Quit Turbulence and Unemployment

From the perspective of a particle physicist*

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Abstract

According to physicist Steven Weinberg (2018): (1) new theories that target new observations should be constrained to agree with observations successfully represented by existing theories; and (2) the constraint to preserve successes of earlier theories is also a guide that can lead to unanticipated understandings of yet other phenomena. We illustrate how Weinberg's advice helps answer the question: in more turbulent times, how do higher risks of skill losses coincident with involuntary layoffs ("layoff turbulence") but also with voluntary quits ("quit turbulence") affect equilibrium unemployment rates? Earlier theories that had included only layoff turbulence had established a positive relationship between turbulence and the unemployment rate within generous welfare states like those in Europe, but the absence of that relationship in countries without much of a welfare state. An influential earlier paper found that even small amounts of quit turbulence would lead to a *negative* relationship between turbulence and unemployment rates. We show that that finding was based on a peculiar calibration of returns to labor mobility that not only made the model miss the positive turbulence-unemployment rate relationship in the post 1970s data for European welfare states, but also miss observations about labor market churning. Repairing the faulty calibration not only brings models with quit turbulence into line with those observations but also guides us to shed light on labor market issues in the US raised by Alan Greenspan (1998).

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"... often the most important constraint on a new theory is ... that it should agree with the whole body of past observations, as crystallized in former theories. ... New theories of course do not agree entirely with any previous theory – otherwise they would not be new – but they must not throw out all the success of former theories. This sort of thing makes the work of the theorist far more conservative than is often thought.

The wonderful thing is that the need to preserve the successes of the past is not only a constraint, but also a guide." Weinberg (2018, p. 197)

1 Introduction and summary

This paper is a macro-labor economics variation on a theme of particle physicist Steven Weinberg (2018, ch. 24), who tells how science constrains new theories to incorporate successful explanations offered by older theories. Besides analyzing a research question of significant practical interest, our inquiry illustrates how to (1) introduce new forces to explain previously neglected phenomena, while (2) being constrained to preserve earlier theories' successes, and (3) reaping unanticipated understandings of other interactions by being constrained by successes of past theories.

Our context is alternative quantitative macroeconomic models of labor market frictions. Macro-labor theory differs from particle physics and cosmology because, unlike those fields, we have no widely accepted "standard model" for studying forces that shape an equilibrium unemployment rate. Instead, there are (at least) three popular frameworks of frictional unemployment each having persuasive advocates and skillful users: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the tradition of framework of Lucas and Prescott (1974). Calibrated models of all three frameworks have been used successfully to reproduce data on labor market flows and to generate plausible responses of unemployment rates to government policies like unemployment insurance and layoff taxes. We have cause to revisit some of these successes in this paper.

Our case study begins with well motivated efforts to add forces and phenomena that had been excluded from a generalized McCall search model of adverse macroeconomic consequences of interactions between microeconomic turbulence and European more generous welfare states (Ljungqvist and Sargent, 1998). Increased turbulence had been modeled as an increase in the rate of human capital loss coincident with an *involuntary* job loss ("layoff turbulence"). That model explained systematically higher and persistent unemployment rates in Europe than in the US since the late 1970s. Significantly, the model had assumed no loss of human capital coincident with a *voluntary* separation from a job. The omission of "quit turbulence" from earlier quantitative explanations of equilibrium unemployment serves as the starting point of our story because in 1998 an astute observer, Alan Greenspan (1998, p. 743), suggested that a more hazardous job market had suppressed mobility among some employed workers and had led to less upward pressure on wages:

> "... the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest."

den Haan, Haefke and Ramey (2005, henceforth DHHR) cite these sentences at the beginning of a paper that constructed and calibrated a matching model containing causal channels capable of representing forces stressed by Greenspan. Their model concisely expressed Greenspan's idea by including quit turbulence in the form of an immediate depreciation of a worker's human capital that in turbulent times could be triggered by a worker's decision to quit a job. DHHR reported a calibrated version of their model that affirmed the quantitative significance of Greenspan's hunch: even a small amount of quit turbulence gave workers strong enough second thoughts about quitting to reduce quits and job reallocation substantially.

But DHHR's success in representing and quantifying Greenspan's intuition had important ramifications for other quantities and prices. In particular, DHHR found and chose to stress that adding even a small amount of quit turbulence to their matching model removed and reversed the unemployment-increasing interactions between layoff turbulence and generosity of welfare states that was the foundation of the earlier work that had explained trans-Atlantic differences in unemployment rates.

Does DHHR's innovative analysis of quit turbulence satisfy Weinberg's constraint both in terms of earlier research on turbulence and more generally in terms of preserving widely shared successes across macro-labor frameworks? With respect to earlier research on turbulence, DHHR actually stress that when they formalize and quantify Greenspan's idea they no longer explain the observed trans-Atlantic unemployment rate differences that earlier theories with turbulence had captured. But playing devil's advocate against Steven Weinberg, could DHHR's analysis be a convincing demonstration of the lack of robustness of that early explanation of those differences to (1) adding quit turbulence, and (2) using a matching model rather than a generalized McCall equilibrium search model? To study these questions, we must establish what accounts for the different outcomes. Was it adding quit turbulence? Or was it abandoning the extended McCall framework in favor of the matching framework? Or might the differences be traced to a peculiar feature of DHHR's analysis? Answering these questions will also shed light on a fatal flaw of DHHR's analysis that makes it impossible to preserve some key quantitative successes widely shared across earlier macro-labor models.

An influential interim answer to these questions was offered in the Handbook of Economic Growth by Hornstein, Krusell and Violante (2005, section 8.3). They interpreted the DHHR finding as indicating lack of robustness of the earlier explanation of those trans-Atlantic unemployment rate differences, leading them to doubt its validity:

> "... once the Ljungqvist and Sargent mechanism is embedded into a model with endogenous job destruction, the comparative statics for increased turbulence are reversed, i.e., unemployment falls. The reason is that as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls."

We say that this is only an "interim" answer to our questions because Hornstein *et al.* (2005) did not isolate the origin of DHHR's finding in terms of specifications of preferences, technologies, and timing protocols that give rise to it. This paper finds that source. The process of doing that illustrates the power of Weinberg's insight "that the need to preserve [earlier successes of macro-labor models] is not only a constraint, but also a guide".

1.1 Stages of inquiry and specific findings

Our formal inquiry consists of three stages. First, as a natural starter, we incorporate quit turbulence into yet another matching model to see if the findings are similar to those of DHHR. A most suitable candidate for that purpose is the matching model of Ljungqvist and Sargent (2007, henceforth LS),[1] which had served as a robustness analysis to show that the positive turbulence-unemployment relationship in the extended McCall search framework of Ljungqvist and Sargent (1998, 2008) also holds up in the matching framework. While that original turbulence theory had only assumed skill loss at involuntary separations, we now also include risk of skill loss at voluntary separations. We then find that the matching model analyses of DHHR and the version of the LS model augmented to incorporate quit turbulence disagree sharply. Instead of DHHR's finding that a small amount of quit turbulence can overturn the positive turbulence-unemployment relationship in the DHHR matching model, it takes large amounts of quit turbulence to have any such dramatic impact on what is evidently a robust relationship in the LS matching model.

Having established that DHHR's remarkable findings are not indigenous to matching models as a class, the second stage of our inquiry is to uncover what explains the puzzling differences in outcomes. Specifically, we note that the LS and DHHR frameworks differ in three dimensions, two in terms of the model structure, and one regarding the parameterization of productivity

[1] LS acknowledged Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code, which LS then augmented in various ways, adding features to be examined here. Also, as indicated in footnotes 8 and 16 below, the current paper was preceded by a related, but distinctly different, exchange of views between den Haan, Haefke and Ramey (2001) and Ljungqvist and Sargent (2004).

distributions. One structural difference is that LS adopt the standard assumptions of free entry of firms and a zero-profit equilibrium condition for posting vacancies, while DHHR assume fixed measures of both firms and workers. The other structural difference is that LS assume that skill upgrades are immediately realized, while DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level. Even though these two structural differences have substantively different consequences for vacancy and wage bargaining outcomes, it turns out that they do not change the qualitative pattern of unemployment dynamics. It turns out that the explanation of our puzzle lies in the third difference, namely, the parameterization of productivity distributions. Specifically, the source of DHHR's exceptional findings about quit turbulence is that they calibrate parameters that set returns to labor mobility so small that even a small mobility cost would shut down voluntary separations.

Having found the source of the puzzle, in order to analyze forces at play further and to strengthen our point about labor mobility, the third stage of our inquiry conducts an analysis of layoff costs. Introducing small layoff costs in tranquil times closes down voluntary separations in the DHHR framework, which illustrates the sensitivity of outcomes to the DHHR parameterization of productivity distributions. Therefore, a small government mandated layoff cost (or any small mobility cost, such as a tiny risk of skill loss when quitting) has counterfactually large effects of suppressing unemployment by shutting down voluntary separations.

1.2 General inference

Our refutation of DHHR's claim extends to DHHR's proposed mechanism in general. The logic here goes as follows. On the one hand, DHHR's hypothesis that small risks of human capital loss at voluntary separations would cause workers to choose not to separate relies on returns to labor reallocation being small. On the other hand, returns to labor reallocation have to be large in any model that has a chance of being consistent with evidence that relatively high layoff costs in European welfare states have had rather small impacts on unemployment outcomes.[2]

These irreconcilable requirements about rates of return to labor reallocation imply a strong quantitative presumption against DHHR's claim that the turbulence theory is not robust to the addition of quit turbulence. Specifically, when returns to labor mobility are calibrated to conform with the historical evidence on layoff costs, there emerges a strong quantitative presumption that the addition of quit turbulence cannot overturn a positive turbulence-unemployment relationship.

Our macro-labor detective project illustrates Steven Weinberg's observation that the need

[2]See e.g. Nickell (1997, p. 66) for an account of substantial measures of employment protection in European welfare states, and the common conclusion of "rather small" impact on unemployment outcomes.

to preserve implications of earlier theories serves as both constraint and guide. Thus, we have managed to incorporate the quit turbulence that was evidently on Alan Greenspan's mind in 1998 while preserving earlier findings of adverse interactions between welfare-state generosity and average unemployment levels. With that extension in hand, we have gone on to show how the same model reconciles the presence of high layoff costs with observed levels of unemployment. Evidently, earlier models exhibit returns to labor reallocation that are large enough to generate observed rates of churning of labor in the presence of some heavy-handed government interventions that raise layoff costs. Thus, in the macro-labor economics project reported here, respecting quantitative successes of earlier theories has indeed functioned both as constraint and guide. Our findings cause us to suggest that Hornstein *et al.* were misled when they accepted DHHR's conclusion about the effects of quit turbulence on unemployment. To the contrary, adding a well calibrated amount of quit turbulence does not reverse the positive turbulence-unemployment relationship in a welfare state. As a further illustration of dividends that accrue from using our model as a guide, in the spirit of Weinberg, we return in our concluding section to have a conversation with Alan Greenspan in light of what our extended model with quit turbulence has taught us.

2 A road map to analysis

A modelling challenge is how to represent the uncontroversial observation that job separators differ in terms of more or less favorable circumstances when separating from their work. For example, workers with valuable skills who separate in order to find better-paying jobs differ from laid-off workers whose skills are no longer in demand due to e.g. changing technology or their type of work 'moving abroad' to low-wage countries.

To capture such differences, DHHR retain the category of involuntary separations from earlier theories, with the continuing interpretation that those represent the most unfavorable circumstances for job separators, i.e., the highest risks of skill loss. In addition to such layoff turbulence, DHHR introduce quit turbulence for workers who voluntarily separate from jobs after draws of poor job-specific productivities at their current employment. Those voluntary separations are thought to represent workers with more favorable circumstances; both in terms of having the opportunity to continue working after shocks to productivity at their current employment, as well as, conditional on separating, facing lower risk of skill loss (as compared to workers suffering involuntary separations).

Given this setup, DHHR investigate whether or not the turbulence theory is robust to the introduction quit turbulence, by measuring how high the risk of skill loss at voluntary separations relative to involuntary separations, need to be in order to generate a negative rather than positive turbulence-unemployment relationship. Hence, the reason for DHHR declaring a lack of robustness is their finding that the turbulence-unemployment is negative already at very low skill loss probabilities for voluntary separators relative to that of involuntary separators:

> "... allowing for a skill loss probability following [voluntary] separation that is only 3% of the probability following [involuntary] separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong *negative* relationship between turbulence and unemployment." (DHHR, p. 1362)

It is surprising and difficult to understand how introducing probabilities of events so small can have such large effects on equilibrium outcomes. In contrast to DHHR's findings, when we introduce skill losses at times of voluntary separations into the LS model, we find only small effects on outcomes: quit turbulence has to be about 50% of layoff turbulence and both kinds of turbulence must be high before quit turbulence can suppress unemployment relative to tranquil zero-turbulence times.

As mentioned above, the LS and DHHR frameworks differ in the following three dimensions.[3] Our strategy to detect the guilty suspect is to start with the LS model and successively make one perturbation at a time, each perturbation being designed to isolate the role of one suspect. In Appendix B, we start from the DHHR model and work through the perturbations in reverse. Both procedures convict the same suspect.

1. **Suspect 1: Labor market tightness.** LS adopt the standard assumptions that free entry of firms and a zero-profit equilibrium condition for posting vacancies determine labor market tightness; while DHHR assume that measures of both firms and workers are fixed and equal, which in turn delivers an exogenous market tightness equal to one.

   Verdict: Not guilty. With endogenous market tightness, higher turbulence decreases market tightness as the "invisible hand" makes adjustments to restore the profitability of firms; lower tightness means lower job finding rates and unemployment increases. This force is not present in the DHHR structure with its exogenous market tightness and thus, no increase in unemployment comes from this channel. Nevertheless, DHHR's omission of this force does not change the qualitative pattern of unemployment dynamics, so changing from endogenous to exogenous market tightness fails to explain the puzzle.

[3] A fourth difference is DHHR's "simplifying assumption" that quit turbulence also applies to job seekers, i.e., after an encounter between an unemployed worker and a vacancy that does not lead to an employment relationship, the worker is exposed to the same risk of skill loss as if she had quit from a job. We omit this assumption since DHHR make it for computational tractability, and it is not part of DHHR's argument. While omitting this assumption does not affect the qualitative pattern of unemployment dynamics in the DHHR model as shown in Appendix B, Appendix C explains how it would significantly suppress unemployment in the LS model.

2. **Suspect 2: Timing of completion of skill upgrade.** LS assume that skill upgrades are immediately realized and, upon skill upgrades, workers draw new productivities. In contrast, DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level,[4] and that a new productivity is drawn from a distribution whose lower support is equal to the endogenous reservation productivity of a worker at that higher skill level (and therefore, a worker who has just received a skill upgrade will remain employed for at least one more period).

