Critique and Consequence*

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> ... economic data are generated by systems of relations that are, in general, stochastic, dynamic, and simultaneous. ... these very relations constitute economic theory and knowledge of them is needed for economic practice.
>
> Marschak (1950)

Abstract

After describing the landscape in macroeconomics and econometrics in Spring 1973 when Robert E. Lucas (1976) first presented his Critique at the inaugural Carnegie-Rochester conference, I add a fourth example based on Sargent and Wallace (1973) to those in section 5 of Lucas's paper. To portray consequences of Lucas's Critique, I use it as a vehicle to describe the time inconsistency of optimal plans and their credibility. A theory of government policy affects chains of influence among money creation and inflation rates at different dates. Different theories of policy bring different state vectors in recursive representations of inflation-money-supply outcomes.

1 The Setting

Lucas (1976) was written in 1973 by a sympathetic and accomplished practitioner of frontier methods for inferring structural parameters of systems of simultaneous stochastic difference equations.[1] Koopmans (1947, 1950) and other economic theorists and statisticians constructed those methods because they wanted to design government policies to avoid business cycle depressions.[2] Marschak had described the challenge and the agenda:[3,4]

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[*] I thank Fernando Alvarez and Greg Kaplan for comprehensive and effective criticisms.

[1] For example, see Griliches et al. (1962).

[2] For insights into how he thought macroeconomic should be done, see Koopmans (1947, 1963).

[3] The Cowles Commission project to construct a statistical theory useful for for quantitative policy problems in dynamic settings was a response to the inadequacy of assuming "fixed regressors in repeated samples" as had been done to study effects of treating plots of agricultural land with fertilizer.

[4] Other contributors to Koopmans (1950) included H. Rubin, R.B. Leipnik, A. Wald, L. Hurwicz, T.

> . . . The economist's objectives are similar to those of an engineer but his data are like those of a meterologist. The economist is often required to estimate the effects of a given (intended or expected) change in the "economic structure," i.e., in the very mechanism that produced his data. None of these changes can he produce beforehand, as in a laboratory experiment; and since some of the changes envisaged have never happened before, the economist often has to estimate the results of changes he has never observed. . . . Thus, practical considerations bring about the economist's concern with economic structure. Hypotheses about economic structure are also known as economic theories. They try to state relations that describe the behavior and environment of men and determine the values taken at any time by economic variables such as prices, output, and consumption of various goods and services, and the prices and amounts of various assets. As there are several variables the economic structure must involve several simultaneous relations to determine them.
>
> Marschak (1950)

Koopmans (1950) and Hood and Koopmans (1953) created statistical methods to infer parameters of systems of simultaneous stochastic difference equations called **structural models** whose **reduced forms** are projections of vectors of endogenous variables on **exogenous variables** and lagged values of **endogenous variables**. A reduced form shifts when historically unprecedented sequences for public policy variables are put into a structural model. Marschak (1953) said that analyzing consequences of historically unprecedented public policy interventions requires knowing structural parameters that are invariant to those interventions. In the 1950s and 1960s, those Cowles Commission tools empowered an enthusiastic cohort of quantitative macroeconomists to infer structural parameters of dynamic stochastic Keynesian models (e.g., marginal propensities to consume, investment accelerators, and interest elasticities of asset demand functions). Lucas admired Klein and Goldberger (1955). Models like theirs were handed off to co-authors and colleagues who could apply the control-theoretic techniques then being imported into economics by Holt et al. (1956), Holt et al. (1960), and others at the Carnegie-Institute of Technology. Edward C. Prescott's 1968 Carnegie PhD thesis, part of which was published in Prescott (1971), studied a frontier aspect of this problem and was well known to Lucas when he wrote Lucas (1976).

John F. Muth's presence in the author list of Holt et al. (1956) and Holt et al. (1960) is significant. Using tools from those sources, he would soon write papers Muth (1960, 1961)

Haavelmo, T.W. Anderson, H. Hotelling, H.B. Mann, and R.L. Anderson. Other creators of modern economics also contributed to Hood and Koopmans (1953).

that would eventually require substantial adjustments and reorientations of the Koopmans-Marschak project. But in 1961, few recognized the ramifications of those two papers for macroeconomics.[5] Bob Lucas foresaw the econometric and policy implications of rational expectations earlier than everyone because he knew more about Cowles Commission structural econometrics and best understood what it could accomplish. It is not the first time in the history of a science that the most effective critic of a paradigm was a young person who thoroughly understood it.

Section 2 adds an example to section 5 of Lucas (1976). Section 3 uses it to illustrate the Lucas Critique and also to set the stage for subsequent sections. A key aspect of this example, as well as the three in Lucas's paper, is that the macroeconomic policy variables in play are *exogenous* in the sense that they are determined outside the model. Sections 4 and 5 describe extensions of the section 3 model in which a government designs a good government policy in the spirit of Marschak (1950). Section 4 analyzes sources of the dynamic inconsistency of that good policy and indicates how private agents in the model believe the policy only because they know that it was chosen once and for all at time 0. Section 5 describes how to make private agents' believe the optimal section 4 policy even when government decision makers choose it one period at a time. Section 6 recounts how difficult it was at the time to understand ramifications of Lucas (1976) for the theory of economic policy, and how long it took for application of a "dynamic programming squared" machinery to sort things out. Section 7 describes different *state* variables that appear in sections 3, 4, and 5, confirming once again that "finding the state is an art." Section 8 describes how a communism of beliefs associated with rational expectations appears in the three versions of the baseline model. This section also discusses whether policy makers should confirm "market anticipations". Section 9 offers old fashioned Hurwicz-Minnesota style "language policing" by recalling how the founders of econometrics defined "reduced form" and "identification" and how reduced forms are tools for estimating theoretically interpretable parameters of structural models, not ends in themselves. Section 10 describes how models like those in sections 4 and 5 make it difficult to partition variables into exogenous and endogenous ones as required to justify a "treatments effects" analysis. Section 11 describes how dynamic interactions render dubious the assumptions of R. A. Fisher's fertilizer "treatment effects" approach. After section 12 concludes, an appendix furnishes details about how section 3 illustrates lessons of Lucas (1976).

