Quit Turbulence and Unemployment

From the perspective of a particle physicist*

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Abstract

Steven Weinberg (2018) says: (1) new theories that target new observations should be constrained to agree with observations successfully represented by existing theories; and (2) preserving successes of earlier theories helps to discover unanticipated understandings of yet other phenomena. Weinberg's advice helps us to answer the question: how do higher risks of skill losses coinciding both with involuntary layoffs ("layoff turbulence") and with voluntary quits ("quit turbulence") affect equilibrium unemployment rates? An earlier analysis that had included only layoff turbulence had established a positive relationship between turbulence and the unemployment rate within generous welfare states, but the absence of that relationship in countries with stingier welfare states. A subsequent influential analysis found that even very *small* amounts of quit turbulence would lead to a *negative* relationship between turbulence and unemployment rates. But that finding was based on a peculiar calibration of a productivity distribution that generates returns to labor mobility that make the model miss the positive turbulence-unemployment rate relationship that has been a theoretical basis for explaining the the persistent trans-Atlantic unemployment divide that emerged in post 1970s data and also miss observations about labor market churning. Repairing the faulty calibration of that productivity distribution not only brings models with quit turbulence into line with those observations but also puts the spotlight on macro-labor calibration strategies and implied returns to labor mobility.

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"... often the most important constraint on a new theory is ... that it should agree with the whole body of past observations, as crystallized in former theories. ... New theories of course do not agree entirely with any previous theory – otherwise they would not be new – but they must not throw out all the success of former theories. This sort of thing makes the work of the theorist far more conservative than is often thought.

The wonderful thing is that the need to preserve the successes of the past is not only a constraint, but also a guide." Steven Weinberg (2018, p. 197)

1 Introduction and summary

This paper is a macro-labor economics variation on the theme of particle physicist Steven Weinberg (2018, ch. 24). Besides analyzing a research question of practical interest, we illustrate how (1) to introduce new forces to explain previously neglected phenomena, while (2) preserving earlier theories' successes, and thereby (3) gathering unanticipated understandings of other interactions.

Our setting is alternative quantitative macroeconomic models of labor market frictions. Unlike particle physics and cosmology, we have no widely accepted "standard model" of forces that shape an equilibrium unemployment rate. Instead, there are (at least) three popular frameworks of frictional unemployment, each having persuasive advocates and skillful users: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the tradition of framework of Lucas and Prescott (1974). Calibrated models of all three frameworks have succeeded in fitting data on labor market flows and generating plausible responses of unemployment rates to government policies like unemployment insurance and layoff taxes. We have cause to revisit some of these successes in this paper.

Our case study begins with a study that added forces and phenomena that Ljungqvist and Sargent (1998) had excluded from a generalized McCall search model of adverse macroeconomic consequences of interactions between microeconomic turbulence and European more generous welfare states. Ljungqvist and Sargent had modeled turbulence in terms of the rate of human capital loss coincident with an *involuntary* job loss ("layoff turbulence"). The model explained systematically higher and persistent unemployment rates in Europe than in the US since the late 1970s. The model included no loss of human capital coincident with a *voluntary* separation from a job. The omission of "quit turbulence" from earlier quantitative explanations of equilibrium unemployment serves as the starting point of our story because in 1998 an astute observer, Alan Greenspan (1998, p. 743), suggested that a more hazardous job market had suppressed mobility among employed workers and had led to less upward pressure on wages:

> "... the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest."

den Haan, Haefke and Ramey (2005, henceforth DHHR) cite Greenspan's words at the beginning of a paper that calibrated a matching model that captures Greenspan's idea by including quit turbulence in the form of an immediate depreciation of a worker's human capital that in turbulent times could be triggered by a worker's decision to quit a job. DHHR reported a calibration that affirmed the quantitative importance of what they interpret as the force Greenspan's had in mind: even a small amount of quit turbulence gave workers strong enough reluctance to quit to reduce both quits and job reallocation substantially. DHHR's success in representing and quantifying Greenspan's intuition had other important ramifications. DHHR go on to stress their finding that adding even a small amount of quit turbulence to their matching model reverses the unemployment-increasing interactions between layoff turbulence and welfare state generosity that Ljungqvist and Sargent (1998) had used to explain trans-Atlantic differences in unemployment rates. DHHR's analysis of quit turbulence seems to violates Weinberg's constraint of preserving successes of earlier models even while rationalizing Greenspan's remarks: thus, DHHR acknowledge that after they formalize and quantify Greenspan's idea, a rise in turbulence can no longer explain the trans-Atlantic unemployment rate differences that Ljungqvist and Sargent's mechanism had captured.

Nevertheless, to play devil's advocate for DHHR against Steven Weinberg, could DHHR's analysis be a convincing demonstration of the lack of robustness of the earlier explanation of those differences to (1) adding quit turbulence and (2) using a matching model rather than a generalized McCall equilibrium search model? To study these questions, we must drill down to discover what accounts for the different outcomes. Was it adding quit turbulence? Or was it abandoning the extended McCall framework in favor of the matching framework? Or might the differences be traced to peculiar features of DHHR's calibration of important exogenous inputs? Answering these questions will illuminate a flaw of DHHR's analysis that prevented them from preserving salient quantitative successes widely shared by earlier macro-labor models and that caused them to miss other important labor market quantities.

An authoritative answer to these questions was offered by Hornstein, Krusell and Violante (2005, section 8.3). They interpreted the DHHR finding as indicating lack of robustness of the earlier explanation of those trans-Atlantic unemployment rate differences, leading them to doubt its validity:

> "... once the Ljungqvist and Sargent mechanism is embedded into a model with endogenous job destruction, the comparative statics for increased turbulence are re-

> versed, i.e., unemployment falls. The reason is that as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls."

However, Hornstein *et al.* (2005) did not claim to have isolated the origin of DHHR's finding in terms of specifications of preferences, technologies, and timing protocols that give rise to it. This paper finds the source and in the process of doing so illustrates the power of Weinberg's insight "that the need to preserve [earlier successes of macro-labor models] is not only a constraint, but also a guide".

We strengthen our findings by interrogating some classic studies – the matching model of Mortensen and Pissarides and the search island model of Alvarez and Veracierto – about what they have to say about the productivity distributions and the returns to labor mobility that are the fulcrum of DDHR's claimed reversal of the LS relationship between turbulence and unemployment. In sections 6 and 7, we shall describe how these studies bring to bear evidence from unemployment incidence and spell durations as well as establishment data on firm and worker turnover that provide overwhelming support for our conclusion that plausibly calibrated quit turbulence is of second-order importance and cannot reverse a positive turbulence-unemployment rate relationship.

1.1 Tactics and findings

First, we incorporate quit turbulence into yet another matching model to see if the findings are similar to those of DHHR. A most suitable candidate for that purpose is the matching model of Ljungqvist and Sargent (2007, henceforth LS)[1] that demonstrated that the positive turbulence-unemployment relationship in the extended McCall search framework of Ljungqvist and Sargent (1998, 2008) also prevails in a matching framework. While that original turbulence theory had assumed skill loss only at times of involuntary separations, we now add risk of skill loss at times of voluntary separations. We discover that the matching model analyses of DHHR and our LS model augmented to incorporate quit turbulence disagree sharply. In place of DHHR's result that a small amount of quit turbulence can overturn the positive turbulence-unemployment relationship in the DHHR matching model, we find that it takes large amounts of quit turbulence to have any such dramatic impact on what is evidently a robust relationship in the LS matching model.

Having established that DHHR's remarkable findings are not endemic in matching models as a class, our second stage is to uncover source of these puzzling differences in outcomes. A taxonomy of differences organizes our investigation. The LS and DHHR frameworks differ along

[1] LS acknowledged Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code, which LS then augmented in various ways, adding features to be examined here. Also, as indicated in footnotes 9 and 16 below, the current paper was preceded by a related, but distinctly different, exchange of views between den Haan, Haefke and Ramey (2001) and Ljungqvist and Sargent (2004).

three dimensions, two in the models' structures, and one in parameterizations of productivity distributions. One structural difference is that LS adopt the standard assumptions of free entry of firms and a zero-profit equilibrium condition for posting vacancies, while DHHR assume fixed measures of both firms and workers. The other structural difference is that LS assume that skill upgrades are immediately realized, while DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level. Even though these two structural differences have consequences for vacancy and wage bargaining outcomes, they do not change the qualitative pattern of unemployment dynamics. It turns out that the explanation of our puzzle lies in the third difference, namely, parameterizations of productivity distributions. The source of DHHR's exceptional findings about quit turbulence is how they calibrate productivity distributions that make returns to labor mobility so small that even a small mobility cost shuts down voluntary separations.

Having found the source of the puzzle, in order to analyze forces at play further, the third stage of our inquiry conducts an analysis of layoff costs. Introducing small layoff costs in tranquil times closes down voluntary separations in the DHHR framework, which illustrates the sensitivity of outcomes to the DHHR parameterization of productivity distributions. Therefore, a small government mandated layoff cost (or any small mobility cost, such as a tiny risk of skill loss when quitting) has implausibly large effects of suppressing unemployment by shutting down voluntary separations.

Our fourth stage maps productivity processes from classic macro-labor studies into our matching model to study their implications for layoff and quit turbulence. The matching model of Mortensen and Pissarides (1999) has a structure that facilitates mapping its productivity process into our model. The search-island model of Alvarez and Veracierto (2001) and the way it enlists establishment data on firm and worker turnover (Davis and Haltiwanger, 1990) offer us different perspectives. In particular, their growth model that intermediates productivity shocks through a neo-classical production function that give rise to large returns to labor mobility that are robust to details of their calibration. In contrast, we have discovered that the Mortensen-Pissarides calibration strategy has a previously undetected fragility that emerges from a ridge of two key parameters that can generate the same targeted unemployment statistic but nevertheless have very different implications for returns to labor mobility. Because Mortensen and Pissarides seemed not to notice that, their calibration teeters on a parameter region of fragility with respect to labor mobility. But actually, since their study focused on employment effects of layoff taxes, outcomes would have made them acutely aware of this issue if their calibration had wandered into the region that implies the extremely low returns to labor mobility associated with DHHR's calibration. That finding probably would have prompted them to explore their parameter space further.

1.2 General inference

Our findings extend beyond details of DHHR's model. On the one hand, DHHR's hypothesis that small risks of human capital loss at voluntary separations would cause workers to choose not to separate relies on returns to labor reallocation being small. On the other hand, returns to labor reallocation have to be large in any model that has a chance of being consistent with evidence that relatively high layoff costs in European welfare states have had rather small impacts on unemployment outcomes.[2]

These irreconcilable requirements about rates of return to labor reallocation imply a strong quantitative presumption against DHHR's claim that the turbulence theory is not robust to the addition of quit turbulence: when returns to labor mobility are calibrated to conform with the historical evidence on layoff costs, a strong quantitative presumption emerges that the addition of quit turbulence cannot overturn a positive turbulence-unemployment relationship.

Our macro-labor detective project illustrates Steven Weinberg's observation that preserving successful implications of earlier theories serves as good guide. Thus, we have managed to incorporate the quit turbulence that was evidently on Alan Greenspan's mind in 1998 while preserving earlier findings of adverse interactions between welfare-state generosity and average unemployment levels. With that extension in hand, we have gone on to show how the same model reconciles the presence of high layoff costs with observed levels of unemployment. Evidently, earlier models exhibit returns to labor reallocation that are large enough to generate observed rates of churning of labor despite some heavy-handed government interventions that raise layoff costs. Thus, in the macro-labor economics project reported here, respecting quantitative successes of earlier theories has guided us well. Our findings imply that Hornstein *et al.* should not have accepted DHHR's conclusions about the effects of quit turbulence on unemployment at face value because, contrary to DHHR, adding a plausibly calibrated amount of quit turbulence does not reverse the positive turbulence-unemployment relationship in a welfare state. As a further illustration of dividends that accrue from using our model as a guide, in the spirit of Weinberg, in our concluding section we have another conversation with Alan Greenspan in light of what our extended model with quit turbulence has taught us.

2 Detective work

How should a model represent the uncontroversial observation that job separators find themselves in different situations? For example, workers with valuable skills who separate in order to find better-paying jobs differ from laid-off workers whose skills are no longer in demand due

[2] See e.g. Nickell (1997, p. 66) for an account of substantial measures of employment protection in European welfare states, and the common conclusion of "rather small" impact on unemployment outcomes.

to, e.g. changing technologies or their types of work 'moving abroad' to low-wage countries.

To capture such differences, DHHR treat involuntary separations as did earlier theories by assuming that they lead to the most unfavorable circumstances for job separators in the sense that they present the highest risks of skill losses. In addition to such layoff turbulence, DHHR introduce quit turbulence for workers who voluntarily separate from jobs after draws of poor job-specific productivities at their current employment. Workers accepting voluntary separations constitute those with more favorable situations both in terms of having an opportunity to continue working after shocks to productivity at their current employment, as well as, conditional on separating, facing a lower risk of skill loss than do workers who suffer involuntary separations.

Within this setup, DHHR investigate whether or not the turbulence theory is robust to introducing quit turbulence by measuring how large the risk of skill loss at times of voluntary separations relative to the risk at times involuntary separations must be in order to generate a negative rather than positive turbulence-unemployment relationship. DHHR assert a lack of robustness because they find that the turbulence-unemployment already becomes negative at very low skill loss probabilities for voluntary separators relative to those for involuntary separators:

> "... allowing for a skill loss probability following [voluntary] separation that is only 3% of the probability following [involuntary] separation eliminates the positive turbulence-unemployment relationship. Increasing this proportion to 5% gives rise to a strong *negative* relationship between turbulence and unemployment." (DHHR, p. 1362)

How can it be that nudging the probability that represents quit turbulence from zero to a tiny positive number can have such large effects on equilibrium outcomes? Isn't it natural to expect to such a small probability wouldn't do much? Indeed, when we introduce skill losses at times of voluntary separations into the LS model, we find only small effects on outcomes: quit turbulence has to be about 50% of layoff turbulence and both kinds of turbulence must be high before quit turbulence can leads to lower unemployment in turbulent times than in tranquil zero-turbulence times. Why do we find such different outcomes than do DDHR? The answer has to be that we have altered something essential either about DDHR's structure or about DDHR's calibrated parameterization. As already mentioned, the LS and DHHR frameworks differ along the three dimensions.[3] To isolate the culprit responsible for the sharply different outcomes between the LS and DDHR structure, our method is to start with the LS model and

[3]A fourth difference is DHHR's "simplifying assumption" that quit turbulence also applies to job seekers, i.e., after an encounter between an unemployed worker and a vacancy that does not lead to an employment relationship, the worker is exposed to the same risk of skill loss as if she had quit from a job. We omit this assumption since DHHR make it for computational tractability, and it is not part of DHHR's argument. While

successively make perturbations one by one, with each perturbation designed to isolate the role of one suspect.[4] Before describing details, here is a summary of the lineup and the outcome of our investigation.

1. **Suspect 1: Labor market tightness.** LS adopt the standard assumptions that free entry of firms and a zero-profit equilibrium condition for posting vacancies determine labor market tightness; while DHHR assume that measures of both firms and workers are fixed and equal, which in turn delivers an exogenous market tightness equal to one.

   Verdict: Not guilty. With endogenous market tightness, higher turbulence decreases market tightness as the "invisible hand" makes adjustments to restore the profitability of firms; lower tightness means lower job finding rates and unemployment increases. This force is not present in the DHHR structure with its exogenous market tightness and thus, no increase in unemployment comes from this channel. Nevertheless, DHHR's omission of this force does not change the qualitative pattern of unemployment dynamics, so changing from endogenous to exogenous market tightness is not the culprit.

2. **Suspect 2: Timing of completion of skill upgrade.** LS assume that skill upgrades are immediately realized and that after skill upgrades workers draw new productivities. In contrast, DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to attain the higher skill level[5] and that a new productivity is drawn from a distribution whose lower support is equal to the endogenous reservation productivity of a worker at that higher skill level (and therefore, a worker who has just received a skill upgrade will remain employed for at least one more period).

   Verdict: Not guilty. The alternative assumptions affect a worker's bargaining position vis-à-vis a firm. Delayed completion effectively erodes the bargaining power of a worker who experiences a skill upgrade. As a result, under delayed completion, wages become negative in periods of skill upgrades (firms extract rents from workers). Nevertheless, these very different outcomes do not change the qualitative pattern of unemployment dynamics and hence, do not resolve the puzzle.

3. **Suspect 3: Different productivity distributions.** LS postulate truncated normal distributions with a wide support, whereas DHHR assume uniform distributions with a

omitting this assumption does not affect the qualitative pattern of unemployment dynamics in the DHHR model as shown in Appendix C. Appendix D explains how it would significantly suppress unemployment in the LS model.

[4]In Appendix C, we start from the DHHR model and work through the perturbations in reverse. Both procedures detect the same culprit.

[5]Contingent on remaining with his present employer, a worker experiencing a skill upgrade in the DHHR model will also realize the higher skill level immediately. To capture this notion of contingency, we use the term 'completion' of a skill upgrade.

narrow support.

Verdict: Guilty! Even in tranquil times, the DHHR parameterization of the productivity distribution delivers much weaker incentives for workers to change jobs. Even a small cost to mobility causes voluntary quits to shut down. After we adjust DHHR's parameterization of the productivity distribution to account for observed unemployment dynamics, a positive relationship between turbulence and unemployment reemerges.

Appendix B describes in detail the computations that lead us to acquit the first two suspects as the culprits that DDHR use to reverse the equilibrium turbulence-unemployment relationship relative to LS.

The paper proceeds as follows. Section 3 develops a matching framework with turbulence that builds on the models of LS and DHHR. Section 4 documents the puzzle, dissects the forces at work, and detects the culprit. Section 5 conducts the layoff tax analysis. Productivity processes calibrated by Mortensen and Pissarides (1999) and Alvarez and Veracierto (2001) are mapped into our matching framework in Sections 6 and 7, respectively. Section 8 offers concluding remarks. Auxiliary material and explorations are relegated to Appendices A–D.

3 A matching framework with turbulence

The LS matching model has 'layoff turbulence' in the form of worse skill transition probabilities for workers who suffer involuntary layoffs. We augment the model to include 'quit turbulence' – worse skill transition probabilities for workers who experience voluntary quits – as in the DHHR model. The following description of our augmented LS model flags where it differs from the DHHR model.

3.1 Environment

Workers There is a unit mass of workers who are either employed or unemployed. Workers are risk neutral, value consumption, and have preferences ordered according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t. \tag{1}$$

They discount future utilities at a rate $\beta \equiv \hat{\beta}(1-\rho^r)$, where $\hat{\beta} \in (0,1)$ is a subjective time discount factor and $\rho^r \in (0,1)$ is a constant probability of retirement. A retired worker exits the economy and is replaced by a newborn worker.

Worker heterogeneity Besides employment status, workers differ along two dimensions: a current skill level i that can be either low (l) or high (h) and a skill level j that determines a worker's entitlement to unemployment benefits. An employed worker has $j = i$, but for an unemployed worker, j is the skill level during his last employment spell. Workers gain or lose skills depending on their employment status and instances of layoffs and quits. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement. In this way, each worker bears two indices (i, j), the first denoting current skill and the second denoting benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, also called market tightness. The probability $\lambda^w(\theta) = M(v, u)/u = M(\theta, 1) \equiv m(\theta)$ that an unemployed worker encounters a vacancy is increasing in market tightness. The probability $M(v, u)/v = m(\theta)/\theta$ that a vacancy encounters an unemployed worker is is decreasing in market tightness.

Difference from DHHR
There is a fixed unit mass of firms. Since there are no costs for posting vacancies, a firm without a worker always chooses to post a vacancy, so $\theta = 1$ always.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a distribution $v_i(z)$ that is indexed by a worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Wages are determined through Nash bargaining, with π and $1 - \pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition probability matrix Q_i, also indexed by the worker's skill level i, where $Q_i(z, z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains his last period productivity with probability $1 - \gamma^s$, but with probability γ^s draws a new productivity from the distribution $v_i(z)$, i.e., the same distribution that a worker of that skill level would face in a new match. Furthermore, a worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with his present employer or quits.

Difference from DHHR
A worker experiencing a skill upgrade draws a new productivity from a transformed version of $v_h(z)$: the distribution is truncated from below at the reservation productivity of a high-skilled worker and rescaled to integrate to one. A skill upgrade is completed only after a worker has remained one period with his present employer.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary in the sense of offering no choice.

