Cross-Phenomenon Restrictions

Unemployment Effects of Layoff Costs and Quit Turbulence*

Isaac Baley† Lars Ljungqvist‡ Thomas J. Sargent§

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Abstract

Cross-phenomenon restrictions associated with returns to labor mobility can inform calibrations of productivity processes in macro-labor models. We exploit how returns to labor mobility influence effects on equilibrium unemployment of changes in (a) layoff costs, and (b) distributions of skill losses coincident with quits ("quit turbulence"). Returns to labor mobility intermediate both effects. Ample labor reallocations observed across market economies that have different layoff costs imply that a turbulence explanation of trans-Atlantic unemployment experiences is robust to adding plausible quit turbulence.

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†Universitat Pompeu Fabra, Barcelona School of Economics, CREI and CEPR, email: isaac.baley@upf.edu

‡Stockholm School of Economics, New York University and CEPR, email: lars.ljungqvist@hhs.se

§New York University and Hoover Institution, email: thomas.sargent@nyu.edu

"... often the most important constraint on a new theory is ... that it should agree with the whole body of past observations, as crystallized in former theories. ... New theories of course do not agree entirely with any previous theory – otherwise they would not be new – but they must not throw out all the success of former theories. This sort of thing makes the work of the theorist far more conservative than is often thought.

"The wonderful thing is that the need to preserve the successes of the past is not only a constraint, but also a guide." Steven Weinberg (2018, p. 197)

1 Introduction

This paper illustrates benefits brought by the calibration approach, codified for macroeconomics by Cooley and Prescott (1995), that Tom Cooley used so well. We aspire to live up to high standards that Tom Cooley set when he practiced what he liked to call "economic science." We present a macro-labor economics application of principles stated by particle physicist Steven Weinberg in the epigraph above. Our research strategy combines essential ingredients of Tom Cooley's approach to research. We study (1) a substantive economic puzzle presented by a structural break in post World War II trans-Atlantic unemployment experiences and (2) an apparently successful turbulence explanation of it that (3) had been challenged for not being robust to a model perturbation that activated a force that the turbulence explanation had neglected.

The challengers provided no direct evidence about that new channel that could help calibrate critical parameters. At this point, Steven Weinberg's rules help: we bring in another phenomenon that earlier macro-labor models had explained well and that is also tied to the challengers' new force. By using an associated "cross-phenomenon restriction" to calibrate critical parameters in the perturbed model, we can resolve the challengers' robustness challenge in favor of the original turbulence explanation of post-World War II differences in trans-Atlantic unemployment outcomes.

After Ljungqvist and Sargent (1998) had used an extended McCall (1970) search model to quantify adverse macroeconomic consequences coming from interactions between microeconomic turbulence and generous unemployment compensation in European welfare states,[1] two complementary studies added forces and phenomena that Ljungqvist and Sargent had excluded. Ljungqvist and Sargent (1998) modeled turbulence in terms of risks of human capital losses coincident with *involuntary* job losses ("layoff turbulence"). While their model explained the persistently higher unemployment rates observed in Europe since the late 1970s, it excluded losses of human capital coincident with *voluntary* separations from jobs. Neglect of such "quit turbulence" risk is the starting point of our story because in 1998 an astute observer, Alan Greenspan (1998, p. 743), suggested that a more hazardous job

[1] Unlike the situations in particle physics and cosmology, there is no "standard model" of forces that shape an equilibrium unemployment rate. Each of three workable classes of models of frictional unemployment has persuasive advocates and skillful users: (1) matching models in the Diamond-Mortensen-Pissarides tradition; (2) equilibrium versions of McCall (1970) search models; and (3) search-island models in the tradition of Lucas and Prescott (1974). Calibrated versions of all three types of models have succeeded in fitting data on labor market flows and generating plausible responses of unemployment rates to government policies like generous unemployment insurance and layoff taxes.

market had suppressed mobility among employed workers and had decreased upward pressures on wages:

> "... the sense of increasing skill obsolescence has also led to an apparent willingness on the part of employees to forgo wage and benefit increases for increased job security. Thus, despite the incredible tightness of labor markets, increases in compensation per hour have continued to be relatively modest."

Greenspan's words inspired den Haan, Haefke and Ramey (2005, henceforth DHHR) to construct a Diamond-Mortensen-Pissarides matching model that they used to represent Greenspan's idea by including possible "quit turbulence" in the form of an immediate stochastic depreciation of a worker's human capital that in turbulent times would be triggered by a worker's decision to quit a job. DHHR's calibrated model implied that even small amounts of quit turbulence made workers reluctant to quit and consequently suppressed both quits and overall job reallocations, thereby

reversing the unemployment-increasing interactions between turbulence and welfare state generosity that Ljungqvist and Sargent (1998) had used to explain trans-Atlantic differences in unemployment rates. DHHR's representation and calibration thus cast doubt on Ljungqvist and Sargent's inference that a rise in turbulence explains the outbreak of high European unemployment in the late 1970s.