Section 2 presents a plain vanilla monetary-fiscal theory of the price level that links the

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[5] Muth's 1960 paper rationalized Milton Friedman's geometric distributed lag model of permanent income by working backwards to find restrictions on the stochastic process for a consumer's non-financial income. Until Lucas (1976), the implications of that finding for econometric practice had not been digested.

price level to the current level and all future rates of growth of the money supply. Each of the section 3, 4, and 5 models restricts a pair $(\vec{\mu}, \vec{\pi})$ of infinite sequences that comprise a joint process for rates of money growth μ and inflation π. These sections subject the section 2 baseline model to distinct "treatments" in the forms of theories about a money growth sequence $\vec{\mu}$. In our section 2 laboratory, a "treatment" is a complete description of a dynamic process for $\vec{\mu}$. When the money growth rate process is endogenous, as it is in the sections 4 and 5 models, supplying a complete description of $\vec{\mu}$ requires also describing an associated process for inflation, perhaps in the form of an "inflation-target" process. In designing optimal treatments in the 4 and 5 models, a public policy authority evaluates consequences of many other treatments. Our experiments tell us that whether components of $\vec{\mu}$ and $\vec{\pi}$ should be classified as exogenous or endogenous depends on a theory of public policy. These experiments remind us that quantitative work with dynamic models of public policy requires methods originated by Koopmans (1947, 1950) and Lucas (1976).

2 A laboratory

We start with Sargent and Wallace's (1973) perfect foresight version of a Cagan (1956) model that Calvo (1978) used as his laboratory. Let m_t be the log of the supply of nominal money balances; $\mu_t = m_{t+1} - m_t$ be the net rate of growth of nominal balances; p_t be the log of the price level; $\pi_t = p_{t+1} - p_t$ be the net rate of inflation between t and $t+1$; and π_t^* be the public's expected rate of inflation between t and $t+1$ based on its time t information. The demand for real balances $\exp\left(\frac{m_t^d}{p_t}\right)$ is governed by a version of the Cagan (1956) demand function:

$$m_t^d - p_t = -\alpha \pi_t^* \, , \; \alpha > 0 \tag{1}$$

for $t \geqslant 0$. Equation (1) asserts that the demand for real balances is inversely related to the public's expected rate of inflation.[6]

Private agents acquire **perfect foresight** by their having solved a forecasting problem. This lets us set

$$\pi_t^* = \pi_t, \tag{2}$$

while equating demand for money to supply lets us set $m_t^d = m_t$ for all $t \geqslant 0$. Equations (1) and (2) then imply

$$m_t - p_t = -\alpha(p_{t+1} - p_t) \, , \; \alpha > 0 \tag{3}$$

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[6] This demand function generates a Laffer curve in the form of a revenue function $R(\pi) = \exp(-\alpha\pi))\,\pi$ from an inflation tax π.

To fill in details about what it means for private agents to have perfect foresight, we subtract equation (3) at time t from the same equation at $t+1$ to get

$$\mu_t - \pi_t = -\alpha \pi_{t+1} + \alpha \pi_t,$$

which we rewrite as a forward-looking first-order linear difference equation in π_s with μ_s as a forcing variable:

$$\pi_t = \frac{\alpha}{1+\alpha}\pi_{t+1} + \frac{1}{1+\alpha}\mu_t, \quad t \geqslant 0 \tag{4}$$

where $0 < \frac{\alpha}{1+\alpha} < 1$.

We want equations (4) for $t \geqslant 0$ to determine the sequence $\vec{\pi} = \{\pi_t\}_{t=0}^{\infty}$ as a function of the sequence $\vec{\mu} = \{\mu_t\}_{t=0}^{\infty}$. Because we want to determine it, we can't take π_0 as an **initial condition**. To determine π_0 we'll require that $\vec{\pi}$ belong to L^2. When a sequence $\vec{\mu}$ is square summable and we insist that $\vec{\pi}$ is also square summable, the linear difference equation (4) can be solved forward in time to get[7]

$$\pi_t = \frac{1}{1+\alpha}\sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^j \mu_{t+j}. \tag{5}$$

If we set $\pi_t^* = \pi_t$ and $m_t^d = m_t$ and plug formula (5) into the money demand function (1), we get a "monetarist" formula for p_t:

$$p_t = m_t + \sum_{j=0}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^{j+1} \mu_{t+j}. \tag{6}$$

3 A Lucas-Critique Model

Equation (5) tells us **what** private agents ultimately want to forecast in an environment described by Cagan's demand function and an exogenous supply of money.[8] To complete this model, Lucas (1976) and Lucas and Sargent (1981) would specify exogenous dynamics

$$s_{t+1} = f(s_t)$$

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[7] For a scalar x_t, let L^2 be the space of sequences $\{x_t\}_{t=0}^{\infty}$ satisfying $\sum_{t=0}^{\infty} x_t^2 < +\infty$. We say that a sequence that belongs to L^2 is **square summable**. Restricting $\vec{\pi}$ to belong to L^2 eliminates "bubbles'' $c\left(\frac{1+\alpha}{\alpha}\right)^t$ that also satisfy the difference equation (4).

[8] Equation (5) and the Sargent and Wallace (1973) model on which it rests provides infrastructure for the "fiscal theory of the price level" originally formulated in equation B18 of Sargent and Wallace (1981, app. B). A Laffer curve in the inflation tax rate is at the center of that model as well as the models presented in sections 4 and 5 below.

$$\mu_t = g(s_t)$$

and from the Euler-like equation (4) deduce that

$$\pi_t^* = h(s_t) = T(f)(s_t),$$

where the operator T embodies cross-equation restrictions that econometrics should impose, but that Lucas (1976) said then standard econometric practice did not impose. Appendix A describes how the function h is not invariant with respect to policy interventions that take the form of a change in the dynamics f of $\vec{s}$. This structure amounts to a fourth example to add to those in section 5 of Lucas (1976), one that again illustrates the hallmark cross-equation restrictions that the T operator of Lucas and Sargent (1981) brings to econometrics.

4 Calvo (1978) as an Inflation Target Model

The section 3 model treated $\vec{\mu} \in L^2$ as **exogenous**. Calvo (1978) made $\vec{\mu}$ **endogenous** by constructing a theory of what $\vec{\mu}$ **should** be, thereby converting $\vec{\mu}$ from a model **input** as it was in section 3 into a model **output**. Calvo analyzed an optimum public policy problem of a type that had motivated Koopmans (1950) and Marschak (1950). As we shall see, this converts π_t into a *state* variable and makes μ_t an outcome that is a function of π_t.