(ii) **Quits:** As a consequence of a new productivity draw on a job, a relationship can continue or be endogenously terminated. We label separation after such an event a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define turbulence as the risk of losing skills after a job separation. High-skilled workers might become low-skilled workers. Two types of turbulence shocks depend on the reason for a job separation, namely, a layoff or a quit. Upon a layoff, a high-skilled worker experiences a skill loss with probability $\gamma^{d,x}$. We label this risk *layoff turbulence*. Upon a quit, a high-skilled worker faces the probability γ^d of a skill loss. We label this risk *quit turbulence*.

Turbulence shocks are timed as follows. At the beginning of a period, exogenous job terminations occur and displaced workers face layoff turbulence. Continuing employed workers can experience new productivity draws on the job and skill upgrades; if workers quit, they are subject to quit turbulence. All separated workers join other unemployed workers in the matching function where they might or might not encounter vacancies next period.

Government policy The government runs a balanced budget. Its revenues come from a flat-rate tax τ on production. Its sole expenditures are benefits paid to the unemployed. An unemployed worker who was low (high) skilled in his last employment receives a benefit b_l (b_h).[6] As mentioned above, newborn workers are entitled to b_l. Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i.

In section 5, the government gets an additional source of revenues by levying a layoff tax Ω on every job termination except for retirements. If the revenues from layoff taxation exceed the expenditures on unemployment benefits, the government sets $\tau = 0$ and returns any surplus as lump-sum transfers to workers.

[6] For simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

3.2 Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j that has drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by the after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period, net of paying the vacancy cost μ. For notational simplicity, we define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.[7]

The match surplus for a new job $s^o_{lj}(z)$ or a continuing job $s_{lj}(z)$ with a low-skilled worker with benefit entitlement j is given by

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we must distinguish between new and continuing jobs. The match surplus when forming a new job with an unemployed high-skilled worker, s^o_{hh}, involves outside values without any risk of skill loss if the match does not result in employment:

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. Since both parties want a positive match surplus, it is mutually agreed whether to start (continue) a job. For a new (continuing) match, the reservation productivity $\underline{z}^o_{ij}$ ($\underline{z}_{ij}$) is the lowest productivity that makes a match profitable and satisfies

$$s^o_{ij}(\underline{z}^o_{ij}) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; 0\Big). \tag{5}$$

Given the reservation productivity $\underline{z}^o_{ij}$ ($\underline{z}_{ij}$), let ν^o_{ij} (ν_{ij}) denote the rejection probability, which is given by the probability mass assigned to all draws from productivity distribution $v_i(y)$ that

[7] Our mathematical formulation and notation follow DHHR closely.

fall below the threshold:

$$\nu_{ij}^{o} \;=\; \int_{-\infty}^{\underline{z}_{ij}^{o}} dv_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

To simplify formulas below, we define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\, dv_i(y). \tag{7}$$

3.3 Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now characterize value functions for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by future layoff turbulence if the worker is laid off or by future quit turbulence if a productivity switch is rejected:

$$\begin{aligned}
&\text{Exogenous separation:} && g_h(z) \;=\; \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^{d,x})\omega_{hh} + \gamma^{d,x}\omega_{lh}}_{\text{layoff turbulence}})\\
&\text{Productivity switch:} && \quad +\; (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}}))\\
&\text{No changes:} && \quad +\; (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big].
\end{aligned} \tag{8}$$

Low-skilled worker The joint continuation value of a match of a firm with a low-skilled worker takes into account the following contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, and a skill upgrade. In the LS model skill upgrades are immediately realized and upon skill upgrades workers immediately become entitled to high unemployment benefits, even if the worker quits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution v_h. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z is

$$\begin{aligned}
&\text{Exogenous separation:} && g_l(z) \;=\; \beta\Big[\rho^x(b_l + \omega_{ll})\\
&\text{Immediate skill upgrade:} && \quad +\; (1-\rho^x)\gamma^u(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^d)\omega_{hh} + \gamma^d\omega_{lh}}_{\text{quit turbulence}}))\\
&\text{Productivity switch:} && \quad +\; (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll} + \nu_{ll}(b_l + \omega_{ll}))\\
&\text{No changes:} && \quad +\; (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big].
\end{aligned} \tag{9}$$

Difference from DHHR

A worker with a skill upgrade must remain with the present employer for one period in order to complete the higher skill level. The new productivity at a skill upgrade is drawn from a distribution $v_u(z) = v_h(z)/(1-v_h(\underline{z}_{hh}))$ having a lower support $\underline{z}_{hh}$, which guarantees a continuation of the employment relationship. Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z in the DHHR model becomes:

$$\begin{array}{llcl} \text{Exogenous separation:} & g_l(z) & = & \beta\Big[\rho^x(b_l+\omega_{ll}) \\ \text{Delayed skill upgrade:} & & + & (1-\rho^x)\gamma^u \int_{\underline{z}_{hh}}^{\infty} [(1-\tau)y + g_h(y)]\ dv_u(y) \\ \text{Productivity switch:} & & + & (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\ \text{No changes:} & & + & (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z))\Big]. \end{array} \quad (10)$$

3.4 Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω_{ij}^w. Recall that the probability that an unemployed worker becomes matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j + \omega_{lj}^w$, where

$$\omega_{lj}^w = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{lj}}^{\infty} \pi s_{lj}^o(y)\ dv_l(y)}_{\text{match + accept}} + \underbrace{b_j + \omega_{lj}^w}_{\text{outside value}}\Big] \qquad j = l, h. \quad (11)$$

A high-skilled unemployed worker with benefit entitlement h, obtains $b_h + \omega_{hh}^w$, where

$$\omega_{hh}^w = \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}_{hh}^o}^{\infty} \pi s_{hh}^o(y)\ dv_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega_{hh}^w}_{\text{outside value}}\Big]. \quad (12)$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda_{ij}^f(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i, j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f = -\mu + \beta\Big[\underbrace{\sum_{(i,j)} \lambda_{ij}^f(\theta)\int_{\underline{z}_{ij}^o}^{\infty} (1-\pi)s_{ij}^o(y)\ dv_i(y)}_{\text{match + accept}} + \underbrace{\omega^f}_{\text{outside value}}\Big]. \quad (13)$$

3.5 Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u = \sum_{i,j} u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy is function only of market tightness θ; the probability $\lambda^f_{ij}(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the worker composition in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be deduced from equation (13) with $w^f = 0$. We summarize these labor market outcomes as follows:

$$\begin{aligned}
\omega^f &= 0 && (14)\\
\mu &= \beta(1-\pi)\sum_{(i,j)} \lambda^f_{ij}(\theta) \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\, dv_i(y) && (15)\\
\lambda^w(\theta) &= m(\theta) && (16)\\
\lambda^f_{ij}(\theta) &= \frac{m(\theta)}{\theta}\frac{u_{ij}}{u} && (17)
\end{aligned}$$

Difference from DHHR

There is a fixed unit mass of firms. Because there are no vacancy costs, a firm without a worker always posts a vacancy. Thus, market tightness equals one, and a firm's value w^f of posting a vacancy can be deduced from equation (13) with $\mu = 0$.

$$\begin{aligned}
\omega^f &= \frac{\beta}{1-\beta}(1-\pi)\sum_{(i,j)} \lambda^f_{ij} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\, dv_i(y) && (18)\\
\theta &= 1 && (19)\\
\lambda^w &= m(1) && (20)\\
\lambda^f_{ij} &= m(1)\frac{u_{ij}}{u} && (21)
\end{aligned}$$

3.6 Wages

Wages are determined through Nash bargaining. Here, we report the equations for the LS model with immediate realization of skill upgrades. We refer readers to den Haan *et al.* (2005, section 2.5) for the DHHR equations under their assumption that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Wage determination Given a productivity draw z in a new match with a positive match surplus, the wage $p^o_{lj}(z)$ of a low-skilled worker with benefit entitlement $j = l, h$ and the wage $p^o_{hh}(z)$ of a high-skilled worker, respectively, solve the following maximization problems:

$$\max_{p^o_{lj}(z)} \quad \left[(1-\tau)z - p^o_{lj}(z) + g^f_l(z) - \omega^f\right]^{1-\pi} \left[p^o_{lj}(z) + g^w_l(z) - b_j - \omega^w_{lj}\right]^{\pi} \tag{22}$$
$$\max_{p^o_{hh}(z)} \quad \left[(1-\tau)z - p^o_{hh}(z) + g^f_h(z) - \omega^f\right]^{1-\pi} \left[p^o_{hh}(z) + g^w_h(z) - b_h - \omega^w_{hh}\right]^{\pi},$$

where $g^w_i(z)$ and $g^f_i(z)$ are future values obtained by the worker and the firm, respectively, from continuing the employment relationship;[8] and ω^f and $b_j + \omega^w_{ij}$ are outside values defined in (11), (12), and (13). The solution to the wage determination problems sets the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$\begin{aligned} p^o_{lj}(z) + g^w_l(z) &= \pi s^o_{lj}(z) + b_j + \omega^w_{lj} \qquad\qquad j = l, h \\ p^o_{hh}(z) + g^w_h(z) &= \pi s^o_{hh}(z) + b_h + \omega^w_{hh}, \end{aligned} \tag{23}$$

where the worker continuation values are

$$\begin{aligned} g^w_l(z) &= \beta(1-\rho^x)\pi\left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z) + \gamma^s\int_{\underline{z}_{ll}}^{\infty} s_{ll}(y)\ dv_l(y)\right] + \gamma^u\int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\ dv_h(y)\right\} \\ &+ \beta(\rho^x + (1-\rho^x)(1-\gamma^u))\left(b_l + \omega^w_{ll}\right) + \beta(1-\rho^x)\gamma^u\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right) \qquad (24) \\ g^w_h(z) &= \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z) + \gamma^s\int_{\underline{z}_{hh}}^{\infty} s_{hh}(y)\ dv_h(y)\right] \\ &+ \beta\rho^x\left(b_h + (1-\gamma^{d,x})\omega^w_{hh} + \gamma^{d,x}\omega^w_{lh}\right) + \beta(1-\rho^x)\left(b_h + (1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}\right). \end{aligned}$$

For ongoing employment relationships, the wages $p_{ll}(z), p_{hh}(z)$ satisfy counterparts of the above equations that use the corresponding match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$\begin{aligned} p_{ll}(z) + g^w_l(z) &= \pi s_{ll}(z) + b_l + \omega^w_{ll} \\ p_{hh}(z) + g^w_h(z) &= \pi s_{hh}(z) + b_h + \underbrace{(1-\gamma^d)\omega^w_{hh} + \gamma^d\omega^w_{lh}}_{\text{quit turbulence}}, \end{aligned} \tag{25}$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

[8]The *joint* continuation values defined in (8) and (9) equal sums of the individual continuation values: $g_i(z) = g^w_i(z) + g^f_i(z),\ i = l, h$.

3.7 Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits amounts to

$$b_l u_{ll} + b_h(u_{lh} + u_{hh}) = \phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})). \tag{26}$$

Income taxes Output is taxed at a constant rate τ. Let $\bar{z}_i$ be the average productivity of employed workers with skill level i. Hence, total tax revenue equals $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the total expenditures described in (26):

$$\phi(\bar{p}_l u_{ll} + \bar{p}_h(u_{lh} + u_{hh})) = \tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}). \tag{27}$$

For computations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$, see Appendix A.2.

3.8 Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlement levels. Here we focus on a group of workers at the center of our analysis: low-skilled unemployed with high benefits. (Appendix A.1 describes flows for other groups of workers.) As in the case of the above wage equations, we report worker flows under the LS assumption of immediate realization of skill upgrades, while referring to den Haan *et al.* (2005, appendix A) for the alternative DHHR assumption.

Inflows to the low-skilled unemployed with high benefits u_{lh} occur in the following situations. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability $\gamma^{d,x}$, they become part of the low-skilled unemployed with high benefit entitlement. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^d of becoming part of the low-skilled unemployed with high benefit entitlement. Outflows from unemployment occur upon successful matching function encounters and retirements. Thus, the net change of low-skilled unemployed with high benefits (equalling zero in a steady state) becomes:

$$\Delta u_{lh} = (1-\rho^r)\Bigg\{ \underbrace{\rho^x \gamma^{d,x} e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^d \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} \Bigg\} - \rho^r u_{lh}. \quad (28)$$

Terms numbered 1 and 3 in expression (28) isolate the forces behind the positive turbulence-unemployment relationship in a welfare state in the LS model. Although more layoff turbulence in term 1 – a higher probability $\gamma^{d,x}$ of losing skills after layoffs – has a small effect on equilibrium unemployment in a laissez-faire economy, it gives rise to a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate on a worker's earnings in her last employment. After a layoff with skill loss, those benefits are high relative to a worker's earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1-\nu^o_{lh})$ in term 3 is low; because of the relatively high outside value of a low-skilled unemployed with high benefits, fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ that a worker encounters a vacancy, which further suppresses successful matches and contributes to the positive turbulence-unemployment relationship.

DHHR reverse this finding by adding the term numbered 2 in expression (28): they assert that if higher turbulence is associated with voluntary quits that are also subject to risks of skill loss, there will be a lower incidence of voluntary quits in turbulent times because the risk of skill loss makes high-skilled workers reluctant to quit. This makes the rejection rate ν_{hh} in term 2 become low in turbulent times. That lower rejection rate causes lower inflows into low-skilled unemployed with high benefits u_{lh} as well as into high-skilled unemployed with high benefits u_{hh}. DHHR argue that even at levels of quit turbulence that are very low relative to layoff turbulence, this force reverses the Ljungqvist and Sargent positive turbulence-unemployment relationship.

3.9 Steady state equilibrium

A steady state equilibrium consists of measures of unemployed u_{ij} and employed e_{ij}; a labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$;

unemployment benefits b_i and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.

b) Free entry of firms implies zero-profit condition (15) in vacancy creation that pins down market tightness in LS. (An exogenous unit mass of firms implies market tightness equal to one in DHHR.)

c) Nash bargaining outcomes (23) and (25) set wages.

d) The tax rate balances the government's budget (27).

e) Net worker flows, such as expression (28), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall\, i, j.$

3.10 Parameterization

Except for the introduction of quit turbulence, we adopt the parameterization of LS. Parameters are divided into two groups, as reported in Table 1 and 2, respectively, to distinguish those that are similar in the DHHR parameterization from those that differ. The Table 1 parameters pin down preferences, sources of risk, and labor market institutions.[9] The Table 2 parameters pin down matching process and productivity distributions that differ markedly between LS and DHHR. Following LS, we assume a semi-quarterly model period.[10]

Preference parameters Given a semi-quarterly model period, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Stochastic processes for productivity Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss.

[9] The similar parameterizations originate from an earlier exchange mentioned in footnote 1. Thus, Ljungqvist and Sargent (2004) advocated modifying the parameterization of den Haan *et al.* (2001) based on calibration targets in the search framework of Ljungqvist and Sargent (1998, 2008), except for one notable *ad hoc* assumption to be discussed in footnote 16. (DHHR adopt Ljungqvist and Sargent's (2004) modification of den Haan et al.'s (2001) parameterization and proceed to introduce quit turbulence.)

[10] DHHR parameterized at a quarterly frequency. The transition probabilities for skill dynamics are the same as those in LS, when taking the different frequencies into account. The only departures from the parameters in Table 1 are the subjective discount factor and the retirement probability, which DHHR set at 0.995 and 0.005, respectively, at a quarterly frequency, or 0.9975 and 0.0025 when converted to a semi-quarterly frequency; these numbers yield an adjusted discount factor of 0.995 at a semi-quarterly frequency. We conducted a sensitivity analysis with respect to the different discount rates and found that adopting the DHHR discount rate in the LS model, while it changes the quantitative findings, does not reverse the sign of the turbulence-unemployment relationship.

Table 1: LS parameterization similar to DHHR

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
$\gamma^{d,x}$	layoff turbulence	$[0,1]$
$\gamma^d = \epsilon\gamma^{d,x}$	quit turbulence	$\epsilon \in [0,1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7

The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years.

Layoff and quit turbulence Following DHHR, we parameterize quit turbulence as a fraction ϵ of layoff turbulence, and we vary it from zero – only layoff turbulence – to one – the two types of turbulence are equal: $\gamma^d = \epsilon\gamma^{d,x}$.

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. We set the replacement rate at $\phi = 0.7$ so that an unemployed worker with benefit entitlement b_j receives 70% of the average wage of employed workers with skill level j.

Matching We assume a Cobb-Douglas matching function $M(v,u) = Au^{\alpha}v^{1-\alpha}$, where A is matching efficiency and α is the elasticity of matches with respect to unemployment. Given this technology, the probability that a worker encounters a vacancy and the probability that a vacancy encounters a particular worker type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{29}$$

When calibrating a matching model to an aggregate unemployment rate, without any calibration targets for vacancy statistics, selecting the parameter pair (A, μ) is a matter of normal-

Table 2: Parameterizations differing between LS and DHHR

Parameter	**Definition**	**LS**	**DHHR**
Matching function			
A	matching efficiency	0.441	0.3
α	elasticity of matches w.r.t. u	0.5	–
μ	flow cost of a vacancy	0.481	–
Productivity distribution			
$v_i(z)$	functional form	Normal	Uniform
$\mathbb{E}[z_l]$	low-skilled mean	1	1
$\mathbb{E}[z_h]$	high-skilled mean	2	2
$support[z_l]$	low-skilled support	[-1, 3]	[0.5, 1.5]
$support[z_h]$	high-skilled support	[0, 4]	[1.5, 2.5]
$std[z_l]$	low-skilled dispersion	1	$\frac{1}{\sqrt{12}}$
$std[z_h]$	high-skilled dispersion	1	$\frac{1}{\sqrt{12}}$

ization. We renormalize LS's setting of (A, μ) so that equilibrium market tightness in tranquil times (no turbulence) with no layoff taxes becomes equal to one.[11] This will facilitate a perturbation exercise in which we shall replace free entry of firms in LS with the DHHR arrangement that exogenously fixes equal masses of firms and workers and a market tightness equal to one.

When assuming DHHR's exogenous matching arrangement in the context of calibrating a Cobb-Douglas matching function, we need only to set parameter A equal to the probability that a worker encounters a vacancy, and so

$$\lambda^w(1) = A, \qquad \lambda^f_{ij}(1) = A\frac{u_{ij}}{u}. \tag{30}$$

Productivity distributions LS assume that productivities are drawn from truncated normal distributions with wide support: $z_l \sim \mathcal{N}(1, 1)$ for low-skilled workers over the support $[-1, 3]$, and $z_h \sim \mathcal{N}(2, 1)$ for high-skilled workers over the support $[0, 4]$.[12] In contrast, DHHR assume uniform distributions with small support: $z_l \sim \mathcal{U}([0.5, 1.5])$ and $z_h \sim \mathcal{U}([1.5, 2.5])$.

[11]Under the original LS parameterization $(A, \mu) = (0.45, 0.5)$, the equilibrium market tightness is equal to $\theta = 0.9618$ in tranquil times and no layoff taxes. We renormalize the parameter pair (A, μ) to attain an equilibrium market tightness of 1 and leave unchanged the probability that a worker encounters a vacancy. Let $(\hat{A}, \hat{\mu})$ be our new parameterization given by $\hat{A} = \kappa^{1-\alpha}A$ and $\hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old parameterization $\kappa = 0.9618$, the new parameterization achieves the desired outcomes.

[12]LS incorrectly implemented the quadrature method at the truncation points of the normal distributions; nevertheless, the constructed distributions are still proper. Therefore, instead of recalibrating the LS model under a correct implementation of the quadrature method, we have chosen to seek to resolve the puzzle in terms of the distributions that were used in the published LS analysis.

4 The Puzzle

If voluntary quits are exposed to a small risk of skill loss, then in the DHHR framework higher turbulence reduces unemployment, while it increases it in the LS framework. What explains the difference?

