U$, we can follow steps analogous to those in section 2 to eliminate $(E-U)$ and J in expression (64),

$$(1-\phi)\left\{\frac{(1-\beta)U \,-\, z}{\beta\,\theta\,q(\theta)} \,-\, \phi\tau\right\} \;=\; \phi\left\{\frac{c}{\beta\,q(\theta)} \,+\, \phi\tau\right\}, \tag{70}$$

which after simplifications yields

$$(1-\beta)U \;=\; z \,+\, \frac{\phi}{1-\phi}\big[\theta\,c \,+\, \beta\,\theta\,q(\theta)\,\tau\big]. \tag{71}$$

After using expression (71) to eliminate $(1-\beta)U$ in expression (69) and simplifying, we obtain our second equation for the equilibrium wage:

$$w \;=\; z \,+\, \phi\left\{y - z \,+\, \theta\,c \,+\, \big[1 - \beta\,(1 - \theta\,q(\theta))\big]\tau\right\}. \tag{72}$$

Next, we equate the right sides of (67) and (72). After moving all terms that involve θ to one side and collecting the remaining terms on the other side, we obtain the following expression for equilibrium market tightness θ:

$$\frac{1-\phi}{c}\left\{y-z-\frac{\beta(\phi r+s)}{1-\phi}\,\tau\right\}=\frac{r+s}{q(\theta)}+\phi\,\theta+\phi\,\beta\,\theta\,q(\theta)\,\tau/c\,. \tag{73}$$

Implicit differentiation of expression (73) yields

$$\begin{aligned}\frac{d\,\theta}{d\,y} &= -\frac{\frac{1-\phi}{c}}{-\left(\frac{-q'(\theta)\,(r+s)}{q(\theta)^2}+\phi+\phi\,\beta\,q(\theta)\,\tau/c+\phi\,\beta\,\theta\,q'(\theta)\,\tau/c\right)}\\
&= \frac{\frac{r+s}{q(\theta)}+\phi\,\theta+\phi\,\beta\,\theta\,q(\theta)\,\tau/c}{\frac{\alpha\,(r+s)}{\theta\,q(\theta)}+\phi+(1-\alpha)\phi\,\beta\,q(\theta)\,\tau/c}\left(y-z-\frac{\beta(\phi r+s)}{1-\phi}\,\tau\right)^{-1}\\
&= \frac{r+s+\phi\,\theta\,q(\theta)\big[1+\beta q(\theta)\tau/c\big]}{\alpha\,(r+s)+\phi\,\theta\,q(\theta)\big[1+(1-\alpha)\beta q(\theta)\tau/c\big]}\;\frac{\theta}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\,\tau}\\
&\equiv \Upsilon^{\mathrm{alt}}\big(\tau/c\big)\,\frac{\theta}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\,\tau}\,,\end{aligned} \tag{74}$$

where the second equality is obtained after using equation (73) to eliminate $(1-\phi)/c$ in the numerator, while in the denominator, we twice invoke the constant elasticity of matching with respect to unemployment, $\alpha=-q'(\theta)\,\theta/q(\theta)$; and the third equality follows from multiplying and dividing by $\theta\,q(\theta)$. We can compute the elasticity of market tightness to be

$$\eta_{\theta,y}=\Upsilon^{\mathrm{alt}}\big(\tau/c\big)\,\frac{y}{y-z-\frac{\beta(\phi r+s)}{1-\phi}\,\tau}\,. \tag{75}$$

The factor $\Upsilon^{\mathrm{alt}}\big(\tau/c\big)$ is bounded from above by $\max\{\alpha^{-1},(1-\alpha)^{-1}\}$. Therefore, and as before, a high elasticity of market tightness requires that the second factor appearing in expression (75) be large, i.e., that the fundamental surplus fraction be small.[27]

[27] If workers' bargaining weight ϕ is zero, the fundamental surplus in expression (75) equals that in (23), i.e., the fundamental surplus is reduced by the annuitized layoff tax, $\beta s\tau$. But if workers' bargaining weight were to be raised, the fundamental surplus would be further reduced in (75) for two reasons as detailed in footnote 9.

A.4 Fixed matching cost after bargaining

Under the assumption that a fixed matching cost H is incurred after the firm and the worker have bargained over the consummation of a match (e.g., a training cost before work commences), the match surplus S becomes

$$S \;=\; \left\{ \sum_{t=0}^{\infty} \beta^t (1-s)^t \left[y - (1-\beta) U \right] \right\} \;-\; H \;\;=\;\; \frac{y - (1-\beta)U - (1-\beta(1-s))H}{1-\beta(1-s)} \,. \qquad (76)$$

By Nash bargaining, the firm receives S_f and the worker S_w of that match surplus, as given by

$$S_f = (1-\phi) S \qquad \text{and} \qquad S_w = \phi S. \qquad (77)$$

A worker's value as unemployed can be written as

$$U \;=\; z \,+\, \beta \left[\theta q(\theta) S_w + U \right],$$

which can be rearranged, and after invoking (77), to read

$$U \;=\; \frac{z \,+\, \beta \theta q(\theta) \frac{\phi}{1-\phi} S_f}{1-\beta} \,. \qquad (78)$$

From equations (76), (77) and (78), a firm's match surplus can be deduced as

$$S_f \;=\; (1-\phi) \, \frac{y - z - \beta(r+s)H}{\beta(r+s) + \beta \theta q(\theta) \phi} \,, \qquad (79)$$

where we have used $\beta = (1+r)^{-1}$ and $1 - \beta(1-s) = \beta(r+s)$.