In the spirit but not the letter of Calvo and Chang (1998), we assume that a Ramsey planner called the government has time t felicity function

$$U(m_t - p_t) = a_0 + a_1(m_t - p_t) - \frac{a_2}{2}(m_t - p_t)^2 \tag{7}$$

where $a_0 > 0, a_1 > 0, a_2 > 0$. When the government changes the stock of nominal money balances at rate μ_t, it incurs social costs $\frac{c}{2}\mu_t^2$. The government wants $\vec{\mu} = \{\mu_t\}_{t=0}^{\infty}$ to maximize

$$\sum_{t=0}^{\infty} \beta^t \left\{ U(m_t - p_t) - \frac{c}{2}\mu_t^2 \right\} \tag{8}$$

subject to monetarist formula (6).

We follow Chang (1998) and note that equations (5) and (6) delineate an equivalence class of continuation sequences $\{\mu_{t+j}\}_{j=0}^{\infty}$ indexed by scalars π_t that attain the same time t real balances $m_t - p_t = -\alpha\pi_t$. Because π_t intermediates $\vec{\mu}_t$ in this way, Chang took π_t as a **state** variable that confronts a **continuation Ramsey planner** at $t \geqslant 1$ and π_0 as a **choice** variable for a time 0 **Ramsey planner**.

To set the stage for our version of Chang's formulation, rewrite equation (4) as

$$\begin{bmatrix} 1 \\ \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1+\alpha}{\alpha} \end{bmatrix} \begin{bmatrix} 1 \\ \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{\alpha} \end{bmatrix} \mu_t, \quad t \geqslant 0$$

or

$$x_{t+1} = Ax_t + B\mu_t. \tag{9}$$

We write (9) so that we can apply an approach that comes from a literature on **recursive contracts** that culminated in Marcet and Marimon (2019) and was initiated by Kydland and Prescott (1980). System (9) is **peculiar** because π_0 is to be chosen by the government; it is *not* an initial condition, as it ordinarily would be in a state-space model.

Chang (1998) recognized that while π_0 is **not** a state variable for a Ramsey planner at $t = 0$, π_t **is** a state variable for a **continuation** Ramsey planner at $t \geqslant 1$. Write a planner's one-period contribution to its social welfare function as

$$\begin{aligned} -s(\pi_t, \mu_t) \equiv -r(x_t, \mu_t) &= \begin{bmatrix} 1 \\ \pi_t \end{bmatrix}' \begin{bmatrix} a_0 & -\frac{a_1\alpha}{2} \\ -\frac{a_1\alpha}{2} & -\frac{a_2\alpha^2}{2} \end{bmatrix} \begin{bmatrix} 1 \\ \pi_t \end{bmatrix} - \frac{c}{2}\mu_t^2 \\ &= -x_t'Rx_t - Q\mu_t^2. \end{aligned}$$

The government's time 0 value is

$$v_0 = -\sum_{t=0}^{\infty} \beta^t r(x_t, \mu_t) = -\sum_{t=0}^{\infty} \beta^t s(\pi_t, \mu_t). \tag{10}$$

Where the government's time t continuation value v_t satisfies

$$v_t = -\sum_{j=0}^{\infty} \beta^j s(\pi_{t+j}, \mu_{t+j}),$$

represent the dependence of v_0 on $(\vec{\pi}, \vec{\mu})$ recursively with the difference equation

$$v_t = -s(\pi_t, \mu_t) + \beta v_{t+1}. \tag{11}$$

Equation (5) maps a **policy** sequence of money growth rates $\vec{\mu} \in L^2$ into an inflation sequence $\vec{\pi} \in L^2$ and then via (11) into a continuation value sequence $\vec{v}$.

Criterion function (10) and constraint system (9) impart the following structure to the government's problem: (i) Setting $\mu_t \neq 0$ imposes costs $\frac{c}{2}\mu_t^2$ only at time t ; but (ii) μ_t affects the government's one-period utilities at all earlier dates $s = 0, 1, \ldots, t$. A single policymaker chooses a sequence $\{\mu_t\}_{t=0}^{\infty}$ once and for all, taking into account how μ_t affects household one-period utilities at all earlier dates $s = 0, 1, \ldots, t-1$.

4.1 Two Bellman Equations

A Ramsey planner chooses $(\vec{\mu}, \vec{\pi})$ to maximize (10) subject to system (9). Subdivide this problem into two stages, as in Ljungqvist and Sargent (2018, ch. 19). In the first stage, take the initial inflation rate π_0 as given and solve an associated dynamic programming problem. In the second stage, choose the best initial inflation rate π_0.

4.1.1 Subproblem 1: Continuation Ramsey Planner

Where $x_t = \begin{bmatrix} 1 \\ \pi_t \end{bmatrix}$, define a feasible set $\Omega(x_0)$ of $(\vec{x}_1, \vec{\mu}_0)$ sequences, both of which must belong to L^2:

$$\Omega(x_0) = \{(\vec{x}_1, \vec{\mu}_0) : x_{t+1} = Ax_t + B\mu_t \, , \ \forall t \geqslant 0\}$$

A value function

$$J(x_0) = \max_{(\vec{x}_1, \vec{\mu}_0) \in \Omega(x_0)} -\sum_{t=0}^{\infty} \beta^t r(x_t, \mu_t)$$

satisfies Bellman equation

$$J(x) = \max_{\mu, x'} \{-r(x, \mu) + \beta J(x')\}$$

where maximization is subject to:

$$x' = Ax + B\mu.$$

This is a linear-quadratic control discounted dynamic programming problem with optimal value function $J(x) = -x'Px$ and optimal decision rule $\mu = -Fx$.