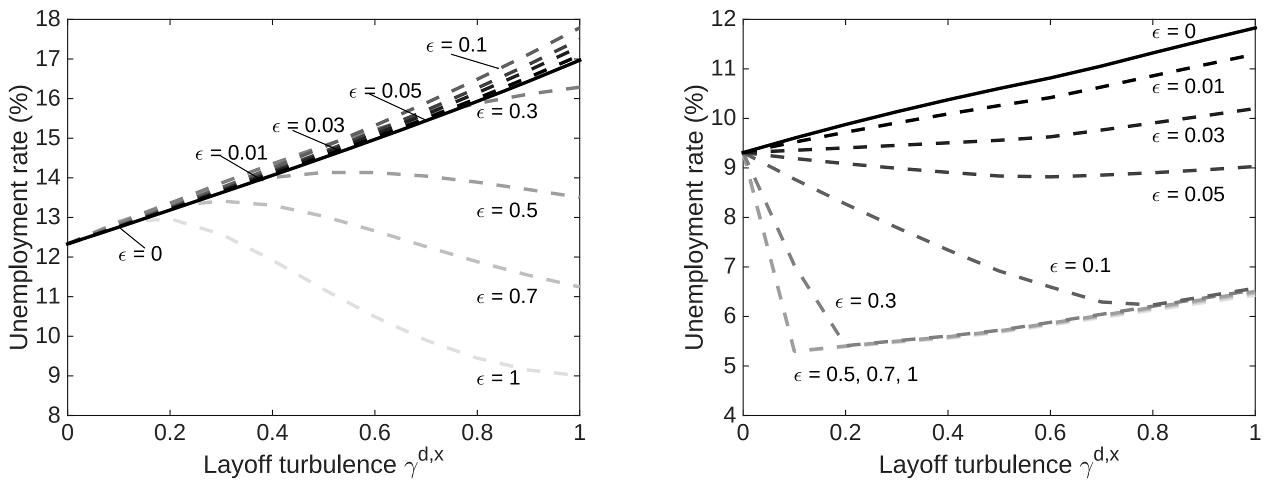
Summary of differences across frameworks The framework of LS features (a) endogenous market tightness with free entry of firms, (b) immediate realization of skill upgrades, and (c) truncated normal productivity distributions with wide support. The framework of DHHR features (a) exogenous market tightness with fixed unit mass of firms, (b) delayed completion of skill upgrades, and (c) uniform productivity distributions with narrow support.

4.1 Two frameworks, opposite conclusions

The two panels in Figure 1 show effects of both layoff and quit turbulence on the unemployment rate in the frameworks of LS and of DHHR, respectively. The x-axis shows layoff turbulence $\gamma^{d,x}$ and the y-axis the unemployment rate in percent. Each line has its own quit turbulence γ^d represented as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$.

Figure 1a shows results for the LS framework. We observe that quit turbulence needs to be high, about 50% of layoff turbulence, before the aggregate unemployment rate starts varying negatively with turbulence, and then only for relatively high levels of layoff turbulence.

Figure 1: Turbulence and unemployment in LS and DHHR



(a) Baseline LS (b) Baseline DHHR

Layoff turbulence $\gamma^{d,x}$ on the x-axis. Each line represents a different quit turbulence γ^d as a fraction ϵ of layoff turbulence, i.e., $\gamma^d = \epsilon\gamma^{d,x}$.

Figure 1b shows the effect of turbulence on the unemployment rate for our baseline DHHR model. This baseline includes two changes to the original DHHR setup that do not alter the results significantly but that facilitate a decomposition that lets us detect the source of the puzzle. Our first modification is that instead of the zero benefits that they receive in the original DHHR setup, we assume that newborn workers are eligible for the same unemployment benefits as are low-skilled workers. This modification reduces the number of worker types while having very small effects on aggregate outcomes. The second modification concerns the risk of losing skills following unsuccessful job market encounters. DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she would be quitting from a job. While DHHR justify this as a "simplifying assumption" made for numerical tractability, we find that it has quantitatively noticeable effects. (See Figure C.1 in Appendix C.) Still, the puzzle remains intact after this second modification – it just takes a somewhat bigger quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship.

The amount of quit turbulence needed to reverse the positive turbulence-unemployment relationship is very small in the DHHR framework. In their original model, the relationship becomes markedly negative at 5% of quit turbulence ($\epsilon = 0.05$), while in our baseline modified DHHR model, quit turbulence needs to be 7% ($\epsilon = 0.07$). The details of a reversal are as follows. The low-skilled workers never quit under DHHR's calibration, so it is high-skilled workers who drive a negative turbulence-unemployment relationship by quitting in fewer numbers.

We also observe in Figure 1b how DHHR's negative turbulence-unemployment relationship can eventually turn positive, as starkly illustrated by a quit turbulence of $\epsilon = 0.3$ and higher. Those high levels of quit turbulence are initially characterized by a steep negative relationship that comes to an abrupt end and kink, followed by a gentler upward-sloping turbulence-unemployment relationship. At the kinks, all endogenous separations have shut down. Reductions in quits, the source of unemployment suppression, have dried up. What leads to that positive turbulence-unemployment relationship is that higher turbulence generates more low-skilled unemployed who have high benefits and are poor prospects for firms seeking to fill vacancies: they have low skills and therefore produce less and, more importantly, are entitled to high benefits that skew bargaining outcomes in their favor. Consequently, equilibrium market tightness must fall to restore firm profitability enough to ensure that the zero-profit condition in vacancy creation holds. Lower market tightness causes higher unemployment.

In Appendix B, we describe in detail the computations that lead us to acquit the first two suspects as the culprits that DDHR use to reverse the equilibrium turbulence-unemployment relationship relative to LS. Let's proceed to the damning evidence against the third suspect.

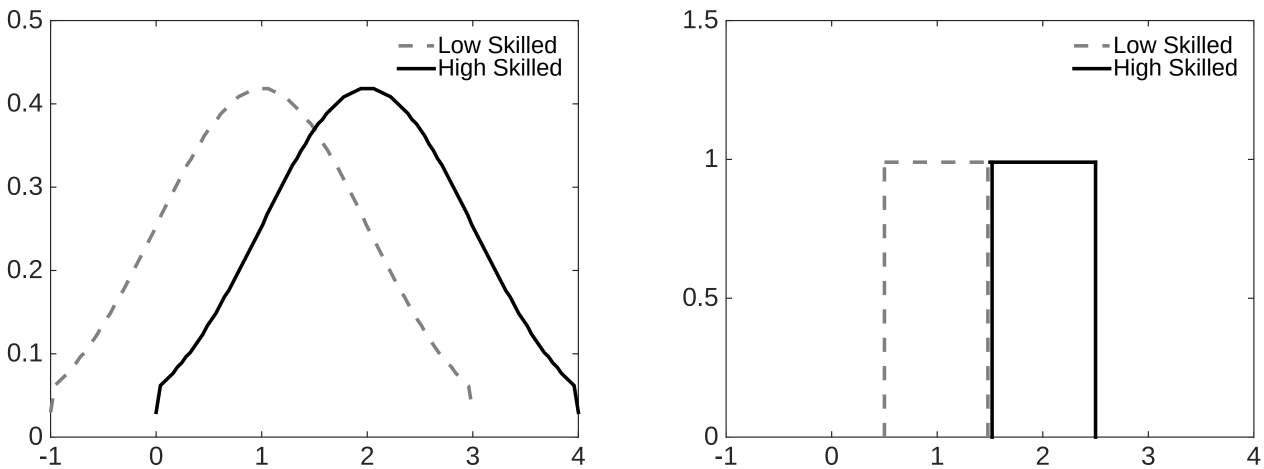
4.2 Third suspect: Different productivity distributions

The third candidate explanation concerns differences in productivity distributions. LS assume truncated normal distributions with wide support while DHHR assume uniform distributions with narrow support, as detailed in Table 2 and depicted in Figure 2.

Perturbation exercise We replace the productivity distributions in the LS model with those of DHHR, i.e., we replace the distributions in Figure 2a with those in Figure 2b.

Results Figure 3 shows results for the alternative productivity distributions. The right panel shows that under the uniform distributions with narrow support, the positive turbulence-unemployment relationship is much weakened and we get DHHR-like outcomes. We conclude that differences in productivity distributions explain the puzzle. Let's drill down further in order to understand how.

Figure 2: DIFFERENT PRODUCTIVITY DISTRIBUTIONS IN LS VS. DHHR

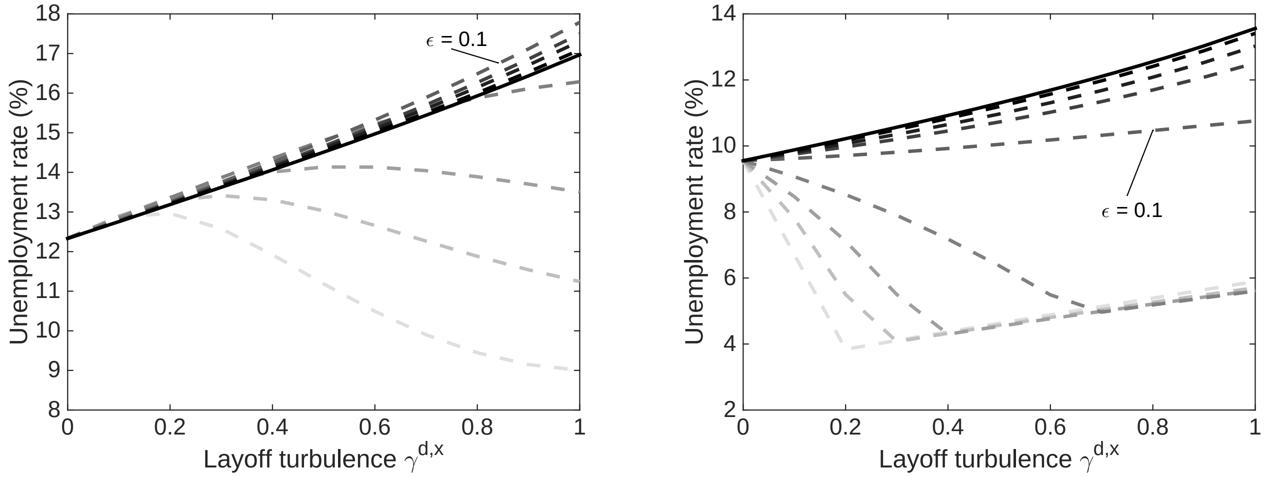


(a) LS (Normal with wide support)

(b) DHHR (Uniform with narrow support)

Discussion: Returns to labor mobility Productivity draws on the job create reasons for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support make returns to labor mobility be very low. As can be seen in Figure 3b, those low returns don't compensate for even rather small amounts of quit turbulence and hence the initially positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative at relatively small levels of quit turbulence. In particular, high-skilled workers choose to remain on the job and accept productivities at the lower end of the support of the productivity distribution rather than quit and have to face even small probabilities of skill loss.

Figure 3: Wide vs. narrow support of productivity distributions in LS

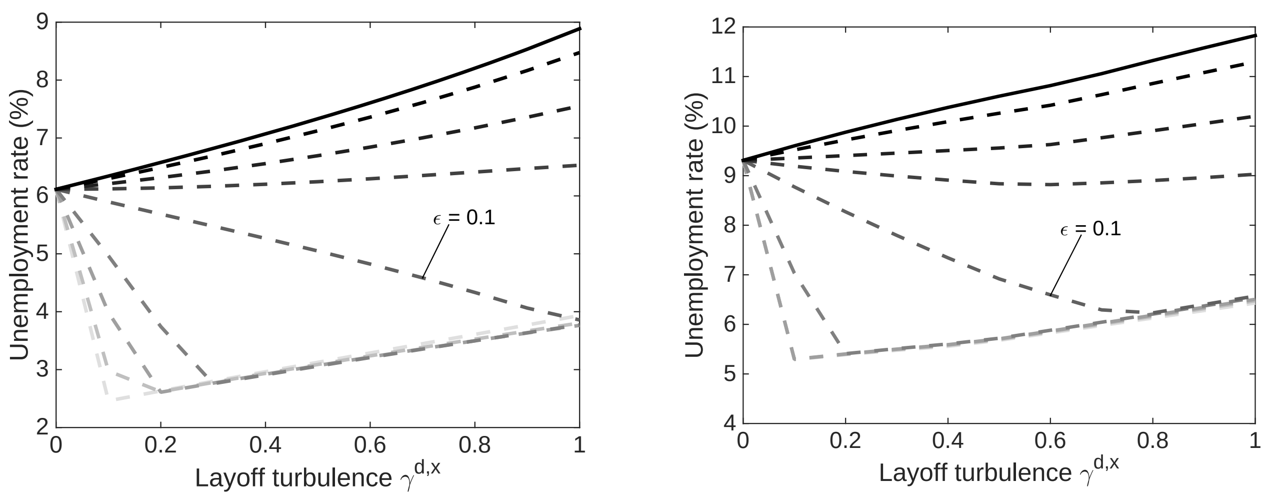


(a) Baseline LS (Wide normal)

(b) LS + Narrow uniform

To confirm that the small dispersion of productivities explains the puzzle, we do an additional perturbation exercise that shrinks the support of the uniform distribution further. Figure 4a shows outcomes in the LS framework when the support of the uniform distribution has width 0.65 instead of 1. Such a shrinkage of the support takes us very close to the outcomes in our baseline DHHR model in Figure 4b. This occurs despite our having retained the other two differences between the frameworks: endogenous versus exogenous market tightness and immediate versus delayed completion of skill upgrades.

Figure 4: Further reduced support of productivity distributions in LS



(a) LS + Even narrower uniform

(b) Baseline DHHR

5 Layoff taxes

To bring out the low return to labor mobility in the DHHR model relative to that of the LS model, we introduce a layoff tax Ω that is levied on every job separation, except for retirement. The layoff tax affects reservation productivities of existing jobs as the match surplus must now fall to the negative of the layoff tax (instead of earlier zero) before a job is terminated:

$$s_{ij}(\underline{z}_{ij}) = -\Omega. \tag{31}$$

When computing wages, we assume standard Nash bargaining between a worker and a firm each getting their shares of the match surplus s_{ij} in every period.[13]

Government revenue With the introduction of a layoff tax, the government's revenue includes revenues from layoff taxes. Let *seps* be total separations excluding retirements, which are equal to

$$seps = (1-\rho^r)\Big[\rho^x(e_{ll}+e_{hh}) + (1-\rho^x)[(1-\gamma^u)\gamma^s\nu_{ll} + \gamma^u\nu_{hh}]e_{ll} + (1-\rho^x)\gamma^s\nu_{hh}e_{hh}\Big]. \tag{32}$$

Then government revenue equals income taxes plus layoff taxes, $\tau(\bar{z}_l e_{ll} + \bar{z}_h e_{hh}) + seps\ \Omega$. The government adjusts the income tax rate τ to set revenue equal to total expenditure on unemployment benefits in expression (26).[14]

5.1 Layoff taxes in LS

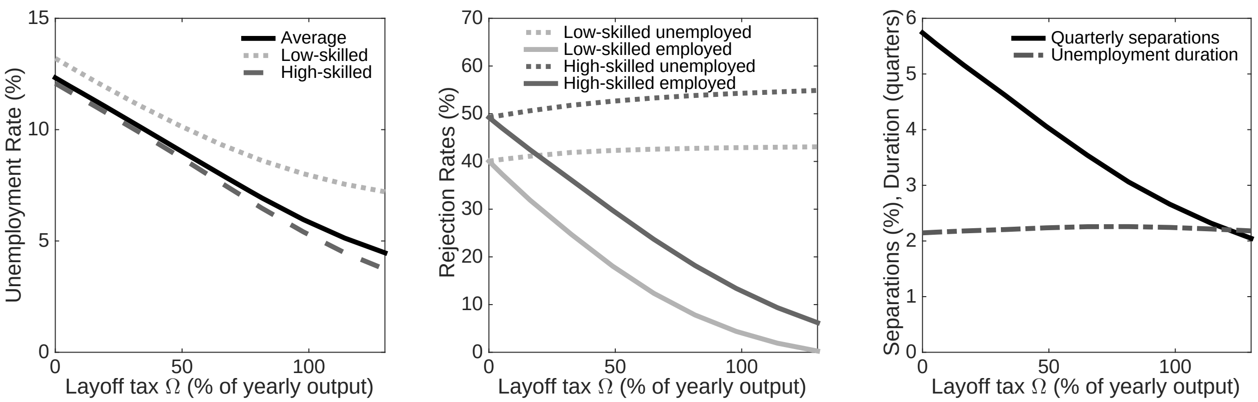
In the LS model without turbulence, Figure 5 shows unemployment and rejection rates by type of worker, as well as aggregate labor flows, as functions of the layoff tax Ω. The layoff tax is expressed as a fraction of the average yearly output per worker in the laissez-faire economy. (See footnote 23.) The unemployment rate falls as the layoff tax increases. Employed workers, both high- and low-skilled, are especially affected by the layoff tax as their rejection rates fall significantly. Nevertheless, these workers remain mobile even with rather large layoff taxes. For example, if the layoff tax reaches the average annual output of a worker, employed high-skilled

[13]An implication of the Nash bargaining assumption is that workers would be paying part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff cost. This generates a two-tier wage system à la Mortensen and Pissarides (1999). Risk neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. As demonstrated by Ljungqvist (2002), the wage profile, not the allocation, is affected by the two-tier wage system. Match surpluses, reservation productivities and market tightness remain the same. Under the two-tier wage system, an initial wage concession by a newly hired worker is equivalent to his posting a bond that equals his share of a future layoff tax.

[14]If layoff tax revenues cover payments of unemployment benefits, i.e., $seps\ \Omega \geq b_l u_{ll} + b_h(u_{lh} + u_{hh})$, then we set $\tau = 0$ and return any government budget surpluses as lump-sum transfers to workers.

workers reject about 12% of offers.

Figure 5: Layoff taxes in LS



Incidentally, Figure 5 illustrates LS's explanation for a welfare state's having lower unemployment than a laissez-faire economy in tranquil times (i.e., before the onset of economic turbulence). In a matching model, countervailing forces emanating from unemployment benefits and layoff taxes can explain why the unemployment rate in a welfare state need not be high (also see Mortensen and Pissarides (1999)). Despite generous unemployment benefits with a replacement rate of $\phi = 0.7$, layoff taxes at the right end of the first panel in Figure 5 cause unemployment to fall below the laissez-fare rate of 5%.

For later use, we note that endogenous separations in the LS model shut down completely when the layoff tax reaches 184% of the average yearly output per worker. This can be discovered by extrapolating the dark solid curve in the middle panel of Figure 5; evidently, high-skilled employees are most resilient before eventually stopping to quit. The corresponding minimum layoff tax required to close down all endogenous separations in the laissez-faire economy with no unemployment insurance is 163%. Without unemployment compensation, the gains from quitting and searching for another job are smaller so that it requires a smaller layoff tax to shut down endogenous separations in the laissez-faire economy.

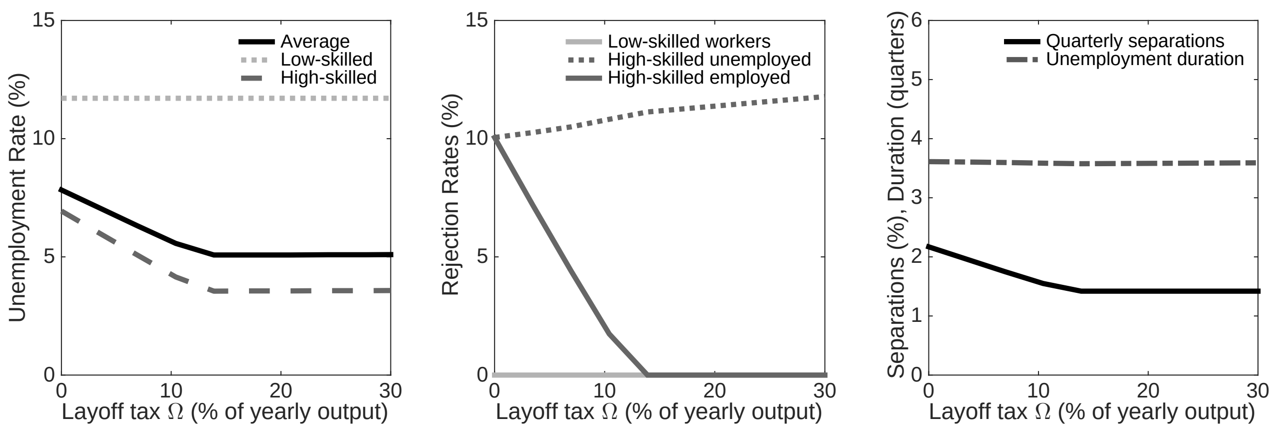
5.2 Layoff taxes in DHHR

We introduce a layoff cost Ω in our baseline DHHR model with the additional inconsequential change that skill upgrades are realized immediately as in the LS framework (Appendix C.2 documents a small impact on equilibrium outcomes of such a change in assumptions). Figure 6 shows how a higher layoff tax affects equilibrium outcomes in the DHHR model without turbulence. Mobility of high-skilled employed completely shuts down at a layoff tax equivalent to 14% of the average annual output per worker in the laissez-faire economy.[15] Above this

[15]In the DHHR laissez-faire economy, a worker's average quarterly output is 1.8 goods.

low level of layoff taxes, the rejection rate of these workers becomes zero and separation rates become constant at exogenous job termination rates. Imposing a small layoff tax eradicates the value of labor mobility. Note that for both employed and unemployed low-skilled workers, the rejection rate is zero for the DHHR parameterization at all levels of the layoff tax.

