The Fundamental Surplus Strikes Again

Lars Ljungqvist Thomas J. Sargent*

August 5, 2020

Abstract

The fundamental surplus isolates parameters that determine the sensitivity of unemployment to productivity in the matching model of Christiano, Eichenbaum, and Trabandt (2016 and in this issue) with alternating-offer wage bargaining. That model thus joins a collection of models in which the fundamental surplus is a useful diagnostic tool.

KEY WORDS: Matching model, alternating offer bargaining, market tightness, fundamental surplus, unemployment, volatility, business cycle.

*Ljungqvist: Stockholm School of Economics and New York University (email: lars.ljungqvist@hhs.se); Sargent: New York University and Hoover Institution (email: thomas.sargent@nyu.edu). We are grateful for excellent research assistance by Quentin Batista and Zejin Shi. Ljungqvist's research was supported by a grant from the Jan Wallander and Tom Hedelius Foundation.

1 Introduction

Christiano, Eichenbaum, and Trabandt (in this issue, CET) assert that their model belies our claim that features that determine the responsiveness of unemployment to productivity in a class of matching models are intermediated through the fundamental surplus.[1] To the contrary, under both of their wage bargaining protocols, their model is a poster child for the power of the fundamental surplus to identify critical parameters that need to be calibrated large enough to make unemployment respond sensitively to productivity. Indeed, although they neglect to point this out, CET's calculations under their Nash bargaining protocol actually provide an excellent example of the fundamental surplus at work. But what about outcomes under their alternating-offer bargaining (AOB)? Here CET's analysis gets derailed by their failure to use the appropriate decomposition of the elasticity of market tightness with respect to productivity.[2] When we deploy the appropriate decomposition, CET's AOB model joins the class of models gathered in Ljungqvist and Sargent (2017) for which the fundamental surplus is the key intermediating quantity.

CET present two candidate decompositions for their AOB model, but choose the wrong one, a wrong turn that could have been avoided if they had understood our approximating version of their model. While CET correctly note that the approximating version postulates identical job destruction rates during bargaining and production spells, their empirical criticism of that simplifying assumption fails to recognize the analytical advantages the assumption brings in isolating the parameters that determine the responsiveness of unemployment to productivity in their AOB model. We can bring out how that simplifying assumption sheds light by presenting stochastic simulations of their model across the unconstrained parameter space (i.e., without the simplifying assumption). We do that in this note. It is remarkable how well the fundamental surplus isolates critical primitives when matching in the labor market is incorporated into CET's DSGE model with habit formation in preferences, adjustment costs in capital formation, capacity utilization costs, Calvo sticky price frictions, and a Taylor rule for monetary policy. Thus, to attain what CET would view to be

[1] In the class of matching models studied by Ljungqvist and Sargent (2017), the fundamental surplus is defined as productivity y minus a quantity x that the *invisible hand* cannot allocate to vacancy creation, a quantity whose economic interpretation differs across models. For example, in a standard matching with Nash bargaining x is the value of leisure. To induce them to work, workers have to receive at least the value of leisure, so the invisible hand cannot allocate that value to vacancy creation. For a fundamental surplus $y - x$, we call $(y - x)/y$ the fundamental surplus fraction.

[2] Market tightness in matching models refers to the endogenous ratio of vacancies to unemployment, movements in which affect the number of matches formed and the resulting level of unemployment.

an empirically plausible model, it is necessary to assemble a combination of high values of the replacement rate in unemployment compensation, the firm's cost to make a counteroffer in wage bargaining, and the fixed cost paid by the firm before bargaining with a newly-found worker. If those parameters are set too low, the fundamental surplus fails to be small enough to make unemployment respond sensitively to productivity.[3]

CET's proposed decomposition of the elasticity of market tightness into two endogenous components, namely, wage inertia and a profit rate, differs in spirit from our two-factor multiplicative decomposition that isolates *parameters*, inputs into the analysis, not outputs. We recognize that the first factor in our multiplicative decomposition contains endogenous objects, but it satisfies a relatively low upper bound that emerged from a professional consensus about values of the exogenous elasticity of matching with respect to unemployment. The second factor in our multiplicative decomposition – the inverse of the fundamental surplus fraction – is an object that is cast solely in terms of parameters. It is this factor that must be large in order for unemployment to respond sensitively to productivity. Hence, so long as parameter values deliver a small enough fundamental surplus fraction, a plethora of matching models that differ in many details can make unemployment respond sensitively to productivity.[4]

For readers who want a model in which the fundamental surplus is *not* central, Kehoe, Lopez, Midrigan, and Pastorino (2019) present a matching model in which they correctly claim to "have abstracted from the standard mechanism of *differential productivity across sectors* [market production versus home production and vacancy creation]." A fruitful way to get under the hood of the Kehoe et al. model is to interpret their assumptions about parameters in terms of our decomposition of the elasticity of market tightness. First, they assume that key quantities are proportional to productivity, in particular, quantities that are deducted from productivity when calculating the fundamental surplus; this causes our

[3]Besides arguing that their AOB model constitutes a counterexample to the channel of the fundamental surplus (which it evidently is not), CET question the predictive power of the fundamental surplus when they impose an exogenous *ad hoc* law of motion for the wage rate. To us this approach throws the baby out with the bath water: how endogenous wage outcomes are influenced by deep parameters characterizing preferences and opportunities are part of the coronary system that makes matching models tick.

[4]In view of earlier communications with them, it mystifies us that CET say that the fundamental surplus downplays wage inertia. In the context of AOB models, we had corrected that misunderstanding when in Ljungqvist and Sargent (2017, p. 2649) we wrote: "we do not doubt that the [AOB model] of Hall and Milgrom (2008) suppresses the influence of the worker's outside value during bargaining [and thereby causes wages to respond less sensitively to unemployment]. But this outcome would be irrelevant had Hall and Milgrom not calibrated a small fundamental surplus fraction." That assertion also applies to CET's version of the AOB model. (Also see footnote 15.)

second multiplicative factor comprising the fundamental surplus to vanish. And since the first multiplicative factor is bounded from above, it follows that the elasticity cannot be large (a point anticipated by Ljungqvist and Sargent (2017, p. 2664, footnote 28)). Second, assuming that a firm's cost of posting a vacancy is proportional to productivity makes the elasticity become zero. As a result of these assumptions, under standard preferences with constant relative risk aversion Kehoe et al. find that unemployment is unresponsive to productivity. To change that outcome, they assume preferences that make risk aversion vary over a business cycle; their baseline model uses a version of Campbell-Cochrane (1999) preferences. Thus, while Kehoe et al. contribute to a literature on how assuming what David Backus mischievously called "exotic preferences" can generate fluctuations,[5] their analysis is not within the general class of matching models with "standard preferences" maintained by CET.