4.1.2 Subproblem 2: Ramsey Planner

The value of the Ramsey problem is

$$v_0^R = \max_{x_0} J(x_0)$$

where v_0 is defined in equation (10) and

$$J(x_0) = -P_{11} - 2P_{21}\pi_0 - P_{22}\pi_0^2.$$

Maximizing $J(x_0)$ with respect to π_0 yields

$$\pi_0^R = -\frac{P_{21}}{P_{22}}.$$

4.1.3 Representation of Ramsey Plan

The preceding calculations provide a recursive representation of a Ramsey plan $\vec{\mu}^R$:

$$\begin{aligned}
\pi_0 &= \pi_0^R && (12)\\
\pi_{t+1} &= d_0 + d_1\pi_t && (13)\\
\mu_t &= b_0 + b_1\pi_t && (14)\\
v_t^R &= g_0 + g_1\pi_t + g_2\pi_t^2. && (15)
\end{aligned}$$

We can compute (d_0, d_1) from $(A - BF)$ and (b_0, b_1) from F, while $g_0 = -P_{11}, g_1 = -2P_{21}, g_2 = -P_{22}$, v_0^R is the value accruing to the Ramsey planner at $t = 0$, and v_t^R is the value accruing to a continuation Ramsey planner at $t \geqslant 1$. Think of $\vec{\pi}$ as a sequence of synthetic **promised inflation rates** that generates a sequence $\vec{\mu}$ of money growth rates. These can be substituted into equation (5) to form **actual** rates of inflation. (It is faithful to the model to call $\vec{\pi}$ a sequence of **inflation targets**.) If we substitute a plan $\vec{\mu} = \{\mu_t\}_{t=0}^{\infty}$ that satisfies these equations into equation (5), we obtain the same sequence $\vec{\pi}$ that is generated by the system (12)–(15), so **promised inflation** equals **actual inflation**.[9]

4.2 Relationship to Fiscal Theory of Price Level

All monetary-fiscal theories of the price level share a version of equation (6). Completing such a theory requires saying how $\vec{\mu}$ is determined. Representation (12), (13), (14) of a Ramsey plan $\vec{\mu}^R$ for the Calvo (1978) model completes a fiscal-monetary theory of the price level by pursuing implications of a timing protocol and a government purpose. There are other ways to complete a monetary-fiscal theory of the price level, one of which we shall describe in section 5.[10]

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[9] Here an application of the Big K, little k trick can be enlightening.

[10] Bassetto (2002, 2005) says more about how to complete a monetary-fiscal theory of the price level.

4.3 Time Inconsistency

Equation (13) implies that under the Ramsey plan

$$\pi_t^R = d_0 \left(\frac{1 - d_1^t}{1 - d_1} \right) + d_1^t \pi_0^R, \tag{16}$$

while μ_t varies over time according to (14) so that

$$\mu_t^R = b_0 + b_1 d_0 \left(\frac{1 - d_1^t}{1 - d_1} \right) + b_1 d_1^t \pi_0^R. \tag{17}$$

Variation of $\vec{\mu}^R$ over time is a symptom of time inconsistency. The Ramsey planner reaps immediate benefits from promising lower inflation by later imposing costly distortions. These benefits are intermediated by reductions in expected inflation that precede reductions in money creation rates that foreshadow them, as indicated by equation (5). A government decision maker offered an opportunity to ignore effects on past utilities and to re-optimize at date $t \geqslant 1$ would want to deviate from a Ramsey plan. A continuation Ramsey plan is not a Ramsey plan.[11]

In settings in which governments actually choose sequentially, private agents would not use $(\vec{\mu}^R, \vec{\pi}^R)$ to forecast $\vec{\pi}$ because they understand that government decision makers at times $t \geqslant 1$ would not choose to continue the plan $\vec{\mu}^R$. Because a sequential timing protocol is realistic, starting with Kydland and Prescott (1977) and Prescott (1977), sceptics have regarded a Ramsey plan as implausible.

4.4 Artful Dodge: "Timeless Perspective"

An occasionally used expedient side-steps the "dynamic consistency problem" by *ex cathedra* setting $\pi_0 = \frac{d_0}{1-d_1}$ instead of to π_0^R in equations (12) and (16), thereby arresting variation over time in both $\vec{\mu}$ and $\vec{\pi}$. This removes the time-variation *symptom* of time inconsistency, but not its *cause*, which is the incentive for continuation government decision makers not to continue plan $\vec{\mu}^R$. Indeed, along plan $\vec{\mu}^R$, a promised inflation rate $\pi = \frac{d_0}{1-d_1}$ is the one from which a continuation Ramsey planner is *most* tempted to deviate. Rather than than "timeless perspective", a better name for value $v_\infty^R = g_0 + g_1 \pi_\infty^R + g_2 (\pi_\infty^R)^2$ is "**worst** continuation Ramsey plan value".

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[11] Besides Calvo (1978), other influential early papers on time inconsistency of optimal macroeconomic policy problems are Kydland and Prescott (1977), Turnovsky and Brock (1980), and Brock and Turnovsky (1981).

5 A Credible Plan

By confronting continuation planners with a different state vector, Abreu (1988), Chari and Kehoe (1990), and Stokey (1989, 1991) showed how to render a Ramsey plan credible under sequential government decision making. They accomplished this by (i) allowing a time t government administrator to choose $\mu_t \in \mathbf{R}$ at t to maximize a continuation that is described by (11); but (ii) for each choice $\mu_t \in \mathbf{R}$, introducing *two* possible continuation values that depend on whether or not the time t government confirms a rate of money creation $\tilde{\mu}_t$ that the public had anticipated at the end of period $t-1$. This timing protocol allows a time t government decision maker to encounter **adverse consequences** from failing to confirm $\tilde{\mu}_t$. Chari and Kehoe (1990) and Stokey (1989, 1991) arranged consequences adverse enough to induce continuation Ramsey planners to adhere to the Ramsey plan.[12]

Decisions unfold as follows: (i) a government decision maker sets μ_t at time t; (ii) private agents' forecasts of $\{\mu_{t+j+1}, \pi_{t+j+1}\}_{j=0}^{\infty}$ respond to whether the government decision maker at t **confirms** or **disappoints** their forecast $\tilde{\mu}_t$ of μ_t brought into period t from period $t-1$; (iii) the government decision maker understands how private agents' forecasts respond to its choice of μ_t; and (iv) the government at t chooses μ_t to maximize a continuation value. For a Ramsey plan $\vec{\mu}^R$ to be credible, it must be true that

$$\mu_t^R = \arg\max_{\mu_t \in \mathbf{R}} \left\{-s(\pi_t^R, \mu_t) + \beta \hat{v}_{t+1}\right\}, \quad t \geqslant 0 \tag{18}$$

where

$$\hat{v}_t = \begin{cases} v_{t+1}^R & \text{if } \mu_t = \mu_t^R \\ v_{t+1}^D & \text{if } \mu_t \neq \mu_t^R \end{cases} \tag{19}$$

where v_{t+1}^D is a continuation value of an alternative plan that continuation government decision makers would want to implement. To provide credibility to a Ramsey plan it is necessary to construct another credible plan with continuation values v_{t+1}^D sufficiently low to satisfy (19) for all t. Next, we apply logic of Abreu (1988) to construct a sufficiently dismal credible plan.[13]

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[12] This is one way to investigate whether outcomes under sequential choices of μ_t can replicate those from a timing protocol in which $\vec{\mu}$ is chosen once and for all at time 0, i.e., whether a "reputation" can substitute for a confining timing protocol.