   Verdict: Not guilty. The alternative assumptions affect a worker's bargaining position vis-à-vis a firm. Delayed completion effectively erodes the bargaining power of a worker who experiences a skill upgrade. As a result, under delayed completion, wages become negative in periods of skill upgrades (firms extract rents from workers). Yet, these very different outcomes do not change the qualitative pattern of unemployment dynamics and hence, do not resolve the puzzle.

3. **Suspect 3: Productivity distributions.** LS postulate truncated normal distributions with a wide support, whereas DHHR assume uniform distributions with a narrow support.

   Verdict: Guilty! The DHHR parameterization delivers much weaker incentives for workers to transit between jobs, even in tranquil times. Hence, any small cost to mobility causes voluntary quits to shut down. After we adjust DHHR's parameterization to account for observed unemployment dynamics, a positive relationship between turbulence and unemployment reemerges.

The paper proceeds as follows. Section 3 develops a matching framework with turbulence that builds on the models of LS and DHHR. Section 4 documents the puzzle, dissects the forces at work, and detects the culprit. Section 5 conducts the layoff tax analysis. Section 6 offers concluding remarks. Auxiliary material and explorations are relegated to Appendices A–C.

3 A matching framework with turbulence

The LS matching model has 'layoff turbulence' in the form of worse skill transition probabilities for workers who suffer involuntary layoffs. We augment the model to include 'quit turbulence' – worse skill transition probabilities for workers who experience voluntary quits – as in the DHHR model. The following description is an account of that augmented LS model that notes carefully where it differs from the DHHR model.

[4] Contingent on remaining with his present employer, a worker experiencing a skill upgrade in the DHHR model will also realize the higher skill level immediately. To capture this notion of contingency, we use the term 'completion' of a skill upgrade.

3.1 Environment

Workers Consider a unit mass of workers who are either employed or unemployed. Workers are risk neutral and value consumption, with preferences ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t. \tag{1}$$

They discount future utilities at a rate $\beta \equiv \hat{\beta}(1-\rho^r)$, where $\hat{\beta} \in (0,1)$ is a subjective time discount factor and $\rho^r \in (0,1)$ is a constant probability of retirement. A retired worker exits the economy and is replaced by a newborn worker.

Worker heterogeneity Besides employment status, workers differ in two dimensions: a current skill level i that can be either low (l) or high (h) and a skill level j during a last employment spell that in turn determines an entitlement to unemployment benefits. Workers gain or lose skills depending on their employment status and instances of layoffs and quits. A worker bears two indices (i, j), the first denoting current skill and the second denoting benefit entitlement. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, also called market tightness. The probability that an unemployed worker encounters a vacancy is $\lambda^w(\theta) = M(v,u)/u = M(\theta, 1) \equiv m(\theta)$, which is increasing in market tightness. The probability that a vacancy encounters an unemployed worker is $M(v,u)/v = m(\theta)/\theta$, which is decreasing in market tightness.

> *Difference from DHHR*
> There is a fixed unit mass of firms. Since there are no costs for posting vacancies, a firm without a worker always chooses to post a vacancy, so $\theta = 1$ always.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a distribution $v_i(z)$ that is indexed by the worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Wages are determined through Nash bargaining, with π and $1-\pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition function Q_i, also indexed by the worker's skill level i, where $Q_i(z, z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains his last period productivity with probability $1 - \gamma^s$, but with probability γ^s a new productivity is drawn from the distribution $v_i(z)$, i.e., the same distribution as a worker of that skill level would face in a new match. Additionally, a worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with his present employer or quits.

Difference from DHHR A worker experiencing a skill upgrade draws a new productivity from a transformed version of $v_h(z)$; namely, the distribution is truncated from below at the reservation productivity of a high-skilled worker, and then the distribution is rescaled to integrate to one. A skill upgrade is completed only after a worker has remained one period with his present employer.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary in the sense of offering no choice.

(ii) **Quits:** As a consequence of a new productivity draw on a job, a relationship may continue or be endogenously terminated. We identify separation after such an event as a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define turbulence as the risk of losing skills after a job separation. High-skilled workers might become low-skilled workers.

Two types of turbulence shocks depend on the reason for a job separation, namely, a layoff or a quit. Upon a layoff, a high-skilled worker experiences a skill loss with probability $\gamma^{d,x}$. We label this risk *layoff turbulence*. Upon a quit, a high-skilled worker faces the probability γ^d of a skill loss. We label this risk *quit turbulence*.

The timing of turbulence shocks are as follows. At the beginning of a period, exogenous job terminations occur and displaced workers face layoff turbulence. Continuing employed workers may experience new productivity draws on the job and skill upgrades; if workers quit, they are subject to quit turbulence. All separated workers join other unemployed workers in the matching function where they might or might not encounter vacancies next period.

Government policy The government runs a balanced budget. Its revenues come from a flat-rate tax τ on production. Its sole expenditures are benefits paid to the unemployed. An unemployed worker who was low (high) skilled in his last employment receives a benefit b_l (b_h).[5] As mentioned above, newborn workers are entitled to b_l. Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i.

In section 5, the government gets an additional source of revenues by levying a layoff tax Ω on every job termination except for retirements. If the revenues from layoff taxation exceed the expenditures on unemployment benefits, the government sets $\tau = 0$ and hands back any surplus as lump-sum transfers to workers.

3.2 Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j who have drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period, net of paying the vacancy cost μ. For notational simplicity, we define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.[6]

The match surplus for a new job $s^o_{lj}(z)$ or a continuing job $s_{lj}(z)$ with a low-skilled worker with benefit entitlement j is given by

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we must distinguish between new and continuing jobs. The match surplus when forming a new job with an unemployed high-skilled worker, s^o_{hh}, involves outside values without any risk of skill loss if the match does not result in employment:

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit

[5]For simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

[6]Our mathematical formulation and notation follow DHHR closely.

turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. Since both parties want a positive match surplus, there is mutual agreement whether or not to start (continue) a job. For a new (continuing) match, the reservation productivity $\underline{z}_{ij}^o$ ($\underline{z}_{ij}$) is the lowest productivity that makes a match profitable and satisfies

$$s_{ij}^o(\underline{z}_{ij}^o) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; 0\Big). \tag{5}$$

Given the reservation productivity $\underline{z}_{ij}^o$ ($\underline{z}_{ij}$), let ν_{ij}^o (ν_{ij}) denote the rejection probability, which is given by the mass of all productivity draws from distribution $v_i(y)$ that fall below the threshold:

$$\nu_{ij}^o \;=\; \int_{-\infty}^{\underline{z}_{ij}^o} dv_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

To simplify formulas below, we define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\; dv_i(y). \tag{7}$$

3.3 Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now characterize this value for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by future layoff turbulence if the worker is laid off or by future quit turbulence if a productivity switch is rejected:

$$\begin{aligned}
&\text{Exogenous separation:} & g_h(z) \;&=\; \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^{d,x})\omega_{hh} + \gamma^{d,x}\omega_{lh}}_{\text{layoff turbulence}}) \\
&\text{Productivity switch:} & &+\; (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{No changes:} & &+\; (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big].
\end{aligned} \tag{8}$$

Low-skilled worker The joint continuation value of a match of a firm with a low-skilled worker takes into account the following contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, or a skill upgrade. Regarding the latter contingency, in the LS model skill upgrades are immediately realized and upon skill upgrades workers immediately become entitled to high unemployment benefits, even if the worker quits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution v_h. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z is given by

$$
\begin{array}{llcl}
\text{Exogenous separation:} & g_l(z) & = & \beta\Big[\rho^x(b_l+\omega_{ll}) \\
\text{Immediate skill upgrade:} & & + & (1-\rho^x)\gamma^u(E_{hh}+\nu_{hh}(b_h+\underbrace{(1-\gamma^d)\omega_{hh}+\gamma^d\omega_{lh}}_{\text{quit turbulence}})) \\
\text{Productivity switch:} & & + & (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\
\text{No changes:} & & + & (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z))\Big].
\end{array}
\tag{9}
$$

Difference from DHHR

A worker with a skill upgrade must remain with the present employer for one period in order to complete the higher skill level. The new productivity at a skill upgrade is drawn from a distribution $v_u(z)=v_h(z)/(1-v_h(\underline{z}_{hh}))$ having a lower support $\underline{z}_{hh}$, which guarantees a continuation of the employment relationship. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z in the DHHR model becomes:

$$
\begin{array}{llcl}
\text{Exogenous separation:} & g_l(z) & = & \beta\Big[\rho^x(b_l+\omega_{ll}) \\
\text{Delayed skill upgrade:} & & + & (1-\rho^x)\gamma^u\displaystyle\int_{\underline{z}_{hh}}^{\infty}[(1-\tau)y+g_h(y)]\,dv_u(y) \\
\text{Productivity switch:} & & + & (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\
\text{No changes:} & & + & (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z))\Big].
\end{array}
\tag{10}
$$

3.4 Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω_{ij}^w. Recall that the probability of an unemployed worker being matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega^w_{lj}$, where

$$\omega^w_{lj} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{lj}}^{\infty} \pi s^o_{lj}(y)\; dv_l(y)}_{\text{match + accept}} \quad + \underbrace{b_j + \omega^w_{lj}}_{\text{outside value}}\Big] \qquad j = l, h. \tag{11}$$

A high-skilled unemployed worker with benefit entitlement h, obtains $b_h + \omega^w_{hh}$, where

$$\omega^w_{hh} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{hh}}^{\infty} \pi s^o_{hh}(y)\; dv_h(y)}_{\text{match + accept}} \quad + \underbrace{b_h + \omega^w_{hh}}_{\text{outside value}}\Big]. \tag{12}$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda^f_{ij}(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i,j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f \;=\; -\mu + \beta\Big[\underbrace{\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} (1-\pi) s^o_{ij}(y)\; dv_i(y)}_{\text{match + accept}} \quad + \underbrace{\omega^f}_{\text{outside value}}\Big]. \tag{13}$$

3.5 Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy is solely a function of market tightness θ; the probability $\lambda^f_{ij}(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the worker composition in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be deduced from equation (13) with $w^f = 0$. We summarize these labor market outcomes as follows.

$$\omega^f \;=\; 0 \tag{14}$$

$$\mu \;=\; \beta(1-\pi)\sum_{(i,j)} \lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\; dv_i(y) \tag{15}$$

$$\lambda^w(\theta) \;=\; m(\theta) \tag{16}$$

$$\lambda^f_{ij}(\theta) \;=\; \frac{m(\theta)}{\theta}\frac{u_{ij}}{u} \tag{17}$$

Difference from DHHR

There is a fixed unit mass of firms. Because there are no vacancy costs, a firm without a worker will always post a vacancy. Thus, market tightness equals one, and a firm's value w^f of posting a vacancy can be deduced from equation (13) with $\mu = 0$.

$$\omega^f = \frac{\beta}{1-\beta}(1-\pi)\sum_{(i,j)} \lambda_{ij}^f \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\, dv_i(y) \tag{18}$$

$$\theta = 1 \tag{19}$$

$$\lambda^w = m(1) \tag{20}$$

$$\lambda_{ij}^f = m(1)\frac{u_{ij}}{u} \tag{21}$$

3.6 Wages

Wages are determined through Nash bargaining. Here, we report the equations for the LS model with immediate realization of skill upgrades. We refer readers to den Haan *et al.* (2005, section 2.5) for the DHHR equations under their assumption that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Wage determination Given a productivity draw z in a new match with a positive match surplus, the wage $p_{lj}^o(z)$ of a low-skilled worker with benefit entitlement $j = l, h$, and the wage $p_{hh}^o(z)$ of a high-skilled worker, respectively, solves the following maximization problems:

$$\max_{p_{lj}^o(z)} \quad \left[(1-\tau)z - p_{lj}^o(z) + g_l^f(z) - \omega^f\right]^{1-\pi} \left[p_{lj}^o(z) + g_l^w(z) - b_j - \omega_{lj}^w\right]^{\pi} \tag{22}$$

$$\max_{p_{hh}^o(z)} \quad \left[(1-\tau)z - p_{hh}^o(z) + g_h^f(z) - \omega^f\right]^{1-\pi} \left[p_{hh}^o(z) + g_h^w(z) - b_h - \omega_{hh}^w\right]^{\pi},$$

where $g_i^w(z)$ and $g_i^f(z)$ are future values obtained by the worker and the firm, respectively, from continuing the employment relationship;[7] and ω^f and $b_j + \omega_{ij}^w$ are the outside values, defined in (11), (12), and (13). The solution to the wage determination problems sets the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$p_{lj}^o(z) + g_l^w(z) = \pi s_{lj}^o(z) + b_j + \omega_{lj}^w \qquad j = l, h \tag{23}$$

$$p_{hh}^o(z) + g_h^w(z) = \pi s_{hh}^o(z) + b_h + \omega_{hh}^w,$$

[7]The *joint* continuation values defined in (8) and (9) equal the sum of the individual continuation values: $g_i(z) = g_i^w(z) + g_i^f(z),\ i = l, h$.

where the worker continuation values are

$$
\begin{aligned}
g_l^w(z) &= \beta(1-\rho^x)\pi \left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z) + \gamma^s \int_{\underline{z}_{ll}}^{\infty} s_{ll}(y)\ dv_l(y)\right] + \gamma^u \int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\ dv_h(y)\right\} \\
&+ \beta(\rho^x + (1-\rho^x)(1-\gamma^u))\left(b_l + \omega_{ll}^w\right) + \beta(1-\rho^x)\gamma^u\left(b_h + (1-\gamma^d)\omega_{hh}^w + \gamma^d\omega_{lh}^w\right) \qquad (24) \\
g_h^w(z) &= \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z) + \gamma^s \int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\ dv_h(y)\right] \\
&+ \beta\rho^x\left(b_h + (1-\gamma^{d,x})\omega_{hh}^w + \gamma^{d,x}\omega_{lh}^w\right) + \beta(1-\rho^x)\left(b_h + (1-\gamma^d)\omega_{hh}^w + \gamma^d\omega_{lh}^w\right).
\end{aligned}
$$

For ongoing employment relationships, the wages $p_{ll}(z), p_{hh}(z)$ satisfy counterparts of the above equations that use the corresponding match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$
\begin{aligned}
p_{ll}(z) + g_l^w(z) &= \pi s_{ll}(z) + b_l + \omega_{ll}^w \qquad (25) \\
p_{hh}(z) + g_h^w(z) &= \pi s_{hh}(z) + b_h + \underbrace{(1-\gamma^d)\omega_{hh}^w + \gamma^d\omega_{lh}^w}_{\text{quit turbulence}},
\end{aligned}
$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

3.7 Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits amounts to

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})). \qquad (26)$$

Income taxes Output is taxed at a constant rate τ. Let $\bar{z}_i$ be the average productivity of employed workers with skill level i. Hence, total tax revenue equals $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the total expenditures described in (26):

$$\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) = \tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}). \qquad (27)$$

For computations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$, see Appendix A.2.