Section 2 puts all the cards on the table and shows that the fundamental surplus works like a charm for CET's models. Sections 3 and 4 drill down into the details of a proper fundamental surplus analysis of CET's AOB model. An example of how to disarm the fundamental surplus channel is presented in Section 5. Section 6 offers a concluding remark.

2 Fundamental surplus works for CET

Our fundamental surplus analysis rests on a two-factor multiplicative decomposition of the elasticity of market tightness with respect to productivity. Because the first factor is bounded from above by what is widely agreed to be a small number, it is the second factor – the inverse of the fundamental surplus fraction – that determines how sensitively unemployment responds to productivity. The first line of Table 1 reproduces the decomposition for CET's estimated Nash bargaining model and the second line presents the decomposition for the

[5] Although Backus et al. (2004) did not list Campbell-Cochrane preferences in their catalogue of "Exotic Preferences for Macroeconomists," in conversations with us, David Backus did include them. Ljungqvist and Uhlig (2015) describe peculiar habit dynamics and welfare implications brought by Campbell-Cochrane preferences. Kehoe et al. acquired those peculiarities when they adopted Campbell-Cochrane preferences with external habit, but they avoided them under their baseline setup in which they assumed that the Campbell-Cochrane habit stock is a function of aggregate productivity instead of consumption. Technically, that makes the shock process do double duty both as productivity shock and as preference shock (intermediated through a Campbell-Cochrane look-alike law of motion). Backus et al. did call "exotic" the Epstein-Zin preferences that Kehoe et al. use to show how with these preferences let their results prevail in setups with: (1) a slow-moving predictable component of productivity as in Bansal and Yaron (2004) and discount factor shocks as in Albuquerque et al. (2016) and Schorfheide et al. (2018); and (2) a slow-moving probability of rare disasters as in Wachter (2013).

Table 1: Elasticity of market tightness with respect to productivity

	Elasticity = $\eta_{\theta,y}$	Υ ×	Inverse of the fundamental surplus fraction
Estimated Nash bargaining	20.4	2.11	9.65
Restricted Nash bargaining	3.97	2.11	1.88
Estimated AOB	24.2	1.65	14.66
Restricted AOB	3.00	1.65	1.82

restricted model, described in the last two lines of CET's Table 2. As first reported by Christiano, Eichenbaum, and Trabandt (2016), their estimated Nash bargaining model fits the data well, but they regard their estimated value of unemployment benefits (88% of the steady-state wage) to be implausibly high. Thus following Christiano et al. (2016), to construct their restricted Nash bargaining model, CET reduce the value of unemployment benefits by 58% in order to make it equal its estimated value from their AOB model. After this change, their restricted Nash bargaining model can no longer explain the data. Through a fundamental surplus lens, the culprit is that smaller unemployment benefits enlarge the fundamental surplus fraction and hence, lower the second factor of the decomposition in Table 1. That makes unemployment much less responsive to productivity. Thus, CET's estimated and restricted Nash bargaining models provide excellent examples of the fundamental surplus at work, a finding that we regret that CET did not bring out.

Turning to the AOB model in Table 1, we adopt the decomposition that had been derived by Christiano et al. (2016, online appendix) and reproduced in the second half of CET's section 3.2.2, but then ignored by CET. Instead, CET frame their arguments in terms of a misleading alternative decomposition in the first half of CET's section 3.2.2. For reasons that we'll soon explain, we use the decomposition of Christiano et al. (2016, online appendix) to compute the quantities in the second two lines of Table 1 rather than the misleading decomposition that CET presented in their Table 2. The quantities reported in our Table 1 indicate that the high elasticity of the estimated AOB model is again attributable to a small fundamental surplus fraction, i.e., a large second factor of the decomposition; the first factor is as usual small. Christiano et al. (2016) conclude that their estimated models with either one of their bargaining protocols explain the data equally well. Thus, both versions generate responses of unemployment to a neutral technology shock that are virtually identical to one another and close to the empirically estimated impulse response function.

Absent from CET's presentation is a recognition that properly configured restricted versions of their models once again illustrate how the fundamental surplus underlies their results. The restricted AOB model in Table 1 perturbs parameters in the fashion suggested by Ljungqvist and Sargent (2017, sec. VII.B) as a way to create an appropriate counterpart to CET's restricted Nash bargaining model. Guided by the parameters that enter the fundamental surplus in the AOB framework, we reduced the value of unemployment compensation and a firm's cost of delay in bargaining both by 58%. When these changes are made, the restricted AOB model experiences the same dramatic deterioration in performance as does CET's restricted Nash bargaining model.[6] Thus, as depicted by Ljungqvist and Sargent (2018, p. 1290, Figure 30.3.6), impulse response functions of unemployment to a neutral technology shock are dramatically suppressed in both restricted models, while still remaining virtually indistinguishable from each other.[7] Table 1 confirms that the associated decline in the elasticity of market tightness is due to a fall in the second factor of the decomposition for the restricted AOB model, i.e., an enlarged fundamental surplus fraction. Hence, the estimated and restricted AOB models both provide outstanding examples of the fundamental surplus at work yet again.