What parts of DHHR's structure are responsible for reversing Ljungqvist and Sargent's inference about the interaction of heightened turbulence and trans-Atlantic differences in unemployment outcomes?

Was it DHHR's adding quit turbulence in the form of skill deterioration risks brought by quitting? Or was it DHHR's decision to replace Ljungqvist and Sargent's extended McCall framework with their version of a Diamond-Mortensen-Pissarides matching model? Or was it something else, such as different calibrations of the processes governing productivity distributions and dynamics that are exogenous to both the DHHR model and the Ljungqvist and Sargent model?

Hornstein, Krusell and Violante (2005, section 8.3) suggested some answers. They accepted that DHHR's finding showed a lack of robustness of Ljungqvist and Sargent's explanation of those trans-Atlantic unemployment rate differences:

> "... once the Ljungqvist and Sargent mechanism is embedded into a model with endogenous job destruction, the comparative statics for increased turbulence are reversed, i.e., unemployment falls. The reason is that as the speed of skill obsolescence rises, workers become more reluctant to separate, and job destruction falls."

Hornstein *et al.* thus concluded that what had allowed DHHR to overturn the Ljungqvist and Sargent inference about how higher turbulence had affected Europe and America differently was Ljungqvist and Sargent's reliance on model that had mostly excluded endogenous job separations.[2] To address that concern, in this paper we too adopt a Diamond-Mortensen-Pissarides matching model by using a version of Ljungqvist and Sargent (2007, henceforth LS) as our benchmark model. We include quit

[2]Learning-by-doing human capital accumulation induces endogenous job separations in the model of Ljungqvist and Sargent (1998). But besides exogenous layoffs, there are no on-the-job shocks to productivity per unit of human capital.

turbulence. In contrast to DHHR, we find that plausible amounts of skill loss at times of voluntary quits have only small effects on outcomes: for quit turbulence to suppress unemployment, it has to be raised to become about 50% of layoff turbulence, not DDHR's 5%, and both kinds of turbulence must also be high.

The big disagreement between the matching model analysis of DHHR and an LS model augmented to incorporate quit turbulence comes from differences in returns to labor mobility that are implied by different widths of the productivity distributions calibrated by DHHR and LS.[3] The spread of the productivity distribution matters because of how it affects returns to labor mobility. Equilibrium returns to labor mobility must be suppressed markedly for the introduction of quit turbulence to be able to reverse the unemployment-increasing interactions between layoff turbulence and welfare state generosity featured by LS. It follows that evidence about returns to labor mobility sheds light on the potential impact of quit turbulence. Where might we find pertinent evidence?

Informative sources include the establishment data on firm and worker turnover assembled by Davis and Haltiwanger (1990), as well as similar data sets from other countries. Taken together they provide compelling evidence that extensive reallocations occur within different market economies that operate under a variety of government policies directed at influencing job separations, some heavy-handed, others light-handed. Central to the present paper is our insistence that calibrated labor productivity processes in macro-labor models have to imply high enough returns to labor mobility if they are to be consistent with the high reallocation rates across economies that have very different public policies toward restraining or promoting resource reallocations. Earlier models that have provided sufficiently high returns to labor mobility to do that despite large cross-economy differences in layoff costs include Alvarez and Veracierto (2001), Mortensen and Pissarides (1999), and Ljungqvist and Sargent (2008).

Taking our cue from the patterns studied in those papers, to infer quantitatively plausible returns to labor mobility, we exploit how they also shape effects on unemployment from the introduction of quit turbulence. Thus, we proceed by first inferring reasonable parameter values for productivity processes from a consensus view about quantitative effects on unemployment from imposing layoff costs. Then we study the associated potential impact of quit turbulence on the relationship between turbulence and unemployment.

Section 2 sets forth a matching model augmented to include DHHR's quit turbulence. The productivity process of LS brings high returns to labor mobility while DHHR's productivity process brings low returns. Section 3 studies effects on unemployment of layoff costs and quit turbulence. Their common dependence on returns to labor mobility ties together the magnitudes of effects of layoff costs and quit turbulence on unemployment. To highlight that link, Section 4 constructs mappings from the parameters of the productivity process to distinct outcome criteria for layoff costs and quit turbulence. respectively. The layoff tax criterion is a minimum layoff tax that serves to shut down all voluntary job separations when turbulence is absent. The quit-turbulence criterion is a minimum level of quit turbulence that suffices to turn the relationship between unemployment and turbulence from positive, as it is according to LS, to negative, as it is according to DHHR. As we vary the width

[3]This finding illustrates an assertion of Baley, Ljungqvist and Sargent (2022) that "returns to labor mobility have too often escaped the attention they deserve as conduits of important forces in macro-labor models." In this paper, we shall use a cross-phenomenon restriction to calibrate those returns.

of the productivity distribution and the on-the-job arrival rate of new productivity draws, the two criteria move together. The criteria reveal that in a parameter vicinity where substantial voluntary separations continue to occur under plausible layoff costs that can be inferred from observed cross-country outcomes, plausible amounts of quit turbulence do not reverse a positive relationship between unemployment and turbulence. Section 5 offers concluding remarks. Auxiliary materials appear in online Appendices.