3.8 Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlement levels. Here we focus on a group of workers at the center of our analysis: low-skilled unemployed with high benefits. (Appendix A.1 presents descriptions of flows for other groups of workers.) As in the case of the above wage equations, we report worker flows under the LS assumption of immediate realization of skill upgrades, while referring to den Haan *et al.* (2005, appendix A) for the alternative DHHR assumption.

Inflows to the low-skilled unemployed with high benefits u_{lh} occur in the following instances. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled unemployed with high benefit entitlement. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^d of becoming part of the low-skilled unemployed with high benefit entitlement. Outflows from unemployment occur upon successful matching function encounters and retirement. Thus, the net change of low-skilled unemployed with high benefits (equalling zero in a steady state) becomes:

$$\begin{aligned}\Delta u_{lh} \quad = \quad (1-\rho^r)\Bigg\{ & \underbrace{\rho^x \gamma^{d,x} e_{hh}}_{\text{1. layoff turbulence}} \quad + \quad \underbrace{(1-\rho^x)\gamma^d \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} \\ & - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}}\Bigg\} - \rho^r u_{lh}. \end{aligned} \tag{28}$$

The terms numbered 1 and 3 in expression (28) isolate the forces behind the positive turbulence-unemployment relationship in a welfare state in the LS model. While more layoff turbulence in term 1 – a higher probability $\gamma^{d,x}$ of losing skills after layoffs – does not have much of an effect on equilibrium unemployment in a laissez-faire economy, it gives rise to a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate on a worker's earnings in her last employment. After a layoff with skill loss, those benefits are high relative to a worker's earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1 - \nu^o_{lh})$ in term 3 is low; because of the relatively high outside value of a low-skilled unemployed with high benefits, fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ of a worker encountering a vacancy, which further suppresses successful matches and contributes to the positive turbulence-unemployment relationship.

DHHR challenge this finding by claiming that it is not robust to adding the term numbered 2 in expression (28): they assert that if higher turbulence is associated with voluntary quits also being subject to risks of skill loss, there would be a lower incidence of voluntary quits in turbulent times because high-skilled workers would become reluctant to quit in order to avoid the risk of skill loss. This would make the rejection rate ν_{hh} in term 2 become low in turbulent times. That lower rejection rate causes lower inflows into low-skilled unemployed with high benefits u_{lh} as well as into high-skilled unemployed with high benefits u_{hh}. DHHR argue quantitatively that even at very low levels of quit turbulence (relative to layoff turbulence), this force is strong enough to overturn the Ljungqvist and Sargent positive turbulence-unemployment relationship.

3.9 Steady state equilibrium

A steady state equilibrium consists of measures of unemployed u_{ij} and employed e_{ij}; a labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$; unemployment benefits b_i and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.

b) Free entry of firms implies zero-profit condition (15) in vacancy creation that pins down market tightness in LS. (An exogenous unit mass of firms implies market tightness equal to one in DHHR.)

c) Nash bargaining outcomes (23) and (25) set wages.

d) The tax rate balances the government's budget (27).

e) Net worker flows, such as expression (28), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall\, i, j$.

3.10 Parameterization

Except for the introduction of quit turbulence, we adopt the parameterization of LS. Parameters are divided into two groups, as reported in Table 1 and 2, respectively, to distinguish those that are similar in the DHHR parameterization from those that differ. The Table 1 parameters pin down preferences, sources of risk, and labor market institutions.[8] The Table 2 parameters

[8]The similar parameterizations originate from an earlier exchange mentioned in footnote 1. Specifically, Ljungqvist and Sargent (2004) argued for modifying the parameterization of den Haan *et al.* (2001) based on calibration targets in the search framework of Ljungqvist and Sargent (1998, 2008), but for one notable *ad hoc* assumption to be discussed in footnote 16. (DHHR adopt Ljungqvist and Sargent's (2004) modification of den Haan et al.'s (2001) parameterization and proceed to introduce quit turbulence.)

pin down the matching process and productivity distributions, which are markedly different between LS and DHHR. Following LS, we assume a semi-quarterly model period.[9]

Preference parameters Given a semi-quarterly model period, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Stochastic processes for productivity Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years.

Table 1: LS parameterization similar to DHHR

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1 - \rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
$\gamma^{d,x}$	layoff turbulence	$[0, 1]$
$\gamma^d = \epsilon\gamma^{d,x}$	quit turbulence	$\epsilon \in [0, 1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7

[9]The parameterization of DHHR is done at a quarterly frequency. The transition probabilities for skill dynamics are the same as those in LS, when taking the different frequencies into account. The only departures from the parameters in Table 1 are the subjective discount factor and the retirement probability, which DHHR set at 0.995 and 0.005, respectively, at a quarterly frequency, or 0.9975 and 0.0025 when converted to a semi-quarterly frequency; these numbers yield an adjusted discount of 0.995 at a semi-quarterly frequency. We conducted a sensitivity analysis with respect to the different discount rates and found that adopting the DHHR discount rate in the LS model, while it changes the quantitative findings, does not overturn the qualitative pattern of a positive turbulence-unemployment relationship.

Layoff and quit turbulence Following DHHR, we parameterize quit turbulence as a fraction ϵ of layoff turbulence, and we vary it from zero – only layoff turbulence – to one – the two types of turbulence are equal: $\gamma^d = \epsilon\gamma^{d,x}$.

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. The replacement rate is set at $\phi = 0.7$ and hence an unemployed worker with benefit entitlement b_j receives 70% of the average wage of employed workers with skill level j.

Matching We assume a Cobb-Douglas matching function $M(v,u) = Au^{\alpha}v^{1-\alpha}$, where A is matching efficiency and α is the elasticity of matches with respect to unemployment. Given this technology, the probability that a worker encounters a vacancy and the probability that a vacancy encounters a particular worker type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{29}$$

When calibrating a matching model to an aggregate unemployment rate, without any calibration targets for vacancy statistics, selecting the parameter pair (A,μ) is a matter of normalization. We renormalize LS's setting of (A,μ) so that equilibrium market tightness in tranquil times (no turbulence) with no layoff taxes becomes equal to one.[10] This will facilitate a perturbation exercise in which we shall replace free entry of firms in LS with the DHHR arrangement that exogenously fixes equal masses of firms and workers and a market tightness equal to one.

When assuming DHHR's exogenous matching arrangement in the context of calibrating a Cobb-Douglas matching function, we only need to set parameter A equal to their parameterized probability of a worker encountering a vacancy, as the corresponding probabilities become:

$$\lambda^w(1) = A, \qquad \lambda^f_{ij}(1) = A\frac{u_{ij}}{u}. \tag{30}$$

Productivity distributions LS assume that productivities are drawn from truncated normal distributions with wide support: $z_l \sim \mathcal{N}(1,1)$ for low-skilled workers over the support $[-1,3]$, and $z_h \sim \mathcal{N}(2,1)$ for high-skilled workers over the support $[0,4]$.[11] In contrast, DHHR assume uniform distributions with small support: $z_l \sim \mathcal{U}([0.5,1.5])$ and $z_h \sim \mathcal{U}([1.5,2.5])$.

[10]Under the original LS parameterization $(A,\mu) = (0.45, 0.5)$, the equilibrium market tightness is equal to $\theta = 0.9618$ in tranquil times and no layoff taxes. We renormalize the parameter pair (A,μ) to attain an equilibrium market tightness of 1 and leave unchanged the probability that a worker encounters a vacancy. Let $(\hat{A},\hat{\mu})$ be our new parameterization given by $\hat{A} = \kappa^{1-\alpha}A$ and $\hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old parameterization $\kappa = 0.9618$, the new parameterization achieves the desired outcomes.

[11]LS incorrectly implemented the quadrature method at the truncation points of the normal distributions; nevertheless, the constructed distributions are still proper. Therefore, instead of recalibrating the LS model under a correct implementation of the quadrature method, we have chosen to seek to resolve the puzzle in terms of the distributions that were used in the published LS analysis.

Table 2: PARAMETERIZATIONS DIFFERING BETWEEN LS AND DHHR

Parameter	Definition	LS	DHHR
Matching function			
A	matching efficiency	0.441	0.3
α	elasticity of matches w.r.t. u	0.5	–
μ	flow cost of a vacancy	0.481	–
Productivity dist.			
$v_i(z)$	functional form	Normal	Uniform
$\mathbb{E}[z_l]$	low-skilled mean	1	1
$\mathbb{E}[z_h]$	high-skilled mean	2	2
$support[z_l]$	low-skilled support	[-1, 3]	[0.5, 1.5]
$support[z_h]$	high-skilled support	[0, 4]	[1.5, 2.5]
$std[z_l]$	low-skilled dispersion	1	$\frac{1}{\sqrt{12}}$
$std[z_h]$	high-skilled dispersion	1	$\frac{1}{\sqrt{12}}$

4 The Puzzle

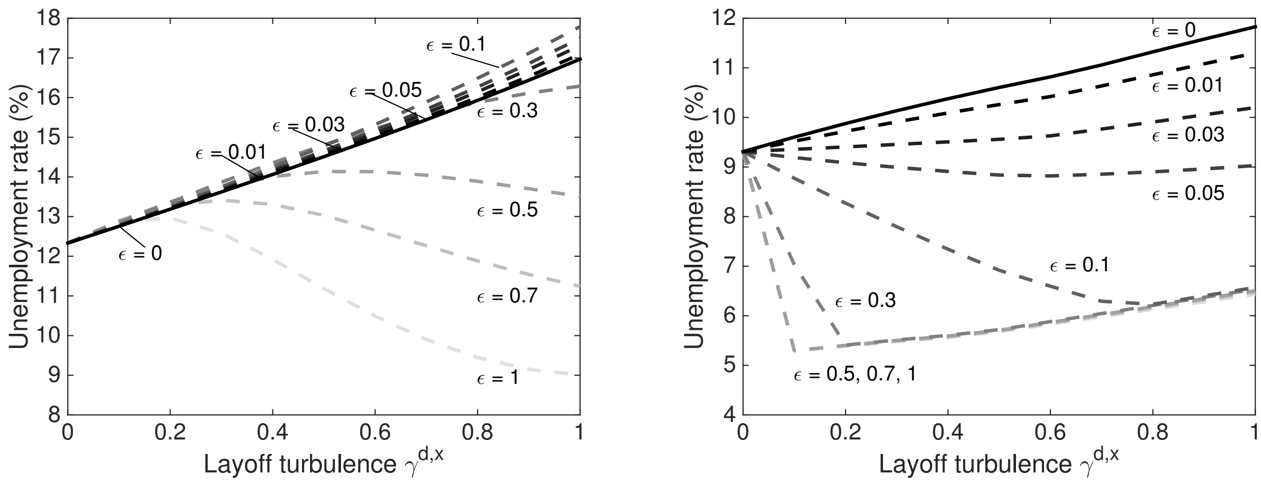
If voluntary quits are exposed to a small risk of skill loss, then higher turbulence leads to a reduction in unemployment in the DHHR framework, while it does not alter the positive turbulence-unemployment relationship in the LS framework. What explains the difference?

Summary of differences across frameworks Recall the differences. The framework of LS features (a) endogenous market tightness with free entry of firms, (b) immediate realization of skill upgrades, and (c) truncated normal productivity distributions with wide support. The framework of DHHR features (a) exogenous market tightness with fixed unit mass of firms, (b) delayed completion of skill upgrades, and (c) uniform productivity distributions with narrow support.

Two frameworks, opposite conclusions The two panels in Figure 1 show effects of both layoff and quit turbulence on the unemployment rate in the frameworks of LS and of DHHR, respectively. The x-axis shows layoff turbulence $\gamma^{d,x}$ and the y-axis the unemployment rate in percent. Each line has its own quit turbulence γ^d represented as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$.

Figure 1a shows results for the LS framework. We observe that quit turbulence needs to be high, about 50% of layoff turbulence, before the aggregate unemployment rate starts varying negatively with turbulence, and then only for relatively high levels of layoff turbulence.

Figure 1: Turbulence and unemployment in LS and DHHR



(a) Baseline LS

(b) Baseline DHHR

Layoff turbulence $\gamma^{d,x}$ on the x-axis. Each line represents a different quit turbulence γ^d as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$.

Figure 1b shows the effect of turbulence on the unemployment rate for our baseline DHHR model. This baseline includes two changes to the original DHHR setup that do not alter the results significantly but that facilitate a decomposition that lets us detect the source of the puzzle.

The first modification is that we assume that newborn workers are eligible for unemployment benefits equivalent to those of low-skilled workers, instead of the zero benefits that they receive in the original DHHR setup. This modification keeps down the number of worker types while having hardly any effect on aggregate outcomes. The second modification concerns the risk of losing skills following unsuccessful job market encounters. DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she would be quitting from a job. While DHHR describe this as a "simplifying assumption" made for numerical tractability, we find that it has quantitatively noticeable effects. (See Figure B.1 in Appendix B.) Still, the puzzle remains intact after this second modification – it just takes a somewhat bigger quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship.

The amount of quit turbulence needed in the DHHR framework to reverse the positive turbulence-unemployment relationship is very small. In the original DHHR model, the relationship becomes markedly negative at 5% of quit turbulence ($\epsilon = 0.05$), while in our baseline DHHR model, quit turbulence needs to be 7% ($\epsilon = 0.07$).

4.1 First suspect: Exogenous market tightness

The first candidate explanation concerns differences in the matching process. In the LS model, market tightness is endogenously determined by a typical free entry of firms assumption. The equilibrium zero-profit condition in vacancy creation pins down market tightness. In contrast, DHHR assume fixed and equal masses of workers and firms so that market tightness is exogenously always equal to one.

Perturbation exercise As described above, our renormalization of parameters (A, μ) in the original LS model yields equilibrium market tightness equal to one at zero turbulence. Our first perturbation exercise is to keep market tightness constant at one as we turn up turbulence. We do that by subsidizing vacancy creation so that the value of a firm posting a vacancy is zero, $w^f = 0$, at market tightness equal to one for any given levels of layoff and quit turbulence. The vacancy subsidies are financed with lump-sum taxation so that government budget constraint (27) is unaffected.

In this exercise where subsidies are used to keep $w^f = 0$ at $\theta = 1$, let $\bar{S}^o(\gamma^{d,x}, \epsilon)$ denote the expected match surplus of a vacancy encountering an unemployed worker, given layoff turbulence $\gamma^{d,x}$ and quit turbulence $\gamma^d = \epsilon\gamma^{d,x}$:

$$\bar{S}^o(\gamma^{d,x}, \epsilon) \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y) \; dv_i(y) \tag{31}$$

where unemployment u_{ij}, reservation productivity $\underline{z}^o_{ij}$, and match surplus $s^o_{ij}(y)$ are understood to be equilibrium values under our particular perturbation exercise.