[13] Abreu (1988) showed how to contain the explosion of plans that, via the second part of constraint (19), lie beneath a single credible plan. He showed us how to construct a credible value vector $\vec{v}^A$ to use as $\vec{v}^D$ in equation (19).

5.1 An Adverse Credible Plan

We assume the following within-period and between-periods timing protocols for each $t \geqslant 0$: (i) at time $t-1$, private agents expect that the government will set $\mu_t = \tilde{\mu}_t$, and more generally that it will set $\mu_{t+j} = \tilde{\mu}_{t+j}$ for all $j \geqslant 0$; (ii) private agents' forecasts $\{\tilde{\mu}_{t+j}\}_{j \geqslant 0}$ determine a $\pi_t = \tilde{\pi}_t$ and an associated log of real balances $m_t - p_t$; (iii) given those expectations and an associated $\pi_t = \tilde{\pi}_t$, at t a government is free to set $\mu_t \in \mathbf{R}$; (iv) if the government at t **confirms** private agents' expectations by setting $\mu_t = \tilde{\mu}_t$, private agents anticipate the continuation government policy $\{\tilde{\mu}_{t+j+1}\}_{j=0}^{\infty}$ and therefore bring expectation $\tilde{\pi}_{t+1}$ into period $t+1$; and (v) if the government **disappoints** private agents by setting $\mu_t \neq \tilde{\mu}_t$, private agents expect the continuation policy at $t+1$ to be $\{\mu_{t+j+1}\}_{j=0}^{\infty} = \{\mu_j^A\}_{j=0}^{\infty}$ and therefore expect an associated π_0^A for $t+1$. To be credible, continuation governments must choose to adhere to $\vec{\mu}^A = \{\mu_j^A\}_{j=0}^{\infty}$.

Temptation to Deviate from a Plan A government's one-period return function $s(\pi, \mu)$ described in equation (15) above has the property that for all π

$$-s(\pi, 0) \geqslant -s(\pi, \mu)$$

Whenever the policy calls for the government to set $\mu \neq 0$, the government could raise its one-period payoff by setting $\mu = 0$. Disappointing private sector expectations by setting $\mu = 0$ would increase the government's **current** payoff but would have adverse consequences for **subsequent** government payoffs because of how the private sector would alter its expectations about future settings of μ. The **temporary** gain constitutes the government's temptation to deviate from a plan. The government at t will resist the temptation to raise its current payoff by setting $\mu = 0$ only if it foresees adverse continuation payoffs.

We call a plan $\vec{\mu}$ **credible** if at each $t \geqslant 0$ the government confirms private agents' prior expectation $\tilde{\mu}_t$ of its setting for μ_t. At each t, a credible plan involves a continuation plan $\tilde{\mu}$ to be followed if the government sets $\mu_t = \tilde{\mu}_t$ and a continuation plan $\hat{\mu}$ to be followed if the government sets $\mu_t \neq \tilde{\mu}_t$. The government chooses to confirm prior expectations only if long-term **losses** from disappointing private sector expectations outweigh immediate *gains*.

Credible plans come in **sets**. At each t, we require (i) a credible (continuation) plan to be followed if a government at t **confirms** private sector expectations; (ii) a credible plan to be followed if a government at t **disappoints** private sector expectations. A huge number of plans seems to be in play because each credible plan itself consists of *two* credible continuation plans. Thus, if a Ramsey plan is to be credible, it must satisfy

$$\begin{aligned} v_t^R &= -s(\pi_t^R, \mu_t^R) + \beta v_{t+1}^R \\ &\geqslant s(\pi_t^R, 0) + \beta v_0^D, \quad t \geqslant 0, \end{aligned} \tag{20}$$

where v_0^D is the continuation value of a *new* credible plan μ^D, π^D that emerges when the government chooses not to confirm expectation π_t^R. But for μ^D, π^D to be credible, v_0^D must also satisfy a system of constraints that are counterparts to (20). Checking whether a plan is credible requires checking whether a larger *set* of plans is credible.

Chang (1998) used technical methods of Abreu et al. (1990) to characterize the set of government values attainable by credible plans. Instead of following Chang's approach, we'll use an approach of Abreu (1988) to construct a credible plan with a low value. We'll use that plan as a continuation plan to be started whenever a government decision maker chooses not to confirm $\vec{\mu}^R$.

A key object in Abreu's approach is a **self-enforcing** plan. A plan $\vec{\mu}^A$ (the superscipt A is for Abreu) is said to be **self-enforcing** if (i) the consequence of disappointing private agents' expectations at time j is to **restart** plan $\vec{\mu}^A$ at time $j+1$; and (ii) consequences of restarting the plan are sufficiently adverse to deter all deviations from the plan. Thus, a government plan $\vec{\mu}^A$ with implied inflation sequence $\vec{\pi}^A$ is **self-enforcing** if

$$\begin{aligned} v_j^A &= -s(\pi_j^A, \mu_j^A) + \beta v_{j+1}^A \\ &\geqslant -s(\pi_j^A, 0) + \beta v_0^A \equiv v_j^{A,D}, \quad j \geqslant 0, \end{aligned} \tag{21}$$

where the identity in the second line of (21) defines the value $v_j^{A,D}$ from deviating from the plan. The second line of (21) states that the consequences of deviating from plan μ^A at time j is simply to restart the plan time $j+1$. It is useful to recall that $\mu = 0$ maximizes the government's one-period return function. The first line tells the consequences of confirming private agents' expectations by following the plan, while the second line tells the consequences of disappointing private agents' expectations. A consequence of inequality (21) is that a self-enforcing plan is credible.