2 A matching model with quit turbulence

Our benchmark is a standard matching model to which we add human capital dynamics that incorporate turbulence. It is a version of the LS (2007) matching model that represents layoff turbulence as more adverse skill transition probabilities for workers who suffer involuntary layoffs. We include DHHR quit turbulence in the form of adverse skill transition probabilities for workers who voluntarily quit.[4]

2.1 Environment

Workers There is a unit mass of workers who are either employed or unemployed. Workers are risk neutral and rank consumption streams according to

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t c_t, \tag{1}$$

where $\beta \equiv \hat{\beta}(1-\rho^r)$, $\hat{\beta} \in (0,1)$ is a subjective time discount factor, and $\rho^r \in (0,1)$ is a constant probability of retirement. Retired workers exit the economy and are replaced by equal measures of newborn workers.

Worker heterogeneity Besides employment status, workers differ along two dimensions: a current skill level i that can be either low (l) or high (h) and an administrative skill level j that determines a worker's entitlement to unemployment benefits. An employed worker has $j = i$; but for an unemployed worker, j is her skill level during her last employment spell. Workers gain or lose skills with transition probabilities that depend on their employment status and instances of layoffs and quits. We assume that all newborn workers enter the labor force with low skills and a low benefit entitlement. Thus, each worker carries along two indices (i, j), the first denoting current skill and the second denoting benefit entitlement.

Firms and matching technology There is free entry of firms who can post vacancies at a cost μ per period. Aggregate numbers of unemployed u and vacancies v are inputs into an increasing, concave and linearly homogeneous matching function $M(v, u)$. Let $\theta \equiv v/u$ be the vacancy-unemployment

[4] LS thanked Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing computer code that LS then modified. Much of our notation and mathematics follow DHHR closely. For an account of differences between the models of LS and DHHR, see Appendix B.

ratio, also called market tightness. The probability $\lambda^w(\theta) = M(v,u)/u = M(\theta,1) \equiv m(\theta)$ that an unemployed worker encounters a vacancy is increasing in market tightness. The probability $M(v,u)/v = m(\theta)/\theta$ that a vacancy encounters an unemployed worker is decreasing in market tightness.

Worker-firm relationships and productivity processes A job opportunity is a productivity draw z from a cumulative distribution function $v_i^o(z)$ that is indexed by a worker's skill level i. We assume that the high-skill distribution first-order stochastically dominates the low-skill distribution: $v_h^o(z) \leq v_l^o(z)$. Wages are set through Nash bargaining, with π and $1-\pi$ as the bargaining weights of a worker and a firm, respectively.

Idiosyncratic shocks within a worker-firm match determine an employed worker's productivities. Productivity in an ongoing job is governed by a first-order Markov process with a transition probability matrix Q_i, also indexed by the worker's skill level i, where $Q_i(z,z')$ is the probability that next period's productivity becomes z', given current productivity z. Specifically, an employed worker retains her last period productivity with probability $1-\gamma^s$, but with probability γ^s draws a new productivity from the distribution $v_i(z)$.

As in the case of the productivity distributions for new matches, the high-skill distribution in ongoing jobs first-order stochastically dominates the low-skill distribution: $v_h(z) \leq v_l(z)$. Furthermore, an employed worker's skills may get upgraded from low to high with probability γ^u. A skill upgrade is accompanied by a new productivity drawn from the high-skill distribution $v_h(z)$. A skill upgrade is realized immediately, regardless of whether the worker remains with her present employer or quits.

We can now define our notions of layoffs and quits.

(i) **Layoffs:** At the beginning of each period, a job is exogenously terminated with probability ρ^x. We call this event a layoff. An alternative interpretation of the job-termination probability ρ^x is that productivity z becomes zero and stays zero forever. A layoff is involuntary.

(ii) **Quits:** After any new on-the-job productivity draw and any skill upgrade, a relationship can continue or be endogenously terminated. We call a separation after such events a voluntary quit because a firm and a worker agree to separate after Nash bargaining.

Turbulence We define layoff and quit varieties of turbulence in terms of risks of losing skills at times of job separations. When a high-skilled worker is laid off, she becomes a low-skilled worker with probability γ^ℓ. We call this risk *layoff turbulence.* When a high-skilled worker *quits*, she becomes a low skilled worker with probability γ^q. We call this risk *quit turbulence.*

At the beginning of a period, exogenous job terminations occur and displaced high-skilled workers face layoff-turbulence risk. Continuing employed workers might receive new on-the-job productivity draws and they might also receive skill upgrades. High-skilled workers face quit turbulence risk whenever they quit. All separated workers join other unemployed workers and wait in the matching function before they have chances to encounter vacancies next period.