A firm's match surplus must also satisfy the zero profit condition in vacancy creation,

$$c \;=\; \beta q(\theta) S_f \qquad \Longrightarrow \qquad S_f \;=\; \frac{c}{\beta q(\theta)} \,. \qquad (80)$$

The two expressions (79) and (80) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c} \left[y - z - \beta(r+s)H \right] \;=\; \frac{r+s}{q(\theta)} \,+\, \phi\, \theta \,. \qquad (81)$$

Paralleling the steps of implicit differentiation in appendix A.1, we arrive at the elasticity of market tightness with respect to productivity as given by (25).

A.5 Fixed matching cost before bargaining

Under the assumption that the firm incurs the fixed matching cost before bargaining with the worker, the match surplus S becomes

$$S = \sum_{t=0}^{\infty} \beta^t (1-s)^t \left[y - (1-\beta)U\right] = \frac{y - (1-\beta)U}{1 - \beta(1-s)}. \tag{82}$$

Nash bargaining outcomes and a worker's value as unemployed are still given by equations (77) and (78), so we can use those expressions together with equation (82) to deduce a firm's match surplus as

$$S_f = (1-\phi)\frac{y - z}{\beta(r+s) + \beta\theta q(\theta)\phi}. \tag{83}$$

Given that the firm bears the fixed matching cost before bargaining with the worker, the zero profit condition in vacancy creation becomes

$$c = \beta q(\theta)[S_f - H] \qquad \Longrightarrow \qquad S_f = \frac{c}{\beta q(\theta)} + H. \tag{84}$$

The two expressions (83) and (84) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c}\left[y - z - \frac{\beta(r+s)}{1-\phi}H\right] = \frac{r+s}{q(\theta)} + \phi\,\theta + \beta\theta q(\theta)\phi H/c. \tag{85}$$

Paralleling the steps of implicit differentiation in appendix A.3, we can compute the elasticity of market tightness with respect to productivity to be

$$\eta_{\theta,y} = \Upsilon^{\mathrm{alt}}\big(H/c\big)\,\frac{y}{y - z - \dfrac{\beta(r+s)}{1-\phi}H}, \tag{86}$$

where the factor $\Upsilon^{\mathrm{alt}}(\cdot)$ is defined in equation (74), and remains bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$. The second factor, the inverse of the fundamental surplus fraction, is larger than in expression (25) for the case of a fixed matching cost *after* bargaining. Because when the fixed matching cost is now incurred by the firm *before* bargaining, it means that the cost must ex ante be financed out of the firm's match surplus and hence, the associated deduction from the fundamental surplus is amplified by a smaller share $1-\phi$ of the match surplus going to the firm.[28]

[28] The reason is the same as in item (i) of footnote 9 in the case of the firm being liable for a layoff tax immediately upon being matched with an unemployed worker.

A.6 Nash-bargained wages in the financial accelerator model

From the case of the firm incurring a fixed matching cost before bargaining in appendix A.5, expression (83) for the firm's match surplus remains the same under Nash-bargained wages in the financial accelerator model (when assuming block-bargaining, i.e., the entrepreneur and the financier form a block when bargaining with the worker). But the zero profit conditions in vacancy creation are very different. While the fixed matching cost H in appendix A.5 was incurred by a firm upon matching with an unemployed worker, the average search cost $K(\sigma^\star)$ for the formation of an entrepreneur-financier pair in the credit market marks the start of that pair's quest to match with a worker in the labor market. Hence, the zero profit condition becomes

$$K(\sigma^\star) \;=\; -\sum_{t=0}^{\infty} \beta^t \big[1 - q(\theta)\big]^t c \;+\; \sum_{t=1}^{\infty} \beta^t \big[1 - q(\theta)\big]^{t-1} q(\theta)\, S_f \;=\; \frac{-c + \beta q(\theta) S_f}{1 - \beta[1 - q(\theta)]}\,,$$

that is,

$$S_f \;=\; \frac{c}{\beta q(\theta)} + \frac{1 - \beta[1 - q(\theta)]}{\beta q(\theta)}\, K(\sigma^\star)\,. \tag{87}$$

The two expressions (83) and (87) for a firm's match surplus jointly determine the equilibrium value of θ:

$$\frac{1-\phi}{c + \beta r K(\sigma^\star)} \left[y - z - \frac{\beta(r+s)}{1-\phi}\, K(\sigma^\star) \right] \;=\; \frac{r+s}{q(\theta)} + \phi\,\theta + \beta\theta q(\theta)\phi\, \frac{K(\sigma^\star)}{c + \beta r K(\sigma^\star)}\,. \tag{88}$$

Paralleling the steps of implicit differentiation in appendix A.3, we can compute the elasticity of market tightness with respect to productivity to be

$$\eta_{\theta,y} \;=\; \Upsilon^{\text{alt}}\left(\frac{K(\sigma^\star)}{c + \beta r K(\sigma^\star)} \right) \frac{y}{y - z - \dfrac{\beta(r+s)}{1-\phi}\, K(\sigma^\star)}\,, \tag{89}$$

where the decomposition resembles expression (86) for the case of a fixed matching cost before bargaining. The earlier fixed matching cost H is replaced by the average search cost $K(\sigma^\star)$ for the formation of an entrepreneur-financier pair in the credit market. Though, there is seemingly one difference regarding the argument of the first factor $\Upsilon^{\text{alt}}(\cdot)$. While the argument had been H divided by c, it is now $K(\sigma^\star)$ divided by $c + \beta r K(\sigma^\star)$. The difference in denominators can be understood when viewing them as the per period cost of a vacancy. In the case of a fixed matching cost, that per period cost is solely the vacancy posting cost c. But in the financial accelerator model, there is also a credit search cost that has to be incurred before posting a vacancy and therefore, there is an additional per period cost associated with a vacancy in the form of an interest cost for the upfront credit search cost, $\beta r K(\sigma^\star)$.