To shed more light on how the fundamental surplus shapes unemployment volatility in CET's stochastic AOB model, we compute a CET version of a figure that Ljungqvist and Sargent (2017, p. 2660, Figure 6) used to analyze the original AOB model by Hall and Milgrom (2008). Starting from CET's estimated AOB model, we perturb a firm's cost of delay in bargaining $\gamma_\triangle$ and the probability δ that a job opportunity is exogenously destroyed between bargaining rounds. Figure 1 shows how different constellations of $(\gamma_\triangle, \delta)$ affect the standard deviation of unemployment. For each pair $(\gamma_\triangle, \delta)$, we adjust a firm's cost of posting a vacancy to make the average unemployment rate stay at 5.5 percent.[8] The circle labelled 'CET' shows the location of CET's estimated AOB model with parameters $\gamma_\triangle = 0.009$ and $\delta = 0.002$. The circle labelled 'restricted' indicates the restricted AOB model except that

[6] Larry Christiano, Marty Eichenbaum and Mathias Trabandt generously conducted this experiment.

[7] For the estimated models, these impulse response functions are also indistinguishable from each other, as shown in the same figure of Ljungqvist and Sargent (2018, p. 1290, Figure 30.3.6).

[8] Ljungqvist and Sargent (2017) instead adjusted the multiplicative efficiency parameter of the matching function to maintain an average unemployment rate of 5.5 percent. Choosing to use a firm's cost of posting a vacancy and/or the multiplicative efficiency parameter of the matching function to attain the unemployment target affects only vacancy outcomes. As explained in detail by Ljungqvist and Sargent (2017, online appendix, footnote 33), without targets for vacancy statistics, the choice is immaterial.

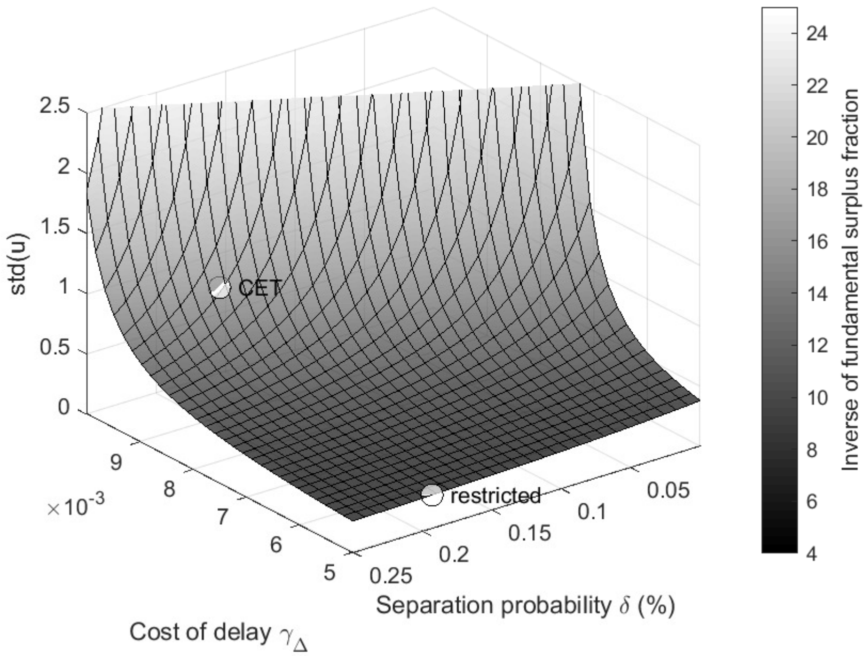

Figure 1: Standard deviation of unemployment in percentage points for different constellations of a firm's cost of delay in bargaining ($\gamma_\triangle$) and the exogenous separation rate while bargaining (δ) in CET's AOB model.

we have reduced $\gamma_\triangle$ only to 0.005 and have left the value of unemployment compensation unchanged.

As indicated by the display to the right of Figure 1, the shade shows the magnitude of the inverse of the fundamental surplus fraction derived by Christiano et al. (2016, online appendix) and used in our Table 1. Evidently, CET's AOB model cannot generate high unemployment volatility without a small fundamental surplus fraction. Furthermore, Figure 1 conveys that a firm's cost of delay in bargaining is a critical parameter for determining the size of the fundamental surplus fraction. In particular, if the constellation of $(\gamma_\triangle, \delta)$ places a model well within the lowlands in Figure 1, attaining a higher standard deviation of unemployment would require setting a higher value of $\gamma_\triangle$. Our next two sections say more about this graph and what it reveals about the fundamental surplus channel in the AOB framework.

Papetti (2019) shows that the fundamental surplus explains a "ridge" of structural parameters in CET's AOB model that are required to fit observed unemployment volatility well. He does this by estimating CET's AOB model conditional on various values of a firm's cost of delay in bargaining. He confirms the presence of an active fundamental surplus force by first uncovering a tight negative linear relationship between the firm's cost of delay in bargaining that he imposes and the value of unemployment compensation that he estimates. Then he computes the fundamental surplus fraction and, as we would expect, finds that it is small and virtually constant across all of his restricted re-estimations of the AOB model. Recall that our multiplicative decomposition of market tightness indicates that the second factor – the inverse of the fundamental surplus fraction – has to be large in order to generate high unemployment volatility. Since Papetti's diverse restricted estimations of the AOB model all require similar high elasticities of market tightness in order to fit observed unemployment volatility, it follows that their associated fundamental surplus fractions must all be small and of similar magnitudes.

The next two sections elaborate on why AOB models pose a special challenge for the fundamental surplus analysis. While some 'art' is required to create appropriate decompositions of the elasticity of market tightness, doing so yields significant insights, as the following application to CET's AOB model shows.

3 An approximating version of CET's AOB model

Hall and Milgrom (2008) were the first to put alternating offer wage bargaining in matching models as a way to enhance business cycle dynamics. Rather than Nash bargaining, a firm and a worker take turns making wage offers. At each bargaining round, the threat is to continue to bargain because doing that has a strictly higher payoff than accepting the outside option. After each unsuccessful bargaining round, the firm incurs a cost of delay $\gamma > 0$ while the worker enjoys the value of leisure z. There is also a probability δ that between bargaining rounds the job opportunity is exogenously destroyed and the worker is sent to the unemployment pool. Hall and Milgrom (2008, p. 1673) emphasize that "the limited influence of unemployment [the outside value of workers] on the wage results in large fluctuations in unemployment under plausible movements in [productivity y]."