At zero turbulence, the operation of the subsidy scheme would not require any payments of subsidies because we have parameterized the matching function so that equilibrium market tightness is then $\theta = 1$, a value of θ at which the zero-profit condition in vacancy creation is satisfied, $w^f = 0$, and by equation (15):

$$\mu = \beta(1-\pi)m(1)\bar{S}^o(0,0). \tag{32}$$

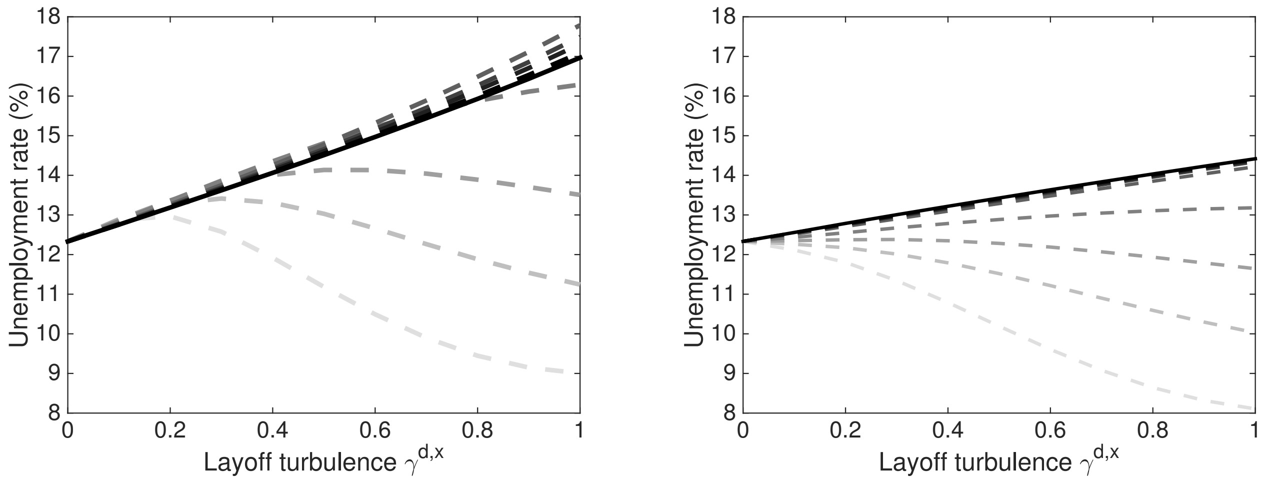
When turbulence is turned on, market tightness would have fallen if it were not for the subsidies to vacancy creation. The subsidy rate makes up for the shortfall of $\beta(1-\pi)m(1)\bar{S}^o(\gamma^{d,x}, \epsilon)$ when compared to the investment of incurring vacancy posting cost μ:

$$1 - subsidy(\gamma^{d,x}, \epsilon) = \frac{\beta(1-\pi)m(1)\bar{S}^o(\gamma^{d,x}, \epsilon)}{\mu} = \frac{\bar{S}^o(\gamma^{d,x}, \epsilon)}{\bar{S}^o(0,0)} \tag{33}$$

where the second equality invokes expression (32).

Results We observe an overall suppression of unemployment rates in Figure 2b as compared to Figure 2a. However, the underlying pattern of unemployment dynamics remains intact, so exogenous market tightness does not explain the puzzle.

Figure 2: ENDOGENOUS VS. EXOGENOUS MARKET TIGHTNESS IN LS



(a) Baseline LS (Endog. market tightness) (b) LS + Exogenous market tightness

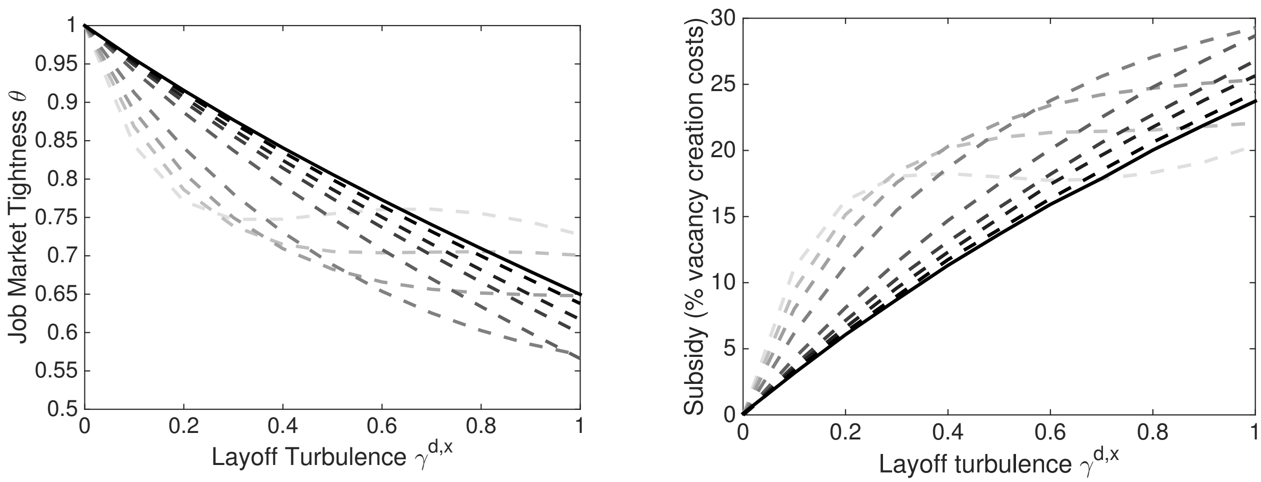
Discussion: Disabling the invisible hand With endogenous market tightness, there is a dramatic decline in market tightness in response to turbulence in Figure 3a. This outcome reflects how an "invisible hand" restores firm profitability so that vacancy creation breaks even. Lower market tightness decreases the probability that a worker encounters a vacancy, which tends to increase unemployment.

Our perturbation exercise disarms those forces by exogenously freezing market tightness at one. Hence, the profitability of vacancies plummets in response to turbulence. Figure 3b plots the subsidy rate for vacancy costs needed to incentivize firms to post enough vacancies to keep market tightness constant at one. At higher levels of turbulence, the subsidy rate becomes quite substantial. The subsidies to vacancy creation contribute to lower unemployment rates. These considerations seem to enhance a suspicion that exogenous market tightness could be the culprit behind the puzzle, so the above vindication was not a foregone conclusion.

4.2 Second suspect: Timing of completion of skill upgrades

The second candidate explanation concerns differences in the timing of completion of skill upgrades. LS assume that skill upgrades are immediately realized while DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Figure 3: Falling market tightness vs. subsidies for vacancy creation



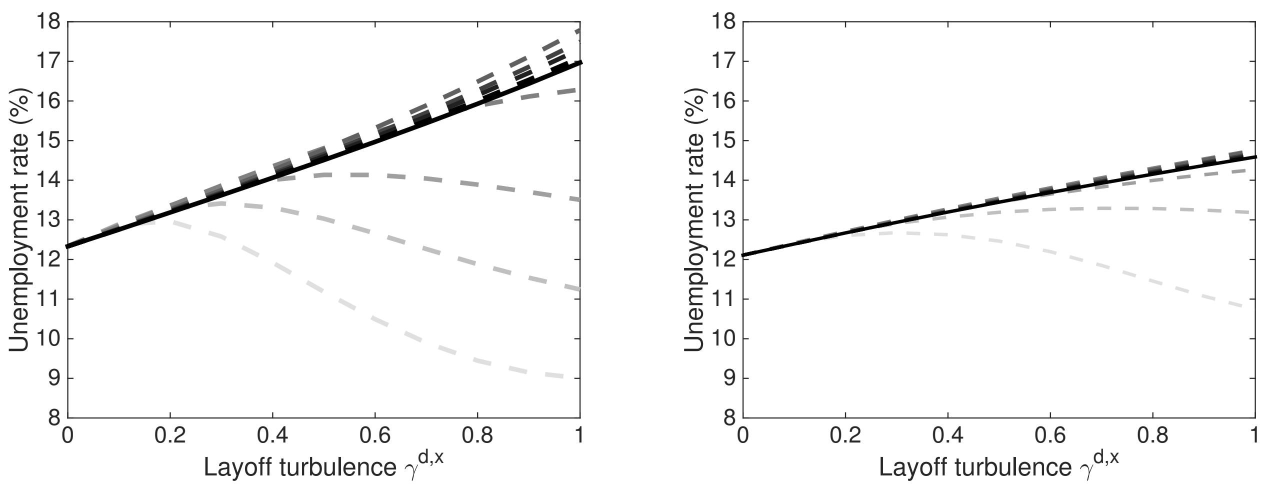
(a) Baseline LS (Endog. market tightness)

(b) LS + Exogenous market tightness

Perturbation exercise We replace immediate realization of skill upgrades in the LS model with delayed completion as in the DHHR model. The change in timing substantially alters the relative bargaining strengths of a worker and a firm.

Results The quantitative outcome in Figure 4b is similar to that of the preceding perturbation exercise in Figure 2b, i.e., it leads to an overall suppression in unemployment rates but without altering the underlying pattern of unemployment dynamics and hence, different timing of completion of skill upgrades does not explain the puzzle.

Figure 4: Immediate vs. delayed completion of skill upgrade in LS



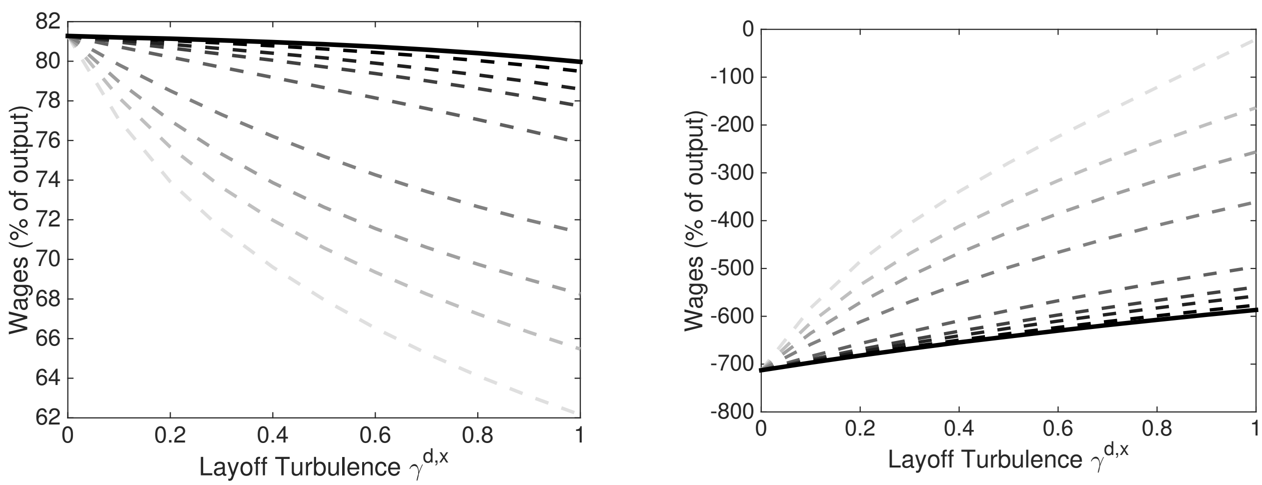
(a) Baseline LS (Immediate upgrade)

(b) LS + Delayed upgrade

Discussion: Delayed completion requires "ransoms" Firms under DHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work. This is possible because the realization of that higher skill level is conditional upon a worker remaining with the present employer for at least one more period, during which the worker can be assessed a "ransom" to secure her human capital gain.

We compare average wages at skill upgrades under immediate completion (Figure 5a) and delayed completion (Figure 5b), expressed in terms of average output per worker in the LS laissez-faire economy at zero turbulence.[12] In Figure 5b, a worker pays the "ransom" in terms of a negative semi-quarterly wage in the period of a skill upgrade, equivalent to the average annual output of a worker. The "ransom" becomes smaller with higher turbulence since the capital value of a skill upgrade is worth less when it is not expected to last long, as well as when quit turbulence locks high-skilled workers into employment relationships and thereby causes a less efficient allocation: fearing skill loss at separations, high-skilled workers accept lower reservation productivities and hence, work on average at lower productivities as compared to an economy in tranquil times with higher labor mobility.

Figure 5: Average wage in period of skill upgrade



(a) Baseline LS (Immediate upgrade)

(b) LS + Delayed upgrade

4.3 Third suspect: Productivity distributions

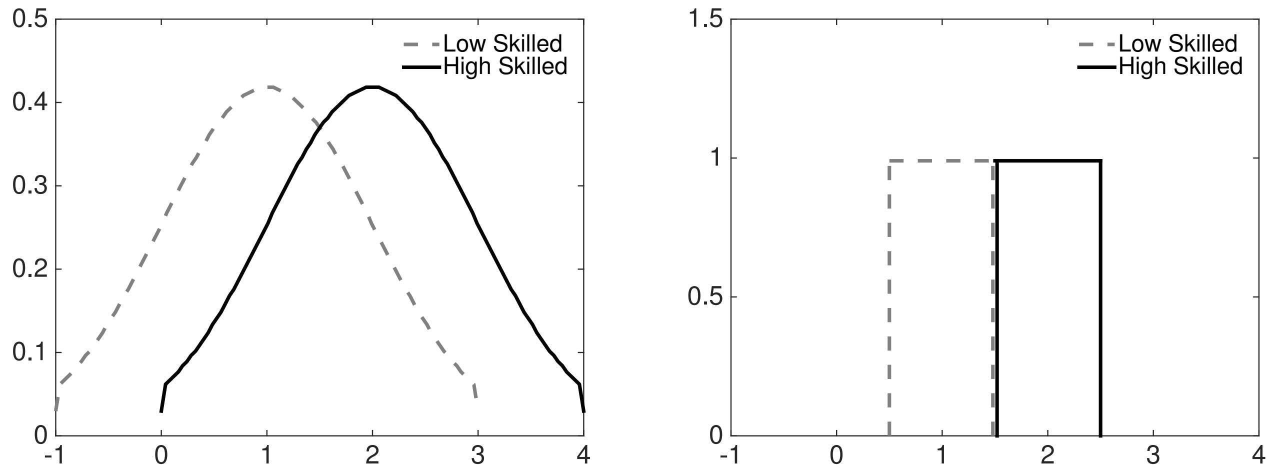
The third candidate explanation concerns differences in productivity distributions. LS assume truncated normal distributions with wide support while DHHR assume uniform distributions with narrow support, as detailed in Table 2 and depicted in Figure 6.

[12] In the LS laissez-faire economy without a government, a worker's average semi-quarterly output is 2.3 goods in tranquil zero-turbulence times.