B Wage elasticity in the standard matching model

To study the determinants of the elasticity of the wage with respect to productivity, we compute the derivative of the wage rate w with respect to productivity y. First, we differentiate wage expression (11) with respect to w, y, and θ,

$$dw \;=\; \phi\, dy \;+\; \phi\, c\, d\theta$$

or

$$\frac{d\,w}{d\,y} \;=\; \phi\left[1 \;+\; c\,\frac{d\,\theta}{d\,y}\right]. \tag{90}$$

Together with the derivative of market tightness with respect to productivity in (54), we arrive at

$$\begin{aligned}\frac{d\,w}{d\,y} \;&=\; \phi\left[1 \;+\; c\,\Upsilon^{\text{Nash}}\,\frac{\theta}{y-z}\right] \;=\; \phi\left[1 \;+\; \frac{(1-\phi)\,q(\theta)}{r+s\;+\;\phi\,\theta\,q(\theta)}\Upsilon^{\text{Nash}}\,\theta\right] \\ &=\; \phi\left[1 \;+\; \frac{(1-\phi)\,\theta\,q(\theta)}{\alpha(r+s)\;+\;\phi\,\theta\,q(\theta)}\right] \;=\; \phi\,\frac{\alpha(r+s)\;+\;\theta\,q(\theta)}{\alpha(r+s)\;+\;\phi\,\theta\,q(\theta)}\,,\end{aligned} \tag{91}$$

where the second equality uses expression (53) to eliminate $c/(y-z)$, and the third equality invokes the definition of Υ^{Nash} in (54).[29] The derivative (91) varies from zero to one as ϕ varies from zero to one. At one extreme, we know that if workers have a zero bargaining weight, $\phi = 0$, the equilibrium wage equals the value of leisure, $w = z$, and hence, the wage does not respond to changes in productivity, $dw/dy = 0$. At the other extreme, if firms have a zero bargaining weight, $\phi = 1$, workers reap all gains from productivity, $w = y$, and hence, the wage responds one-for-one to changes in productivity, $dw/dy = 1$. But of course,

[29] Another approach to compute dw/dy (and $d\theta/dy$) is to express the total differential of wage equations (5) and (11) in matrix form,

$$\begin{bmatrix} q(\theta) & (w-y)q'(\theta) \\ 1 & -\phi\, c \end{bmatrix} \begin{bmatrix} dw \\ d\theta \end{bmatrix} \;=\; \begin{bmatrix} q(\theta)\, dy \\ \phi\, dy \end{bmatrix},$$

and then apply Cramer's rule, followed by substitutions analogous to those above. As an alternative to any one of the two wage equations, we can also use equation (53) in this calculation.

in the latter case, there would be no vacancy creation in the first place so no one would be employed.

Next, we use equation (5) to derive an expression for the wage as a fraction of productivity:

$$\begin{aligned} \frac{w}{y} &= 1 - \frac{r+s}{q(\theta)}\,\frac{c}{y} \;=\; 1 - \frac{(1-\phi)(r+s)}{r+s\,+\,\phi\,\theta\,q(\theta)}\,\frac{y-z}{y} \\ &= \frac{\big[\phi + (1-\phi)\,z/y\big](r+s)\,+\,\phi\,\theta\,q(\theta)}{r+s\,+\,\phi\,\theta\,q(\theta)}\,, \end{aligned} \tag{92}$$

where the second equality uses equation (53) to eliminate $c/q(\theta)$. In the Hagedorn-Manovskii calibration with an extremely small fundamental surplus fraction, $(y-z)/y \approx 0$, it follows immediately from the second equality of expression (92) that $w/y \approx 1$. This outcome is a necessary consequence of the fact that if workers are to be willing to work the wage rate cannot fall below z, nor can it exceed y if firms are to be motivated to post vacancies. As discussed in section 3.1, within more common calibrations of the matching model $\phi\theta q(\theta)$ is usually high relative to $(r+s)$, and hence, the third equality of expression (92) explains why w/y can still be close to one even when the value of leisure is not close to productivity.

Using expressions (91) and (92), the elasticity of the wage with respect to productivity, $\eta_{w,y} \equiv (dw/dy)/(w/y)$, becomes

$$\eta_{w,y} \;=\; \phi\,\frac{\alpha(r+s)\,+\,\theta\,q(\theta)}{\alpha(r+s)\,+\,\phi\,\theta\,q(\theta)}\;\frac{r+s\,+\,\phi\,\theta\,q(\theta)}{\big[\phi + (1-\phi)\,z/y\big](r+s)\,+\,\phi\,\theta\,q(\theta)}\,. \tag{93}$$

As just discussed, under the Hagedorn-Manovskii calibration, the second fraction in expression (93) must approximately equal one, and hence, the wage elasticity coincides with the derivative of the wage with respect to productivity in expression (91). This observation sheds light on how Hagedorn and Manovskii (2008) designed their calibration. The worker's bargaining weight ϕ in equation (91) is used to attain a target wage elasticity, while the fundamental surplus fraction, $(y-z)/y$, in equation (15) is used to attain a particular elasticity of market tightness that via equation (14) is linked to a target elasticity of unemployment. Therefore, as also argued by Hagedorn and Manovskii, the choice of wage elasticity is incidental to the outcome that unemployment is highly sensitive to productivity changes, which instead is driven by their calibration of a small fundamental surplus fraction.

As mentioned above, Hagedorn and Manovskii argue that the high wage elasticity in a common calibration of the matching model can be lowered without changing the implication that unemployment is not very sensitive to productivity changes. Such a perturbed calibration also involves modifying the wage elasticity by altering the worker's bargaining weight ϕ, though a complication is that both fractions in expression (93) are at play when $z << y$. Nevertheless, within some bounds, the workers' bargaining power can be lowered in a common calibration of the matching model to reduce the wage elasticity substantially, as argued by Hagedorn and Manovskii and illustrated by our Figure 3(b). And so long as the fundamental surplus fraction is kept high, the elasticity of market tightness remains low in equation (15) so that unemployment does not respond very much to productivity, as illustrated in Figure 3(a).