It is an analytical advantage that the second factor in the two-factor decomposition of the elasticity of market tightness, the primary determinant of the magnitude of the elasticity, is cast solely in terms of parameters. But for AOB, Ljungqvist and Sargent (2017, sec. IV.C) point out that this is true only when the exogenous job destruction probabilities under bargaining and production are assumed to be equal. Proceeding under that simplifying assumption, we analyzed what can be called an "approximating version" of Hall and Milgrom's AOB model in order to arrive at the desired two-factor decomposition of the elasticity. We now do the same for CET's AOB, including their assumption of a fixed cost κ paid by the firm before bargaining with a newly-found worker.

Instead of Hall and Milgrom's (2008) assumption that alternating offers are made in successive periods and can continue indefinitely, CET assume that bargaining proceeds within a period and that there is a maximum number of rounds. Specifically, the firm and the worker take turns extending offers across an even number M sub-periods. The firm goes first and makes a wage offer w_1^f in the first sub-period and, so long as there is no agreement, continues to make offers w_j^f in subsequent odd sub-periods $j = 3, 5, \ldots, M-1$. Likewise, the worker makes wage offers w_j^w in even sub-periods $j = 2, 4, \ldots, M$. An offer w_M^w in the last sub-period is assumed to be take-it-or-leave-it, i.e., the match is broken up if the offer is not accepted.[9]

A sub-period wage rate refers to compensation for work performed in one sub-period. For example, if the first wage offer w_1^f were to be accepted, as it will be in an equilibrium, the match produces output throughout the entire period and the worker's total compensation would be $Mw_1^f \equiv w$. Analogously, on a sub-period basis, a worker's output, the value of leisure and a firm's cost of delay in bargaining, are denoted $y_\triangle \equiv y/M$, $z_\triangle \equiv z/M$, and $\gamma_\triangle \equiv \gamma/M$, respectively. We will now show that the determinant of the fundamental surplus fraction and hence, of the elasticity of market tightness with respect to productivity, is a linear combination of the value of leisure z, a firm's cost of delay in bargaining γ, and the fixed cost κ paid by a firm before bargaining with a newly-found worker.

Since it is optimal for both bargaining parties to make barely acceptable wage offers, the following indifference conditions must hold across two successive equilibrium wage offers.[10] In an odd sub-period j, the equilibrium wage offer w_j^f extended by the firm would make the worker indifferent between accepting the offer or waiting until the next sub-period to make the equilibrium counteroffer w_{j+1}^w:

$$(M-j+1)w_j^f = z_\triangle + (M-j)w_{j+1}^w, \qquad \text{for } j = 1, 3, \ldots, M-1, \tag{1}$$

where the left side is the worker's labor income for the remaining number of sub-periods

[9] As just mentioned, we adopt the assumption that job destruction probabilities are the same during bargaining and production. Thus, since there is no intraperiod job destruction during production, we assume no job destruction between bargaining rounds within a period.

[10] When indifferent between accepting and declining, an agent is assumed to accept an offer.

$M - j + 1$ when accepting and working at the wage rate w_j^f, while the right side is the worker's value upon declining the offer which consists of the value of leisure $z_\triangle$ in the current sub-period and subsequent labor income when working for one less sub-period $M - j$ at the counteroffer wage rate w_{j+1}^w (that the firm would accept, in accordance with the indifference condition to which we turn next). Symmetrically, in an even sub-period j, the equilibrium wage offer w_j^w extended by the worker would be such that the firm is indifferent between accepting the offer or waiting until the next sub-period to make the equilibrium counteroffer w_{j+1}^f:

$$(M - j + 1)\left[y_\triangle - w_j^w\right] = -\gamma_\triangle + (M - j)\left[y_\triangle - w_{j+1}^f\right], \quad \text{for } j = 2, 4, \ldots, M - 2, \tag{2}$$

where the left side is the firm's profits during the remainder of the period when accepting the wage offer, while the right side is the firm's value upon declining the offer, i.e., the incurrence of a cost of delay in bargaining $\gamma_\triangle$ and the subsequent earning of profits based on the firm's counteroffer (that the worker would accept, in accordance with indifference condition (1)).

After recursively substituting wages in equations (1) and (2), the first wage offer can be expressed as

$$w_1^f = \frac{z_\triangle + \left(\frac{M}{2} - 1\right)\left[z_\triangle + \gamma_\triangle + y_\triangle\right] + w_M^w}{M}. \tag{3}$$

Regarding the worker's take-it-or-leave-it offer w_M^w extended to the firm with a zero outside value, the worker would ask for the entire output $y_\triangle$ in that sub-period M and the expected present value of all future profits in the match,

$$w_M^w = y_\triangle + \sum_{i=1}^{\infty} \tilde{\beta}^i M\left(y_\triangle - w_1^f\right) = y_\triangle + \tilde{\beta} M \frac{y_\triangle - w_1^f}{1 - \tilde{\beta}}, \tag{4}$$

where $\tilde{\beta} \equiv \beta(1 - s)$, and β is workers' and firms' discount factor, and s is an exogenous job destruction probability between periods. After substituting (4) into (3), the first wage offer can be expressed solely in terms of primitives,

$$w_1^f = y_\triangle - \frac{1}{2}(1 - \tilde{\beta})\left[y_\triangle - z_\triangle - \frac{M - 2}{M}\gamma_\triangle\right]. \tag{5}$$

This is the wage that a firm would immediately offer a worker when first matched; the offer would be accepted. Hence, a firm's value of a filled job becomes

$$J = \sum_{i=0}^{\infty} \tilde{\beta}^i (y - w) = \frac{y - M w_1^f}{1 - \tilde{\beta}} = \frac{1}{2}\left[y - z - \frac{M-2}{M}\gamma\right], \tag{6}$$

where we have substituted $M w_1^f$ for the per-period wage rate w and expression (5) for w_1^f.