Government policy The government provides unemployment compensation. An unemployed worker who was low (high) skilled in her last employment receives a benefit b_l (b_h).[5] Unemployment benefit b_i is calculated as a fraction ϕ of the average wage of employed workers with skill level i. The government imposes a layoff tax Ω on every job termination except for retirements.

The government levies a flat-rate tax τ on production and runs a balanced budget. If layoff tax revenues fully cover payments of unemployment benefits, the government sets $\tau = 0$ and returns the surplus as lump-sum transfers to workers. Since surpluses typically don't arise in our analyses, we choose to omit such lump-sum transfers in various equations below.[6]

2.2 Match surpluses

A match between a firm and a worker with skill level i and benefit entitlement j that has drawn productivity z will form an employment relationship, or continue an existing one, if a match surplus is positive. The match surplus for a new job $s^o_{ij}(z)$ or a continuing job $s_{ij}(z)$ is given by after-tax productivity $(1-\tau)z$ plus the future joint continuation value $g_i(z)$ minus the outside values of the match that consist of the worker's receiving unemployment benefit b_j and a future value ω^w_{ij} associated with entering the unemployment pool in the current period; and the firm's value ω^f from entering the vacancy pool in the current period, net of paying the vacancy cost μ. Define $\omega_{ij} \equiv \omega^w_{ij} + \omega^f$.

The match surplus $s^o_{lj}(z)$ for a new job or $s_{lj}(z)$ for a continuing job for a low-skilled worker with benefit entitlement j equal to

$$s^o_{lj}(z) \;=\; s_{lj}(z) \;=\; (1-\tau)z + g_l(z) - [b_j + \omega_{lj}], \qquad j = l, h. \tag{2}$$

To compute the match surplus for jobs with high-skilled workers, we distinguish between new and continuing jobs. The match surplus s^o_{hh} for forming a new job with an unemployed high-skilled worker involves outside values without risk of skill loss if the match does not result in employment is

$$s^o_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \omega_{hh}]. \tag{3}$$

In contrast, the match surplus for a continuing job with a high-skilled worker or for a job with an earlier low-skilled worker who gets a skill upgrade that is immediately realized involves quit turbulence:

$$s_{hh}(z) \;=\; (1-\tau)z + g_h(z) - [b_h + \underbrace{(1-\gamma^q)\omega_{hh} + \gamma^q \omega_{lh}}_{\text{quit turbulence}}]. \tag{4}$$

Reservation productivities and rejection rates A worker and a firm split the match surplus through Nash bargaining with outside values as threat points. The splitting of match surpluses ensures mutual agreement whether to start (continue) a job. For a new (continuing) match, the reservation

[5] As mentioned above, newborn workers are entitled to b_l. Also, for simplicity, we assume that a worker who receives a skill upgrade and chooses to quit, is entitled to high benefits.

[6] The exceptional case in which a government surplus has to be returned to workers as lump-sum transfers is described in our reference in Section 3.2 to a layoff tax analysis in a version of the LS economy without unemployment benefits.

productivity $\underline{z}^o_{ij}$ ($\underline{z}_{ij}$) is the lowest productivity that makes a match profitable and satisfies

$$s^o_{ij}(\underline{z}^o_{ij}) \;=\; 0 \qquad \Big(s_{ij}(\underline{z}_{ij}) \;=\; -\Omega\Big). \tag{5}$$

Note that in a continuing match, the surplus must fall to the negative of the layoff tax before a job is terminated.

Given the reservation productivity $\underline{z}^o_{ij}$ ($\underline{z}_{ij}$), let ν^o_{ij} (ν_{ij}) denote the rejection probability, which is given by the probability mass assigned to all draws from productivity distribution $v^o_i(y)$ ($v_i(y)$) that fall below a threshold:

$$\nu^o_{ij} \;=\; \int_{-\infty}^{\underline{z}^o_{ij}} dv^o_i(y) \qquad \left(\nu_{ij} \;=\; \int_{-\infty}^{\underline{z}_{ij}} dv_i(y)\right). \tag{6}$$

Define

$$E_{ij} \equiv \int_{\underline{z}_{ij}}^{\infty} [(1-\tau)y + g_i(y)]\; dv_i(y). \tag{7}$$

2.3 Joint continuation values

Consider a match between a firm and a worker with skill level i. Given a current productivity z, $g_i(z)$ is the joint continuation value of the associated match. We now describe value functions for low- and high-skilled workers.