Finally, for a given workers' bargaining weight ϕ, we take the limit of the wage elasticity as y approaches z, which also implies that a worker's probability of finding a job $\theta q(\theta)$ approaches zero:

$$\lim_{y \to z, \theta q(\theta) \to 0} \eta_{w,y} \;=\; \lim_{\theta q(\theta) \to 0} \left[\phi \, \frac{\alpha(r+s) \,+\, \theta\, q(\theta)}{\alpha(r+s) \,+\, \phi\, \theta\, q(\theta)} \right] \;=\; \phi\,. \tag{94}$$

This limit is discernible in Figure 3(b) as productivity approaches $z = 0.6$. In Figure 2, we temporarily arrest this convergence by recalibrating three unemployment schedules for $\phi = 0.5$ to increase the efficiency parameter A in the matching function so that the unemployment rate is 5 percent at productivities 0.61, 0.63, and 0.65, respectively. This recalibration increases a worker's probability of finding a job $\theta q(\theta)$, which by the logic of our limiting calculation in expression (94) arrests the convergence to the limit described in (94) and explains why all three wage elasticities in Figure 2 are approximately 0.97 as compared to 0.83, 0.91, and 0.93 for productivities 0.61, 0.63 and 0.65, respectively, along the solid line for $\phi = 0.5$ in Figure 3(b).

C Adjustments to Hall's (2005) parameterization

We alter Hall's value of the vacancy cost because, regrettably, his calibration implies that the job filling probability exceeds unity for all productivity levels except for the highest one.[30] By lowering the vacancy cost to $c = 0.1$, and making a corresponding adjustment of

[30] When eyeballing Hall's (2005) Figure 2, there are two ways of inferring market tightness at the lowest productivity level, for example. First, measures of vacancies and unemployment are $v \approx 0.025$ and $u \approx 0.082$. Second, the probability of finding a job is $\theta q(\theta) \approx 0.38$. Given Hall's calibration of the efficiency parameter of the matching function, $A = 0.947$, both ways imply a market tightness of around 0.30 at the lowest productivity level, which in turn implies a probability of filling a vacancy of $q(\theta) \approx 1.25$.

the efficiency parameter A of the matching function, we can preserve the same equilibrium unemployment outcomes reported by Hall.[31]

To facilitate our sensitivity analysis, we also alter Hall's model period from one month to one day because a shorter model period fosters the existence of equilibria with vacancy creation. (See footnote 6.) To accomplish our conversion from a monthly to a daily frequency, we compute a daily version $\hat{\Pi}$ of the monthly transition probability matrix Π by minimizing the sum of squared elements from the matrix operation $(\hat{\Pi}^{30} - \Pi)$ so that the monthly transition probabilities implied by $\hat{\Pi}$ are close to those of Π. Because of the high persistence in productivity, this target is approximated well with a daily value of $\hat{\rho} = 0.9996$. Other parameters that need to be converted are the efficiency parameter of the matching function and the separation rate, now with daily values of $A/30$ and $s/30$, and also the discount factor that becomes $\beta^{1/30}$ at a daily frequency.

D Fundamental surplus in complex environments

D.1 Vintage-capital growth analysis

In Figure 7, the dashed and solid lines, meant to refer to stylized versions of Europe and the U.S., respectively, are almost replicas of corresponding lines in panels A and B of Hornstein et al.'s (2007, hereafter HKV) figure 4. They depict steady-state unemployment rates

[31] As noted in section 4.1, aside from any targets for vacancy statistics, the joint parameterization of c and A is a choice of normalization. Specifically, given a calibration c and A with an associated market tightness θ determined by equation (53), the same equilibrium unemployment rate and job finding probability can be attained with an alternative parameterization $\hat{c} = \zeta c$ and $\hat{A} = \zeta^{1-\alpha} A$, for any $\zeta \in (0, \theta^{\alpha}/A]$. To verify this claim, solve for the new market tightness $\hat{\theta}$ from an appropriate version of equation (53),

$$\frac{1-\phi}{[\zeta c]}(y-z) \;=\; \frac{r+s}{[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha}} + \phi\,\hat{\theta}\,.$$

After multiplying both sides by ζ and comparing to equilibrium expression (53) for the initial parameterization, the new solution satisfies $\hat{\theta} = \theta/\zeta$. At this new market tightness $\hat{\theta}$, the unemployment rate is unchanged because the job finding probability is unchanged, $\hat{\theta}[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha} = \theta\, A\theta^{-\alpha} = \theta\, q(\theta)$. The new probability of filling a vacancy is $[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha} = \zeta A\theta^{-\alpha} = \zeta\, q(\theta) \in (0,1]$, where the bounds follow from the range for ζ. The value of a filled job remains unchanged, as can be verified from expression (4), the no-profit condition in vacancy creation

$$\hat{J} \;=\; \frac{\hat{c}}{\beta[\zeta^{1-\alpha}A]\hat{\theta}^{-\alpha}} \;=\; \frac{\zeta c}{\beta[\zeta^{1-\alpha}A][\theta/\zeta]^{-\alpha}} \;=\; \frac{c}{\beta A\theta^{-\alpha}} \;=\; J\,,$$

where the second equality invokes the alternative parameterization $\hat{c} = \zeta c$ and the equilibrium outcome $\hat{\theta} = \theta/\zeta$.