Perturbation exercise We replace the productivity distributions in the LS model with those of DHHR, i.e., we replace the distributions in Figure 6a with those in Figure 6b.

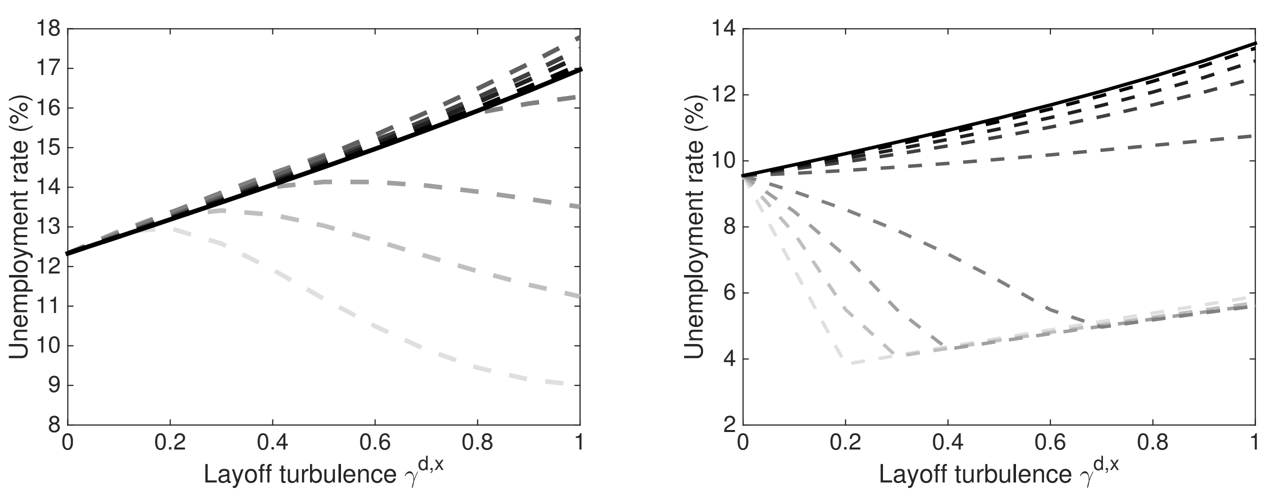
Results Figure 7 shows results for the alternative productivity distributions. The right panel shows that under the uniform distributions with narrow support, the positive turbulence-unemployment relationship is much weakened and we get DHHR-like outcomes. We conclude that differences in productivity distributions explain the puzzle.

Figure 6: Different productivity distributions in LS vs. DHHR



(a) LS (Normal with wide support) (b) DHHR (Uniform with narrow support)

Figure 7: Wide vs. narrow support of productivity distributions in LS

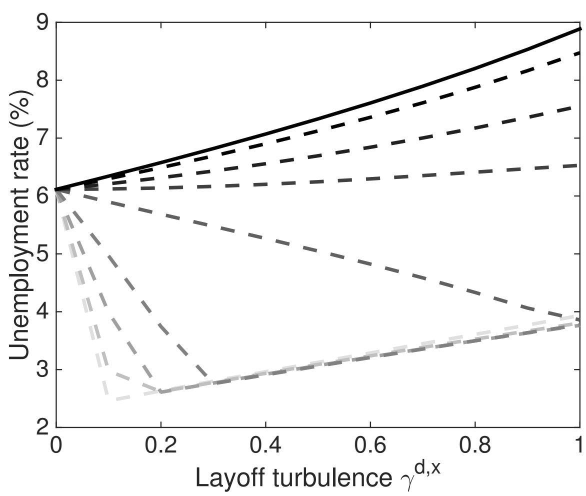


(a) Baseline LS (Wide normal) (b) LS + Narrow uniform

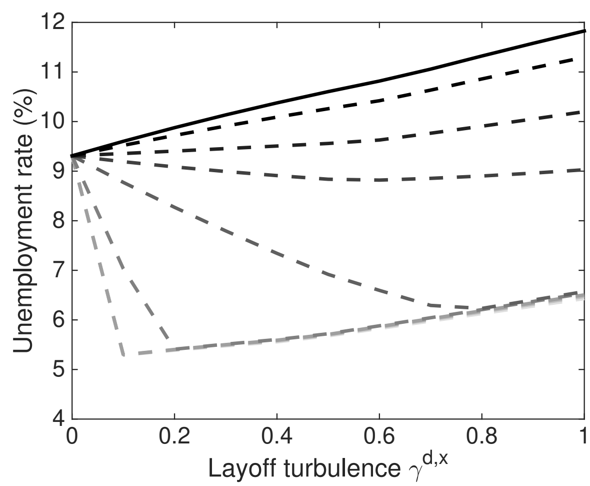
Discussion: Returns to labor mobility Productivity draws on the job create reasons for workers to change employer in search for higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support make returns to labor mobility be very low. As can be seen in Figure 7b, those low returns are outweighed by rather small amounts of quit turbulence and hence the initially positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative at relatively small levels of quit turbulence. In particular, high-skilled workers choose to remain on the job and accept productivities at the lower end of the support of the productivity distribution rather than quit and have to face even small probabilities of skill loss.

To confirm that the small dispersion of productivities explains the puzzle, we do an additional perturbation exercise that shrinks the support of the uniform distribution further. Figure 8a shows outcomes in the LS framework when the support of the uniform distribution is of width 0.65 instead of 1. Such a shrinkage of the support takes us very close to the outcomes in our baseline DHHR model in Figure 8b, in spite of retaining the other two differences between the frameworks: endogenous versus exogenous market tightness and immediate versus delayed completion of skill upgrades.

Figure 8: Further reduced support of productivity distributions in LS



(a) LS + Even narrower uniform



(b) Baseline DHHR

5 Layoff taxes

To bring out the low return to labor mobility in the DHHR model relative to that of the LS model, we introduce a layoff tax Ω that is levied on every job separation, except for retirement. The layoff tax affects reservation productivities of existing jobs as the match surplus must now fall to the negative of the layoff tax (instead of earlier zero) before a job is terminated:

$$s_{ij}(\underline{z}_{ij}) = -\Omega. \tag{34}$$

When computing wages, we assume standard Nash bargaining with a worker and a firm getting their shares of the match surplus s_{ij} in every period.[13]

Government revenue With the introduction of a layoff tax, the government's revenue includes revenues from layoff taxes. Let *seps* be total separations excluding retirements, which are equal to

$$seps = (1-\rho^r)\Big[\rho^x(e_{ll}+e_{hh}) + (1-\rho^x)[(1-\gamma^u)\gamma^s\nu_{ll} + \gamma^u\nu_{hh}]e_{ll} + (1-\rho^x)\gamma^s\nu_{hh}e_{hh}\Big]. \tag{35}$$

Then government revenue equals income taxes plus layoff taxes, $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}) + seps\ \Omega$. The government adjusts the income tax rate τ to set revenue equal to total expenditure on unemployment benefits in expression (26).[14]

5.1 Layoff taxes in LS

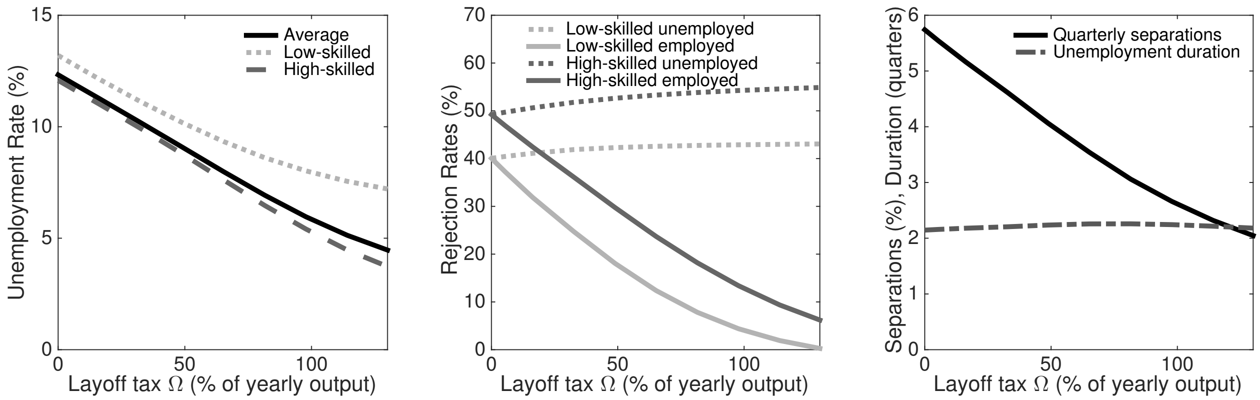
In the LS model without turbulence, Figure 9 shows unemployment and rejection rates by type of worker, as well as aggregate labor flows, as functions of the layoff tax Ω. The layoff tax is expressed as a fraction of the average yearly output per worker in the laissez-faire economy. (See footnote 12.) The unemployment rate falls as the layoff tax increases. Employed workers, both high- and low-skilled, are particularly affected by the layoff tax as their rejection rates fall significantly. Still, the mobility of these workers remains operative even for rather large layoff taxes. For example, if the layoff tax reaches the average annual output of a worker, employed high-skilled workers reject about 12% of offers.

Incidentally, Figure 9 shows LS's explanation for why a welfare state having lower unemployment than a laissez-faire economy in tranquil times (i.e., before the onset of economic turbulence). As noted by Mortensen and Pissarides (1999), in a matching model countervailing forces emanating from unemployment benefits and layoff taxes can explain why the unemploy-

[13] An implication of the Nash bargaining assumption is that workers would be paying part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff cost. This generates a two-tier wage system à la Mortensen and Pissarides (1999). Risk neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. As demonstrated by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Match surpluses, reservation productivities and market tightness remain the same. Under the two-tier wage system, an initial wage concession of a newly hired worker is equivalent to his posting a bond that equals his share of a future layoff tax.

[14] If layoff tax revenues cover payments of unemployment benefits, i.e., $seps\ \Omega \geq b_l u_{ll} + b_h(u_{lh} + u_{hh})$, then we set $\tau = 0$ and return any government budget surpluses as lump-sum transfers to workers.

Figure 9: Layoff taxes in LS



ment rate in a welfare state need not be high. In spite of generous unemployment benefits with a replacement rate of $\phi = 0.7$, layoff taxes at the right end of the first panel in Figure 9 cause unemployment to fall below the laissez-fare rate of 5%.

5.2 Layoff taxes in DHHR

We introduce a layoff cost Ω in our baseline DHHR model with the additional inconsequential change that skill upgrades are realized immediately as in the LS framework (Appendix B.2 documents a small impact on equilibrium outcomes of such a change in assumptions). Figure 10 shows how a higher layoff tax affects equilibrium outcomes in the DHHR model without turbulence.

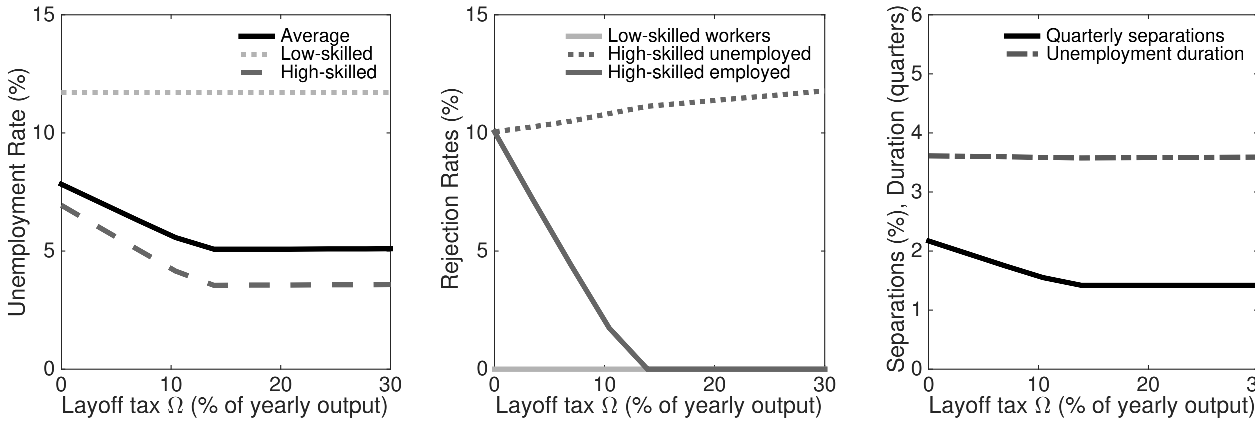
Mobility of high-skilled employed completely shuts down at a layoff tax equivalent to 14% of the average annual output per worker in the laissez-faire economy.[15] Above this low level of layoff taxes, the rejection rate of these workers becomes zero and separation rates become constant at exogenous job termination rates. Imposing a small layoff tax makes the value of labor mobility evaporate. Note that for low-skilled workers, both employed and unemployed, the rejection rate is zero for the DHHR parameterization at all levels of the layoff tax.

This exercise confirms that the productivity distributions of DHHR imply small incentives for labor mobility in tranquil times.[16] A small government mandated layoff cost has counterfactually large effects of suppressing unemployment by shutting down all quits. So it should

[15] In the DHHR laissez-faire economy, a worker's average quarterly output is 1.8 goods.

[16] The productivity distributions of DHHR also emerged from an earlier exchange discussed in footnote 8. Specifically, Ljungqvist and Sargent (2004) criticized den Haan *et al.* (2001) for making low- and high-skilled workers almost indistinguishable from one another because of nearly overlapping productivity distributions for the two types of workers. As a remedy, by moving the uniform distributions apart and ending up with the disjoint supports in Figure 6b, Ljungqvist and Sargent (2004) succeeded in making low- and high-skilled workers distinct from one another; but as shown here that fails to generate returns to labor mobility consistent with historical observations. In the subsequent matching analysis of LS, layoff costs were introduced and productivity distributions had to be properly calibrated, as shown in subsection 5.1.

Figure 10: Layoff taxes in DHHR



come as no surprise that other small costs to mobility, such as a tiny risk of skill loss when quitting, cause unemployment to fall and thereby, can reverse Ljungqvist and Sargent's positive turbulence-unemployment relationship.

6 Listening more carefully to Alan Greenspan

In light of our findings about the effects of recalibrating quit turbulence in order to respect the Weinberg constraint, we rejoin and extend the conversation with Alan Greenspan with which DHHR began their paper. In the passage that DHHR cite, reproduced in section 1 above, Greenspan does indeed seem to be concerned with the DHHR's quit turbulence force as well as the sort of effect in reducing job mobility that comes with DHHR's calibration. But if we listen to all that seems to have been on Greenspan's mind, we hear that Greenspan did not emphasize such possible effects of increased turbulence more broadly. To the contrary, earlier in the very same paragraph cited by DHHR, Greenspan (1998, p. 743) says that it is not decreased but high labor mobility (i.e., "churning") that is mainly on his mind:

> "... the perception of increased churning of our workforce in the 1990s has understandably increased the sense of accelerated job-skill obsolescence among a significant segment of our workforce, especially among those most closely wedded to older technologies. The pressures are reflected in a major increase in on-the-job training and a dramatic expansion of college enrollment, especially at community colleges. As a result, the average age of full-time college students has risen dramatically in recent years as large numbers of experienced workers return to school for skill upgrading."