High-skilled worker The joint continuation value of a match of a firm with a high-skilled worker with current productivity z, denoted $g_h(z)$, is affected by prospects of future layoff turbulence if by chance the worker is laid off and of by chance experiencing future quit turbulence should the worker decide to quit after receiving an unacceptable on-the-job new productivity draw:

$$\begin{aligned}
&\text{Exogenous separation:} && g_h(z) \;=\; \beta\Big[\rho^x(b_h + \underbrace{(1-\gamma^\ell)\omega_{hh} + \gamma^\ell\omega_{lh}}_{\text{layoff turbulence}}) \\
&\text{Productivity switch:} && \qquad +\;\; (1-\rho^x)\gamma^s(E_{hh} + \nu_{hh}(b_h + \underbrace{(1-\gamma^q)\omega_{hh} + \gamma^q\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{Status quo:} && \qquad +\;\; (1-\rho^x)(1-\gamma^s)((1-\tau)z + g_h(z))\Big].
\end{aligned} \tag{8}$$

Low-skilled worker The joint continuation value of a firm match with a low-skilled worker accounts for these contingencies: no changes in productivity or skills, an exogenous separation, a productivity switch, and a skill upgrade. When a skill upgrade occurs, even if the worker chooses to quit, the worker immediately becomes entitled to high unemployment benefits. Furthermore, a skill upgrade coincides with a new draw from the high-skill productivity distribution v_h.

Thus, the joint continuation value of a match between a firm and a low-skilled worker with current productivity z, denoted by $g_l(z)$, is

$$\begin{aligned}
&\text{Exogenous separation:} && g_l(z) &=& \beta\Big[\rho^x(b_l+\omega_{ll}) \\
&\text{Immediate skill upgrade:} && &+& (1-\rho^x)\gamma^u(E_{hh}+\nu_{hh}(b_h+\underbrace{(1-\gamma^q)\omega_{hh}+\gamma^q\omega_{lh}}_{\text{quit turbulence}})) \\
&\text{Productivity switch:} && &+& (1-\rho^x)(1-\gamma^u)\gamma^s(E_{ll}+\nu_{ll}(b_l+\omega_{ll})) \\
&\text{Status quo:} && &+& (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z))\Big]. \qquad (9)
\end{aligned}$$

2.4 Outside values

Value of unemployment An unemployed worker with current skill level i and benefit entitlement j receives benefits b_j and has a future value ω^w_{ij}. Recall that the probability that an unemployed worker becomes matched next period is $\lambda^w(\theta)$.

A low-skilled unemployed worker with benefit entitlement j obtains $b_j+\omega^w_{lj}$, where

$$\omega^w_{lj} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{lj}}^{\infty}\pi s^o_{lj}(y)\;dv^o_l(y)}_{\text{match + accept}} \;+\; \underbrace{b_j+\omega^w_{lj}}_{\text{outside value}}\Big] \qquad j=l,h. \qquad (10)$$

A high-skilled unemployed worker with benefit entitlement h obtains $b_h+\omega^w_{hh}$, where

$$\omega^w_{hh} \;=\; \beta\Big[\underbrace{\lambda^w(\theta)\int_{\underline{z}^o_{hh}}^{\infty}\pi s^o_{hh}(y)\;dv^o_h(y)}_{\text{match + accept}} \;+\; \underbrace{b_h+\omega^w_{hh}}_{\text{outside value}}\Big]. \qquad (11)$$

Value of a vacancy A firm that searches for a worker pays an upfront cost μ to enter the vacancy pool and thereby obtains a fraction $(1-\pi)$ of the match surplus if an employment relationship is formed next period. Let $\lambda^f_{ij}(\theta)$ be the probability of filling the vacancy with an unemployed worker of type (i,j). Then a firm's value ω^f of entering the vacancy pool is:

$$\omega^f \;=\; -\mu+\beta\Big[\underbrace{\sum_{(i,j)}\lambda^f_{ij}(\theta)\int_{\underline{z}^o_{ij}}^{\infty}(1-\pi)s^o_{ij}(y)\;dv^o_i(y)}_{\text{match + accept}} \;+\; \underbrace{\omega^f}_{\text{outside value}}\Big]. \qquad (12)$$

2.5 Market tightness and matching probabilities

Let u_{ij} be the number of unemployed workers with current skill i and benefit entitlement j. The total number of unemployed workers is $u=\sum_{i,j}u_{ij}$. The probability $\lambda^w(\theta)$ that an unemployed worker encounters a vacancy depends only on market tightness θ; the probability $\lambda^f_{ij}(\theta)$ that a vacancy encounters an unemployed worker with skill level i and benefit entitlement j also depends on the mix of workers in the unemployment pool. Free entry of firms implies that a firm's expected value of posting a vacancy is zero. Equilibrium market tightness can be inferred from equation (12) with

$w^f = 0$. In summary, labor market outcomes are:

$$\omega^f = 0 \tag{13}$$

$$\mu = \beta(1-\pi)\sum_{(i,j)} \lambda^f_{ij}(\theta) \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv^o_i(y) \tag{14}$$

$$\lambda^w(\theta) = m(\theta) \tag{15}$$

$$\lambda^f_{ij}(\theta) = \frac{m(\theta)}{\theta}\frac{u_{ij}}{u}. \tag{16}$$

2.6 Wages

We assume Nash bargaining between a worker and a firm, each getting a share of the match surplus every period.[7] Given a productivity draw z in a new match with a positive match surplus, wage $p^o_{lj}(z)$ of a low-skilled worker with benefit entitlement $j = l, h$ and wage $p^o_{hh}(z)$ of a high-skilled worker, respectively, solve