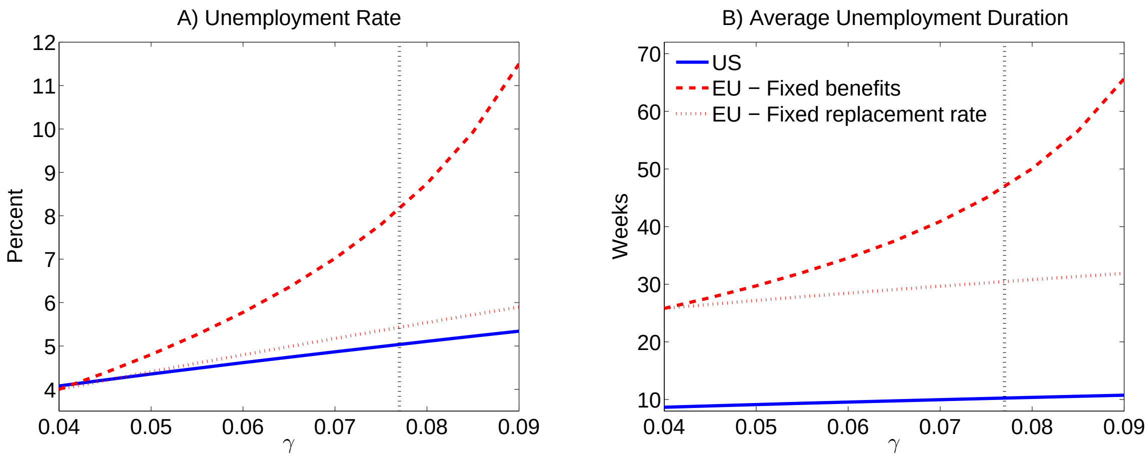

Figure 7: Unemployment rates and average durations of unemployment for different rates of capital-embodied technological change $\triangle$, where the dashed and solid lines refer to Europe and the U.S., respectively. The dotted horizontal lines depict what European outcomes would be if the replacement rate in unemployment compensation had been kept constant (rather than the quantity $\hat{b}^{\mathrm{EU}}$).

and average durations of unemployment at different rates of capital-embodied technological change $\triangle$. HKV suggest that pre-1970 and post-1990 are characterized by $\triangle = 0.04$ and $\triangle = 0.077$, respectively, i.e., the leftmost ends of the panels vis-à-vis the dotted vertical lines. Hence, starting from the same unemployment rate of 4% in pre-1970, panel A shows that the unemployment rate increases in post-1990 by over four percentage points in Europe but by just one percentage point in the U.S., with corresponding changes in average unemployment duration in panel B. These different outcomes are due to HKV's assumptions about government policies and also about exogenous separation rates that differ across Europe and the U.S. but remain fixed over time.

HKV calibrate government policies to be more active in Europe than in the U.S.: unemployment benefits $b^{\mathrm{EU}} = 0.33$ versus $b^{\mathrm{US}} = 0.05$, which correspond to replacement rates of 75% and 10% of average wages in pre-1970 (when $\triangle = 0.04$) in Europe and the U.S., respectively; a layoff tax $\tau^{\mathrm{EU}} = 0.45$ in Europe, which is equivalent to one year of average wages in pre-1970 (when $\triangle = 0.04$) versus no layoff tax in the U.S.; and a pair of European income and payroll taxes $\{24\%, 21\%\}$ versus a U.S. pair $\{17\%, 8\%\}$.[32] To attain the same 4% unemployment rate in pre-1970 (when $\triangle = 0.04$), HKV assume a significantly

[32] To study balanced growth paths, HKV assume that unemployment benefits b and layoff taxes τ (as well as the investment cost for machines) change at the economy's growth rate.

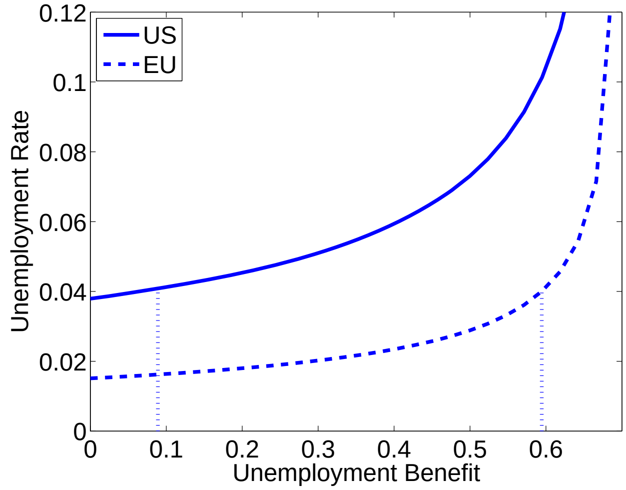

Figure 8: Unemployment rates for different unemployment benefits in pre-1970 (when $\triangle = 0.04$) in the U.S. (solid line) and Europe (dashed line).

lower exogenous separation rate in Europe than in the U.S. Based on these calibrations, the steady-state outcomes in Figure 7 emerge when varying $\triangle$. HKV (2007, p. 1110) also solve their model for Europe by activating one government policy at a time and conclude that "the technology–policy interaction is much starker when the three policies are considered together; as $\triangle$ increases, if one estimated the total role of policy by merely summing the effects of the individual policies, one would only account for less than one-third of the total technology–policy interaction predicted by the model with all policies jointly considered."