Self-enforcing plans can be used to construct other credible plans, including ones with better values. Thus, where $\vec{v}^A$ is the value associated with $\vec{\mu}^A$, a sufficient condition for another plan $\vec{\mu}$ associated with inflation $\vec{\pi}$ and value $\vec{v}$ to be **credible** is that

$$\begin{aligned} v_j &= -s(\pi_j, \mu_j) + \beta v_{j+1} \\ &\geqslant -s(\pi_j, 0) + \beta v_0^A \quad \forall j \geqslant 0 \end{aligned} \tag{22}$$

For this condition to be satisfied it is necessary and sufficient that

$$-s(\pi_j, 0) - (-s(\pi_j, \mu_j)) < \beta(v_{j+1} - v_0^A)$$

The left side of the above inequality is the government's **gain** from deviating from the plan, while the right side is the government's **loss**. A key step in Abreu's (1988) approach is first to construct a self-enforcing plan that has a **low** time 0 value. To construct a self-enforcing plan $\vec{\mu}$ with a low time 0 value, we proceed as follows. We insist that future government decision makers initially set μ_t to a value yielding low one-period utilities to the household for a long time, after which government decisions yield high one-period utilities. Low one-period utilities early are a **stick**. High one-period utilities later are a **carrot** that induces earlier governments to swallow bad-tasting medicine. The bad-tasting medicine is high rates of money creation that temporarily induce high rates of inflation. Thus, consider a candidate plan $\vec{\mu}^A$ that sets $\mu_t^A = \bar{\mu}^A$ (a high positive number) for $T^A - 1$ periods, and then reverts to the Ramsey plan so that

$$\mu_t^A = \begin{cases} \bar{\mu}^A, & t = 0, \ldots, T^A - 1 \\ \mu_{t-T^A}^R, & t \geqslant T^A. \end{cases} \tag{23}$$

Denote this sequence by $\vec{\mu}^A = \{\mu_t^A\}_{t=0}^{\infty}$. A sequence of inflation rates implied by this plan, $\{\pi_t^A\}_{t=0}^{\infty}$, can be calculated to be

$$\pi_t^A = \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^{j+1} \mu_{t+j}^A$$

The value of $\{\pi_t^A, \mu_t^A\}_{t=0}^{\infty}$ at time 0 is

$$v_0^A = -\sum_{t=0}^{T_A - 1} \beta^t s(\pi_t^A, \mu_t^A) + \beta^{T_A} J(\pi_0^R),$$

and the continuation value of the plan at time $t \geqslant 1$ can be represented recursively by

$$\begin{aligned} v_{T^A}^A &= g_0 + g_1 \pi_0^R + g_2 (\pi_0^R)^2 \\ v_t^A &= -s(\pi_t^A, \mu_t^A) + \beta v_{t+1}^A. \end{aligned}$$

For big enough $\bar{\mu}^A$ and T_A, this plan is self-enforcing.

To check whether a Ramsey plan can be sustained with v_0^A as the continuation value of deviating from it, it is useful to compute $v_\infty^R \equiv \lim_{t\to\infty} v_t$. We compute $\pi_\infty^R = \lim_{t\to\infty} \pi_t^R$ to be $\pi^R_\infty = \frac{d_0}{1-d_1}$ and find

$$v^R_\infty = g_0 + g_1\pi^R_\infty + g_2(\pi^R_\infty)^2.$$

To check sustainability of a Ramsey plan it is sufficient to check the most stringent condition:

$$\begin{aligned} v^R_\infty &= -s(\pi^R_\infty, \mu^R_\infty) + \beta v^R_\infty \\ &\geqslant -s(\pi^R_\infty, 0) + \beta v^A_0 \end{aligned} \tag{24}$$

or

$$\beta(v^R_\infty - v^A_0) \geqslant s(\pi^R_\infty, \mu^R_\infty) - s(\pi^R_\infty, 0). \tag{25}$$

6 Nested Dynamic Programs

When Lucas (1976) appeared it was unclear to many readers what its implications would be either for econometric practice or for the project of applying control theoretic techniques like dynamic programming to design macroeconomic policies in the tradition of Marschak (1950, 1953). In a Carnegie-Rochester conference paper, Prescott (1977) interpreted Kydland and Prescott (1977) as having shown that rational expectations rendered dynamic programming inapplicable because of how it presented the government with a non-serial dynamic problem in which future control values affect current government payoffs.[14] It took several years before Kydland and Prescott (1980) rescinded that criticism by artfully deploying nested dynamic programs and redefining the state that confronts a planner. The Kydland and Prescott (1980) apparatus and the theory of credible plans that we have applied to Calvo's model are both applications of **dynamic programming squared** problems in which a value that emerges from one Bellman equation appears as an argument in another Bellman equation. Thus, our models have involved two Bellman equations: (i) equation (4) expresses how π_t depends on μ_t and π_{t+1}; and (ii) equation (11) expresses how value v_t depends on (μ_t, π_t) and v_{t+1}, so that a value π from one Bellman equation appears as an argument of a second Bellman equation that restricts another value v. These Bellman equations enter as nested Bellman equations for constructing a Ramsey plan: (i) a value function for **continuation** Ramsey planners; and (ii) a value function for a Ramsey planner.

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[14] Because he regarded time inconsistent plans as incredible and therefore impractical, Prescott advocated studying the operating characteristics of alternative exogenous government policy rules, along lines of Lucas (1972).

7 Finding the State as an Art

We have completed a Sargent and Wallace (1973) version of the Cagan (1956) model with three distinct theories about money creation plans $\vec{\mu}$.

- In the section 3 Lucas (1976) version, $\vec{\mu}$ is described by a state-space representation with exogenous Markov state s_t driving μ_t.
- In the section 4 version of Calvo (1978) and Chang (1998), $\vec{\mu}$ is chosen once and for all by a Ramsey planner at time 0.
- In the section 5 version inspired by Chari and Kehoe (1990), Abreu (1988), Stokey (1989, 1991), and Chang (1998), a time t government decision maker chooses μ_t.

Each version has a recursive representation cast in terms of one or more Bellman equations. State vectors differ across the three versions.

- The Markov s_t driving the exogenous $\vec{\mu}$ process is the state in the Lucas Critique version of section 3.
- The state is a promised inflation rate π_t in the section 4 Ramsey plan version.
- In the section 5 model of a credible Ramsey plan, the state at t is a triple (μ_t^R, v_t^R, v_t^D), where μ_t^R, v_t^R emerge from the section 4 Ramsey plan and $\vec{v}_t^D$ is a sequence of continuation values associated with a credible plan with a low value.

These different state variables confront the time t government decision maker to whom the model assigns responsibility for choosing μ_t.