$$\max_{p^o_{lj}(z)} \quad \left[(1-\tau)z - p^o_{lj}(z) + g^f_l(z) - \omega^f\right]^{1-\pi} \left[p^o_{lj}(z) + g^w_l(z) - b_j - \omega^w_{lj}\right]^{\pi} \tag{17}$$

$$\max_{p^o_{hh}(z)} \quad \left[(1-\tau)z - p^o_{hh}(z) + g^f_h(z) - \omega^f\right]^{1-\pi} \left[p^o_{hh}(z) + g^w_h(z) - b_h - \omega^w_{hh}\right]^{\pi},$$

where $g^w_i(z)$ and $g^f_i(z)$ are future values obtained by the worker and the firm, respectively, from continuing an employment relationship;[8] and ω^f and $b_j + \omega^w_{ij}$ are outside values defined in (10), (11), and (12). Solutions to these wage determination problems set the sum of the worker's wage and continuation value equal to the worker's share π of the match surplus plus her outside value:

$$p^o_{lj}(z) + g^w_l(z) = \pi s^o_{lj}(z) + b_j + \omega^w_{lj} \qquad j = l, h \tag{18}$$

$$p^o_{hh}(z) + g^w_h(z) = \pi s^o_{hh}(z) + b_h + \omega^w_{hh},$$

[7] Nash bargaining implies that workers pay part of the layoff tax upon a job separation. An alternative assumption is that once a worker is hired, firms are the only ones liable for the layoff tax. That generates a two-tier wage system à la Mortensen and Pissarides (1999). Risk-neutral firms and workers would be indifferent between adhering to period-by-period Nash bargaining or a two-tier wage system. Ljungqvist (2002) showed that the wage profile, but not the allocation, is affected by the two-tier wage system. Match surpluses, reservation productivities, and market tightness remain the same. Under the two-tier wage system, an initial wage concession by a newly hired worker is equivalent to her posting a bond that equals her share of a future layoff tax.

[8] *Joint* continuation values defined in (8) and (9) equal sums of the individual continuation values: $g_i(z) = g^w_i(z) + g^f_i(z),\ i = l, h$.

where worker continuation values are

$$
\begin{aligned}
g_l^w(z) &= \beta(1-\rho^x)\pi\left\{(1-\gamma^u)\left[(1-\gamma^s)s_{ll}(z)+\gamma^s\int_{\underline{z}_{ll}}^{\infty}s_{ll}(y)\ dv_l(y)\right]+\gamma^u\int_{\underline{z}_{hh}}^{\infty}s_{hh}(y)\ dv_h(y)\right\}\\
&+ \beta(\rho^x+(1-\rho^x)(1-\gamma^u))\left(b_l+\omega_{ll}^w\right)+\beta(1-\rho^x)\gamma^u\left(b_h+(1-\gamma^q)\omega_{hh}^w+\gamma^q\omega_{lh}^w\right) \qquad (19)\\
g_h^w(z) &= \beta(1-\rho^x)\pi\left[(1-\gamma^s)s_{hh}(z)+\gamma^s\int_{\underline{z}_{hh}}^{\infty}s_{hh}(y)\ dv_h(y)\right]\\
&+ \beta\rho^x\left(b_h+(1-\gamma^\ell)\omega_{hh}^w+\gamma^\ell\omega_{lh}^w\right)+\beta(1-\rho^x)\left(b_h+(1-\gamma^q)\omega_{hh}^w+\gamma^q\omega_{lh}^w\right).
\end{aligned}
$$

For ongoing employments, wages $p_{ll}(z)$ and $p_{hh}(z)$ satisfy counterparts of the above equations that use appropriate match surpluses $s_{ll}(z)$ and $s_{hh}(z)$:

$$
\begin{aligned}
p_{ll}(z)+g_l^w(z) &= \pi s_{ll}(z)+b_l+\omega_{ll}^w \qquad (20)\\
p_{hh}(z)+g_h^w(z) &= \pi s_{hh}(z)+b_h+\underbrace{(1-\gamma^q)\omega_{hh}^w+\gamma^q\omega_{lh}^w}_{\text{quit turbulence}},
\end{aligned}
$$

where the latter expression for the high-skilled wage now involves quit turbulence on the right side.

2.7 Government budget constraint

Unemployment benefits Benefit entitlement j awards an unemployed worker benefit b_j equal to a fraction ϕ of the average wage $\bar{p}_j$ of employed workers with skill level j. Therefore, total government expenditure on unemployment benefits are

$$
b_l u_{ll}+b_h(u_{lh}+u_{hh})=\phi(\bar{p}_l u_{ll}+\bar{p}_h(u_{lh}+u_{hh})). \qquad (21)
$$

Layoff taxes The measure Ξ of total separations excluding retirements equals

$$
\Xi=(1-\rho^r)\Big[\rho^x(e_{ll}+e_{hh})+(1-\rho^x)[(1-\gamma^u)\gamma^s\nu_{ll}+\gamma^u\nu_{hh}]e_{ll}+(1-\rho^x)\gamma^s\nu_{hh}e_{hh}\Big]. \qquad (22)
$$

Then government revenue from layoff taxation equals $\Omega\,\Xi$.