Figure 6: Layoff taxes in DHHR



This exercise confirms that the productivity distributions of DHHR imply small incentives for labor mobility in tranquil times.[16] A small government mandated layoff cost has counterfactually large effects of suppressing unemployment by shutting down all quits. So it also makes sense that other small costs to mobility, such as a tiny risk of skill loss when quitting, cause unemployment to fall and thereby, can reverse Ljungqvist and Sargent's positive turbulence-unemployment relationship.

It is noteworthy that there are no endogenous separations at all in the corresponding laissez-faire economy of DHHR. So endogenous separations occur in our baseline DHHR model only because they are encouraged by a generous replacement rate of $\phi = 0.7$.

6 Mortensen-Pissarides (1999) productivity process

Mortensen and Pissarides (1999, henceforth MP) also study how skill dynamics can interact with welfare-state institutions in a matching model. But in contrast to LS and DHHR, MP assume that individual workers are permanently attached to their skill levels and focus on

[16]The productivity distributions of DHHR also emerged from an earlier exchange discussed in footnote 9. Specifically, Ljungqvist and Sargent (2004) criticized den Haan *et al.* (2001) for making low- and high-skilled workers almost indistinguishable from one another because of nearly overlapping productivity distributions for the two types of workers. As a remedy, by moving the uniform distributions apart and ending up with the disjoint supports in Figure 2b, Ljungqvist and Sargent (2004) succeeded in making low- and high-skilled workers distinct from one another; but as shown here that fails to generate returns to labor mobility consistent with historical observations. In the subsequent matching analysis of LS, layoff costs were introduced and productivity distributions had to be properly calibrated, as shown in subsection 5.1.

effects of a mean preserving spread of the cross section distribution of skills across workers. To capture 'directed search,' MP assume a separate matching function for each skill level.

For us, a key object of the MP model is a probability distribution of idiosyncratic productivities that multiply workers' skills in ongoing matches. MP assume that distribution function is uniform on support $[z^{min}, 1]$ so that the cumulative density is $F(z) = (z - z^{min})/(1 - z^{min})$ for all $z \in [z^{min}, 1]$. As in LS and DHHR, productivity shocks in ongoing matches arrive at an exogenous rate γ^s. But in contrast to LS and DHHR, new matches have productivity equal to the upper support of the distribution.

Table 3: MP's parameter values (central to our study)

Parameter	**Definition**	**Value**
z^{min}	minimum productivity	0.64
γ^s	productivity switch probability (at a quarterly frequency)	0.1

MP's parameterization in Table 3 gives the same arrival rate of productivity switches as LS and DHHR, i.e., MP's quarterly probability $\gamma^s = 0.1$ is consistent with the semi-quarterly probability $\gamma^s = 0.05$ in Table 1. Because the narrow range of the support of MP's uniform distribution $[0.64, 1]$ is in the same ballpark as DHHR in Table 2, one might expect returns to labor mobility in the MP model to be as small as those of DHHR. However, all new matches in the MP model have productivity equal to the upper support of the distribution, which enhances returns to labor mobility as compared to DHHR's assumption that a new match draws a productivity from the same distribution as ongoing matches. Thus, the question is a quantitative one – a question that will also compel us to investigate the calibration approach chosen by MP.

6.1 Mapping MP's productivity process into LS

Our criterion for faithfully mapping the MP productivity process into the LS economy is how closely the resulting LS economy resembles MP's (1999, Table 2a) findings on how unemployment responds to unemployment insurance and layoff taxes as reproduced in the first panel of our Table 4. The fit cannot be perfect since, for example, the LS economy has two skill levels while MP choose to conduct their calculations for the case of a single skill level equal to 1. Another difference is that MP assume a training cost while LS have none.

As an intermediate step, we compute outcomes in a perturbed version of the LS model with several features modified to be the same as in MP. Specifically, the perturbed LS model has only

Table 4: Unemployment Rate Effects of the UI Replacement Ratio (ϕ) and Layoff Tax (Ω)

Mortensen and Pissarides (1999, Table 2a)

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$
$\Omega = 0.0$	4.8	5.5	6.2	7.3	9.0	11.9
$\Omega = 0.5$	3.7	4.3	5.0	5.9	7.5	10.3
$\Omega = 1.0$	2.5	2.9	3.5	4.4	5.7	8.4
$\Omega = 1.5$	1.1	1.5	1.9	2.6	3.6	5.9
$\Omega = 2.0$	0.0	0.0	0.0	0.0	1.3	2.9

Perturbed version of LS with only low-skilled workers

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$
$\Omega = 0.0$	5.0	5.5	6.2	7.2	8.6	11.0
$\Omega = 0.5$	4.2	4.6	5.2	6.0	7.2	9.2
$\Omega = 1.0$	3.2	3.6	4.1	4.8	5.9	7.6
$\Omega = 1.5$	2.2	2.5	2.9	3.5	4.4	5.9
$\Omega = 2.0$	1.1	1.3	1.7	2.1	2.8	3.9
$\Omega = 2.5$	0.4	0.5	0.5	0.6	1.0	1.8

A perturbed version of the LS model with only low-skilled workers, no exogenous breakups $\rho^x = 0$, an added value of leisure equal to 0.28, and MP's productivity process with $z^{min} = 0.64$. Matching efficiency is calibrated to $A = 0.66$. Layoff taxes Ω are expressed in terms of quarterly output.

LS model with the MP productivity process

	$\phi = 0.0$	$\phi = 0.1$	$\phi = 0.2$	$\phi = 0.3$	$\phi = 0.4$	$\phi = 0.5$	$\phi = 0.6$	$\phi = 0.7$
$\Omega = 0.0$	5.0	5.4	5.8	6.4	7.0	7.8	8.8	10.2
$\Omega = 1.0$	3.9	4.2	4.5	5.0	5.5	6.2	7.1	8.4
$\Omega = 2.0$	3.0	3.3	3.6	4.0	4.5	5.1	5.9	7.0
$\Omega = 3.0$	2.1	2.3	2.6	3.0	3.4	3.9	4.5	5.5
$\Omega = 4.0$	1.3	1.3	1.5	1.8	2.2	2.6	3.1	3.9
$\Omega = 5.0$	1.3	1.3	1.4	1.5	1.6	1.7	1.8	2.3

The LS model with MP's productivity process with $z^{min} = 0.6$. Matching efficiency is calibrated to $A = 0.37$. Layoff taxes Ω are expressed in terms of quarterly output.

low-skilled workers (with skills equal to one), no exogenous breakups $\rho^x = 0$, an added value of leisure equal to 0.28, and MP's productivity process with $z^{min} = 0.64$. The efficiency factor on the matching function is calibrated to be $A = 0.66$ in order to keep our target of 5 percent unemployment in the laissez-faire economy. The unemployment outcomes of the perturbed LS model in the second panel of Table 4 are almost the same as those of MP in our first panel. However, a noticeable difference is that LS unemployment cannot become zero since there is exogenous retirement with probability $\rho^r = 0.0031$. Hence, the influx of new workers in the LS model means that the unemployment rate can never fall below 3 percent and will be higher if the average time to find a job for newcomers exceeds one semi-quarterly model period.

Encouraged by the success of our intermediate step in approximating MP's unemployment outcomes, we turn to the full-fledged version of the LS model with two skill levels, low-skilled and high-skilled workers with skills equal to 1 and 2, respectively. We restore the exogenous breakup probability $\rho^x = 0.005$ and set the value of leisure to zero. In short, we adopt the exact parameterization of the LS model in Tables 1 and 2 but with the MP productivity process with $z^{min} = 0.6$.[17] Also, we re-calibrate the efficiency factor on the matching function to be $A = 0.37$ in order to have 5 percent unemployment in the laissez-faire economy.

The third panel of Figure 4 contains outcomes of our full-fledged version of the LS model with the MP productivity process. Now our comparison to MP's outcomes in the first panel has to be more subtle and bring to bear adjustments beyond those to the retirement rate deployed in our intermediate step. First, in our two-skill economy, the steady-state labor force consists of 20 percent low-skilled and 80 percent high-skilled workers. Thus, the layoff tax numbers in the third panel would have to be cut approximately in half to be comparable to the first two panels when expressing layoff taxes relative to workers' output since high-skilled workers who make up the vast majority of the labor force in the third panel are twice as productive as the workers of the first two panels. Because the layoff taxes reported in the third panel are twice as high as those reported in the first two panels, we can compare outcomes line-by-line across panels. Second, the assumption of a value of leisure equal to 0.28 for workers with skill level one in the first two panels lets us convert that into an extra replacement rate in unemployment insurance of 0.3 in the third panel. Thus, a replacement rate ϕ in the first two panels would correspond to a replacement rate of $\phi + 0.3$ in the third panel. Third, the last panel can be thought of as having calibrated a laissez-faire unemployment rate of 6.4 percent, as given by column $\phi = 0.3$ (and no layoff tax), because a replacement rate $\phi = 0.3$ would represent only the value of leisure according to our conversion argument. A way to correct for this concocted elevated unemployment rate of the laissez-faire calibration is to deduct from each computed

[17]Since there is no pretense of trying to exactly reproduce MP's unemployment outcomes, we have rounded off the parameter value $z^{min} = 0.6$. A parameter that we will subject to a sensitivity analysis in the next subsection.

unemployment rate an adjustment equal to the difference between the third panel's column $\phi = 0.3$ and column $\phi = 0$, i.e., a single adjustment for each value of the layoff tax. As an illustration, these adjustments would turn the unemployment rates in column $\phi = 0$ into the new numbers of column $\phi = 0.3$.

The preceding three adjustments intended to make the third panel comparable to the first two panels are implemented in Table 5, including a re-labelling of replacement rates to become $\hat{\phi} = \phi - 0.3$ and layoff taxes to become $\hat{\Omega} = 0.5\,\Omega$. Evidently, our mapping of MP into LS is quite successful when comparing Table 5 to the MP outcomes in the first panel of Figure 4. However, differences appear at higher layoff taxes at which the higher unemployment rates of the LS model can largely be attributed to its exogenous rates of retirements $\rho^r = 0.0031$ and of breakups $\rho^x = 0.005$. Since our intermediate step includes the retirement rate but not the exogenous breakup rate, it is understandable that unemployment outcomes at higher layoff taxes in the second panel of Table 4 fall between the lower and higher unemployment rates of MP in the first panel of Table 4 and LS in Table 5, respectively. Apparently, at such high layoff taxes, endogenous separations have either shut down or are about to in all of the economies so that unemployment becomes driven mostly by exogenous shocks of separation.

Table 5: Assessing the success of mapping MP into LS

Adjusted version of the LS economy with the MP productivity process						
	$\hat{\phi} = 0.0$	$\hat{\phi} = 0.1$	$\hat{\phi} = 0.2$	$\hat{\phi} = 0.3$	$\hat{\phi} = 0.4$	Adj. factor
$\hat{\Omega} = 0.0$	5.0	5.6	6.4	7.4	8.8	1.4
$\hat{\Omega} = 0.5$	3.9	4.4	5.1	6.0	7.3	1.1
$\hat{\Omega} = 1.0$	3.0	3.5	4.1	4.9	6.0	1.0
$\hat{\Omega} = 1.5$	2.1	2.5	3.0	3.6	4.6	0.9
$\hat{\Omega} = 2.0$	1.3	1.7	2.1	2.6	3.4	0.5
$\hat{\Omega} = 2.5$	1.3	1.4	1.5	1.6	2.1	0.2

6.2 Fragility of MP's calibration

In conducting the quantitative analysis of the preceding subsection, we encountered a fragility in how MP had restricted the calibration of a key parameter that affects returns to labor mobility, namely, the lower support z^{min} of the productivity distribution. We describe that fragility by conducting a quantitative sensitivity analysis with respect to the parameter z^{min} after first describing MP's calibration strategy.

MP (1999, pp. 256-257) describe their calibration strategy as follows:

> "The policy parameters are chosen to reflect the US case. All other structural parameters, except for the value of leisure b and minimum match product [z^{min}] which are chosen so that the steady state unemployment rate and the average duration of an unemployment spell match the average experience in the United States over the past twenty years, are similar to those assumed and justified in Mortensen (1994) Mortensen (1994) and Millard and Mortensen (1997) Millard and Mortensen (1997)."

That calibration of values of leisure and z^{min} is confirmation by Millard and Mortensen (1997, p. 555) who say:

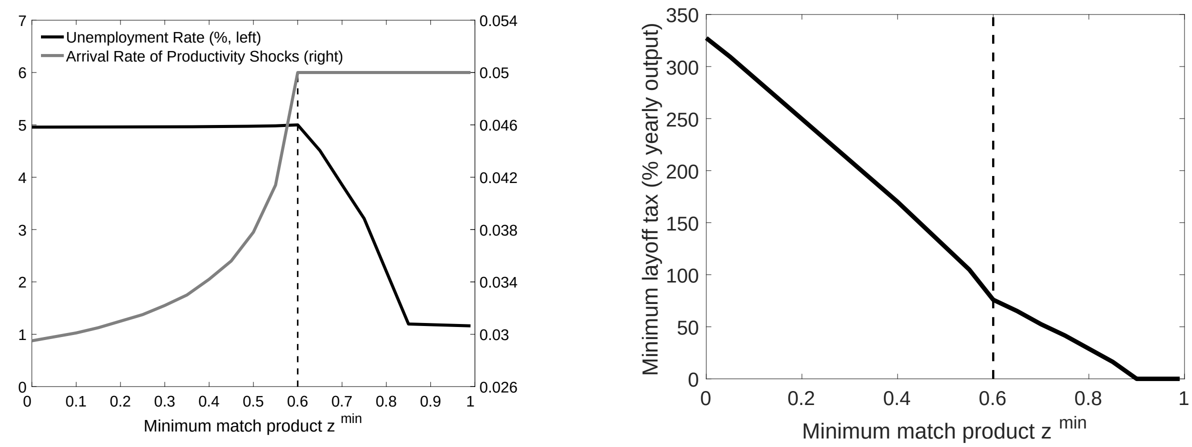
> "... two parameters for which there is no direct evidence, the forgone value of leisure b and a measure of dispersion in the idiosyncratic shock denoted as [z^{min}], are chosen to match the average duration of unemployment and incidence of unemployment experienced over the 1983-92 period."

For a given steady-state unemployment rate, calibrations of the average duration of an unemployment spell and the incidence of unemployment are two sides of the same coin. Below, we calibrate to target the incidence of unemployment. However, our most important move is to put another parameter for which we have no direct evidence on the table, namely, the arrival rate γ^s of productivity shocks.

We use the laissez-faire parameter configuration of the LS model with the MP productivity process in the third panel of Table 4 to explain this important tradeoff associated with the choice of a pair (z^{min}, γ^s). Recall that the economy is parameterized to have $z^{min} = 0.6$ and a productivity switch probability $\gamma^s = 0.005$ in the LS semi-quarterly model period (which corresponds to MP's quarterly probability 0.1 in Table 3). Now, in accordance with MP's target of a particular incidence of unemployment (or, on the flip side, a particular average duaration of an unemployment spell), we 'freeze' the laissez-faire economy's quarterly separation rate of 6.77 percent. Specifically, for each value of $z^{\min} \leq 0.6$, we find an associated value of γ^s that implies an unchanged quarterly separation rate. The lighter curve in panel (a) of Figure 7 traces out the pairs of (z^{min}, γ^s) that attain the targeted quarterly separation rate of 6.77 percent. In our 'normal' parameter range, there is a positive relationship between z^{min} and γ^s, because a higher z^{min} means smaller dispersion of productivities and therefore fewer shocks that call forth endogenous quits so the exogenous arrival rate of shocks γ^s has to be raised to keep the separation rate unchanged. The darker line shows that the laissez-faire unemployment rate remains constant at 5 percent throughout these calculations for $z^{\min} \leq 0.6$.

We can also extend these calculations for $z^{\min} > 0.6$ (not shown); but after 0.64 no γ^s can be found to generate as high a quarterly separation rate as 6.77 percent. To see why, notice that

Figure 7: Calibration of LS model with MP productivity z^{min}



(a) Arrival rate of productivity shocks γ^s

(b) Minimum layoff tax to shut down quits

the lighter curve in panel (a) of Figure 7 becomes ever steeper as it approaches $z^{\min} = 0.6$ from below. Evidently, this arithmetic must eventually come to a stop, since it would be impossible to maintain *any* endogenous separations as the parameter z^{min} approaches the upper support of 1 where the productivity distribution would become degenerate as a single mass point. Instead of depicting the breakdown of our algorithm, we simply freeze all the parameters of the economy at $z^{min} = 0.6$, except for the parameter itself as we compute equilibria for higher values of z^{min}. As depicted in panel (a) of Figure 7 for $z^{\min} > 0.6$ and a constant productivity switch probability $\gamma^s = 0.05$, the unemployment starts falling until all endogenous separations come to a halt and the unemployment curve becomes horizontal to reflect exogenous rates of retirement $\rho^r = 0.0031$ and breakups $\rho^x = 0.005$.

Panel (b) of Figure 7 refers to the welfare-state configuration of the model with replacement rate $\phi = 0.7$ in the third panel of Table 4. The figure depicts the minimum layoff tax required to shut down all endogenous separations measured in terms of an average worker's annual output in the laissez-faire economy. As discussed in section 5.1, the welfare state needs a higher layoff tax to shut down than does the laissez-faire economy. That is also true when comparing the far-right flattening of the layoff-tax curve at zero for the welfare state in panel (b) and the flattening of the laissez-faire unemployment curve in panel (a) (which is trivially associated with no layoff tax required to shut down endogenous separations because they have already come to a halt). This tiny slice of the z^{min}-range where endogenous separations have shut down in the laissez-faire economy and are barely present in the welfare state would be the counterpart to the baseline DHHR model, as discussed in section 5.2. In contrast, the baseline LS model discussed in section 5.1 requires a minimum layoff tax of circa 180 percent of a worker's annual output so that the counterpart in panel (b) of Figure 7 would be a LS economy with a MP productivity process with $z^{\min}$ of around 0.35.

A final take-away from Figure 7 is that MP unnecessarily constrained themselves by postulating a quarterly productivity switch probability 0.1 in Table 3. That caused MP to back into a treacherous region of the parameter space in which any perceived need to increase $z^{\min}$ further would instead have rendered MP's calibration targets unattainable because there would then just not be big enough returns to labor mobility when the range of the uniform distribution becomes too small.[18]

6.3 Turbulence under MP productivity process

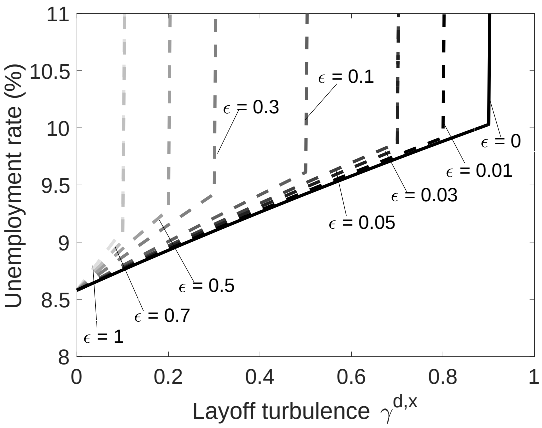
Figure 8 depicts how unemployment respond to turbulence in four of the calibrated economies from Figure 7, indexed by $z^{min} \in \{0, 0.2, 0.4, 0.6\}$. The two top panels show robust positive turbulence-unemployment relationships for any combination of layoff and quit turbulence.

A new feature is the possibility of a spike indicating a 'meltdown' that occurs when the unemployment rate soars to a level of 55-60 percent (outside of the graphs). The following forces cause the meltdown. Under MP's assumption that all new jobs start with a productivity equal to the upper support of the distribution, a reservation productivity can take only one of two possible values: either the productivity z^{min} is acceptable to a worker-vacancy encounter or it is not. This creates a possible a 'tipping point' at which a change in turbulence moves the economy *from* an equilibrium in which all worker-vacancy encounters result in matches *to* an equilibrium in which there is no Nash-bargaining solution for some worker-vacancy encounters. This happens at the meltdowns in Figure 8: firms cannot afford to pay a wage to low-skilled workers with high benefits that is high enough to compensate them for surrendering their high benefits. When turbulence reaches that tipping point, the stochastic steady state becomes one in which skill loss leads to an absorbing state of unemployment until retirement – a 'turbo-charged' positive turbulence-unemployment relationship.