8 RE Communism Confounds Roles

A rational expectations assumption asserts that everyone inside a model shares joint probability distributions with the model builder. In deterministic models like those in this paper, rational expectations means that everyone inside the model knows a model's input and output sequences. The models in sections 3, 4, and 5 all exploit a shared model communism. Each ruthlessly eradicates the free parameters and extra variables that would be required to carry along heterogeneities of beliefs. Such model sharing means that variables must play multiple roles.

In the Calvo-Chang model of section 4, the inflation rate π_t plays three roles: (i) in equation (5), π_t is the actual rate of inflation between t and $t+1$; (ii) in equations (1) and (2), π_t is also the public's expected rate of inflation between t and $t+1$; and (iii) in system (12)–(15), π_t is a promised rate of inflation chosen by the Ramsey planner at time 0. In the section 5 model, a credible government plan $\vec{\mu}$ plays two roles: (i) it is a sequence of actions chosen by the government; and (ii) it is a sequence of private agents' forecasts of government actions. Thus, $\vec{\mu}$ is both a government policy and a collection of private agents' forecasts of government policy.

In a rational expectations model, a government policy rule is a private sector forecasting rule about government decisions. Does the government *choose* policy actions or does it simply *confirm* prior private sector forecasts? An argument in favor of the *government chooses* interpretation comes from noting that the theory of credible plans builds in a theory that the government each period chooses the action that it wants. An argument in favor of the *simply confirm* interpretation emerges from staring at inequality (22) that defines a credible policy. Coexistence of these two interpretations disturbed Blinder (1999, ch. 3) when he discussed whether the FOMC should ever "disappoint the market".

9 Recovering Structural from Descriptive Parameters

Koopmans (1947) interpreted purely descriptive models as data-compression devices and recommended them, not as ends in themselves, but as inputs into estimating structural models with parameters that could be trusted to remain invariant with respect to historically unprecedented policy interventions. Koopmans illustrated distinct roles of descriptive and structural models with examples from the history of physics, e,g., Galilleo and Kepler produced descriptive models on the basis of which Newton constructed his structural model.

We can imagine a descriptive model that represents the pair of infinite sequences $(\vec{\mu}, \vec{\pi})$ by a vector ψ that consists of a small number of curve-fitting parameters:

$$(\vec{\mu}, \vec{\pi}) = G(\psi).$$

Our section 3, 4, and 5 models are intended to be structural. Each represents a pair of sequences $(\vec{\mu}, \vec{\pi})$ as a function of a vector θ of economically interpretable parameters. Thus, we can write the section 3 plain vanilla Lucas Critique model as

$$(\vec{\mu}, \vec{\pi}, \vec{s}) = F(\theta), \tag{26}$$

where

$$\theta = (\alpha, f, g).$$

By recycling notation by letting F represent another function, we can write the section 4 and 5 models[15] as

$$(\vec{\mu}, \vec{\pi}) = F(\theta) \tag{27}$$

where now the parameter vector is

$$\theta = (\alpha, a_0, a_1, a_2, \gamma, \beta).$$

The classic Cowles Commission[16] notion of identification relates parameters of descriptive and structural models. A vector of parameters θ of a structural model is said to be identified if it can be inferred from a vector of parameters ψ of an associated descriptive model. Koopmans and his colleagues refined this definition by introducing a special type of descriptive model G that they called the “reduced form of a structural model”.[17] They created that reduced form by using population moments implied by the structural model to construct an implied vector autoregression.[18] That let them represent parameters ψ of the reduced form with a function

$$\psi = r(\theta). \tag{28}$$

Structural parameters θ are said to be (globally) identified when r has a well defined inverse and

$$\theta = r^{-1}(\psi). \tag{29}$$

10 Experiments, Exogeneity, and Endogeneity

Studies patterned on R. A. Fisher’s “fixed regressors in repeated samples” fertilizer experiments want exogenous variations. Our section 3, 4, and 5 models differ in whether they deliver neat partitions into exogenous and endogenous variables. The section 3 model hardwires a partition by assuming the $\vec{\mu}$ is exogenous.[19] Such a partition is more tenuous in the

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[15] That outcomes $\vec{\mu}, \vec{\pi}$ are described by the same function F and the same parameter vector θ implies that it impossible to use those outcomes alone to distinguish between the 4 and 5 models: by themselves, the outcomes are silent about whether they are consequences of the once-and-for-all timing protocol for the choice of $\vec{\mu}$ in the Calvo model of section 4 or the sequential timing protocol coupled with the system of beliefs present in the Chari-Kehoe model of section 5 model.

[16] Again see Koopmans (1950) and Hood and Koopmans (1953).

[17] Gallant and Tauchen (1996) describe how to identify and estimate parameters of a structural model from a “auxiliary” descriptive model that is not the reduced form of the structural model. They explain why good estimators of structural model parameters require that the auxiliary model fits well.

[18] The Classic Cowles Commission concept of a reduced form model differs from the widespread contemporary debased “reduced form” that denotes a descriptive model that is not explicitly tied to a structural model whose parameters describe the situations and preferences of people inside the model.

[19] A perturbation of the section 3 model that allows feedback to μ_t from past π_t’s like that documented by Sargent (1977) would render that partition tenuous.

sections 4 and 5 models where Marschak-style government planners choose pairs of sequences $(\vec{\mu}, \vec{\pi})$ jointly, building in dependencies across and within entire inflation $\vec{\pi}$ and money growth $\vec{\mu}$ processes. Those dependencies preclude partitioning variables neatly into exogenous and endogenous ones. Instead, true to the intention of the Koopmans-Marschak project, cross-equation, cross-frequency restrictions come from taking **time series** to be the objects of interest.[20]

11 Treatments

Koopmans (1950) and Marschak (1950, 1953) and other giants of post World War II economic theory and statistics created modern quantitative economics because the R. A. Fisher fertilizer treatment effects approach hadn't helped them understand causal chains in dynamic stochastic models. "Occasionally, economic experience (e.g., international comparisons) provides "natural" experimental information on $T(f)$ for many different environments, but such good fortune cannot be relied on in general (Lucas and Sargent (1981, p. xiii)". Mussa (1986) offered a good example, but Mussa's data included only a few alternative f's.[21] Someone who wants to carry out a Marschak style analysis of public policy must think through a vast number of historically unprecedented policies. "On a little reflection, it is difficult to feel any general optimism as to the solubility of this problem … If any success is to be possible, it will clearly involve some boldness in the use of economic theory" (Lucas and Sargent (1981, p. xiv)).