We read Greenspan as writing here about US workers who have been hit by the type of human capital destruction shock that LS used to capture increased turbulence. Greenspan says that those workers have ways of rebuilding their human capital in addition to the ways that are open to them in the LS model, thereby suggesting interesting ramifications of increased turbulence for other observations not on the table in either the DHHR or the LS framework. It would be interesting to add such activities to the model environment that succeeded in explaining trans-Atlantic unemployment experiences, while playing by Weinberg's rules.

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A Equilibrium computation

A.1 General algorithm structure

Here we outline the structure of the algorithm that we used to compute equilibria.[17] It centers around approximating the joint continuation values $g_i(z)$ by using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.
2. Guess initial values for:
   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - ω^f: firms' outside value (in LS, $\omega^f = 0$)
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers
3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.
4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).
5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:

$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}}^{\infty} s_{ij}^o(y)\ dv_i(y).$$

6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1\ \ z]$.
7. Update the value of posting a vacancy, market tightness, and matching probabilities:
   - under LS's endogenous market tightness,

$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u};$$

[17] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

- under DHHR's exogenous market tightness, compute

$$\omega^f = \frac{\beta}{1-\beta} A(1-\pi)\bar{s}, \qquad \theta = 1, \qquad \lambda^w = A, \qquad \lambda^f_{ij} = A\frac{u_{ij}}{u}.$$

8. Update values ω^w_{ij} of being unemployed using (11) and (12).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned} \Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s\nu_{ll}\right\} e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{ll})u_{ll} \end{aligned} \tag{A.1}$$

$$\begin{aligned} \Delta u_{lh} &= (1-\rho^r)\left\{\rho^x\gamma^{d,x}e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x})e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \end{aligned} \tag{A.3}$$

$$\begin{aligned} \Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\left\{(1-\nu^o_{ll})u_{ll} + (1-\nu^o_{lh})u_{lh}\right\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll} \end{aligned} \tag{A.4}$$

$$\begin{aligned} \Delta e_{hh} &= (1-\rho^r)\left\{\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh} \end{aligned} \tag{A.5}$$

   These expressions embed LS's assumption of immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see the corresponding expressions for worker flows in den Haan *et al.* (2005, appendix A).

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, in order to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (27), respectively.

11. Adjust tax rate τ in (27) to balance government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to 2 and use as guesses the last values computed.

A.2 Average wages and productivities

The following computations refer to the LS model with immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see den Haan *et al.* (2005, appendices A–C).

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p^o_{lh}(z) > p^o_{ll}(z) = p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} = \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll}+e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty}\left[\frac{e'_{ll}}{e_{ll}}p^o_{lh}(y)+\frac{e''_{ll}}{e_{ll}}p_{ll}(y)\right]\frac{dv_l(y)}{1-v_l(\underline{z}^o_{lh})} \;+\; \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}p_{ll}(y)\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}}\int_{\underline{z}^o_{lh}}^{\infty}y\,\frac{dv_l(y)}{1-v_l(\underline{z}^o_{lh})} \;+\; \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}y\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed. For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$\begin{aligned} e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\ e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_h &= \int_{\underline{z}^o_{hh}}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p^o_{hh}(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y)\right] \frac{dv_h(y)}{1-v_h(\underline{z}^o_{hh})} + \frac{e_{hh}-e'_{hh}-e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y)\, \frac{dv_h(y)}{1-v_h(\underline{z}_{hh})} \\ \bar{z}_h &= \frac{e'_{hh}+e''_{hh}}{e_{hh}} \int_{\underline{z}^o_{hh}}^{\infty} y\, \frac{dv_h(y)}{1-v_h(\underline{z}^o_{hh})} + \frac{e_{hh}-e'_{hh}-e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y\, \frac{dv_h(y)}{1-v_h(\underline{z}_{hh})}. \end{aligned}$$

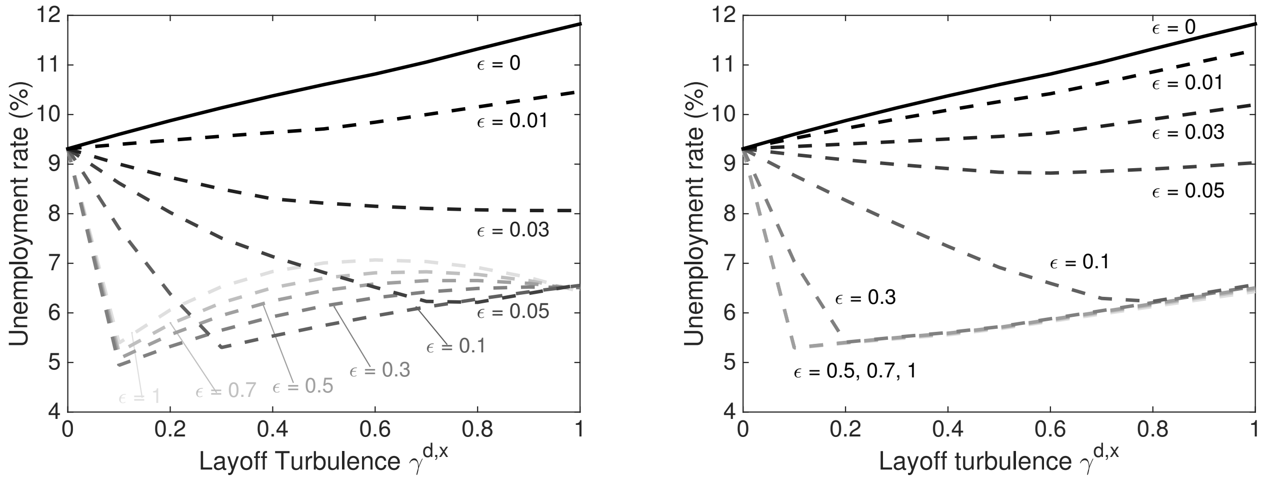
B Starting from DHHR framework

We now reverse the analysis by starting from the DHHR framework and investigating the consequences of three perturbations. The features in the original DHHR framework to be perturbed are (i) exogenous labor market tightness, (ii) delayed completion of skill upgrade, and (iii) uniform productivity distributions with narrow support. But before that, we eliminate two auxiliary assumptions in the DHHR analysis.

Eliminate auxiliary assumption of zero benefits for newborn workers Instead of DHHR's assumption of no benefits during the initial unemployment spells of newborn workers, we assume that they are eligible for unemployment benefits equivalent to those of low-skilled workers. This change has hardly any effect on aggregate outcomes.

Eliminate auxiliary assumption of turbulence for unemployed DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she had instead quit a job. DHHR describe this as an auxiliary assumption that they justify in terms of its computational tractability, but we find that it has noticeable quantitative consequences. Thus, figure B.1 presents outcomes for the original DHHR framework with turbulence for unemployed workers and our baseline DHHR model without that kind of turbulence. While the outcomes are not as stark in latter model, the underlying pattern of unemployment dynamics remains intact – it just takes some more quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship.

Figure B.1: With vs. without turbulence for unemployed in DHHR



(a) Original DHHR (Turbulence unemployed) (b) Baseline DHHR (No turbulence unempl.)

An assumption that mere encounters between vacancies and unemployed workers are associated with risks of losing skills unless employment relationships are formed directly suppresses returns to labor mobility. But as can be inferred from Figure B.1, such an exposure of job seekers to skill loss does not have much impact on unemployment outcomes since, as Appendix B.3 will teach us, compressed productivity distributions in DHHR already reduce returns to labor mobility. However, the substantial incentives for labor mobility under LS's parameterization of productivity distributions are significantly affected and suppressed by that auxiliary assumption of DHHR. Appendix C discusses this in detail.

B.1 First suspect: Exogenous market tightness

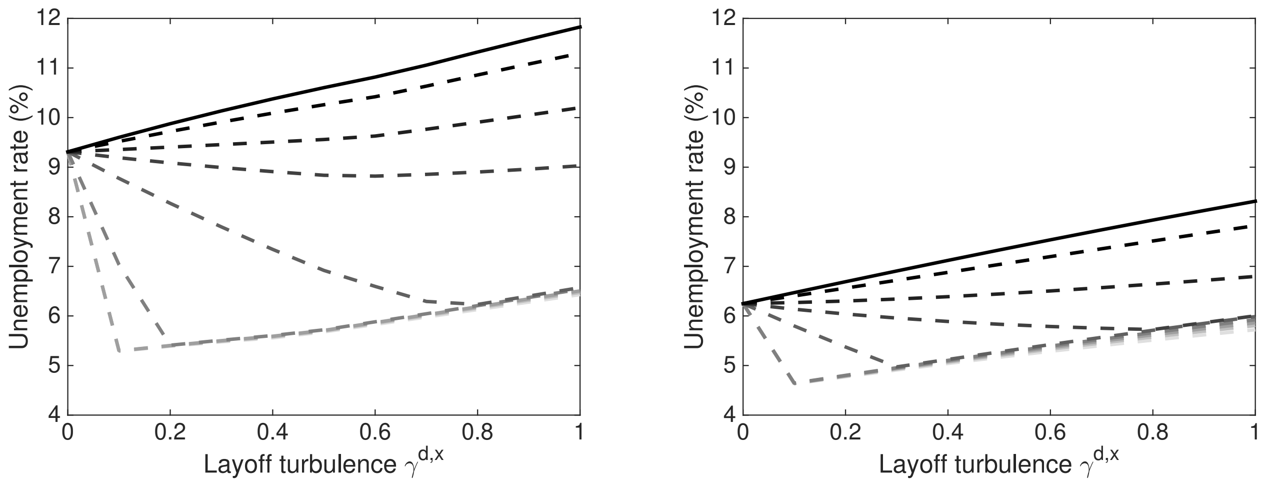
Perturbation exercise In the DHHR framework, there is an exogenous mass of firms and there are no costs for posting vacancies. Hence the value w^f of a firm posting a vacancy is trivially positive. We now perturb DHHR to feature free entry of firms, $w^f = 0$ in equilibrium, and an endogenous market tightness determined by (15). In order to implement that perturbation, we must introduce and assign values to two additional parameters, α and μ. Following LS, we assume that the elasticity of the matching function with respect to unemployment equals $\alpha = 0.5$, a fairly common parameterization.

Lacking an obvious way to parameterize the vacancy posting cost μ in this perturbation, we solve the model for different values of $\mu > 0$.[18] We find that for values of μ above 0.7,

[18]The vacancy posting cost μ must be positive to have an equilibrium with free entry of firms. The discrete

all voluntary quits vanish. Therefore, since DHHR's challenge to a Ljungqvist-Sargent positive turbulence-unemployment relationship is based on changes in the incidence of quits, we consider $\mu \in (0, 0.7)$ to be the permissible range. As an illustration, Figure B.2b depicts equilibrium outcomes for the midpoint of that parameter range, $\mu = 0.35$.

Figure B.2: Exogenous vs. endogenous market tightness in DHHR



(a) Baseline DHHR (Exog. market tightness) (b) DHHR + Endogenous market tightness

Results Except for the very top end of the parameter range $\mu \in (0, 0.7)$, the qualitative pattern of Figure B.2 represents the unemployment-turbulence relationship for the DHHR framework under the two alternative matching assumptions. In both cases, rather small amounts of quit turbulence reduce unemployment. Therefore, exogenous versus endogenous market tightness does not explain the puzzle.

In the vicinity of parameter value $\mu = 0.7$, the curve for $\epsilon = 0.1$ in the corresponding version of Figure B.2b (not shown here) takes on a positive slope, i.e., outcomes become LS-like with a positive turbulence-unemployment relationship. This might have been anticipated. As mentioned above, $\mu = 0.7$ is also the parameterization at which all voluntary quits vanish, which would seem to disarm the DHHR quit turbulence argument.[19]

model period and the Cobb-Douglas matching function call for an additional caveat. As the value of μ approaches zero, the equlibrium probability of filling a vacancy goes to zero. That creates a problem when the associated probability of a worker encountering a vacancy exceeds the permissible value of unity. Therefore, we only compute equilibria for μ greater than 0.0063. If one would like to compute equilibria for lower values of μ, it could be done by augmenting the match technology to allow for corner solutions at which the short end of the market determines the number of matches; e.g., in the present case, by freezing the job finding probability at unity while randomly allocating the unemployed across all vacancies that draw an "encounter." (See Ljungqvist and Sargent (2007, section 7.2).)

[19]For a more nuanced reasoning about the equilibrium forces at work under the threat of losing skills in a matching model, see the discussion of an "allocation channel" and a "bargaining channel" in section C.2. While

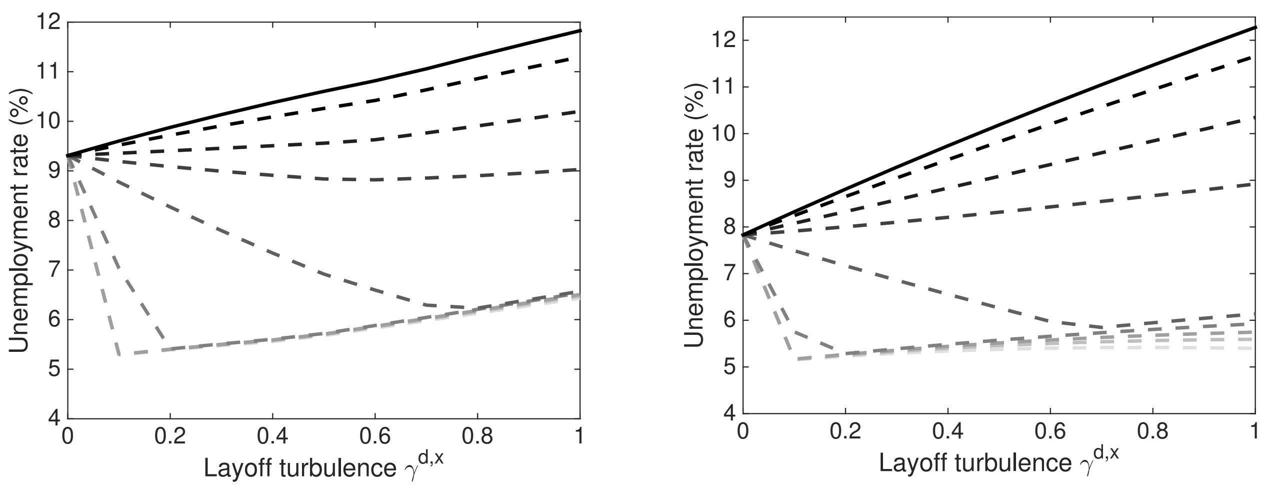
Incidentally, as we will learn in Appendix B.3, the raw fact that voluntary quits vanish at a relatively low value of the vacancy posting cost $\mu = 0.7$ is indicative of low returns to labor mobility in the DHHR model that come from compressed productivity distributions.