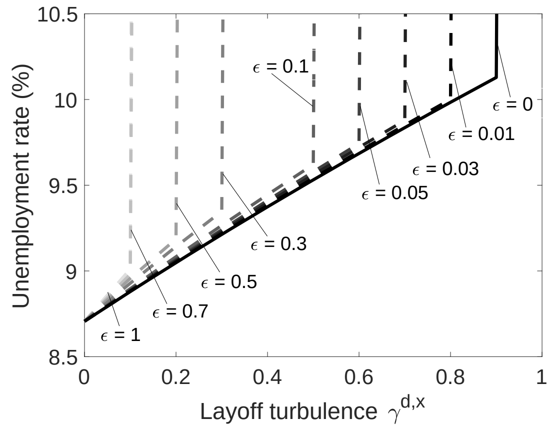
In the preceding subsection, we mapped our baseline LS model into a corresponding economy with the MP productivity process in Figure 7. We argued that the corresponding economy would be one with $z^{min} = 0.35$. Interestingly, the turbulence outcomes in Figure 8c with $z^{min} = 0.4$ resemble those of Figure 1a for our baseline LS model. In particular, only high levels of quit

[18] In personal communications with us, Stephen Millard described how he and Dale Mortensen used evidence on firing costs that they gleaned from data on the experience rating feature of the U.S. unemployment insurance system to calibrate parameters z^{min}, γ^s and the value of leisure to match targets for the unemployment rate (6.5%), unemployment incidence (7%), and the elasticity of unemployment incidence with respect to firms' firing cost (0.09). They calibrated these three parameters by solving three simultaneous equations, conditional on the other parameters. (See also Mortensen (1994, p. 203)). Evidently, the resulting quarterly value $\gamma^s = 0.1$ was imported to MP (1999, pp. 256-257) who calibrated the value of leisure and z^{min} to the steady state unemployment rate and the average duration of an unemployment spell.

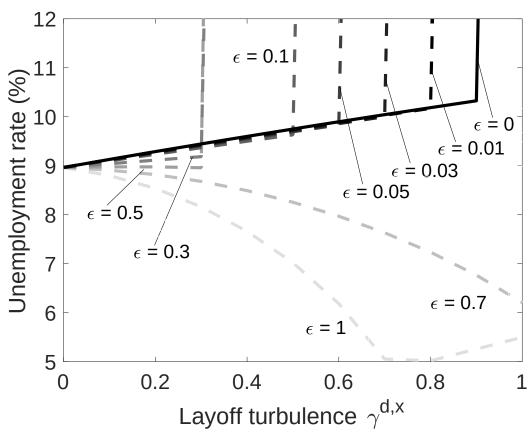
Figure 8: Turbulence with MP productivity z^{min}



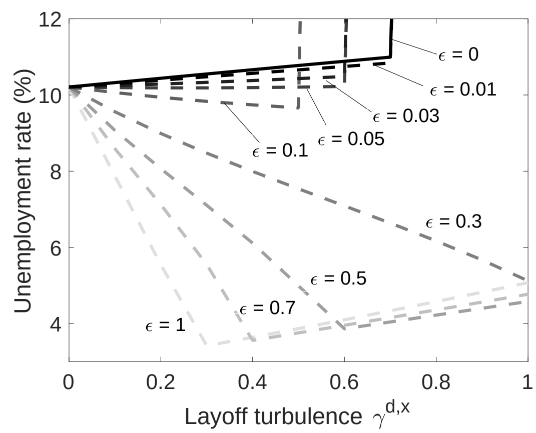
(a) $z^{min} = 0$



(b) $z^{min} = 0.2$



(c) $z^{min} = 0.4$



(d) $z^{min} = 0.6$

turbulence can cause a negative turbulence-unemployment relationship.

Regarding the turbulence outcomes in Figure 8d with $z^{min} = 0.6$, there is a close resemblence to that present in our earlier Figure 3b. The latter figure refers to our experiment that imports the DHHR productivity distribution into the baseline LS model and finds returns to labor mobility that are suppressed but not so low as in the baseline DHHR model. Since Figure 3b can be thought of as an intermediate step in moving from high to very low returns to mobility, the same can be said about Figure 8d with $z^{min} = 0.6$. Since the latter model is also our LS model with the MP productivity process that reproduces MP's unemployment outcomes in Table 4, it becomes another way of expressing what we said earlier about MP teetering on the edge of a treacherous region of the parameter space.

7 Alvarez-Veracierto (2001) productivity process

To study effects of firing costs and severance payments in an incomplete markets setting in which rigid wages don't depend on individual firms' states and risk-averse agents self-insure against income risk, Alvarez and Veracierto (2001, henceforth AV) formulate a search-island model in the tradition of framework of Lucas and Prescott (1974).[19] A state-independent wage and an incentive to self-insure are features that are absent from the LS and DHHR environments in which workers are risk neutral and wages are determined in Nash bargaining between a worker and a firm. For our present purposes, the object of the Alvarez-Veracierto model that interests us is the stochastic process governing idiosyncratic productivities that, intermediated through a production function, determine workers' outputs. AV calibrate a productivity distribution that they coax from establishment data on job creation and destruction (Davis and Haltiwanger, 1990) cast within a model in which outcomes are shaped by a neo-classical production function.

An individual firm's output y_t at time t is given by the production function

$$y_t = x_t \, k_t^{\xi} \, n_t^{\psi} \,, \tag{33}$$

where $\xi > 0$, $\psi > 0$, $\xi + \psi < 1$, k_t is capital, n_t is labor, and x_t is an idiosyncratic productivity shock. The idiosyncratic shock x_t can take one of three values $\{0, x^{low}, x^{high}\}$ and follows a first-order Markov process with a transition probability matrix Q. Zero productivity is an absorbing state that indicates death of a firm.

[19] Because they calibrate their model to Davis and Haltiwanger's (1990) establishment data, AV use the term "establishment" instead of "firm".

The transition probability matrix Q takes the following form:

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ \eta & \omega(1-\eta) & (1-\omega)(1-\eta) \\ \eta & (1-\omega)(1-\eta) & \omega(1-\eta) \end{bmatrix}, \tag{34}$$

where $\eta \in (0,1)$ is the probability of a firm's death and, conditional on surviving, $\omega \in (0,1)$ is the probability that a firm's productivity is unchanged from last period. The transition probability matrix Q in (34) treats low and high productivity shocks symmetrically. In addition, initial productivities drawn by new firms have equal probabilities of being low and high. Under these assumptions, there are as many firms with low productivity as with high productivity in a stochastic steady state.

Table 6 lists parts of AV's parameterization that are central to us. The production function is calibrated in a standard way to match commonly used targets: AV calibrate the capital share parameter ξ to match the U.S. capital-output ratio and the labor share parameter ψ to replicate a labor share in national income of 0.64. For a semi-quarterly model period and normalization $x_1 = 1$, AV (2001, p. 488)

> "select the parameters [η], ω and [x_2] to reproduce observations on job creation and job destruction reported by Davis and Haltiwanger (1990): the average job creation and job destruction rates due to births and deaths are both about 0.73 percent a quarter, the average job creation and job destruction rates due to continuing establishments are about 4.81 percent a quarter, and the annual persistence of both job creation and destruction is about 75 percent. We obtained these observations by selecting [x_2] = 2.12, [η]= 0.0037, and $\omega = 0.973$."[20]

Note that AV's empirical targets for quarterly job churning sum to 5.5 percent – 0.73 percent due to births and deaths of establishments and 4.81 percent from job creation and job destruction due to continuing establishments. This total rate of 5.5 percent lines up well with outcomes in the LS economy without a layoff tax in the rightmost panel of Figure 5, in which the quarterly separation rate is around 5.7 percent. Also, there is a quantitatively close overlap between the empirical 0.73 percent a quarter attributed to establishment turnover, modelled as an exogenous firm failure rate by AV (i.e., twice the semi-quarterly rate $\eta = 0.0037$ in Table 6), and the exogenous breakup/layoff rate of 1 percent assumed by LS and DHHR (i.e., twice the semi-quarterly rate $\rho^x = 0.005$ in Table 1). It remains for us to describe how to map the AV productivity process pertaining to production functions with both capital and labor into our matching framework and the productivities of one-worker firms with no physical capital.

[20]We have corrected AV's (2001, p. 488) erroneous reference to "[η] = 0.037" with the correct number 0.0037, as reported in Table 1 of AV's 1998 working paper (Federal Reserve Bank of Chicago, WP 98-2).

Table 6: AV's PARAMETER VALUES (CENTRAL TO OUR STUDY)

Parameter	**Definition**	**Value**
Technology		
ξ	capital share	0.19
ψ	labor share	0.58
Productivity		
x_2	high productivity	2.12
ω	persistence of productivity	0.973
η	death of firm	0.0037

7.1 A simplified AV model

We simplify AV's benchmark economy by assuming an endowment of perpetual firms, and by eliminating a minor firing tax. First, instead of AV's costly creation of new establishments, suppose that the economy is endowed with a fixed measure of firms equal to the steady-state measure in AV's benchmark economy. And whenever a firm dies with probability η, it is replaced by a new firm as in AV's steady state, but now without any cost of creation. We retain AV's assumption that a banking sector owns both the establishments and the capital that they rent. Second, we eliminate a minor firing tax in AV's (2001, p. 487) benchmark economy that represents employers' experience-rated tax to finance the unemployment benefit system, motivated by AV's argument that "these taxes work approximately as firing taxes". Instead, the government could marginally increase the payroll tax by the annuitized expected value of that minor firing tax.[21]

With the firm creation cost and the firing tax gone, a firm's problem is purely static. A firm maximizes profits renting enough capital and labor in spot markets to equate their marginal products to the rental rate on capital r and the before-payroll-tax wage $w^{\star}$, respectively. In a steady state, there are only two types of firms: firms with low (high) productivity, of which each one rents k_1 (k_2) units of capital and hires n_1 (n_2) workers. In this stationary equilibrium, we can switch from a time subscript on variables to a state subscript: state 1 stands for low productivity, $x_1 = x^{low}$, and state 2 for high productivity, $x_2 = x^{high}$.

In an equilibrium, the marginal product of labor in both types of firms equals the wage $w^{\star}$,

$$w^{\star} = \psi \, x_1 \, k_1^{\xi} \, n_1^{\psi-1} = \psi \, x_2 \, k_2^{\xi} \, n_2^{\psi-1} \, . \tag{35}$$

[21] According to AV's 1998 working paper (Federal Reserve Bank of Chicago, WP 98-2), the firing tax is equal to only 30 percent of the semi-quarterly before-payroll-tax wage rate.

After dividing both sides of the last equality by $\psi\, x_1\, k_1^{\xi}\, n_1^{\psi}\, n_2^{-1}$, we have

$$\frac{n_2}{n_1} = \frac{x_2}{x_1}\left(\frac{k_2}{k_1}\right)^{\xi}\left(\frac{n_2}{n_1}\right)^{\psi}. \tag{36}$$

Likewise, the marginal product of capital equals the rental rate r,

$$r = \xi\, x_1\, k_1^{\xi-1}\, n_1^{\psi} = \xi\, x_2\, k_2^{\xi-1}\, n_2^{\psi}. \tag{37}$$

After dividing both sides of the last equality by $\xi\, x_1\, k_1^{\xi}\, n_1^{\psi}\, k_2^{-1}$, we have

$$\frac{k_2}{k_1} = \frac{x_2}{x_1}\left(\frac{k_2}{k_1}\right)^{\xi}\left(\frac{n_2}{n_1}\right)^{\psi}. \tag{38}$$

Since the right-hand sides of expressions (36) and (38) are the same, the capital-labor ratio is the same across all firms,

$$\frac{n_2}{n_1} = \frac{k_2}{k_1} \qquad \Rightarrow \qquad \frac{k_1}{n_1} = \frac{k_2}{n_2}. \tag{39}$$

By substituting (39) into expression (36), the ratio of labor employed by the two types of firms is

$$\frac{n_2}{n_1} = \frac{x_2}{x_1}\left(\frac{n_2}{n_1}\right)^{\xi}\left(\frac{n_2}{n_1}\right)^{\psi} \qquad \Rightarrow \qquad \frac{n_2}{n_1} = \left(\frac{x_2}{x_1}\right)^{\frac{1}{1-\xi-\psi}}. \tag{40}$$

When using AV's parameterization in Table 6 to evaluate expression (40), a low-productivity firm employs only 3.81 percent as many workers as a high-productivity firm does. Furthermore, since there are equal numbers of the two types of firms, it follows that high-productivity firms account for more than 96 percent of aggregate employment.

7.2 Mapping AV's productivity process into LS

We use two steps to map AV's productivity process into LS. First, for our simplified AV model in the preceding section, we construct a hypothetical wage schedule of a firm that experiences a switch from high to low productivity, but offers all its workers to remain in the firm at a schedule of different pay. Second, we re-interpret that hypothetical wage schedule as a probability distribution of productivities in our matching framework with one-worker firms.

For the first step, consider a high-productivity firm that has just experienced a shock of low productivity, but instead of reducing its employment by $n_2 - n_1$ workers, the firm randomly orders its current employees and offers the following wage schedule. The first n_1 workers are offered the wage rate $w^{\star}$, i.e., the market-determined wage rate that all firms pay to their workers and n_1 is the employment level of other low-productivity firms. Then, under a pledge to keep the capital-labor ratio unchanged, the firm offers each successive worker in the randomly arranged order a wage equal to her marginal product. Thus, the wage offered to the worker in position $n \in (n_1, n_2]$ is given by

$$\begin{aligned}\psi\, x_1\, k^\xi\, n^{\psi-1} &= \psi\, x_1\, k^\xi\, n^{\psi-1} \frac{w^\star}{\psi\, x_2\, k_2^\xi\, n_2^{\psi-1}} = \frac{x_1\, \left(\frac{k}{n}\, n\right)^\xi\, n^{\psi-1}}{x_2\, \left(\frac{k_2}{n_2}\, n_2\right)^\xi\, n_2^{\psi-1}}\, w^\star \\ &= \frac{x_1}{x_2}\left(\frac{n}{n_2}\right)^{-(1-\xi-\psi)} w^\star \;\equiv\; \Gamma_{w^\star}\left(\frac{n}{n_2}\right) \qquad \text{for } \frac{n}{n_2} \in \left(\frac{n_1}{n_2}, 1\right], \qquad (41)\end{aligned}$$

where the first equality multiplies and divides by the same quantity $w^\star$ while in the denominator imposing that $w^\star$ equals the marginal product of labor in a high-productivity firm, as given by expression (35), and the third equality uses the firm's pledge to keep the capital-labor ratio unchanged; hence, in the numerator and denominator the capital-labor ratio cancels.

The search frictions that workers face in a search-island model would make some workers in our simplified AV model choose to accept wage offers below $w^\star$. But under AV's parameterization, the vast majority would decline such offers and instead enter the pool of unemployed. However, for our purposes, it is useful to proceed as if all workers choose to remain with the firm. Since the argument of wage schedule $\Gamma_{w^\star}(n/n_2)$ is employment position n relative to the employment level of a high-productivity firm, the inverse function $\Gamma_{w^\star}^{-1}(w)$ gives the fraction of workers earning a wage greater than or equal to w and hence, the fraction of workers earning less than or equal to w is given by

$$F_{w^\star}(w) = 1 - \Gamma_{w^\star}^{-1}(w) = 1 - \left[\frac{x_1 w^\star}{x_2 w}\right]^{\frac{1}{1-\xi-\psi}} \qquad \text{for } = w \in \left[\frac{x_1 w^\star}{x_2},\, w^\star\right), \qquad (42)$$

and the fraction of workers at the mass point $w = w^\star$ is equal to

$$1 - \lim_{w \to w^\star} F_{w^\star}(w) = \Gamma_{w^\star}^{-1}(w^\star) = \left[\frac{x_1}{x_2}\right]^{\frac{1}{1-\xi-\psi}} \qquad (43)$$

which is indeed the same as the equilibrium value of n_1/n_2 in expression (40).

In the second step of our mapping of AV into LS, we re-interpret the shocks of AV as follows. AV's probability η that a firm dies becomes our probability ρ^x of an exogenous breakup. AV's probability $1 - \omega$ that a firm receives a productivity shock becomes our probability γ^s that a productivity switch hits a continuing firm-worker match. At such a switch, a new productivity z is now drawn from a skill-specific distribution $F_{z_i^{max}}(z)$ where $i = l$ and $i = h$ for a low-skilled and a high-skilled worker, respectively, with cumulative density

$$F_{z_i^{max}}(z) = 1 - \Gamma^{-1}_{z_i^{max}}(z) = 1 - \left[\frac{x_1\, z_i^{max}}{x_2\, z}\right]^{\frac{1}{1-\xi-\psi}} \qquad \text{for } = z \in \left[\frac{x_1\, z_i^{max}}{x_2},\, z_i^{max}\right), \tag{44}$$

and the probability of mass point $z = z_i^{max}$ is given by expression (43). We take AV's variable $w^{\star}$ as the upper bound z_i^{max} of our skill-specific productivity distribution. It is a rather direct analogue to the above hypothetical wage schedule in the simplified AV model, but instead of workers being randomly assigned along a wage offer schedule, continuing firm-worker matches in LS draw productivities from a corresponding distribution. In accordance with AV and similar to MP in the preceding section, the productivity of a newly formed firm-worker match is equal to the upper support of the productivity distribution.

Figure 9: AV productivity distributions

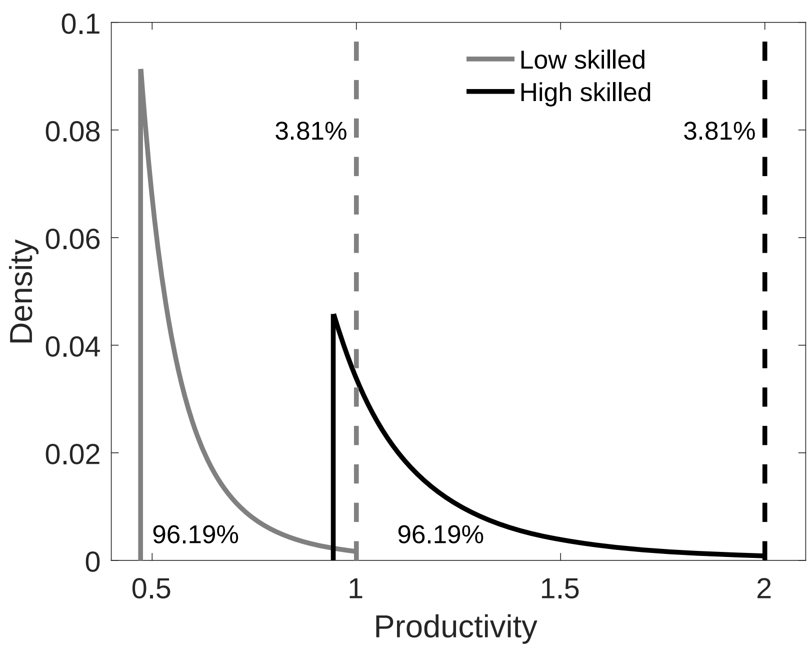


Figure 9 depicts the densities of our two skill-specific productivity distributions when blending AV's parameterization in Table 6 with the assumption of LS and DHHR that a low-skilled worker has half the earnings potential of a high-skilled worker, $z_l^{max} = 1$ and and $z_h^{max} = 2$. The shape of a density reflects the concavity of AV's production function. In particular, since we imposed a constant capital-labor ratio in the employment perturbations away from an efficient level of operation, the concavity of a firm's output with respect to employment arises from AV's assumption of decreasing returns to scale. The lowest productivity of a distribution in Figure 9 reflects an excessively high employment level of a firm that has not shed its labor force after switching from high to low productivity. Hence, the excessively high employment is far up on a flattening concave production function where a rather small increase in the marginal product of labor would be associated with a relatively long journey down the production surface to significantly lower employment levels that explains the high densities at those low productivities. The reasoning is the opposite for productivities just below the efficient employment level, where the steeper curvature of the concave production function means that a small increase in the marginal product of labor does not have much of an associated change in employment, providing the low densities at high productivities just below the efficient level. The mass point at the upper support reflects that all workers employed at that efficient level are paid the marginal product of labor evaluated at that efficient employment level.