In an equilibrium, the zero-profit condition in vacancy creation must hold,

$$c = \beta\, q(\theta)\, [J - \kappa]\,, \tag{7}$$

where the left side is the cost of posting a vacancy c, and the right side is the firm's expected discounted payoff. The probability $q(\theta)$ that a vacancy encounters an unemployed worker is a function of equilibrium market tightness θ: $q(\theta) = A\theta^{-\alpha}$ where $A > 0$ and $\alpha \in (0, 1)$. Upon encountering a worker, the firm pays the fixed cost κ before bargaining. Thus, the firm's payoff from an encounter is the value of a filled job J minus the fixed cost κ. After substituting expression (6) into (7), an equilibrium condition for market tightness is

$$\frac{c}{\beta q(\theta)} = \frac{1}{2}\left[y - z - \frac{M-2}{M}\gamma - 2\kappa\right]. \tag{8}$$

As in Ljungqvist and Sargent (2017), we can use implicit differentiation to compute the elasticity of market tightness with respect to productivity:

$$\eta_{\theta,y} = \frac{1}{\alpha}\, \frac{y}{y - z - \frac{M-2}{M}\gamma - 2\kappa} \equiv \Upsilon^{\text{sticky}}\, \frac{y}{y - z - \frac{M-2}{M}\gamma - 2\kappa}, \tag{9}$$

where the second factor is the inverse of the fundamental surplus fraction. Recall from footnote 1 that the fundamental surplus is the difference between productivity y and model-specific quantities that the *invisible hand* cannot allocate to vacancy creation. In addition to the value of leisure z deducted in a standard Nash bargaining model, deductions now include a firm's cost γ of delay in bargaining. This item captures the worker's prospective gains from exploiting the cost that delay imposes on the firm. The third deduction is twice the value of the fixed cost κ that a firm must pay before bargaining with a newly-found worker.[11]

[11] For an equilibrium to exist, the fundamental surplus in the denominator of the second factor in decomposition (9) must be positive, i.e., parameters must satisfy

$$y > z + \frac{M-2}{M}\gamma + 2\kappa\,.$$

To shed further light on decomposition (9), we compute a corresponding expression for the elasticity of market tightness in the original AOB model of Hall and Milgrom (2008),[12]

$$\eta_{\theta,y}^{\mathrm{HM}} = \Upsilon^{\mathrm{sticky}} \frac{y}{y - z - \tilde{\beta}\gamma - (1-\tilde{\beta}^2)\kappa} = \Upsilon^{\mathrm{sticky}} \frac{y}{y - z - \tilde{\beta}\gamma - (1+\tilde{\beta})(1-\tilde{\beta})\kappa} . \tag{10}$$

The deductions in the fundamental surpluses in expressions (9) and (10) are the same for the value of leisure z and approximately the same for the cost of delay in bargaining γ, but there is a large difference with respect to the fixed matching cost κ. We have here factored the coefficient on κ into $(1+\tilde{\beta})$ times $(1-\tilde{\beta})$ in expression (10). Evidently, the factor $(1+\tilde{\beta})$ corresponds to the coefficient 2 on κ in CET's AOB model, in which, between bargaining rounds, there is neither discounting nor, in the approximating version, job destruction. (Hence, the factor $(1+\tilde{\beta})$ in expression (10) becomes $(1+1)$ in (9).) The remaining part of the κ term in (10) is the annuitized value of the fixed cost, $(1-\tilde{\beta})\kappa$, i.e., the fixed cost is amortized over the expected duration of a match.[13] Why instead does the full value κ appear in the remaining κ term of CET's AOB model? The answer lies in CET's assumption of a finite bargaining horizon of one model period, at the end of which, if bargaining runs its full course, the last wage offer will be extended by the worker as a take-it-or-leave-it offer to the firm. Consequently, CET's equilibrium bargaining outcome is one in which the firm is *essentially* confined to a single model period to recover its costs of forming a match, including the fixed cost κ. Therefore, the capital value κ rather than any amortized value is deducted to arrive at the fundamental surplus in CET's AOB model.

For yet another comparison, we turn to the corresponding elasticity of market tightness

---

[12] After invoking the equilibrium wage in the approximating version of the Hall-Milgrom (2008) AOB model in Ljungqvist and Sargent (2017, p. 2648, eq. (35)), a firm's value of a filled job is

$$J = \frac{y-w}{1-\tilde{\beta}} = \frac{y - \dfrac{z+\tilde{\beta}(y+\gamma)}{1+\tilde{\beta}}}{1-\tilde{\beta}} = \frac{1}{1-\tilde{\beta}^2}\left[(1+\tilde{\beta})y - z - \tilde{\beta}(y+\gamma)\right] = \frac{1}{1-\tilde{\beta}^2}\left[y - z - \tilde{\beta}\gamma\right],$$

and hence, expression (7) for the zero-profit condition in vacancy creation becomes

$$\frac{c}{\beta q(\theta)} = J - \kappa = \frac{1}{1-\tilde{\beta}^2}\left[y - z - \tilde{\beta}\gamma - (1-\tilde{\beta}^2)\kappa\right].$$

We can use implicit differentiation to compute the elasticity of market tightness in expression (10).

[13] Let ψ be an annuity that, when paid for the duration of a match, has the same expected present value as a firm's fixed matching cost κ so that $\sum_{i=0}^{\infty} \beta^i (1-s)^i \psi = \kappa \quad \Rightarrow \quad \psi = (1-\tilde{\beta})\kappa$.

in a Nash bargaining model,

$$\eta_{\theta,y}^{\text{Nash}} = \Upsilon^{\text{alt}}(\cdot)\ \frac{y}{y - z - \frac{1}{1-\phi}(1-\tilde{\beta})\,\kappa}\,, \tag{11}$$

where $1-\phi$ is the firm's Nash bargaining weight, and the factor $\Upsilon^{\text{alt}}(\cdot)$ satisfies a relatively low upper bound that emerged from a professional consensus about values of the exogenous elasticity of matching with respect to unemployment.[14] The analogue to expressions (9) and (10) for the AOB models is the deduction of the value of leisure z while there is no cost of delay in bargaining γ in the Nash bargaining model. Just as in the Hall-Milgrom AOB model, the annuitized value of the fixed matching cost, $(1-\tilde{\beta})\kappa$, appears in expression (11). Regarding the other factor of the coefficient on κ, there is now the inverse of the firm's bargaining weight, $(1-\phi)^{-1}$. As explained by Ljungqvist and Sargent (2017, online appendix A.5), this comes from the timing protocol that the firm pays the fixed cost *before* rather than *after* bargaining with a newly-found worker. In particular, when the fixed matching cost is paid before bargaining in the Nash bargaining model, the incidence of the cost falls *essentially* on the firm alone as it has become a sunk cost by the time bargaining occurs, which amplifies its suppressing the fundamental surplus. Specifically, it manifests itself as the amortized value of the fixed matching cost being divided by the firm's bargaining weight – an effect that is absent when the fixed matching cost is paid after bargaining in the Nash bargaining model. (See Ljungqvist and Sargent (2017, sec. III.C).)