Income taxes Output is taxed at a constant rate τ. Where $\bar{z}_i$ is average productivity of employed workers with skill level i, total tax revenue equals $\tau(\bar{z}_l e_{ll}+\bar{z}_h e_{hh})$, where e_{ll} (e_{hh}) is the number of employed workers with low skills and low benefit entitlement (high skills and high benefit entitlement).

Balanced budget The government runs a balanced budget. The tax rate τ on output is set to cover the expenditures on unemployment benefits described in (21) net of layoff tax revenues $\Omega\,\Xi$:

$$
\phi(\bar{p}_l u_{ll}+\bar{p}_h(u_{lh}+u_{hh}))-\Omega\,\Xi=\tau(\bar{z}_l e_{ll}+\bar{z}_h e_{hh}). \qquad (23)
$$

Calculations of average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ appear in Appendix A.2.

2.8 Worker flows

Workers move across employment and unemployment states, skill levels, and benefit entitlements. Here we focus on low-skilled unemployed with high benefits, workers at the center of our analysis. (Appendix A.1 describes flows for other groups of workers.)

Inflows to the pool of low-skilled unemployed with high benefits u_{lh} come from the following sources. Layoff turbulence affects high-skilled workers e_{hh} who get laid off; with probability γ^{ℓ}, they become low-skilled unemployed workers entitled to high unemployment benefits. Quit turbulence affects high-skilled workers e_{hh} who reject productivity switches, as well as low-skilled workers e_{ll} who get skill upgrades and then reject their new productivity draws. All of those quitters face probability γ^q of entering the pool of low-skilled unemployed workers entitled to high unemployment benefits.

Outflows from unemployment coincide with successful matches and retirements. Thus, the net change of low-skilled unemployed with high benefits (equalling zero in a steady state) is

$$\begin{aligned} \Delta u_{lh} \quad = \quad (1-\rho^r)\Bigg\{ & \underbrace{\rho^x \gamma^{\ell} e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^q \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} \\ & - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} \Bigg\} - \rho^r u_{lh}. \end{aligned} \tag{24}$$

Terms numbered 1 and 3 in expression (24) identify sources of a positive layoff-turbulence, unemployment relationship in a welfare state in the LS model. Although more layoff turbulence in term 1 – a higher probability γ^{ℓ} of losing skills after layoffs – has a small effect on equilibrium unemployment in a "laissez-faire" environment in which $\phi = 0, \Omega = 0$, it provokes a strong turbulence-unemployment relationship in a welfare state that offers a generous unemployment benefit replacement rate for a worker's earnings in her last job. After a layoff with skill loss, those benefits are high relative to a worker's earnings prospects at her now diminished skill level. As a consequence, the acceptance rate $(1 - \nu^o_{lh})$ in term 3 is low; the relatively high outside value of a low-skilled unemployed with high benefits implies that fewer matches have positive match surpluses, as reflected in a high reservation productivity z^o_{lh}. Moreover, given those suppressed match surpluses, equilibrium market tightness θ falls to restore firm profitability enough to make vacancy creation break even. Lower market tightness, in turn, reduces the probability $\lambda^w(\theta)$ that a worker encounters a vacancy, which further decreases the fraction of successful matches and thereby contributes to a positive layoff-turbulence, unemployment relationship.

Presence of quit turbulence adds the term numbered 2 in expression (24). On the one hand, an additional source of turbulence $\gamma^q > 0$ can further increase the equilibrium unemployment rate since there is one more channel for high-skilled workers to lose skills and become low-skilled unemployed with high benefits. On the other hand, quit turbulence also exerts a countervailing force that could attenuate or even reverse a positive turbulence-unemployment relationship. When voluntary quits are also subject to risks of skill loss, there will be fewer voluntary quits in turbulent times; exposing themselves to a risk of skill loss makes high-skilled workers more reluctant to quit, lowering the rejection rate ν_{hh} in term 2. That lower rejection rate causes lower inflows u_{lh} into the pool of low-skilled unemployed who are entitled to high benefits as well as inflows u_{hh} into the pool of high-skilled unemployed who are entitled to high unemployment benefits . This is the force activated by DHHR to reverse a positive turbulence-unemployment relationship.