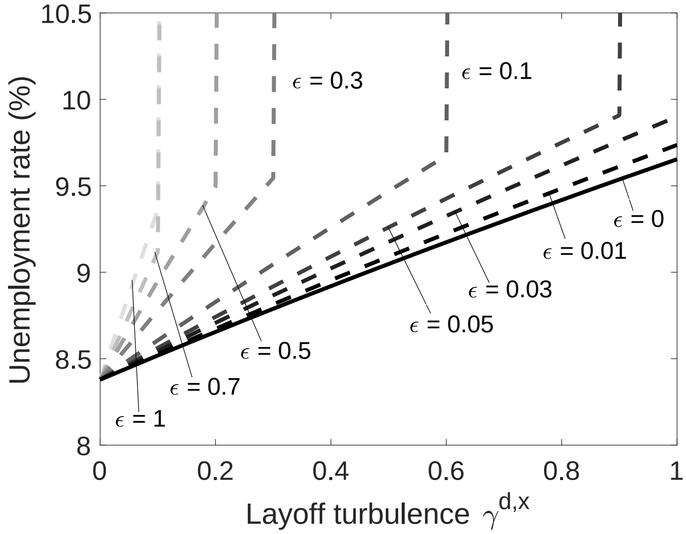
7.3 Turbulence under AV productivity process

As in section 6.3, we import the AV productivity process into the baseline LS model to study how unemployment respond to turbulence. Thus, we adopt the AV productivity process as parameterized in Table 6 with the adapted productivity distribution in expression (44) while keeping the rest of the parameterization of the baseline LS model in Tables 1 and 2, except for the matching efficiency A that we calibrate to target a laissez-faire unemployment rate of 5 percent in tranquil times.

The turbulence outcomes under the AV productivity process in Figure 10 resemble those under the versions of the MP process in the top two panels of Figure 8 and indicate a strong positive relationship between unemployment and turbulence. Actually, the relationship is even stronger under the AV productivity process given the functional form of the AV probability distribution with densities depicted in Figure 9. That functional form reflects AV's underlying growth model as mirrored in its neo-classical production function. The theoretical structure makes it difficult to imagine how any plausibly parameterized quit turbulence could ever suppress the strong forces for reallocation of workers across establishments that are present in the AV model.

The establishment data on firm and worker turnover (Davis and Haltiwanger, 1990) that AV use to calibrate their model, and data sets from other countries, provide overwhelming empirical evidence of extensive reallocations across diverse market economies with different types of government policies. Our present study of the consequences of alternative labor productivity processes in macro-labor models conveys a message consistent with that evidence: explaining observations on firm turnover, labor mobility, and prevalent government policies that aim to arrest firm-worker separations requires theoretical constructs calibrated with ample returns to labor mobility. Quantitative models that have meager returns to labor mobility cannot explain these observations. For example, as pointed out in section 5.2, returns to labor mobility in the

Figure 10: Turbulence with AV productivity



laissez-faire economy of the DHHR model are so low that there are no quits in equilibrium.

8 Hearing Alan Greenspan

In light of our findings about the effects of recalibrating quit turbulence in order to respect the Weinberg constraint, we rejoin and extend the conversation with Alan Greenspan with which DHHR began their paper. In the passage that DHHR cite, reproduced in section 1 above, Greenspan does indeed seem to be concerned with the DHHR's quit turbulence force as well as the sort of effect in reducing job mobility that comes with DHHR's calibration. But if we listen to all that seems to have been on Greenspan's mind, we hear that Greenspan did not emphasize such possible effects of increased turbulence more broadly. To the contrary, earlier in the very same paragraph cited by DHHR, Greenspan (1998, p. 743) says that it is not lower but higher labor mobility (i.e., "churning") that is mainly on his mind:

> "... the perception of increased churning of our workforce in the 1990s has understandably increased the sense of accelerated job-skill obsolescence among a significant segment of our workforce, especially among those most closely wedded to older technologies. The pressures are reflected in a major increase in on-the-job training and a dramatic expansion of college enrollment, especially at community colleges. As a result, the average age of full-time college students has risen dramatically in recent years as large numbers of experienced workers return to school for skill upgrading."

We read Greenspan as writing here about US workers who have been hit by the type of human capital destruction shock that LS used to capture increased turbulence. Greenspan says that those workers have ways of rebuilding their human capital in addition to the ways that are open to them in the LS model, thereby suggesting interesting ramifications of increased turbulence for other observations not on the table in either the DHHR or the LS framework. It would be interesting to add such activities to the model environment that succeeded in explaining trans-Atlantic unemployment experiences, while playing by Weinberg's rules.

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A Equilibrium computation

A.1 General algorithm structure

Here we outline the structure of the algorithm that we used to compute equilibria.[22] It centers around approximating the joint continuation values $g_i(z)$ by using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.

2. Guess initial values for:

   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - ω^f: firms' outside value (in LS, $\omega^f = 0$)
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers

3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.

4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).

5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:

$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}}^{\infty} s_{ij}^o(y)\ dv_i(y).$$

6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1\ \ z]$.

7. Update the value of posting a vacancy, market tightness, and matching probabilities:

   - under LS's endogenous market tightness,

$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u};$$

[22] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

- under DHHR's exogenous market tightness, compute

$$\omega^f = \frac{\beta}{1-\beta} A(1-\pi)\bar{s}, \qquad \theta = 1, \qquad \lambda^w = A, \qquad \lambda^f_{ij} = A\frac{u_{ij}}{u}.$$

8. Update values ω^w_{ij} of being unemployed using (11) and (12).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned} \Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s \nu_{ll}\right\} e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{ll})u_{ll} \end{aligned} \tag{A.1}$$

$$\begin{aligned} \Delta u_{lh} &= (1-\rho^r)\left\{\rho^x \gamma^{d,x} e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^{d,x}) e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \end{aligned} \tag{A.3}$$

$$\begin{aligned} \Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\left\{(1-\nu^o_{ll})u_{ll} + (1-\nu^o_{lh})u_{lh}\right\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s \nu_{ll}]e_{ll} \end{aligned} \tag{A.4}$$

$$\begin{aligned} \Delta e_{hh} &= (1-\rho^r)\left\{\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s \nu_{hh}]e_{hh} \end{aligned} \tag{A.5}$$

   These expressions embed LS's assumption of immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see the corresponding expressions for worker flows in den Haan *et al.* (2005, appendix A).

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, in order to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (27), respectively.

11. Adjust tax rate τ in (27) to balance government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to 2 and use as guesses the last values computed.

A.2 Average wages and productivities

The following computations refer to the LS model with immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see den Haan *et al.* (2005, appendices A–C).

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p^o_{lh}(z) > p^o_{ll}(z) = p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} = \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll}+e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty}\left[\frac{e'_{ll}}{e_{ll}}p^o_{lh}(y)+\frac{e''_{ll}}{e_{ll}}p_{ll}(y)\right]\frac{dv_l(y)}{1-v_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty} p_{ll}(y)\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}}\int_{\underline{z}^o_{lh}}^{\infty} y\,\frac{dv_l(y)}{1-v_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty} y\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed. For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$
\begin{aligned}
e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\
e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}.
\end{aligned}
$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$
\begin{aligned}
\bar{p}_h &= \int_{\underline{z}^o_{hh}}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p^o_{hh}(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y)\right] \frac{dv_h(y)}{1-v_h(\underline{z}^o_{hh})} + \frac{e_{hh}-e'_{hh}-e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y)\, \frac{dv_h(y)}{1-v_h(\underline{z}_{hh})} \\
\bar{z}_h &= \frac{e'_{hh}+e''_{hh}}{e_{hh}} \int_{\underline{z}^o_{hh}}^{\infty} y\, \frac{dv_h(y)}{1-v_h(\underline{z}^o_{hh})} + \frac{e_{hh}-e'_{hh}-e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y\, \frac{dv_h(y)}{1-v_h(\underline{z}_{hh})}.
\end{aligned}
$$

B Two acquitted suspects

B.1 First suspect: Exogenous market tightness

The first candidate explanation concerns differences in the matching process. In the LS model, market tightness is endogenously determined by a typical free entry of firms assumption. The equilibrium zero-profit condition in vacancy creation pins down market tightness. In contrast, DHHR assume fixed and equal masses of workers and firms so that market tightness is exogenously always equal to one.

Perturbation exercise As described above, our renormalization of parameters (A,μ) in the original LS model yields equilibrium market tightness equal to one at zero turbulence. Our first perturbation exercise is to keep market tightness constant at one as we turn up turbulence. We do that by subsidizing vacancy creation so that the value of a firm posting a vacancy is zero, $w^f=0$, at market tightness equal to one for any given levels of layoff and quit turbulence. The vacancy subsidies are financed with lump-sum taxation so that government budget constraint (27) is unaffected.

In this exercise where subsidies are used to keep $w^f=0$ at $\theta=1$, let $\bar{S}^o(\gamma^{d,x},\epsilon)$ denote the expected match surplus of a vacancy encountering an unemployed worker, given layoff turbulence $\gamma^{d,x}$ and quit turbulence $\gamma^d = \epsilon\gamma^{d,x}$:

$$\bar{S}^o(\gamma^{d,x}, \epsilon) \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\, dv_i(y) \tag{B.6}$$

where unemployment u_{ij}, reservation productivity $\underline{z}_{ij}^o$, and match surplus $s_{ij}^o(y)$ are understood to be equilibrium values under our particular perturbation exercise.

At zero turbulence, the operation of the subsidy scheme would not require any payments of subsidies because we have parameterized the matching function so that equilibrium market tightness is then $\theta = 1$, a value of θ at which the zero-profit condition in vacancy creation is satisfied, $w^f = 0$, and by equation (15):

$$\mu = \beta(1-\pi)m(1)\bar{S}^o(0,0). \tag{B.7}$$

When turbulence is turned on, market tightness would have fallen if it were not for the subsidies to vacancy creation. The subsidy rate makes up for the shortfall of $\beta(1-\pi)m(1)\bar{S}^o(\gamma^{d,x}, \epsilon)$ when compared to the investment of incurring vacancy posting cost μ:

$$1 - subsidy(\gamma^{d,x}, \epsilon) = \frac{\beta(1-\pi)m(1)\bar{S}^o(\gamma^{d,x}, \epsilon)}{\mu} = \frac{\bar{S}^o(\gamma^{d,x}, \epsilon)}{\bar{S}^o(0,0)} \tag{B.8}$$

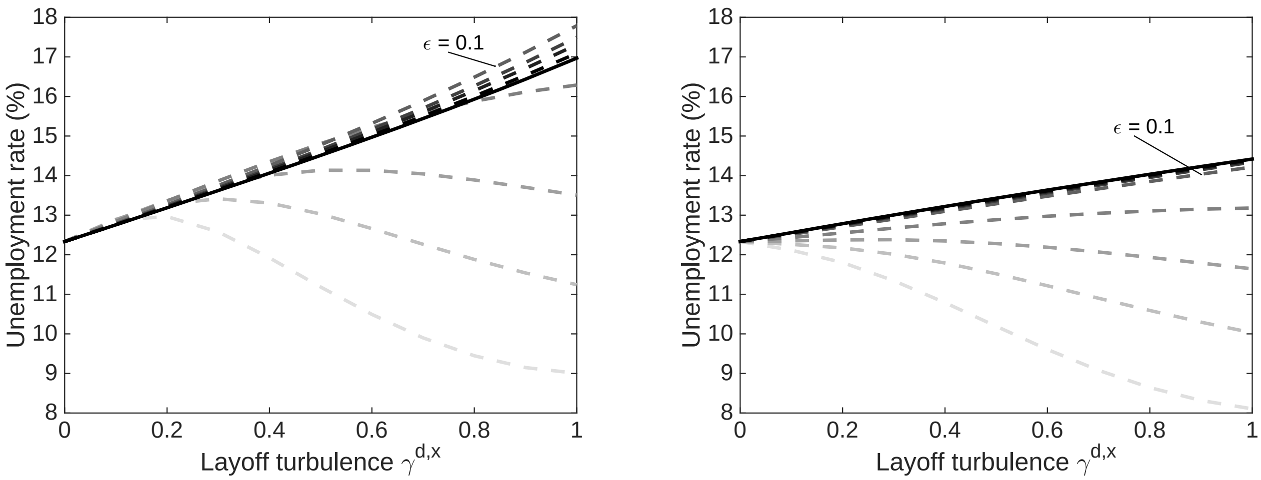
where the second equality invokes expression (B.7).

Results We observe an overall suppression of unemployment rates in Figure B.1b as compared to Figure B.1a. However, the underlying pattern of unemployment dynamics remains intact, so exogenous market tightness does not explain the puzzle.

Discussion: Disabling the invisible hand With endogenous market tightness, there is a dramatic decline in market tightness in response to turbulence in Figure B.2a. This outcome reflects how an "invisible hand" restores firm profitability so that vacancy creation breaks even. Lower market tightness decreases the probability that a worker encounters a vacancy, which tends to increase unemployment.

Our perturbation exercise disarms those forces by exogenously freezing market tightness at one. Hence, the profitability of vacancies plummets in response to turbulence. Figure B.2b plots the subsidy rate for vacancy costs needed to incentivize firms to post enough vacancies to keep market tightness constant at one. At higher levels of turbulence, the subsidy rate becomes quite substantial. The subsidies to vacancy creation contribute to lower unemployment rates. These considerations seem to enhance a suspicion that exogenous market tightness could be the culprit behind the puzzle, so the above vindication was not a foregone conclusion.

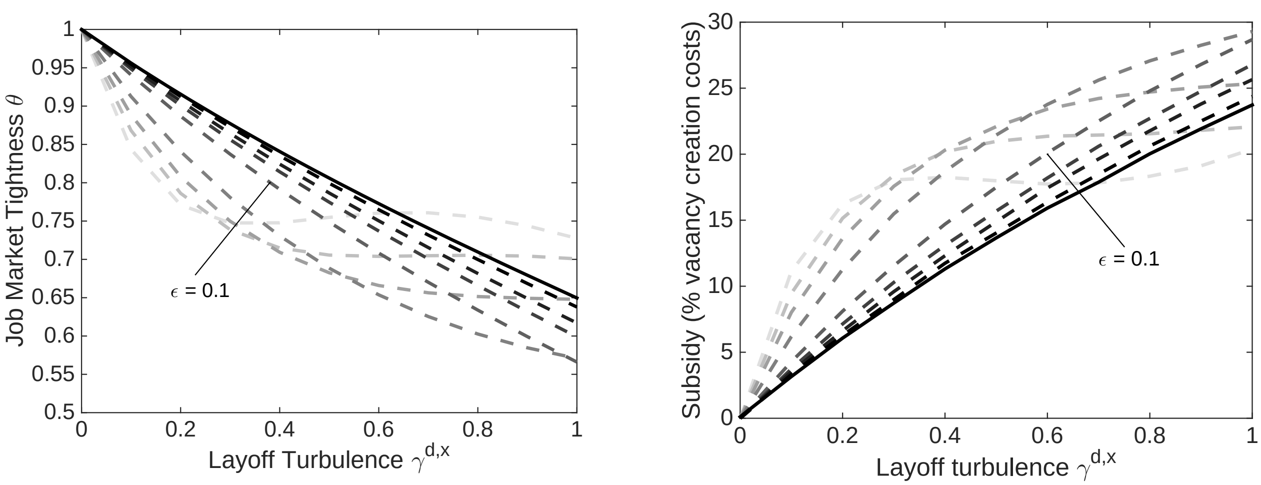
Figure B.1: Endogenous vs. exogenous market tightness in LS



(a) Baseline LS (Endog. market tightness)

(b) LS + Exogenous market tightness

Figure B.2: Falling market tightness vs. subsidies for vacancy creation



(a) Baseline LS (Endog. market tightness)

(b) LS + Exogenous market tightness

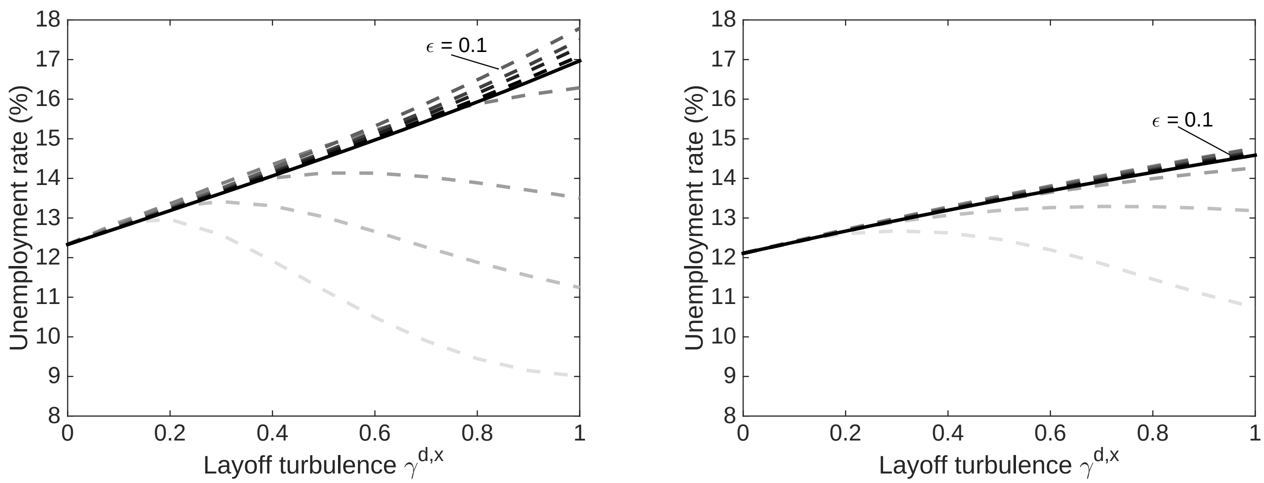
B.2 Second suspect: Timing of completion of skill upgrades

The second candidate explanation concerns differences in the timing of completion of skill upgrades. LS assume that skill upgrades are immediately realized while DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Perturbation exercise We replace immediate realization of skill upgrades in the LS model with delayed completion as in the DHHR model. The change in timing substantially alters the relative bargaining strengths of a worker and a firm.

Results The quantitative outcome in Figure B.3b is similar to that of the preceding perturbation exercise in Figure B.1b, i.e., it leads to an overall suppression in unemployment rates but without altering the underlying pattern of unemployment dynamics and hence, different timing of completion of skill upgrades does not explain the puzzle.

Figure B.3: Immediate vs. delayed completion of skill upgrade in LS



(a) Baseline LS (Immediate upgrade)

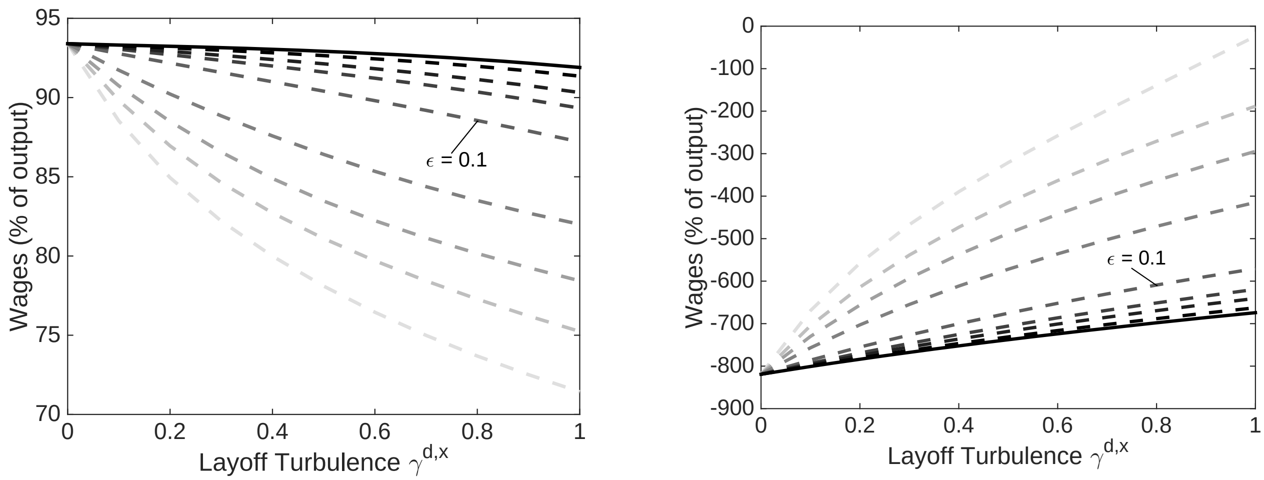
(b) LS + Delayed upgrade

Discussion: Delayed completion requires "ransoms" Firms under DHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work. This is possible because the realization of that higher skill level is conditional upon a worker remaining with the present employer for at least one more period, during which the worker can be assessed a "ransom" to secure her human capital gain.