To make things more precise, we explain how we constructed Figure 7. Selecting one of HKV's possible government policies, we consider only unemployment benefits, and find that our alternative setting of $\hat{b}^{\mathrm{EU}} = 0.594$ and $\hat{b}^{\mathrm{US}} = 0.089$ can reproduce the outcomes of HKV's bundle of policies (as mentioned, the dashed and solid lines in Figure 7 are virtually the same as those of HKV's figure 4).[33] It is instructive to examine how the unemployment rate depends on pre-1970 benefits (when $\triangle = 0.04$), as depicted in Figure 8. Following HKV, we assume a much smaller exogenous separation rate in Europe than in the U.S. and hence

[33]With unemployment benefits as the sole policy, our algorithm for reproducing HKV's unemployment outcomes is as follows. First, for each value of unemployment benefits, we find an exogenous separation rate that produces HKV's targeted unemployment rate of 4% in pre-1970. Next, among all such pairs of benefits and exogenous separation rates, we select the pair that best reproduces HKV's relationship between unemployment and the exogenous rate of capital-embodied technological change (exhibited in panel A of HKV's figure 4). While this algorithm induces us to lower HKV's parameterization of the exogenous separation rate in Europe from 0.0642 to 0.0356, it leaves HKV's value for the U.S. unchanged at 0.2117.

the dashed line for Europe lies much below the solid line for the U.S. The vertical dotted lines mark benefits, $\hat{b}^{\mathrm{US}}$ and $\hat{b}^{\mathrm{EU}}$, respectively, at which the U.S. and Europe attain the same unemployment rate of 4% in pre-1970. A change to a higher $\triangle$ with its implied decline in fundamental surplus fractions is like rightward movements along the curves in Figure 8. Evidently, before 1970 Europe was poised to experience a larger increase in unemployment because a higher benefit level had decreased its fundamental surplus fraction. Thus, what matters is how far an economy is situated to the right along the curve in Figure 8 relative to where the unemployment relationship becomes much steeper. This is a way of expressing HKV's observation cited above that "the technology–policy interaction is much starker when [all] policies are considered together."

It is instructive to investigate the role of a factor contributing to the diminished post-1990 fundamental surplus fraction in HKV's theory of the outbreak of high European unemployment. Our HKV version in Figure 7 with a single policy of unemployment benefits, $\hat{b}^{\mathrm{EU}} = 0.594$, implies a replacement rate of 85% in pre-1970, while the same fixed quantity $\hat{b}^{\mathrm{EU}}$ implies a higher replacement rate of more than 93% post-1990. If instead we let $\hat{b}^{\mathrm{EU}}$ vary with $\triangle$ enough to keep the replacement rate constant at the value that prevails in pre-1970, we obtain the equilibrium relationship presented by the dotted curve in Figure 7. The resulting much smaller rise in European unemployment post-1990 suggests that HKV's explanation of higher European unemployment is not just about higher capital-embodied technological change but instead must come from their implicit assumptions that unemployment benefits became more generous and that the layoff tax became more burdensome.[34] If they had wanted to keep the unemployment benefit and the layoff tax policy constant over time in relation to the average wage, HKV would have had to calibrate an even smaller fundamental surplus fraction for pre-1970 Europe in order to generate a larger response of unemployment to higher capital-embodied technological change post-1990.

D.2 DSGE analysis

Christiano et al. (2015, hereafter CET) report on a perturbation of their Nash model by which the estimated replacement rate in unemployment insurance is cut by 58% from 0.88

[34] In HKV's analysis of Europe, the unemployment benefit, $b^{\mathrm{EU}} = 0.33$, corresponds to a replacement rate of 75% (83%), and the layoff tax, $\tau^{\mathrm{EU}} = 0.45$, is equal to 1.0 (1.14) annual average wages in pre-1970 (post-1990). To keep European policy unchanged over time in terms of the average wage, post-1990 values of the unemployment benefit and the layoff tax would have to be $b^{\mathrm{EU}} = 0.295$ and $\tau^{\mathrm{EU}} = 0.40$, respectively, resulting in a post-1990 European unemployment rate of merely 5.4%. Once again, unemployment outcomes in our augmented HKV analysis are very close to those of our HKV version with only unemployment benefits.

to 0.37, i.e., a reduction to the estimated level of their alternating-offer bargaining (AOB) model. From the perspective of the fundamental surplus as argued in section 8.2, we requested the authors to conduct a corresponding perturbation of their AOB model, namely, to cut by 58% both the level of unemployment insurance (from the original replacement rate of 0.37) and a firm's cost to make a counteroffer (from an original fraction 0.6 of a firm's daily revenue per worker) at the postulated daily frequency of a firm and a worker, alternatingly, making counteroffers. To keep the targeted unemployment rate at 5.5%, the vacancy posting cost is adjusted.[35]

CET generously conducted the perturbation of their AOB model to compute impulse response functions for a monetary policy shock, a neutral technology shock and an investment-specific technology shock, respectively, as reported in Figures 9, 10 and 11. As CET had found virtually identical impulse responses across their estimated Nash and AOB models, we note that the perturbed models also generate almost identical impulse responses (when comparing Figures 9, 10 and 11 for the perturbed AOB model to those of the perturbed Nash model in Christiano et al. (2015, figures 4, 5 and 6)). As a result, both perturbed models experience the same "dramatic deterioration in performance" as CET had found for the perturbed Nash model. Seen through the lens of the fundamental surplus, those same outcomes for the perturbed AOB model were to be expected. It is the common channel of enlarged fundamental surplus fractions that mutes the elasticity of market tightness in both models. For example, given a large fundamental surplus fraction in the AOB model, it is immaterial that the alternative bargaining protocol of Hall and Milgrom (2008) suppresses the influence of the worker's outside value during bargaining.

[35] Initially, we sought to adjust the multiplicative efficiency parameter of the matching function to keep the targeted unemployment rate at 5.5%, but the computation algorithm broke down because of poor numerical properties when the resulting vacancy filling rate fell too close to zero. In any case, as shown in footnote 31, it is just a question of normalization whether we adjust the vacancy posting cost or the efficiency parameter of the matching function to target a particular unemployment rate.