B.2 Second suspect: Timing of completion of skill upgrades

Perturbation exercise DHHR assume that after a skill upgrade a worker must remain with the present employer for one period in order to complete the higher skill level. In this section, we introduce immediate completion of skill upgrades as in LS.

Results Figure B.3 shows that there is no substantial difference in the turbulence-unemployment relationship for the alternative timings in the DHHR model. Hence, delayed versus immediate completion of skill upgrades does not explain the puzzle.

Figure B.3: Delayed vs. immediate completion of skill upgrade in DHHR



(a) Baseline DHHR (Delayed upgrade)

(b) DHHR + Immediate upgrade

B.3 Third suspect: Productivity distributions

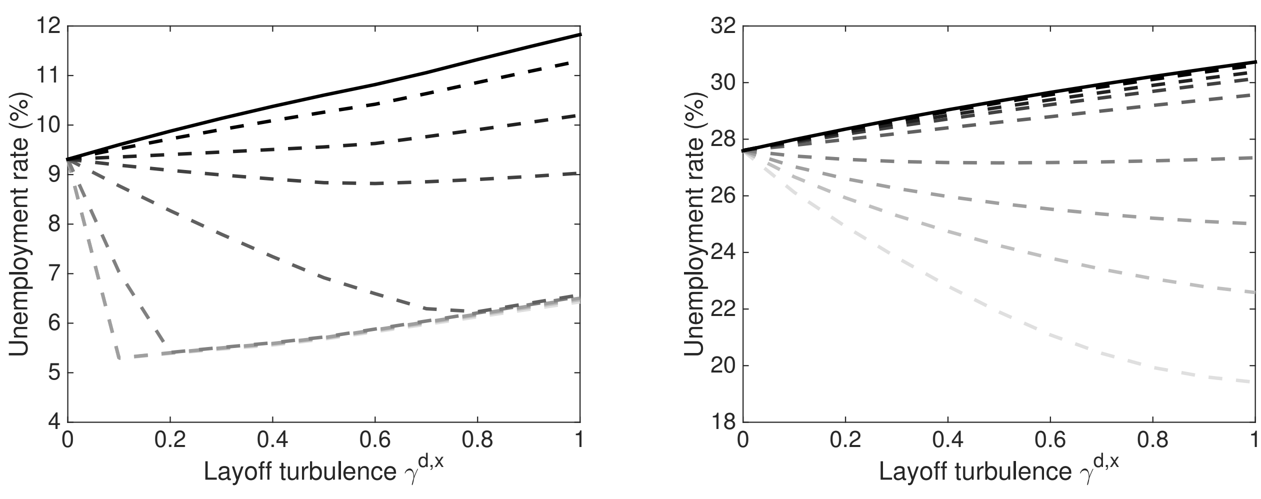
Perturbation exercise DHHR assume uniform distributions with narrow support: $z_l \sim \mathcal{U}([0.5, 1.5])$ and $z_h \sim \mathcal{U}([1.5, 2.5])$. In this section we replace those distributions in the DHHR model by the truncated normal distributions assumed by LS: $z_l \sim \mathcal{N}(1, 1)$ for low-skilled workers over the support $[-1, 3]$, and $z_h \sim \mathcal{N}(2, 1)$ for high-skilled workers over the support $[0, 4]$. Notice

that section pertains to the introduction of turbulence facing unemployed workers in terms of a risk of losing skills after an encounter between a firm and a worker that does not result in employment, similar reasoning can be applied to quit turbulence for employed workers.

the big difference is the dispersion in productivities: the standard deviation is $1/\sqrt{12}$ in the uniform case of DHHR and 1 in the normal case of LS.

Results Figure B.4 shows how the turbulence-unemployment relationship is altered in the DHHR model when we switch from DHHR's productivity distributions to those of LS. First, the larger variances of the LS distributions exert upward pressures on reservation productivities and labor reallocation rates, but DHHR's assumption that an exogenously given market tightness equals one means that the relative number of vacancies cannot expand, so overall unemployment rates become higher. Second, and critical to our inquiry, the inference to be drawn from Figure B.4 agrees with what we inferred after studying the obverse perturbation of the LS model in Figure 7; namely, differences in productivity distributions are key to explaining the puzzle. When we import the LS distributions into the DHHR model, small amounts of quit turbulence no longer unduly dissuade high-skilled workers with poor productivity draws to quit and seek better employment opportunities. Hence, the present perturbation disarms DHHR's argument for suppressed quit rates and allows the Ljungqvist-Sargent turbulence force to operate unimpeded. The right panel of Figure B.4 shows how turbulence and unemployment are positively related until quit turbulence reaches about 30% of layoff turbulence after which the relationship becomes negative.

Figure B.4: NARROW VS. WIDE SUPPORT OF PRODUCTIVITY DISTRIBUTIONS IN DHHR



(a) Baseline DHHR (Narrow uniform)

(b) DHHR + Wide normal

C Turbulence affecting job market encounters

DHHR assume that after an encounter between a firm and an unemployed worker that does not result in employment, the worker faces the same risk of losing skills as if she had quit from a job. They justify this assumption only for its tractability in allowing them to reduce the number of worker types that they must track. In Figure B.1 of Appendix B, we confirm that the assumption does not make much of a difference for DHHR's inference about the turbulence-unemployment relationship in their model. But when we pursue a parallel analysis in the LS model as we do here, we find that DHHR's simplifying assumption has a large impact. We show this in subsection C.1. To shed light on the forces at work, subsection C.2 undertakes yet another perturbation exercise that limits the exposure to such risk to the first $\bar{k}$ periods of an unemployment spell, after which there is no risk of skill loss during the rest of an unemployment spell.

To allow for a more general formulation, we assume a distinct probability γ^e of skill loss after an unsuccessful job market encounter, while γ^d continues to denote the probability of skill loss when quitting from an employment relationship.

C.1 Introducing turbulence for unemployed workers in LS

When unemployed high-skilled workers face a probability γ^e of losing skills after unsuccesful job market encounters, the match surplus in (3) of a new job with a high-skilled worker changes to

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^e)\omega_{hh} + \gamma^e\omega_{lh}], \tag{C.6}$$

where the outside value in brackets reflects the risk of skill loss if the firm and worker do not enter an employment relationship. The net change of the mass of low-skilled unemployed with high benefits in (28) changes to

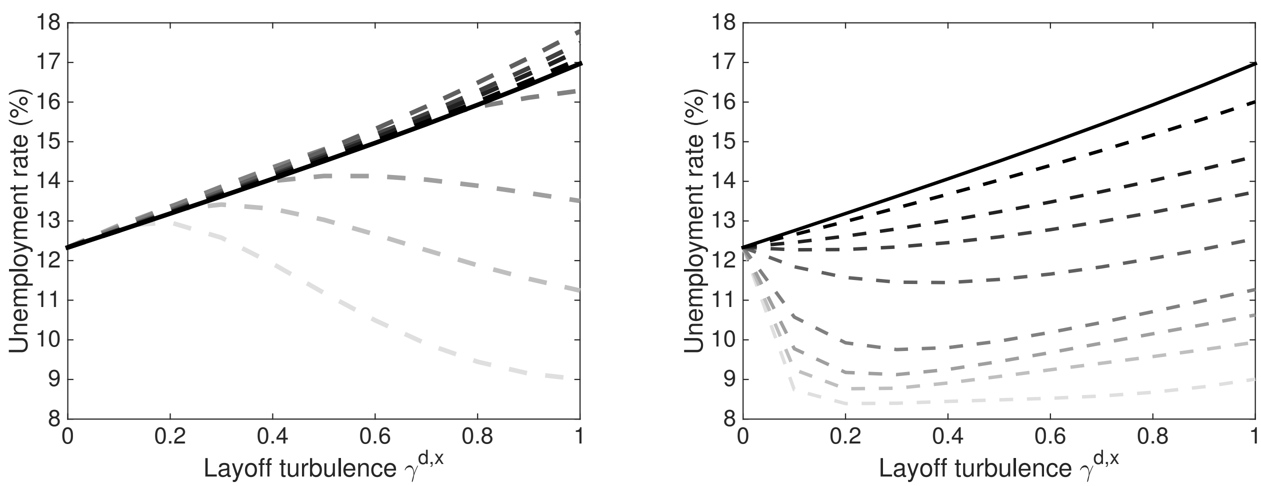
$$\begin{aligned}\Delta u_{lh} \;=\; (1-\rho^r)\Bigg\{ & \underbrace{\rho^x\gamma^{d,x}e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^d\nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} \\ & - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} + \underbrace{\lambda^w(\theta)\gamma^e\nu^o_{hh}u_{hh}}_{\text{4. turbulence unempl.}} \Bigg\} - \rho^r u_{lh},\end{aligned} \tag{C.7}$$

where the new term numbered 4 is the inflow of unemployed high-skilled workers who have just lost their skills after job market encounters that did not lead to employment.

Turning to a quantitative assessment of turbulence for unemployed workers in the LS model, we have to take a stand on the different lengths of a model period used in the parameterizations of LS and DHHR. In the case of the exogenously given layoff risk, the probability of a layoff at the semi-quarterly frequency in LS's model is half of the probability at the quarterly frequency in DHHR's model, as discussed in footnote 9. Analogously, but less obviously, for the risk of skill loss after endogenously determined unsuccessful job market encounters we assume that $\gamma^e = 0.5\gamma^d$ in LS's semi-quarterly model as compared to DHHR's assumption that $\gamma^e = \gamma^d$ in their quarterly model. However, for the record, our conclusion from Figure C.1 remains the same with or without the latter adjustment. That is, with or without this adjustment, adding exposure of unemployed workers to risks of skill loss after unsuccessful job market encounters has sizeable effects on the turbulence-unemployment relationship in the LS model.

As discussed in footnote 3, risk of skill loss after unsuccessful job market encounters was not part of DHHR's use of quit turbulence to challenge a Ljungqvist-Sargent positive turbulence-unemployment relationship. Rather, they adopted it for computational tractability. Hence, we feel justified in discarding this auxiliary feature of DHHR's original analysis in order to focus more sharply on the key explanation to the puzzle – different productivity distributions. But it is nevertheless tempting to turn on and off their auxiliary assumption in order to shed further light on the mechanics of our particular matching model, and matching frameworks more generally. Therefore, we offer the following suggestive decomposition of forces at work.

Figure C.1: Without vs. with turbulence for the unemployed in LS



(a) Baseline LS (No turbulence unempl.)

(b) LS + Turbulence unemployed

C.2 Decomposition of forces at work

We seek to isolate two interrelated forces acting when job seekers are exposed to risk of skill loss after unsuccessful job market encounters in a matching model. First, the mere risk of losing skills when turning down job opportunities suppresses the return to labor mobility in many frictional models of labor markets, including the basic McCall (1970) search model where wages are drawn from an exogenous offer distribution. Such risks would render job seekers more prone to accept employment opportunities. We call this the "allocation channel." Second, the matching framework contains yet another force when risk of skill loss after an unsuccessful job market encounter weakens the bargaining position of a worker vis-à-vis a firm and accordingly affects match surpluses received by firms. That in turn affects vacancy creation via the equilibrium condition that vacancy posting must break even. We call this the "bargaining channel."

It presents a challenge to isolate these two channels because everything is related to everything else in an equilibrium. Here we study how equilibrium outcomes change as we vary the horizon over which the risk of skill loss prevails during an unemployment spell. Thus, after an unsuccessful job market encounter, let an unemployed worker be exposed to risks of skill losses for the first $\bar{k}$ periods of being unemployed and thereafter to suffer no risk of skill loss for the remainder of that unemployment spell. To illustrate the allocation channel, consider the basic McCall search model. Starting from $\bar{k} = 0$, equilibrium unemployment would initially be significantly suppressed for each successive increase in the parameter $\bar{k}$ because workers anticipate ever longer periods of effective exposure to risk of skill loss when unemployed; but eventually, the value of $\bar{k}$ is so high that it is most unlikely that a worker remains unemployed for such an extended period of time and hence, a worker's calculation of the payoff from quitting a job would hardly be affected by any additional increase in $\bar{k}$. Thus, in a McCall search model, via the allocation channel, equilibrium unemployment would hardly change for higher values of $\bar{k}$. In contrast, we will find in the LS matching model that unemployment suppression effects that occur in response to increases in $\bar{k}$ don't die out beyond such high values of $\bar{k}$. We then argue that those equilibrium outcome effects can be attributed to the bargaining channel.

Notation Let u^0_{hh} denote the mass of high-skilled workers who become unemployed in each period without losing skills, and let u^k_{hh} be the mass of those workers who remain high-skilled and unemployed after an unemployment duration of $k = 1, \ldots, \bar{k} - 1$ periods. A final category $u^{\bar{k}}_{hh}$ includes all workers who remain high-skilled and unemployed after unemployment spells of *at least* $\bar{k}$ periods, i.e., $u^{\bar{k}}_{hh}$ is the mass of unemployed high-skilled workers who no longer face any risk of skill loss in their current unemployment spells.

Using the same superscript convention, let $\omega^{w,k}_{hh}$ for $k = 0, \ldots, \bar{k}$ be the future value of unemployment of an unemployed high-skilled worker in category u^k_{hh}, with $\underline{z}^k_{hh}$ and ν^k_{hh} denoting the worker's reservation productivity and rejection probability next period, and for any match accepted next period, the match surplus is $s^k_{hh}(z)$ and the initial wage is $p^k_{hh}(z)$.