We compare average wages at skill upgrades under immediate completion (Figure B.4a) and delayed completion (Figure B.4b), expressed in terms of average output per worker in the

LS laissez-faire economy at zero turbulence.[23] In Figure B.4b, a worker pays the "ransom" in terms of a negative semi-quarterly wage in the period of a skill upgrade, equivalent to the average annual output of a worker. The "ransom" becomes smaller with higher turbulence since the capital value of a skill upgrade is worth less when it is not expected to last long, as well as when quit turbulence locks high-skilled workers into employment relationships and thereby causes a less efficient allocation: fearing skill loss at separations, high-skilled workers accept lower reservation productivities and hence, work on average at lower productivities as compared to an economy in tranquil times with higher labor mobility.

Figure B.4: Average wage in period of skill upgrade



(a) Baseline LS (Immediate upgrade)

(b) LS + Delayed upgrade

C Starting from DHHR framework

We now reverse the analysis by starting from the DHHR framework and investigating the consequences of three perturbations. The features in the original DHHR framework to be perturbed are (i) exogenous labor market tightness, (ii) delayed completion of skill upgrade, and (iii) uniform productivity distributions with narrow support. But before that, we eliminate two auxiliary assumptions in the DHHR analysis.

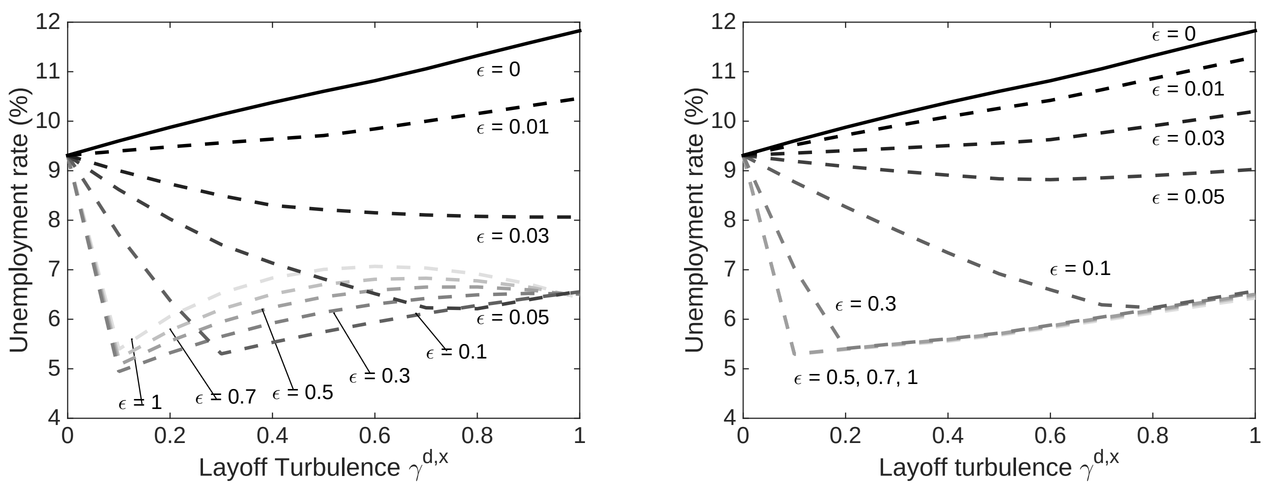
Eliminate auxiliary assumption of zero benefits for newborn workers Instead of DHHR's assumption of no benefits during the initial unemployment spells of newborn workers,

[23] In the LS laissez-faire economy without a government, a worker's average semi-quarterly output is 2.3 goods in tranquil zero-turbulence times.

we assume that they are eligible for unemployment benefits equivalent to those of low-skilled workers. This change has hardly any effect on aggregate outcomes.

Eliminate auxiliary assumption of turbulence for unemployed DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she had instead quit a job. DHHR describe this as an auxiliary assumption that they justify in terms of its computational tractability, but we find that it has noticeable quantitative consequences. Thus, figure C.1 presents outcomes for the original DHHR framework with turbulence for unemployed workers and our baseline DHHR model without that kind of turbulence. While the outcomes are not as stark in latter model, the underlying pattern of unemployment dynamics remains intact – it just takes some more quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship.

Figure C.1: WITH VS. WITHOUT TURBULENCE FOR UNEMPLOYED IN DHHR



(a) Original DHHR (Turbulence unemployed) (b) Baseline DHHR (No turbulence unempl.)

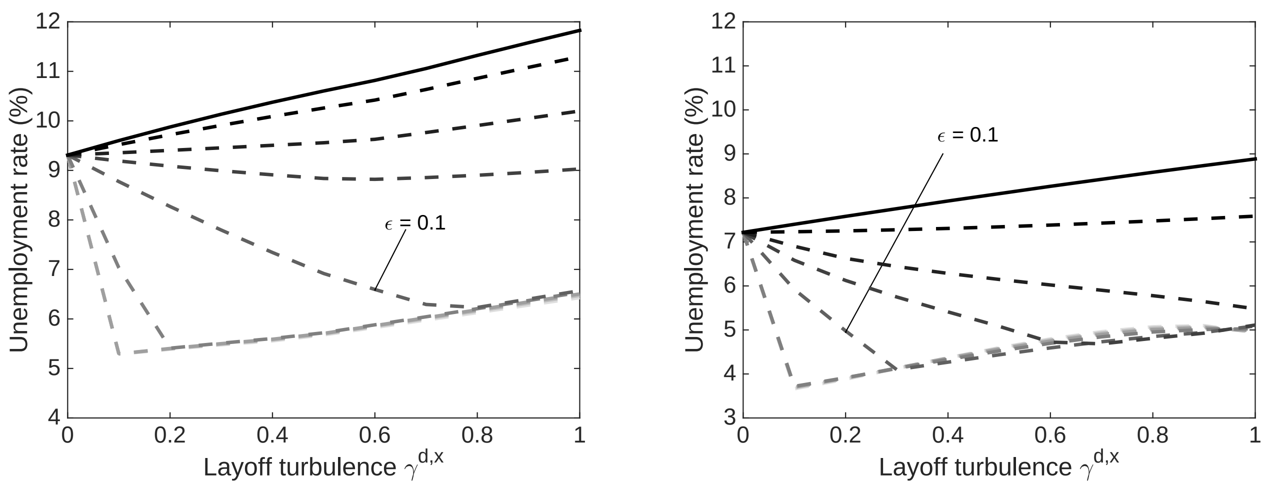
An assumption that mere encounters between vacancies and unemployed workers are associated with risks of losing skills unless employment relationships are formed directly suppresses returns to labor mobility. But as can be inferred from Figure C.1, such an exposure of job seekers to skill loss does not have much impact on unemployment outcomes since, as Appendix C.3 will teach us, compressed productivity distributions in DHHR already reduce returns to labor mobility. However, the substantial incentives for labor mobility under LS's parameterization of productivity distributions are significantly affected and suppressed by that auxiliary assumption of DHHR. Appendix D discusses this in detail.

C.1 First suspect: Exogenous market tightness

Perturbation exercise In the DHHR framework, there is an exogenous mass of firms and there are no costs for posting vacancies. Hence the value w^f of a firm posting a vacancy is trivially positive. We now perturb DHHR to feature free entry of firms, $w^f = 0$ in equilibrium, and an endogenous market tightness determined by (15). In order to implement that perturbation, we must introduce and assign values to two additional parameters, α and μ. Following LS, we assume that the elasticity of the matching function with respect to unemployment equals $\alpha = 0.5$, a fairly common parameterization.

Lacking an obvious way to parameterize the vacancy posting cost μ in this perturbation, we solve the model for different values of $\mu > 0$.[24] We find that for values of μ above 0.7, all voluntary quits vanish. Therefore, since DHHR's challenge to a Ljungqvist-Sargent positive turbulence-unemployment relationship is based on changes in the incidence of quits, we consider $\mu \in (0, 0.7)$ to be the permissible range. As an illustration, Figure C.2b depicts equilibrium outcomes for the midpoint of that parameter range, $\mu = 0.35$.

Figure C.2: Exogenous vs. endogenous market tightness in DHHR



(a) Baseline DHHR (Exog. market tightness) (b) DHHR + Endogenous market tightness

[24] The vacancy posting cost μ must be positive to have an equilibrium with free entry of firms. The discrete model period and the Cobb-Douglas matching function call for an additional caveat. As the value of μ approaches zero, the equlibrium probability of filling a vacancy goes to zero. That creates a problem when the associated probability of a worker encountering a vacancy exceeds the permissible value of unity. Therefore, we only compute equilibria for μ greater than 0.0063. If one would like to compute equilibria for lower values of μ, it could be done by augmenting the match technology to allow for corner solutions at which the short end of the market determines the number of matches; e.g., in the present case, by freezing the job finding probability at unity while randomly allocating the unemployed across all vacancies that draw an "encounter." (See Ljungqvist and Sargent (2007, section 7.2).)

Results Except for the very top end of the parameter range $\mu \in (0, 0.7)$, the qualitative pattern of Figure C.2 represents the unemployment-turbulence relationship for the DHHR framework under the two alternative matching assumptions. In both cases, rather small amounts of quit turbulence reduce unemployment. Therefore, exogenous versus endogenous market tightness does not explain the puzzle.

In the vicinity of parameter value $\mu = 0.7$, the curve for $\epsilon = 0.1$ in the corresponding version of Figure C.2b (not shown here) takes on a positive slope, i.e., outcomes become LS-like with a positive turbulence-unemployment relationship. This might have been anticipated. As mentioned above, $\mu = 0.7$ is also the parameterization at which all voluntary quits vanish, which would seem to disarm the DHHR quit turbulence argument.[25]

Incidentally, as we will learn in Appendix C.3, the raw fact that voluntary quits vanish at a relatively low value of the vacancy posting cost $\mu = 0.7$ is indicative of low returns to labor mobility in the DHHR model that come from compressed productivity distributions.

C.2 Second suspect: Timing of completion of skill upgrades

Perturbation exercise DHHR assume that after a skill upgrade a worker must remain with the present employer for one period in order to complete the higher skill level. In this section, we introduce immediate completion of skill upgrades as in LS.

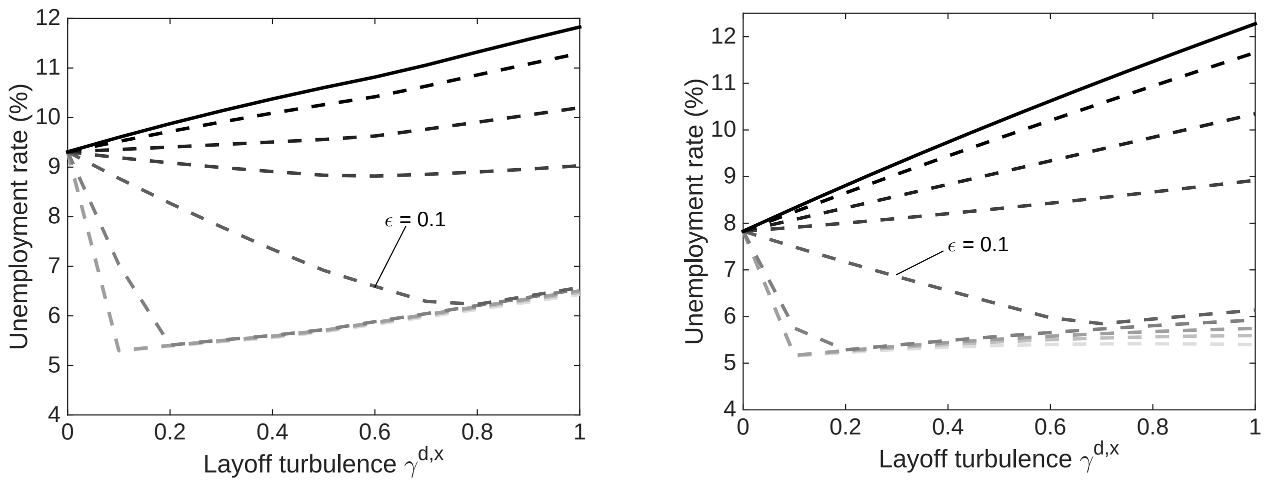
Results Figure C.3 shows that there is no substantial difference in the turbulence-unemployment relationship for the alternative timings in the DHHR model. Hence, delayed versus immediate completion of skill upgrades does not explain the puzzle.

C.3 Third suspect: Productivity distributions

Perturbation exercise DHHR assume uniform distributions with narrow support: $z_l \sim \mathcal{U}([0.5, 1.5])$ and $z_h \sim \mathcal{U}([1.5, 2.5])$. In this section we replace those distributions in the DHHR model by the truncated normal distributions assumed by LS: $z_l \sim \mathcal{N}(1, 1)$ for low-skilled workers over the support $[-1, 3]$, and $z_h \sim \mathcal{N}(2, 1)$ for high-skilled workers over the support $[0, 4]$. Notice the big difference is the dispersion in productivities: the standard deviation is $1/\sqrt{12}$ in the uniform case of DHHR and 1 in the normal case of LS.

[25] For a more nuanced reasoning about the equilibrium forces at work under the threat of losing skills in a matching model, see the discussion of an "allocation channel" and a "bargaining channel" in section D.2. While that section pertains to the introduction of turbulence facing unemployed workers in terms of a risk of losing skills after an encounter between a firm and a worker that does not result in employment, similar reasoning can be applied to quit turbulence for employed workers.

Figure C.3: Delayed vs. immediate completion of skill upgrade in DHHR



(a) Baseline DHHR (Delayed upgrade)

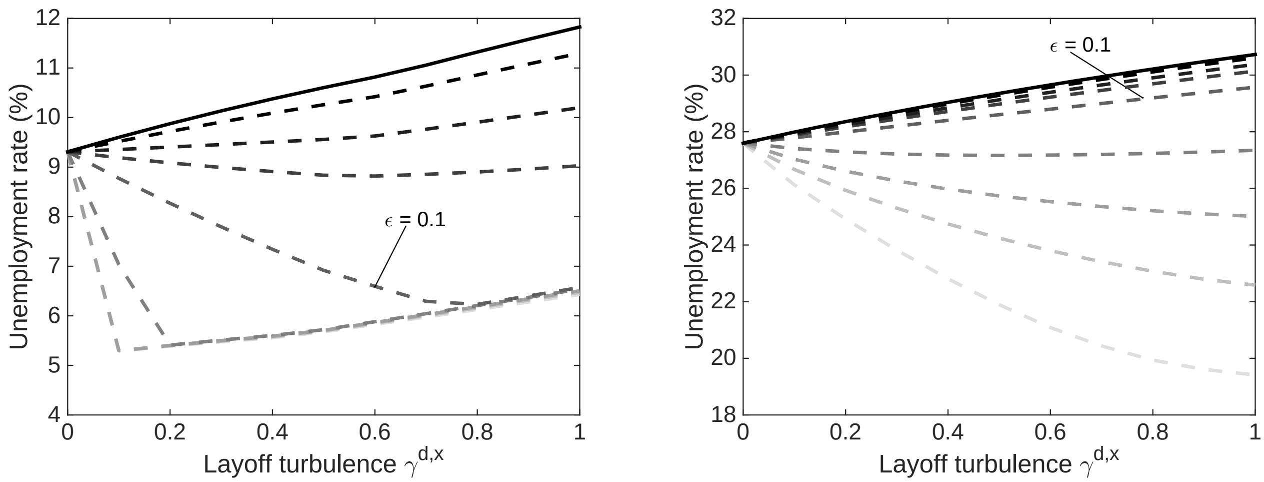
(b) DHHR + Immediate upgrade

Results Figure C.4 shows how the turbulence-unemployment relationship is altered in the DHHR model when we switch from DHHR's productivity distributions to those of LS. First, the larger variances of the LS distributions exert upward pressures on reservation productivities and labor reallocation rates, but DHHR's assumption that an exogenously given market tightness equals one means that the relative number of vacancies cannot expand, so overall unemployment rates become higher. Second, and critical to our inquiry, the inference to be drawn from Figure C.4 agrees with what we inferred after studying the obverse perturbation of the LS model in Figure 3; namely, differences in productivity distributions are key to explaining the puzzle. When we import the LS distributions into the DHHR model, small amounts of quit turbulence no longer unduly dissuade high-skilled workers with poor productivity draws to quit and seek better employment opportunities. Hence, the present perturbation disarms DHHR's argument for suppressed quit rates and allows the Ljungqvist-Sargent turbulence force to operate unimpeded. The right panel of Figure C.4 shows how turbulence and unemployment are positively related until quit turbulence reaches about 30% of layoff turbulence after which the relationship becomes negative.

D Turbulence affecting job market encounters

DHHR assume that after an encounter between a firm and an unemployed worker that does not result in employment, the worker faces the same risk of losing skills as if she had quit from a job. They justify this assumption only for its tractability in allowing them to reduce the number of worker types that they must track. In Figure C.1 of Appendix C, we confirm that

Figure C.4: Narrow vs. wide support of productivity distributions in DHHR



(a) Baseline DHHR (Narrow uniform)

(b) DHHR + Wide normal

the assumption does not make much of a difference for DHHR's inference about the turbulence-unemployment relationship in their model. But when we pursue a parallel analysis in the LS model as we do here, we find that DHHR's simplifying assumption has a large impact. We show this in subsection D.1. To shed light on the forces at work, subsection D.2 undertakes yet another perturbation exercise that limits the exposure to such risk to the first $\bar{k}$ periods of an unemployment spell, after which there is no risk of skill loss during the rest of an unemployment spell.

To allow for a more general formulation, we assume a distinct probability γ^e of skill loss after an unsuccessful job market encounter, while γ^d continues to denote the probability of skill loss when quitting from an employment relationship.

D.1 Introducing turbulence for unemployed workers in LS

When unemployed high-skilled workers face a probability γ^e of losing skills after unsuccesful job market encounters, the match surplus in (3) of a new job with a high-skilled worker changes to

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^e)\omega_{hh} + \gamma^e\omega_{lh}], \tag{D.9}$$

where the outside value in brackets reflects the risk of skill loss if the firm and worker do not enter an employment relationship. The net change of the mass of low-skilled unemployed with high benefits in (28) changes to

$$\Delta u_{lh} = (1-\rho^r)\Bigg\{ \underbrace{\rho^x \gamma^{d,x} e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^d \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} + \underbrace{\lambda^w(\theta)\gamma^e \nu^o_{hh} u_{hh}}_{\text{4. turbulence unempl.}} \Bigg\} - \rho^r u_{lh}, \tag{D.10}$$

where the new term numbered 4 is the inflow of unemployed high-skilled workers who have just lost their skills after job market encounters that did not lead to employment.

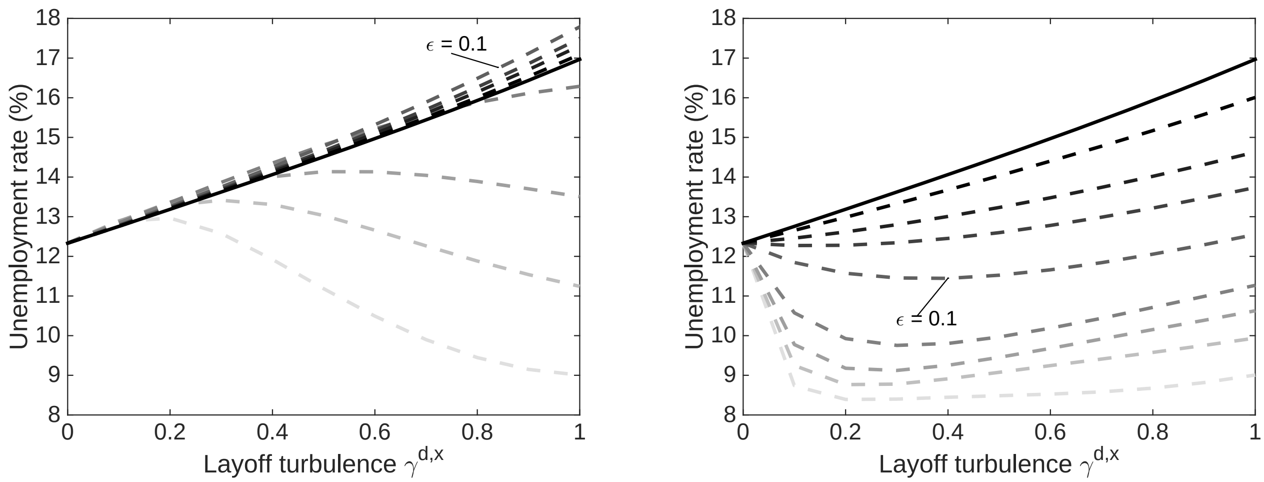
Turning to a quantitative assessment of turbulence for unemployed workers in the LS model, we have to take a stand on the different lengths of a model period used in the parameterizations of LS and DHHR. In the case of the exogenously given layoff risk, the probability of a layoff at the semi-quarterly frequency in LS's model is half of the probability at the quarterly frequency in DHHR's model, as discussed in footnote 10. Analogously, but less obviously, for the risk of skill loss after endogenously determined unsuccessful job market encounters we assume that $\gamma^e = 0.5\gamma^d$ in LS's semi-quarterly model as compared to DHHR's assumption that $\gamma^e = \gamma^d$ in their quarterly model. However, for the record, our conclusion from Figure D.1 remains the same with or without the latter adjustment. That is, with or without this adjustment, adding exposure of unemployed workers to risks of skill loss after unsuccessful job market encounters has sizeable effects on the turbulence-unemployment relationship in the LS model.