In AOB models, the inverse of the fundamental surplus fraction naturally governs how sensitively unemployment responds to productivity. Recall how zero-profit condition (7) in vacancy creation ensures that firms expect to break even when they post vacancies. A consequence is that a firm's gain from matching with an unemployed worker $(J-\kappa)$ will on average have been spent on the vacancy posting costs incurred to fill that job. Interestingly, the firm's gain on the right side of expression (8) is exactly half of the fundamental surplus in the denominator of the second factor in decomposition (9). That constant fraction is the heart of the matter. First, since the fundamental surplus $y-x$ is a part of productivity y,

[14]Ljungqvist and Sargent (2017, online appendix A.5) derive expression (11) as

$$\eta_{\theta,y}^{\text{Nash}} = \Upsilon^{\text{alt}}\big(\kappa/c\big)\ \frac{y}{y - z - \frac{1}{1-\phi}\beta(r+s)\,\kappa}\,,$$

where $r \equiv \beta^{-1}-1$ is the net subjective rate of discounting and hence, $\beta(r+s) = 1-\tilde{\beta}$. The factor $\Upsilon^{\text{alt}}(\cdot)$ is shown to be bounded from above by $\max\{\alpha^{-1}, (1-\alpha)^{-1}\}$.

it follows that a given percentage change in productivity translates into a larger percentage change in the fundamental surplus via the multiplier $y/(y-x)$, i.e., the inverse of the fundamental surplus fraction. Second, a consequence of the firm on average spending half of the fundamental surplus on filling a job is that a small fundamental surplus fraction magnifies the effect of a productivity change on the equilibrium quantity of resources spent on vacancy creation. Third, that magnified change on vacancy creation causes a large change in market tightness, as conveyed by decomposition (9), and hence, unemployment responds sensitively to small changes in productivity.[15]

4 Pleasant fundamental surplus arithmetic for CET

CET confirm that our approximating version of the AOB model in the preceding section is the same as their AOB structure under an assumption of equal job destruction probabilities during bargaining and production ($\delta = 0$). From the perspective of the fundamental surplus, it is important that consensus then prevails on our decomposition (9) of the elasticity of market tightness, which is identical to CET's derivation of their corresponding expression (27).[16] While strictly speaking our formula (9) only applies to the far-right-end curvature of Figure 1 along which $\delta = 0$, we can show that it is informative for the entire graph. Formula (9) is the multi-dimensional representation of a family of graphs like that of Figure 1 mapping out the importance of parameters z, γ and κ in the determination of how sensitively

[15]CET propose an alternative decomposition of the elasticity of market tightness as given by CET's expression (15) that we reproduce here in the first equality of

$$\eta_{\theta,y} = \frac{1}{\alpha} \underbrace{\frac{y}{y - w - (1-(1-s)\beta)\kappa}}_{1/(\text{Profit rate})} \underbrace{\left[1 - \frac{dw}{dy}\right]}_{\text{Wage inertia term}} = \frac{1}{\alpha} \underbrace{\frac{y}{\frac{1}{2}(1-\tilde{\beta})\left[y - z - \frac{M-2}{M}\gamma - 2\kappa\right]}}_{1/(\text{Profit rate})} \underbrace{\frac{1-\tilde{\beta}}{2}}_{\text{Wage inertia term}} \qquad (\star)$$

where we use our notation for variables and parameters (as detailed in footnote 16). The second equality invokes equilibrium expression (5) for the wage rate $w = Mw_1^f$ and computes its derivative with respect to productivity y. Evidently, under the simplifying assumption of equal job destruction probabilities under bargaining and production, the elasticity is a function only of parameters.

To illustrate the limitations of CET's exclusive focus on the wage inertia term, consider two economies with different values of $0.5(1-\tilde{\beta})$, and ask which one of those economies has a higher elasticity of market tightness. Simplifying expression ($\star$) to become our decomposition in (9) shows that the economy with the smaller fundamental surplus fraction has a higher elasticity of market tightness. For a discussion of how the fundamental surplus in the Nash bargaining or other matching models is related to CET's decomposition of the elasticity of market tightness in terms of profits and wage inertia, see Ljungqvist and Sargent (2017, sec. V).

[16]We adopt our notation from Ljungqvist and Sargent (2017), so our quantities α, y, z, $\gamma/M = \gamma_\triangle$ and $1-s$ correspond to CET's σ, ϑ, D, γ and ρ.

unemployment responds to productivity in CET's AOB model. Furthermore, formula (9) explains the tight negative linear relationship between γ and z that Papetti (2019) uncovers in his suite of restricted re-estimations of CET's AOB model, conditional on different values of γ. Since for producing a particular value of the elasticity of market tightness in formula (9) the two parameters are linearly related, variations in the postulated firms' cost of delay in bargaining γ are offset by estimated changes in the value of unemployment compensation z.