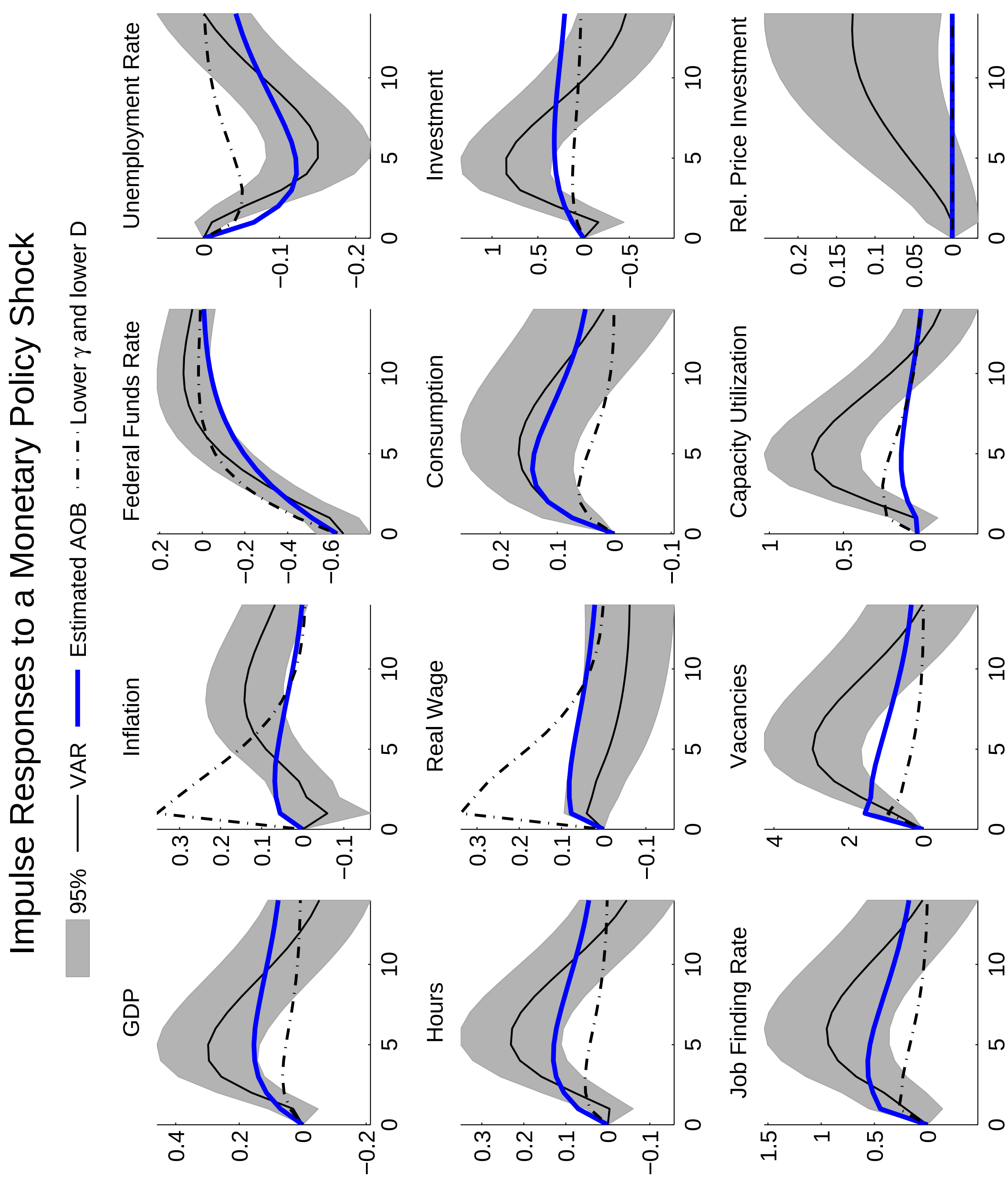

Figure 9: Impulse responses to a monetary policy shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.