As discussed in footnote 3, risk of skill loss after unsuccessful job market encounters was not part of DHHR's use of quit turbulence to challenge a Ljungqvist-Sargent positive turbulence-unemployment relationship. Rather, they adopted it for computational tractability. Hence, we feel justified in discarding this auxiliary feature of DHHR's original analysis in order to focus more sharply on the key explanation to the puzzle – different productivity distributions. But it is nevertheless tempting to turn on and off their auxiliary assumption in order to shed further light on the mechanics of our particular matching model, and matching frameworks more generally. Therefore, we offer the following suggestive decomposition of forces at work.

D.2 Decomposition of forces at work

We seek to isolate two interrelated forces acting when job seekers are exposed to risk of skill loss after unsuccessful job market encounters in a matching model. First, the mere risk of losing skills when turning down job opportunities suppresses the return to labor mobility in many frictional models of labor markets, including the basic McCall (1970) search model where wages are drawn from an exogenous offer distribution. Such risks would render job seekers more prone to accept employment opportunities. We call this the "allocation channel." Second, the matching

Figure D.1: Without vs. with turbulence for the unemployed in LS



(a) Baseline LS (No turbulence unempl.)

(b) LS + Turbulence unemployed

framework contains yet another force when risk of skill loss after an unsuccessful job market encounter weakens the bargaining position of a worker vis-à-vis a firm and accordingly affects match surpluses received by firms. That in turn affects vacancy creation via the equilibrium condition that vacancy posting must break even. We call this the "bargaining channel."

It presents a challenge to isolate these two channels because everything is related to everything else in an equilibrium. Here we study how equilibrium outcomes change as we vary the horizon over which the risk of skill loss prevails during an unemployment spell. Thus, after an unsuccessful job market encounter, let an unemployed worker be exposed to risks of skill losses for the first $\bar{k}$ periods of being unemployed and thereafter to suffer no risk of skill loss for the remainder of that unemployment spell. To illustrate the allocation channel, consider the basic McCall search model. Starting from $\bar{k} = 0$, equilibrium unemployment would initially be significantly suppressed for each successive increase in the parameter $\bar{k}$ because workers anticipate ever longer periods of effective exposure to risk of skill loss when unemployed; but eventually, the value of $\bar{k}$ is so high that it is most unlikely that a worker remains unemployed for such an extended period of time and hence, a worker's calculation of the payoff from quitting a job would hardly be affected by any additional increase in $\bar{k}$. Thus, in a McCall search model, via the allocation channel, equilibrium unemployment would hardly change for higher values of $\bar{k}$. In contrast, we will find in the LS matching model that unemployment suppression effects that occur in response to increases in $\bar{k}$ don't die out beyond such high values of $\bar{k}$. We then argue that those equilibrium outcome effects can be attributed to the bargaining channel.

Notation Let u_{hh}^0 denote the mass of high-skilled workers who become unemployed in each period without losing skills, and let u_{hh}^k be the mass of those workers who remain high-skilled and unemployed after an unemployment duration of $k = 1, \ldots, \bar{k}-1$ periods. A final category $u_{hh}^{\bar{k}}$ includes all workers who remain high-skilled and unemployed after unemployment spells of *at least* $\bar{k}$ periods, i.e., $u_{hh}^{\bar{k}}$ is the mass of unemployed high-skilled workers who no longer face any risk of skill loss in their current unemployment spells.

Using the same superscript convention, let $\omega_{hh}^{w,k}$ for $k = 0, \ldots, \bar{k}$ be the future value of unemployment of an unemployed high-skilled worker in category u_{hh}^k, with $\underline{z}_{hh}^k$ and ν_{hh}^k denoting the worker's reservation productivity and rejection probability next period, and for any match accepted next period, the match surplus is $s_{hh}^k(z)$ and the initial wage is $p_{hh}^k(z)$.

Laws of motion The laws of motion for worker categories u_{hh}^k, for $k = 0, \ldots, \bar{k}-1$, have in common that all workers leave the category next period. The inflow to the initial category u_{hh}^0 consists of employed high-skilled workers who experience non-turbulent layoffs or quits, including low-skilled employed workers who have just received a skill upgrade. Each successive category u_{hh}^k, for $k = 1, \ldots, \bar{k}-1$, receives its inflow from not retired workers in the preceding category u_{hh}^{k-1}, those who did not match or experienced non-turbulent rejections of matches:

$$\Delta u_{hh}^k = \begin{cases} (1-\rho^r)\Big[\underbrace{\rho^x(1-\gamma^{d,x})e_{hh}}_{\text{non-turbulent layoff}} + \underbrace{(1-\rho^x)\nu_{hh}(1-\gamma^d)(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{non-turbulent quit}}\Big] \; - \; u_{hh}^k & \text{if } k = 0 \\ \\ (1-\rho^r)\Big[\underbrace{(1-\lambda^w(\theta))}_{\text{no match}} + \underbrace{\lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)}_{\text{non-turbulent rejected match}}\Big]u_{hh}^{k-1} \; - \; u_{hh}^k & \text{if } 0 < k < \bar{k}. \end{cases}$$

The final category $u_{hh}^{\bar{k}}$ also receives inflows from the preceding category $u_{hh}^{\bar{k}-1}$, but now outflows are only partial. The workers who leave are the retirees and those with accepted matches (those with rejected matches are no longer affected by turbulence and thus always remain):

$$\Delta u_{hh}^{\bar{k}} = (1-\rho^r)\Big[(1-\lambda^w(\theta)) + \lambda^w(\theta)\nu_{hh}^{\bar{k}-1}(1-\gamma^e)\Big]u_{hh}^{\bar{k}-1} \; - \; \Big[\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{hh}^{\bar{k}})\Big]\,u_{hh}^{\bar{k}}.$$

The law of motion for u_{lh} workers is modified to receive the inflow from the different u_{hh}^k categories that suffered turbulent rejections in their first $\bar{k}$ periods of unemployment:

$$\begin{aligned} \Delta u_{lh} \;=\; & (1-\rho^r)\Big[\underbrace{\rho^x\gamma^{d,x}e_{hh} + (1-\rho^x)\nu_{hh}\gamma^d(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{turbulent separations}} + \underbrace{\lambda^w(\theta)\gamma^e\sum_{k=0}^{\bar{k}-1}\nu_{hh}^k u_{hh}^k}_{\text{turbulent rejections}}\Big] \\ & - \; [\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu_{lh}^o)]\,u_{lh}. \end{aligned}$$

The law of motion for high-skilled employed workers e_{hh} is adjusted to include those gaining employment from the different u^k_{hh} categories:

$$\begin{aligned}\Delta e_{hh} &= (1-\rho^r)\Big[\underbrace{\lambda^w(\theta)\sum_{k=0}^{\bar{k}}(1-\nu^k_{hh})u^k_{hh}}_{\text{accepted new matches}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}}_{\text{accepted upgrades}}\Big] \\ &- \left[\rho^r + (1-\rho^r)(\rho^x + (1-\rho^x)\gamma^s\nu_{hh})\right]e_{hh}.\end{aligned}$$

High-skilled unemployed: match surplus, initial wage, and value of unemployment For a high-skilled worker who remains unemployed after $k < \bar{k}$ periods, the match surplus of any job opportunity next period reflects an outside option with risk γ^e of losing skills if the employment relationship is not formed; but after $\bar{k}$ periods there is no such risk:

$$s^k_{hh}(z) = \begin{cases} (1-\tau)z + g_h(z) - \left[b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e\omega^w_{lh} + \omega^f\right] & \text{if } k < \bar{k} \\ (1-\tau)z + g_h(z) - \left[b_h + \omega^{w,k}_{hh} + \omega^f\right] & \text{if } k = \bar{k}. \end{cases}$$

Reservation productivities and rejection probabilities satisfy

$$s^k_{hh}(\underline{z}^k_{hh}) = 0\,, \qquad \nu^k_{hh} = \int_{-\infty}^{\underline{z}^k_{hh}} dv_h(y)\,.$$

The wage in the first period of employment of such a high-skilled worker is

$$\begin{aligned} p^k_{hh}(z) + g^w_h(z) &= \pi s^k_{hh}(z) + b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e\omega^w_{lh} \qquad \text{if } k < \bar{k} \\ p^k_{hh}(z) + g^w_h(z) &= \pi s^k_{hh}(z) + b_h + \omega^{w,k}_{hh} \qquad \text{if } k = \bar{k}. \end{aligned}$$

The value of unemployment for a high-skilled worker in his k:th period of unemployment is equal to $b_h + \omega^{w,k}_{hh}$, where

$$\omega^{w,k}_{hh} = \begin{cases} \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty}\pi s^k_{hh}(y)\,dv_h(y)}_{\text{match + accept}} + \underbrace{\lambda^w(\theta)(b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e\omega^w_{lh})}_{\text{outside value with match}} \\ \qquad + \underbrace{(1-\lambda^w(\theta))(b_h + \omega^{w,k+1}_{hh})}_{\text{outside value without match}}\Big] & \text{if } k < \bar{k} \\ \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty}\pi s^k_{hh}(y)\,dv_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega^{w,k}_{hh}}_{\text{outside value}}\Big] & \text{if } k = \bar{k}. \end{cases}$$

High-skilled employed: match surplus, wage, and joint continuation value The match surplus for continuing employment of a high-skilled worker reflects the risk of layoffs and quits that can be affected by turbulence in the form of skill loss. A non-turbulent separation falls into the initial category of high-skilled unemployed, u^0_{hh}. We adjust match surpluses, wages, and joint continuation values of these workers to include the new outside value $\omega^{w,0}_{hh}$.

The match surplus of a continuing job with a high-skilled worker is

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d \omega^w_{lh} + \omega^f]$$

and the wage equals

$$p_{hh}(z) + g^w_h(z) = \pi s_{hh}(z) + b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d \omega^w_{lh}.$$

The joint continuation value of a job with a high-skilled worker is

$$\begin{aligned} g_h(z) &= \beta\Big[\rho^x \left(b_h + (1-\gamma^{d,x})\omega^{w,0}_{hh} + \gamma^{d,x}\omega^w_{lh} + \omega^f\right) \\ &+ (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z)) \\ &+ (1-\rho^x)\gamma^s \left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh} + \omega^f\right)\right)\Big]. \end{aligned}$$

Since a low-skilled worker faces the possibility of a skill upgrade, we also need to update the joint continue value of an employed low-skilled worker as follows:

$$\begin{aligned} g_l(z) &= \beta\Big[\rho^x(b_l + \omega^w_{ll} + \omega^f) \\ &+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z + g_l(z)) \\ &+ (1-\rho^x)(1-\gamma^u)\gamma^s \left(E_{ll} + \nu_{ll}(b_l + \omega^w_{ll} + \omega^f)\right) \\ &+ (1-\rho^x)\gamma^u \left(E_{hh} + \nu_{hh}\left(b_h + (1-\gamma^d)\omega^{w,0}_{hh} + \gamma^d\omega^w_{lh} + \omega^f\right)\right)\Big]. \end{aligned}$$

Vacancy creation Free entry of firms make a firm's value ω^f of entering the vacancy pool be zero. With more types of unemployed high-skilled workers, zero-profit condition (15) changes to become

$$\mu = \beta\frac{m(\theta)}{\theta}(1-\pi)\left[\frac{u_{ll}}{u}\int_{\underline{z}^o_{ll}}^{\infty} s^o_{ll}(y)\ dv_l(y) + \frac{u_{lh}}{u}\int_{\underline{z}^o_{lh}}^{\infty} s^o_{lh}(y)\ dv_l(y) + \sum_{k=0}^{\bar{k}} \frac{u^k_{hh}}{u}\int_{\underline{z}^k_{hh}}^{\infty} s^k_{hh}(y)\ dv_h(y)\right],$$

where $u = u_{ll} + u_{lh} + \sum_{k=0}^{\bar{k}} u^k_{hh}$.

High-skilled unemployment spells terminated within $\bar{k}$ periods In each period, a mass u^0_{hh} of high-skilled workers flows into unemployment. Let $\phi^{\bar{k}}$ denote the fraction of these who will experience unemployment spells of no longer duration than $\bar{k}$ periods. To enable a recursive computation, define m_h^k as the mass of workers who remain high-skilled and unemployed after k periods, and let m_l^k be the accompanying mass that remain unemployed but who have experienced skill loss by that kth period of unemployment. Given initial conditions $m_h^0 = u_{hh}^0$ and $m_l^0 = 0$, we compute

$$\begin{aligned} m_h^k &= (1-\rho^r)\left[1-\lambda^w(\theta)+\lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)\right] m_h^{k-1} \\ m_l^k &= (1-\rho^r)\left[(1-\lambda^w(\theta)+\lambda^w(\theta)\nu_{lh})m_l^{k-1}+\lambda^w(\theta)\nu_{hh}^{k-1}\gamma^e m_h^{k-1}\right], \end{aligned}$$

for $k = 1, \ldots, \bar{k}$;[26] and

$$\phi^{\bar{k}} = \frac{u_{hh}^0 - m_h^{\bar{k}} - m_l^{\bar{k}}}{u_{hh}^0}. \tag{D.11}$$

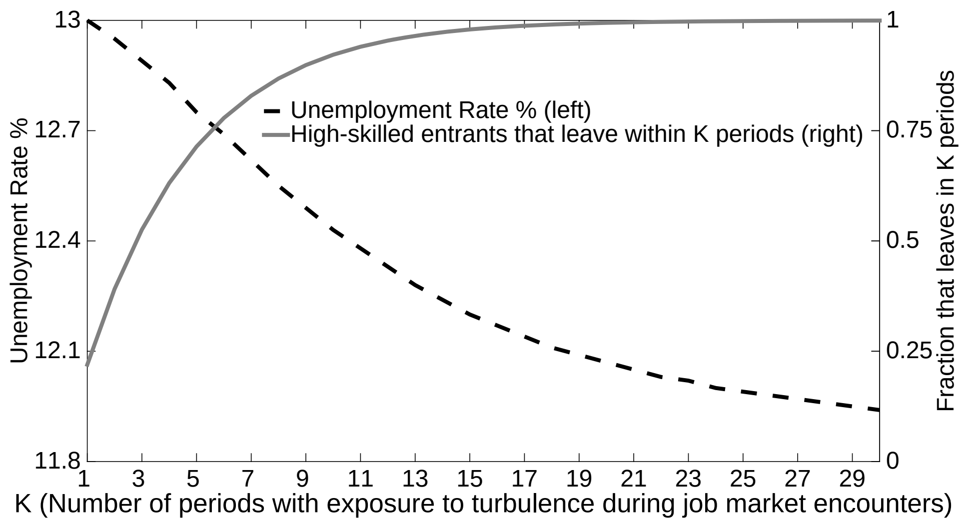
Numerical example To illustrate and decompose the forces at work, we set layoff turbulence equal to $\gamma^{d,x} = 0.2$ and quit turbulence to $\gamma^d = \epsilon\gamma^{d,x} = 0.1 \cdot \gamma^{d,x} = 0.02$. As discussed above, turbulence for unemployed workers in LS's semi-quarterly model is assumed to be half of quit turbulence, i.e., $\gamma^e = 0.5\gamma^d = 0.01$.

Figure D.2 depicts two unemployment outcomes in distinct economies that differ only with respect to the parameter value of $\bar{k}$, i.e., the length of time over which an unemployed worker is exposed to the risk of losing skills due to unsuccessful job market encounters. The two outcomes are the unemployment rate u and the fraction $\phi^{\bar{k}}$ of high-skilled entrants into unemployment who will see their unemployment spells terminated within $\bar{k}$ periods by either finding employment or retiring. For each economy indexed by $\bar{k}$, the value of u can be read off from the dashed line (in percent on the left scale), and $\phi^{\bar{k}}$ from the solid line (as a fraction on the right scale).

As anticipated from our above discussion of the allocation channel, the unemployment rate in Figure D.2 is lower in economies with a higher $\bar{k}$ since longer exposure to risk of skill loss reduces the return to labor mobility. Hence, fewer high-skilled workers quit their jobs, and those who do quit will on average move back into employment more quickly. For example, when $\bar{k}$ increases from 1 to 9, the unemployment rate falls by half a percentage point. As noted earlier, the allocation channel would also be operating in the basic McCall search model, and the unemployment effects of further increases in $\bar{k}$ there should become muted when the value of $\bar{k}$ is set so high that the vast majority of unemployment spells are shorter than $\bar{k}$ in durations. But, as can be seen in Figure D.2 at $\bar{k} = 9$, 90 percent of all unemployment spells by high-skilled entrants are terminated within $\bar{k}$ periods, yet the unemployment rate falls another half a percentage point after further increases in $\bar{k}$. According to our earlier discussion of the bargaining channel, there is a force in matching models that is not present in McCall

[26] Note that $m_h^k = u_{hh}^k$ for $k = 0, \ldots, \bar{k}-1$, while $m_h^{\bar{k}}$ is merely a subset of $u_{hh}^{\bar{k}}$.

Figure D.2: Exposures of unemployed workers in LS to turbulence



models. This other force makes it possible for skill losses at unlikely long unemployment spells to have substantial effects on equilibrium outcomes through its impact on bargaining. The reason is that even though realizations of such long unemployment spells are rare, the extended risk of skill loss will weaken the bargaining position of a worker vis-à-vis a firm throughout an unemployment spell.[27]

Figure D.3 depicts additional statistics that summarize outcomes across alternative values of $\bar{k}$. The positive relationship between $\bar{k}$ and market tightness indicates how the bargaining channel tilts match surpluses to firms when the risk of skill loss after unsuccessful job market encounters weakens the bargaining position of workers. Recall that the equilibrium zero-profit condition for vacancy posting funnels expected present values of firms' match surpluses into vacancy creation. The resulting higher market tightness implies a higher probability that an unemployed worker encounters a vacancy. Evidently, a worker's higher match probability induces low-skilled unemployed workers (as well as employed ones), both those with low and those with high benefits, to choose higher reservation productivities. The net result is still a shorter average duration of unemployment spells. And with not much change in a mildly U-shaped relationship for the job separation rate, we arrive at an unemployment rate that

[27]For another stark example of unlikely events having large effects on equilibrium outcomes through the bargaining channel, see Ljungqvist and Sargent's (2017) analysis of alternating-offer wage bargaining as one way to make unemployment respond sensitively to movements in productivity in matching models. A general result is that the elasticity of market tightness with respect to productivity is inversely related to a model-specific "fundamental surplus," Under alternating-offer bargaining the fundamental surplus is approximately equal to the difference between productivity and the sum of the value of leisure and a firm's cost of delay in bargaining. Thus, the magnitude of the latter cost is a critical determinant of the volatility of unemployment in response to productivity shocks, even though no such cost will ever be incurred because in equilibrium there will be no delay in bargaining.

continues to fall over most of the range in Figure D.2. From these intricacies, we conclude that the bargaining channel already operates in tandem with the allocation channel over the first range of $\bar{k}$ in that figure, but that it operates mostly on its own over the second range where most entrants of high-skilled workers into unemployment expect to terminate their unemployment spells well before $\bar{k}$ periods.

Figure D.3: More statistics pointing to the "bargaining channel"