What implications does our simplifying $\delta = 0$ assumption have more generally for CET's AOB model? As Hall and Milgrom (2008) pointed out, the reason for high unemployment volatility in the AOB framework is the limited influence of unemployment (the outside value of workers) on wage outcomes. Hence, any increase in the probability δ that a job opportunity evaporates between bargaining rounds and sends the worker back to the unemployment pool will weaken the enhancement mechanism provided by the AOB framework by increasing the influence of a worker's outside value on wages. In Figure 1 this manifests itself as a widening of the lowlands at higher values of δ, so that it requires higher values of γ before reaching the steep slopes of the standard deviation of unemployment as a function of γ. From the perspective of the fundamental surplus, δ acts like a nuisance parameter that obscures the authentic determinants of how sensitively unemployment responds to productivity, i.e., the parameters z, γ and κ in the second factor of formula (9) for the approximating version. Admittedly, at small enough fundamental surplus fractions, perturbations of δ toward smaller values will significantly affect and increase what would already be a relatively high standard deviation of unemployment in Figure 1. Using the general decomposition of Christiano et al. (2016, online appendix) that allows any value of δ, this would appear as a further increase in an already high second factor, i.e., an already small fundamental surplus fraction would become even smaller. But once again, this would obscure the fact that the authentic determinants of the elasticity of market tightness in the decomposition of Christiano et al. would continue to be the parameters z, γ and κ: unless their combined values are large enough, the economy would be located far within the lowlands of Figure 1, and the value of δ would not matter much.[17]

[17] In their critique of the approximating version ($\delta = 0$), CET evaluate formula (9) at the parameter values of their estimated AOB model and find a negative elasticity. That negative number indicates that no equilibrium exists since parameters fail to satisfy the restriction presented in footnote 11. CET use this finding to dismiss the approximating version. We prefer to emphasize the approximating version's strength as a tool for revealing the parameters that determine the elasticity of market tightness in CET's AOB model. Thus, looking at Figure 1, while CET walk up along the far-right-end curvature where $\delta = 0$ and fall off a precipice at which no equilibrium exists, our panoramic view of the surface instead indicates how curvature at the far-right-end offers an informative overall topography of the relevant mountain walls and lowlands.

The simple decomposition in (9) derived from the approximating version, and the general decomposition of Christiano et al. (2016, online appendix) each have strengths. The simple decomposition is enlightening about how to choose between candidates for a general decomposition, and its transparent revelation of critical parameters and forces of the fundamental surplus channel. First, a strength of the simple decomposition is that a consensus prevails about its derivation under its simplifying assumption of equal job destruction probabilities during bargaining and production spells. While a good guiding principle might seem to be to find a general decomposition cast in the image of formula (9), how to accomplishing this is not obvious, to which the present exchange of views with CET testifies. CET do not tell us what criteria they used to select a decomposition from the two possiblities from their section 3.2.2, but maybe it was because the second factor of the decomposition that they unfortunately chose has no endogenous labor market objects, in contrast to the decomposition that they derived by Christiano et al. (2016, online appendix) and that we think they should have chosen.[18] The important issue here is whether the chosen general decomposition captures the essence of the decompositions of Ljungqvist and Sargent (2017), including our new formula (9), in which the first factor is subject to a low upper bound and hence, the effects of different parameterizations on how sensitively unemployment responds to productivity are transmitted through changes in the second factor – the inverse of the fundamental surplus fraction. Our section 2 observations confirm the virtues of the appropriate decomposition while what we judge to be an inappropriate decomposition in CET's Table 2 in which its first factor is evidently not constrained by any low upper bound utterly fails to shed light on the question under study here. Heuristically, the critical parameters z, γ and κ identified by the approximating version are "bleeding" into the first factor in the inappropriate decomposition and hence, no inference can be drawn from such a mixed-up and misleading decomposition. This insight from the approximating version and others discussed above demonstrate the strengths of the simple decomposition transparently to convey the essential components of the fundamental surplus channel. Furthermore, while implications from the general decomposition can be drawn out only numerically, the numerical studies we have read indicates that it performs as illustrated by, for example, Papetti's (2019) re-estimations of CET's AOB model.

[18] As shown for the simple decomposition in equation (9) and in Ljungqvist and Sargent (2017), it is analytically advantageous if the second factor of an *appropriate* decomposition can be expressed solely in terms of parameters.

5 Disarming the fundamental surplus channel

We can use CET's AOB model as an example to show how the fundamental surplus channel is disarmed when quantities deducted in the computation of the fundamental surplus covary procyclically with productivity. Let I^j, for $j \in \{z, \gamma, \kappa\}$, be an indicator equal to 0 if quantity j is either parametrically given or equal to 1 if quantity j is a fraction of productivity y, as given by $\bar{j} \cdot y$ where $\bar{j} \in (0,1)$. Under these assumptions, the zero-profit condition (8) becomes adjusted to

$$\begin{aligned} \frac{c}{\beta q(\theta)} &= \frac{1}{2}\Bigg[\left(1 - I^z\bar{z} - I^\gamma\,\frac{M-2}{M}\,\bar{\gamma} - I^\kappa \cdot 2\bar{\kappa}\right) y \\ &\qquad -(1-I^z)z - (1-I^\gamma)\frac{M-2}{M}\,\gamma - (1-I^\kappa)\cdot 2\kappa\Bigg]. \end{aligned} \tag{12}$$

We can use implicit differentiation to compute the elasticity of market tightness with respect to productivity $\bar{\eta}_{\theta,y}$. To make it comparable to earlier elasticity reported in (9), we evaluate the expression at the steady-state calibration $\bar{z}y = z$, $\bar{\gamma}y = \gamma$ and $\bar{\kappa}y = \kappa$,[19]

$$\bar{\eta}_{\theta,y} = \Upsilon^{\text{sticky}}\,\frac{y - I^z z - I^\gamma\,\dfrac{M-2}{M}\,\gamma - I^\kappa\cdot 2\kappa}{y - z - \dfrac{M-2}{M}\,\gamma - 2\kappa}. \tag{13}$$

We can verify that if no deductions vary with productivity, i.e., for $I^z = I^\gamma = I^\kappa = 0$, the elasticity $\bar{\eta}_{\theta,y}$ is identical to the earlier elasticity $\eta_{\theta,y}$ in (9). For each deduction j that is assumed to vary with productivity, setting $I^j = 1$ serves to suppress the elasticity $\bar{\eta}_{\theta,y}$. By setting $I^z = I^\gamma = I^\kappa = 1$, the smallest elasticity is attained at $\bar{\eta}_{\theta,y} = \Upsilon^{\text{sticky}}$, an outcome that completely disarms the fundamental surplus channel and that was anticipated by Ljungqvist