Laws of motion The laws of motion for worker categories u^k_{hh}, for $k = 0, \ldots, \bar{k} - 1$, have in common that all workers leave the category next period. The inflow to the initial category u^0_{hh} consists of employed high-skilled workers who experience non-turbulent layoffs or quits, including low-skilled employed workers who have just received a skill upgrade. Each successive category u_{hh}^k, for $k = 1, \ldots, \bar{k}-1$, receives its inflow from not retired workers in the preceding category u_{hh}^{k-1}, those who did not match or experienced non-turbulent rejections of matches:

$$\Delta u_{hh}^k = \begin{cases} (1-\rho^r)\Big[\underbrace{\rho^x(1-\gamma^{d,x})e_{hh}}_{\text{non-turbulent layoff}} + \underbrace{(1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{non-turbulent quit}}\Big] \; - \; u_{hh}^k & \text{if } k = 0 \\ (1-\rho^r)\Big[\underbrace{(1-\lambda^w(\theta))}_{\text{no match}} + \underbrace{\lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)}_{\text{non-turbulent rejected match}}\Big] u_{hh}^{k-1} \; - \; u_{hh}^k & \text{if } 0 < k < \bar{k}. \end{cases}$$

The final category $u_{hh}^{\bar{k}}$ also receives inflows from the preceding category $u_{hh}^{\bar{k}-1}$, but now outflows are only partial. The workers who leave are the retirees and those with accepted matches (those with rejected matches are no longer affected by turbulence and thus always remain):

$$\Delta u_{hh}^{\bar{k}} = (1-\rho^r)\Big[(1-\lambda^w(\theta)) + \lambda^w(\theta)\nu_{hh}^{\bar{k}-1}(1-\gamma^e)\Big] u_{hh}^{\bar{k}-1} \; - \; \Big[\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^{\bar{k}})\Big] u_{hh}^{\bar{k}}.$$

The law of motion for u_{lh} workers is modified to receive the inflow from the different u_{hh}^k categories that suffered turbulent rejections in their first $\bar{k}$ periods of unemployment:

$$\begin{aligned} \Delta u_{lh} \;&=\; (1-\rho^r)\Big[\underbrace{\rho^x\gamma^{d,x}e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{turbulent separations}} + \underbrace{\lambda^w(\theta)\gamma^e \sum_{k=0}^{\bar{k}-1} \nu_{hh}^k u_{hh}^k}_{\text{turbulent rejections}}\Big] \\ &\quad - \; [\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)]\, u_{lh}. \end{aligned}$$

The law of motion for high-skilled employed workers e_{hh} is adjusted to include those gaining employment from the different u_{hh}^k categories:

$$\begin{aligned} \Delta e_{hh} \;&=\; (1-\rho^r)\Big[\underbrace{\lambda^w(\theta)\sum_{k=0}^{\bar{k}}(1-\nu_{hh}^k)u_{hh}^k}_{\text{accepted new matches}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}}_{\text{accepted upgrades}}\Big] \\ &\quad - \; [\rho^r + (1-\rho^r)(\rho^x + (1-\rho^x)\gamma^s\nu_{hh})]\, e_{hh}. \end{aligned}$$

High-skilled unemployed: match surplus, initial wage, and value of unemployment For a high-skilled worker who remains unemployed after $k < \bar{k}$ periods, the match surplus of any job opportunity next period reflects an outside option with risk γ^e of losing skills if the employment relationship is not formed; but after $\bar{k}$ periods there is no such risk:

$$s_{hh}^{k}(z) = \begin{cases} (1-\tau)z + g_h(z) - \Big[b_h + (1-\gamma^e)\omega_{hh}^{w,k+1} + \gamma^e \omega_{lh}^{w} + \omega^f\Big] & \text{if } k < \bar{k} \\ \\ (1-\tau)z + g_h(z) - \Big[b_h + \omega_{hh}^{w,k} + \omega^f\Big] & \text{if } k = \bar{k}. \end{cases}$$

Reservation productivities and rejection probabilities satisfy

$$s_{hh}^{k}(\underline{z}_{hh}^{k}) = 0\,, \qquad \nu_{hh}^{k} = \int_{-\infty}^{\underline{z}_{hh}^{k}} dv_h(y)\,.$$

The wage in the first period of employment of such a high-skilled worker is

$$\begin{aligned} p_{hh}^{k}(z) + g_h^w(z) &= \pi s_{hh}^{k}(z) + b_h + (1-\gamma^e)\omega_{hh}^{w,k+1} + \gamma^e \omega_{lh}^{w} && \text{if } k < \bar{k} \\ p_{hh}^{k}(z) + g_h^w(z) &= \pi s_{hh}^{k}(z) + b_h + \omega_{hh}^{w,k} && \text{if } k = \bar{k}. \end{aligned}$$

The value of unemployment for a high-skilled worker in his k:th period of unemployment is equal to $b_h + \omega_{hh}^{w,k}$, where

$$\omega_{hh}^{w,k} = \begin{cases} \beta \Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{hh}^{k}}^{\infty} \pi s_{hh}^{k}(y)\; dv_h(y)}_{\text{match + accept}} + \underbrace{\lambda^w(\theta)(b_h + (1-\gamma^e)\omega_{hh}^{w,k+1} + \gamma^e\omega_{lh}^{w})}_{\text{outside value with match}} \\ \qquad + \underbrace{(1-\lambda^w(\theta))(b_h + \omega_{hh}^{w,k+1})}_{\text{outside value without match}} \Big] & \text{if } k < \bar{k} \\ \\ \beta \Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{hh}^{k}}^{\infty} \pi s_{hh}^{k}(y)\; dv_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega_{hh}^{w,k}}_{\text{outside value}} \Big] & \text{if } k = \bar{k}. \end{cases}$$

High-skilled employed: match surplus, wage, and joint continuation value The match surplus for continuing employment of a high-skilled worker reflects the risk of layoffs and quits that can be affected by turbulence in the form of skill loss. A non-turbulent separation falls into the initial category of high-skilled unemployed, u_{hh}^{0}. We adjust match surpluses, wages, and joint continuation values of these workers to include the new outside value $\omega_{hh}^{w,0}$.

The match surplus of a continuing job with a high-skilled worker is

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^d)\omega_{hh}^{w,0} + \gamma^d \omega_{lh}^{w} + \omega^f]$$

and the wage equals

$$p_{hh}(z) + g_h^w(z) = \pi s_{hh}(z) + b_h + (1-\gamma^d)\omega_{hh}^{w,0} + \gamma^d \omega_{lh}^w.$$

The joint continuation value of a job with a high-skilled worker is

$$\begin{aligned} g_h(z) &= \beta\Big[\rho^x \left(b_h + (1-\gamma^{d,x})\omega_{hh}^{w,0} + \gamma^{d,x}\omega_{lh}^w + \omega^f\right) \\ &+ (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z)) \\ &+ (1-\rho^x)\gamma^s \left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega_{hh}^{w,0} + \gamma^d\omega_{lh}^w + \omega^f\right)\right)\Big]. \end{aligned}$$

Since a low-skilled worker faces the possibility of a skill upgrade, we also need to update the joint continue value of an employed low-skilled worker as follows:

$$\begin{aligned} g_l(z) &= \beta\Big[\rho^x(b_l + \omega_{ll}^w + \omega^f) \\ &+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z)) \\ &+ (1-\rho^x)(1-\gamma^u)\gamma^s \left(E_{ll} + \nu_{ll}(b_l + \omega_{ll}^w + \omega^f)\right) \\ &+ (1-\rho^x)\gamma^u \left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega_{hh}^{w,0} + \gamma^d\omega_{lh}^w + \omega^f\right)\right)\Big]. \end{aligned}$$

Vacancy creation Free entry of firms make a firm's value ω^f of entering the vacancy pool be zero. With more types of unemployed high-skilled workers, zero-profit condition (15) changes to become

$$\mu = \beta\frac{m(\theta)}{\theta}(1-\pi)\left[\frac{u_{ll}}{u}\int_{\underline{z}_{ll}^o}^{\infty} s_{ll}^o(y)\ dv_l(y) + \frac{u_{lh}}{u}\int_{\underline{z}_{lh}^o}^{\infty} s_{lh}^o(y)\ dv_l(y) + \sum_{k=0}^{\bar{k}}\frac{u_{hh}^k}{u}\int_{\underline{z}_{hh}^k}^{\infty} s_{hh}^k(y)\ dv_h(y)\right],$$

where $u = u_{ll} + u_{lh} + \sum_{k=0}^{\bar{k}} u_{hh}^k$.

High-skilled unemployment spells terminated within $\bar{k}$ periods In each period, a mass u_{hh}^0 of high-skilled workers flows into unemployment. Let $\phi^{\bar{k}}$ denote the fraction of these who will experience unemployment spells of no longer duration than $\bar{k}$ periods. To enable a recursive computation, define m_h^k as the mass of workers who remain high-skilled and unemployed after k periods, and let m_l^k be the accompanying mass that remain unemployed but who have experienced skill loss by that kth period of unemployment. Given initial conditions $m_h^0 = u_{hh}^0$ and $m_l^0 = 0$, we compute

$$\begin{aligned} m_h^k &= (1-\rho^r)\left[1-\lambda^w(\theta) + \lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)\right]m_h^{k-1} \\ m_l^k &= (1-\rho^r)\left[(1-\lambda^w(\theta) + \lambda^w(\theta)\nu_{lh})m_l^{k-1} + \lambda^w(\theta)\nu_{hh}^{k-1}\gamma^e m_h^{k-1}\right], \end{aligned}$$

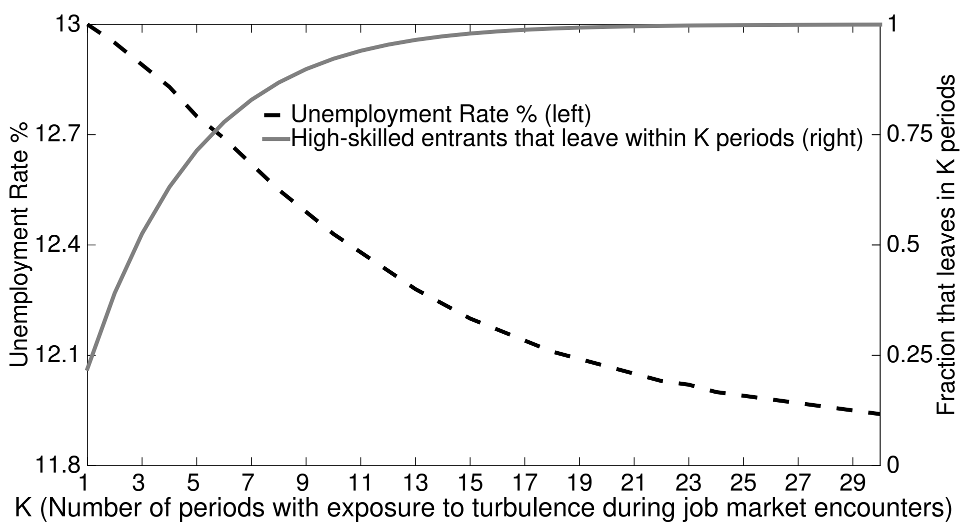
for $k = 1, \ldots, \bar{k}$;[20] and

$$\phi^{\bar{k}} = \frac{u_{hh}^0 - m_h^{\bar{k}} - m_l^{\bar{k}}}{u_{hh}^0}. \tag{C.8}$$

Numerical example To illustrate and decompose the forces at work, we set layoff turbulence equal to $\gamma^{d,x} = 0.2$ and quit turbulence to $\gamma^d = \epsilon\gamma^{d,x} = 0.1 \cdot \gamma^{d,x} = 0.02$. As discussed above, turbulence for unemployed workers in LS's semi-quarterly model is assumed to be half of quit turbulence, i.e., $\gamma^e = 0.5\gamma^d = 0.01$.

Figure C.2 depicts two unemployment outcomes in distinct economies that differ only with respect to the parameter value of $\bar{k}$, i.e., the length of time over which an unemployed worker is exposed to the risk of losing skills due to unsuccessful job market encounters. The two outcomes are the unemployment rate u and the fraction $\phi^{\bar{k}}$ of high-skilled entrants into unemployment who will see their unemployment spells terminated within $\bar{k}$ periods by either finding employment or retiring. For each economy indexed by $\bar{k}$, the value of u can be read off from the dashed line (in percent on the left scale), and $\phi^{\bar{k}}$ from the solid line (as a fraction on the right scale).

Figure C.2: Exposures of unemployed workers in LS to turbulence



As anticipated from our above discussion of the allocation channel, the unemployment rate in Figure C.2 is lower in economies with a higher $\bar{k}$ since longer exposure to risk of skill loss reduces the return to labor mobility. Hence, fewer high-skilled workers quit their jobs, and those who do quit will on average move back into employment more quickly. For example, when $\bar{k}$ increases from 1 to 9, the unemployment rate falls by half a percentage point. As noted earlier, the allocation channel would also be operating in the basic McCall search model, and the unemployment effects of further increases in $\bar{k}$ there should become muted when the

[20]Note that $m_h^k = u_{hh}^k$ for $k = 0, \ldots, \bar{k} - 1$, while $m_h^{\bar{k}}$ is merely a subset of $u_{hh}^{\bar{k}}$.

value of $\bar{k}$ is set so high that the vast majority of unemployment spells are shorter than $\bar{k}$ in durations. But, as can be seen in Figure C.2 at $\bar{k} = 9$, 90 percent of all unemployment spells by high-skilled entrants are terminated within $\bar{k}$ periods, yet the unemployment rate falls another half a percentage point after further increases in $\bar{k}$. According to our earlier discussion of the bargaining channel, there is a force in matching models that is not present in McCall models. This other force makes it possible for skill losses at unlikely long unemployment spells to have substantial effects on equilibrium outcomes through its impact on bargaining. The reason is that even though realizations of such long unemployment spells are rare, the extended risk of skill loss will weaken the bargaining position of a worker vis-à-vis a firm throughout an unemployment spell.[21]

Figure C.3 depicts additional statistics that summarize outcomes across alternative values of $\bar{k}$. The positive relationship between $\bar{k}$ and market tightness indicates how the bargaining channel tilts match surpluses to firms when the risk of skill loss after unsuccessful job market encounters weakens the bargaining position of workers. Recall that the equilibrium zero-profit condition for vacancy posting funnels expected present values of firms' match surpluses into vacancy creation. The resulting higher market tightness implies a higher probability that an unemployed worker encounters a vacancy. Evidently, a worker's higher match probability induces low-skilled unemployed workers (as well as employed ones), both those with low and those with high benefits, to choose higher reservation productivities. The net result is still a shorter average duration of unemployment spells. And with not much change in a mildly U-shaped relationship for the job separation rate, we arrive at an unemployment rate that continues to fall over most of the range in Figure C.2. From these intricacies, we conclude that the bargaining channel already operates in tandem with the allocation channel over the first range of $\bar{k}$ in that figure, but that it operates mostly on its own over the second range where most entrants of high-skilled workers into unemployment expect to terminate their unemployment spells well before $\bar{k}$ periods.

[21] For another stark example of unlikely events having large effects on equilibrium outcomes through the bargaining channel, see Ljungqvist and Sargent's (2017) analysis of alternating-offer wage bargaining as one way to make unemployment respond sensitively to movements in productivity in matching models. A general result is that the elasticity of market tightness with respect to productivity is inversely related to a model-specific "fundamental surplus," Under alternating-offer bargaining the fundamental surplus is approximately equal to the difference between productivity and the sum of the value of leisure and a firm's cost of delay in bargaining. Thus, the magnitude of the latter cost is a critical determinant of the volatility of unemployment in response to productivity shocks, even though no such cost will ever be incurred because in equilibrium there will be no delay in bargaining.

Figure C.3: More statistics pointing to the "bargaining channel"