Some thoughtful researchers have asserted that *too much* boldness would be required. That makes them pessimistic about prospects for success. Early efforts to estimate parameters of dynamic models in ways that "respect the Lucas Critique"[22] provoked Sims (1980) to argue that structural macroeconomic models embrace identifying restrictions that are so implausible that they render incredible any associated estimates of structural parameters. Sims concluded that the best practice is to estimate vector autoregressions and to interpret associated shocks and impulse response functions. For many structural models in the Koopmans-Marschak-Lucas tradition, reduced forms *are* vector autoregressions. Sims argued against pretending to go beyond those reduced forms by implementing (29), thus bringing back to life troublesome arguments of Liu (1960) that practitioners of macroeconometrics

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[20] This is the message of Sargent (1981).

[21] Note how for Mussa (1986), different "experiments" involve different $\vec{\mu}$ sequences, not just one or two different μ_t's. Alvarez et al. (2022) exploit some experiments to help them infer structural parameters. After reading about outcomes of some "experiments" in Latin America described by Kehoe and Nicolini, one might think that Lucas and Sargent's "good fortune" phrase is in bad taste.

[22] For example, see various contributions in the Lucas and Sargent (1981) collection.

had set aside for two decades.[23]

12 Concluding Remarks

While I confess occasionally sympathizing with identification doubts summarized in the last section, (see the title of Sargent and Sims (1977)), I prefer to end with grateful and optimistic notes. Muth (1960, 1961) and Lucas (1976) ignited a 1970s-1980s project that brought rational expectations into time series econometrics and applied macro and microeconomic theory. By refining our appreciation of what a structural parameter is and why it is useful, that project brought us closer to realizing the promise held out by the creators of an econometrics suitable for systems of dynamic stochastic difference equations. After learning and sympathizing with what Koopmans (1950) and Marschak (1950, 1953) intended, Lucas paid their approach the compliment of criticizing it and opening ways to improve it.[24]

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[23] Schorfheide (2000), Del Negro and Schorfheide (2004), and Christiano et al. (2005) explore and exploit connections between structural model and associated reduced forms that take the form of vector autoregressions. The success of applications like Christiano et al. (2005) has probably led Christopher Sims to become less pessimistic about the rational expectations econometrics project.

[24] Remarkable progress has been made in carrying out the project opened up by Lucas (1976). For just a few contributions, see Hall and Rust (2000), Rust and Hall (2003), Todd and Wolpin (2006), Leeper et al. (2013), Leeper et al. (2017), and Gillingham et al. (2022), and Todd and Wolpin (2023). Chemla and Hennessy (2020), Hennessy and Strebulaev (2020), and Galiani and Pantano (2021) discuss studies that aim to infer "causes and effects" from descriptive statistical models that include minimal dynamic interactions.

Appendix

A Fourth Lucas Critique Example

To fit the section 3 version of the Cagan (1956)-Sargent and Wallace (1973) model into the Lucas and Sargent (1981) framework, let $\vec{\mu}$ be governed by the linear dynamic system

$$\begin{aligned} s_{t+1} &= \tilde{A} s_t \\ \mu_t &= G s_t \end{aligned} \tag{30}$$

where s_t is an $n \times 1$ vector of variables that carry information about future rates of money creation. Then $\mu_{t+j} = G\tilde{A}^j s_t$, so equations (2) and (5) imply

$$\pi_t^* = \frac{1}{1+\alpha} G \sum_{j=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^j \tilde{A}^j s_t$$

or by applying a Neumann series formula

$$\pi_t^* = \frac{1}{1+\alpha} G \left[I - \left(\frac{\alpha}{1+\alpha} \right) \tilde{A} \right]^{-1} s_t. \tag{31}$$

Substituting this equation into (6) gives

$$p_t = m_t + \frac{\alpha}{1+\alpha} G \left[I - \left(\frac{\alpha}{1+\alpha} \right) \tilde{A} \right]^{-1} s_t$$

Thus, the joint process $\vec{p}, \vec{m}$ is governed by the state-space system

$$\begin{aligned} \begin{bmatrix} s_{t+1} \\ m_{t+1} \end{bmatrix} &= \begin{bmatrix} \tilde{A} & 0 \\ G & 1 \end{bmatrix} \begin{bmatrix} s_t \\ m_t \end{bmatrix} \\ \begin{bmatrix} m_t \\ p_t \end{bmatrix} &= H \begin{bmatrix} s_t \\ m_t \end{bmatrix} \end{aligned} \tag{32}$$

where

$$H = \begin{bmatrix} 0 & 1 \\ \frac{\alpha}{1+\alpha} G \left[I - \left(\frac{\alpha}{1+\alpha} \right) \tilde{A} \right]^{-1} & 1 \end{bmatrix}. \tag{33}$$

In terms of the Lucas and Sargent (1981) structure, set $z_t = \begin{bmatrix} s_t \\ m_t \end{bmatrix}, x_t = p_t, u_t = \pi_t^*$, where m_t is a component of z_t. Then system (32)-(33) is an instance of Lucas and Sargent's (1981) structure in which

$$\begin{aligned} z_t &= f(z_t, \epsilon_{t+1}) \\ 0 &= g(x_t, z_t, u_t) \\ u_t &= h(z_t) \\ h &= T(f) \end{aligned} \tag{34}$$

Evidently we can set

$$\begin{aligned} f &: s_{t+1} = \tilde{A} s_t + \tilde{C} \epsilon_{t+1} \\ h &: \pi_t^* = \frac{1}{1+\alpha} G \left[I - \left(\frac{\alpha}{1+\alpha} \right) \tilde{A} \right]^{-1} s_t, \end{aligned} \tag{35}$$

the second equation of which exhibits an instance the hallmark cross-equation restrictions carried by the operator $T(f)$.[25]

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[25] In the first equation of (35) I have added a term $\tilde{C}\epsilon_{t+1}$. That the presence of this term does not affect the decision rule for π_t^* is an implication of a Theil-Simon **certainty equivalence** outcome that prevails in linear models. See Lucas and Sargent (1981) for a discussion of how this principle brought insights about distinct contributions to dynamic decision rules made by **optimization** and **forecasting**.

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