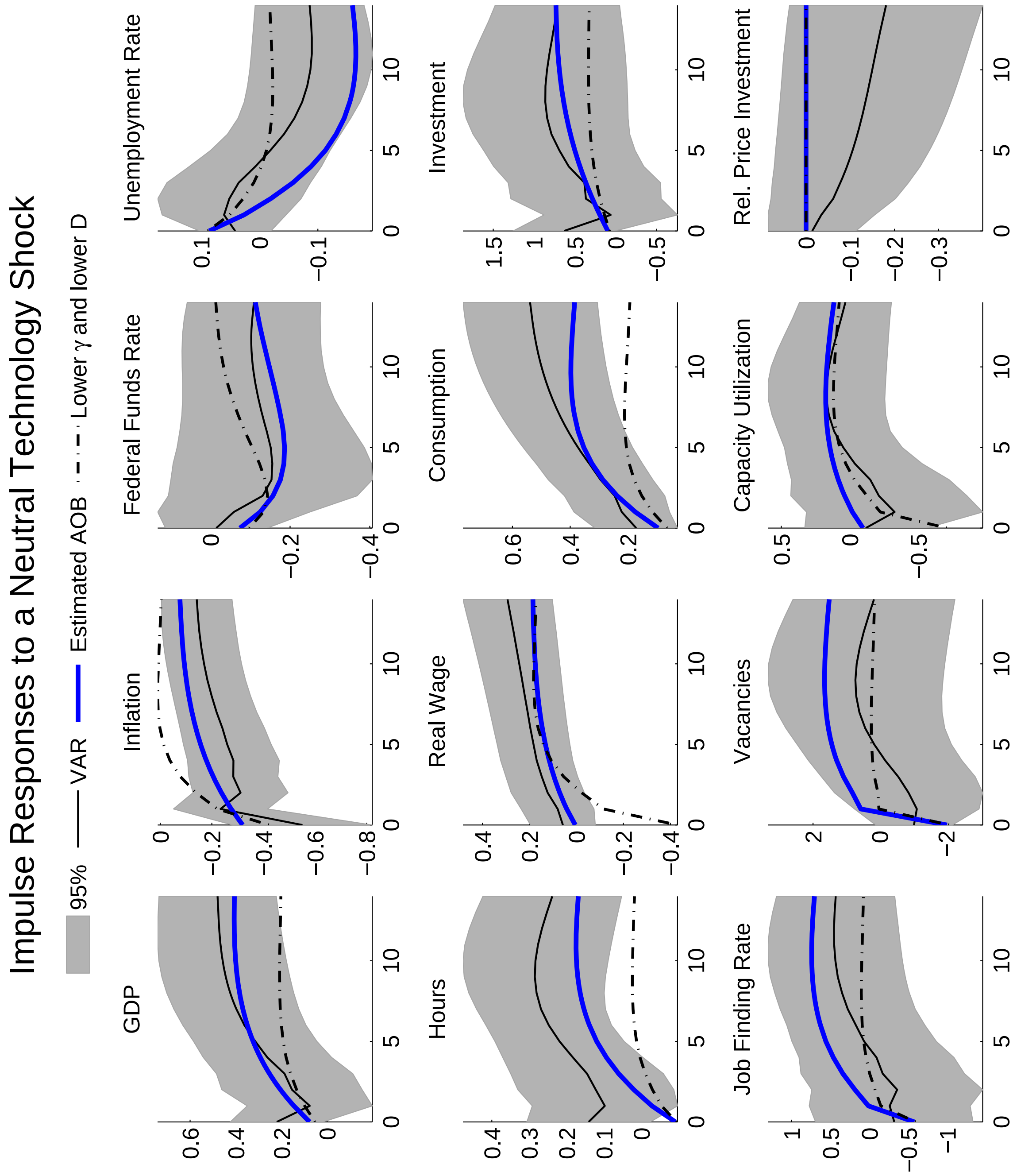

Figure 10: Impulse responses to a neutral technology shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.

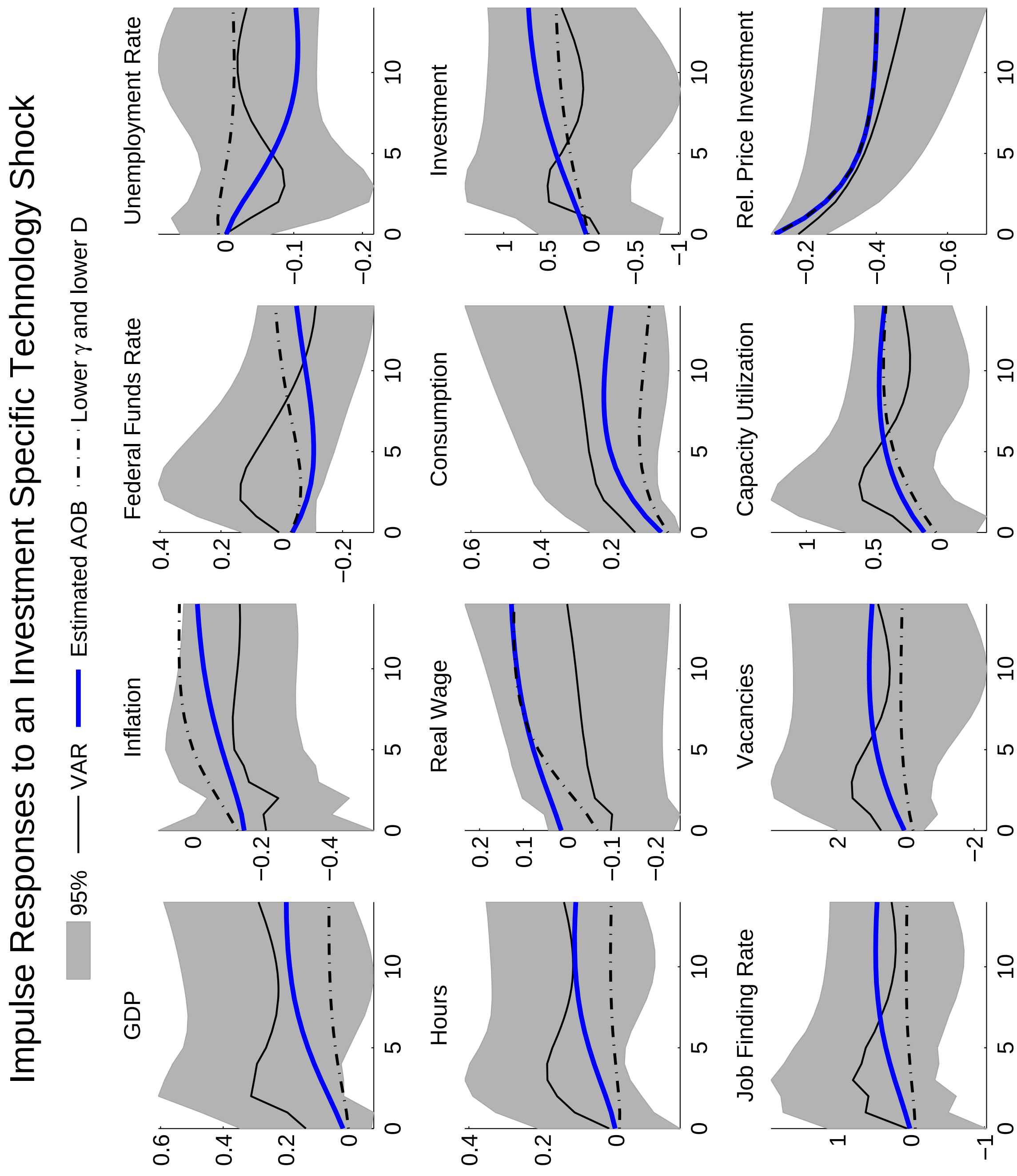

Figure 11: Impulse responses to an investment-specific technology shock in the AOB model. The dashed lines refer to the perturbed model in which the replacement rate in unemployment insurance and a firm's cost to make a counteroffer are both cut by roughly one half.