[19] Implicit differentiation of (12) yields

$$\frac{d\theta}{dy} = -\frac{\frac{1}{2}\left(1 - I^z\bar{z} - I^\gamma\,\frac{M-2}{M}\,\bar{\gamma} - I^\kappa\cdot 2\bar{\kappa}\right)}{-\dfrac{-q'(\theta)\,c}{q(\theta)^2\,\beta}} = \frac{\dfrac{c}{\beta q(\theta)}\left(1 - I^z\bar{z} - I^\gamma\,\frac{M-2}{M}\,\bar{\gamma} - I^\kappa\cdot 2\bar{\kappa}\right)}{\dfrac{\alpha\,c}{\theta\,q(\theta)\,\beta}\left(y - z - \frac{M-2}{M}\,\gamma - 2\kappa\right)},$$

where the second equality is obtained after using expression (12) to eliminate the quantity $\frac{1}{2}$ in the numerator, while in the denominator, we invoke the constant elasticity of matching with respect to unemployment, $\alpha = -q'(\theta)\,\theta/q(\theta)$. Note that in substituting for $\frac{1}{2}$, we use the steady-state calibration $\bar{z}y = z$, $\bar{\gamma}y = \gamma$ and $\bar{\kappa}y = \kappa$ so that the *value* of expression (12) becomes identical to that of (8) and so that the two can be interchanged. Furthermore, that calibration is used again when computing the elasticity in (13).

and Sargent (2017, p. 2664, footnote 28):

> Extending a challenge that Chodorow-Reich and Karabarbounis (2016) presented to the matching literature, suppose that movements in productivity are associated with offsetting comovements in factors that affect deductions from productivity in the fundamental surplus. By arresting the fundamental surplus fraction, those offsetting changes would make unemployment unresponsive to productivity changes. While Chodorow-Reich and Karabarbounis investigated only the consequences of a procyclical value of leisure, similar consequences would flow from procyclicality in, e.g., fixed matching costs, a firm's cost of delay in alternating-offer wage bargaining, ...

In addition to setting $I^z = I^\gamma = I^\kappa = 1$, suppose that the cost of posting a vacancy c also varies with productivity, as given by $\bar{c}\,y$ where $\bar{c} > 0$. Zero-profit condition (12) then becomes

$$\frac{\bar{c}\,y}{\beta q(\theta)} = \frac{1}{2}\left(1 - \bar{z} - \frac{M-2}{M}\,\bar{\gamma} - 2\bar{\kappa}\right) y. \tag{14}$$

Since y cancels in equation (14), market tightness θ does not depend on productivity; hence, the elasticity of market tightness with respect to productivity is zero. This is the route taken by Kehoe et al. (2019) who correctly claim to "have abstracted from the standard mechanism of *differential productivity across sectors* [market production versus home production and vacancy creation]." Indeed, Kehoe et al. confirm that under standard preferences with constant relative risk aversion, unemployment is unresponsive to productivity in their model, so alternative "exotic preferences" are required to explain volatility. Thus, even here the fundamental surplus serves as a handy tool for understanding ingredients needed to arrest "the standard mechanism of *differential productivity across sectors*" in matching models.

6 Concluding remark

The fundamental surplus is a diagnostic technique that yields insights about the parameters that make unemployment respond sensitively to productivity across a diversity of matching models, including CET's.

References

Albuquerque, Rui, Martin Eichenbaum, Victor X. Luo, and Sergio Rebelo. 2016. Valuation Risk and Asset Pricing. *Journal of Finance* 71 (6):2861–2904.

Backus, David K., Bryan R. Routledge, and Stanley E. Zin. 2004. Exotic Preferences for Macroeconomists. In *NBER Macroeconomics Annual 2004*, edited by Mark Gertler and Kenneth Rogoff. Cambridge, Mass.: MIT Press.

Bansal, Ravi and Amir Yaron. 2004. Risk for the Long Run: A Potential Resolution of Asset Pricing Puzzles. *Journal of Finance* 59 (4):1481–1509.

Campbell, John Y. and John H. Cochrane. 1999. By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy* 107 (2):205–251.

Chodorow-Reich, Gabriel and Loukas Karabarbounis. 2016. The Cyclicality of the Opportunity Cost of Employment. *Journal of Political Economy* 124 (6):1563–1618.

Christiano, Lawrence J., Martin S. Eichenbaum, and Mathias Trabandt. 2016. Unemployment and Business Cycles. *Econometrica* 84 (4):1523–1569.

———. 2021. Why is Unemployment so Countercyclical? *Review of Economic Dynamics* XX (X):xx–xx.

Hall, Robert E. and Paul R. Milgrom. 2008. The Limited Influence of Unemployment on the Wage Bargain. *American Economic Review* 98 (4):1653–1674.

Kehoe, Patrick J., Pierlauro Lopez, Virgiliu Midrigan, and Elena Pastorino. 2019. Asset Prices and Unemployment Fluctuations. Working Paper no. 26580. Cambridge, Mass.: National Bureau of Economic Research.

Ljungqvist, Lars and Thomas J. Sargent. 2017. The Fundamental Surplus. *American Economic Review* 107 (9):2630–2665.

———. 2018. *Recursive Macroeconomic Theory,* Fourth Edition. Cambridge, Mass.: MIT Press.

Ljungqvist, Lars and Harald Uhlig. 2015. Comment on the Campbell-Cochrane Habit Model. *Journal of Political Economy* 123 (5):1201–1213.

Papetti, Andrea. 2019. Unemployment and Business Cycles in the Euro Area. Unpublished manuscript.

Schorfheide, Frank, Dongho Song, and Amir Yaron. 2018. Identifying Long-Run Risks: A Bayesian Mixed-Frequency Approach. *Econometrica* 86 (2):617–654.

Wachter, Jessica A. 2013. Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? *Journal of Finance* 68 (3):987–1035.