2.9 Steady state equilibria

A steady state equilibrium consists of measures u_{ij} of unemployed workers and e_{ij} employed workers; labor market tightness θ, probabilities $\lambda^w(\theta)$ that workers encounter vacancies and $\lambda^f_{ij}(\theta)$ that vacancies encounter workers; reservation productivities $\underline{z}^o_{ij}, \underline{z}_{ij}$, match surpluses $s^o_{ij}(z), s_{ij}(z)$, future values of an unemployed worker ω^w_{ij} and of a firm posting a vacancy ω^f; wages $p^o_{ij}(z), p_{ij}(z)$; unemployment benefits b_i and a tax rate τ; such that

a) Match surplus conditions (5) determine reservation productivities.

b) Free entry of firms implies zero-profit condition (14) in vacancy creation that pins down market tightness.

c) Nash bargaining outcomes (18) and (20) set wages.

d) The tax rate balances the government's budget (23).

e) Net worker flows, such as expression (24), are all equal to zero: $\Delta u_{ij} = \Delta e_{ij} = 0, \quad \forall\, i, j.$

2.10 Parameterization

Apart from considering alternative assumptions about the productivity process and different values of the layoff tax, the benchmark model shares the remaining parameterization with LS, in conjunction with DHHR's codification of quit turbulence, as reported in Table 1.[9] The model period is half a quarter.

Preference parameters In light of our semi-quarterly model with its eight periods per year, we specify a discount factor $\hat{\beta} = 0.99425$ and a retirement probability $\rho^r = 0.0031$, which together imply an adjusted discount of $\beta = \hat{\beta}(1 - \rho^r) = 0.991$. The retirement probability implies an average time of 40 years in the labor force.

Stochastic processes for productivity Exogenous layoffs occur with probability $\rho^x = 0.005$, on average a layoff every 25 years. We set a probability of upgrading skills $\gamma^u = 0.0125$ so that it takes on average 10 years to move from low to high skill, conditional on no job loss. The probability of a productivity switch on the job equals $\gamma^s = 0.05$, so a worker expects to retain her productivity for 2.5 years.

[9]Subject to the caveat of DHHR assuming a fixed population of firms of the same measure as that of workers and hence, an exogenous market tightness equal to 1, the remaining parameterization in Table 1 is identical or similar to that of DHHR. For a detailed account, see Appendix B.

Table 1: PARAMETERIZATION OF BENCHMARK MODEL

Parameter	Definition	Value
Preferences		
$\hat{\beta}$	discount factor	0.99425
ρ^r	retirement probability	0.0031
$\beta = \hat{\beta}(1-\rho^r)$	adjusted discount	0.991
Sources of risk		
ρ^x	exogenous breakup probability	0.005
γ^u	skill upgrade probability	0.0125
γ^s	productivity switch probability	0.05
γ^ℓ	layoff turbulence	$[0,1]$
$\gamma^q = \epsilon\gamma^\ell$	quit turbulence	$\epsilon \in [0,1]$
Labor market institutions		
π	worker bargaining power	0.5
ϕ	replacement rate	0.7
Ω	layoff tax	0
Matching function		
A	matching efficiency	0.45
α	elasticity of matches w.r.t. u	0.5
μ	cost of posting a vacancy	0.5

Layoff and quit turbulence Following DHHR, we parameterize quit turbulence as a fraction ϵ of layoff turbulence so that $\gamma^q = \epsilon\gamma^\ell$. We vary ϵ from zero – denoting complete absence of quit turbulence – to one – in which case layoff and quit turbulence risks are equal.

Labor market institutions We set a worker's bargaining power to be $\pi = 0.5$. We set the replacement rate in unemployment compensation at $\phi = 0.7$ and initially set the layoff tax $\Omega = 0$. When we study the effects of layoff taxes on unemployment in section 4, we'll set $\Omega > 0$.

Matching We assume a Cobb-Douglas matching function $M(v,u) = Au^\alpha v^{1-\alpha}$, which implies that the probability that a worker encounters a vacancy and that the probability that a vacancy encounters a worker of a particular type, respectively, are:

$$\lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda^f_{ij}(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u}. \tag{25}$$

The elasticity of matches with respect to unemployment is $\alpha = 0.5$, in line with a consensus that plausible values fall in the mid range of the unit interval (e.g., see Petrongolo and Pissarides (2001)). We adopt LS's parameterization of the matching efficiency $A = 0.45$ and the cost of posting a vacancy $\mu = 0.5$.

3 High (LS) and low (DHHR) returns to labor mobility

This section describes implications for returns to labor mobility of disparate productivity distributions calibrated by LS and DHHR. We start with two models, one representing LS's specification, the other representing DHHR's. In subsection 3.4 we proceed to project each of these models into a common benchmark model and verify that projected versions of the two models do good jobs of representing outcomes in the original models. We then use two calibrations of the productivity distribution in the benchmark model to isolate their effects on outcomes.

Parameterizations of LS and DHHR reported in the first two columns of Table 2 and depicted in Figure 1 provide examples of different productivity distributions that imply different returns to labor mobility. LS and DHHR both assume that productivity distributions are the same for new and ongoing matches, so that $v_i^o(z) = v_i(z)$. LS parameterize truncated normal distributions in Figure 1a whereas DHHR in Figure 1b assume uniform distributions with narrow ranges.[10] These probability distributions imply different returns to labor mobility that in turn affect how much equilibrium unemployment respond to changes in either layoff taxes or in quit turbulence. As indicated in the previous paragraph, we start by studying these effects in the original models of LS and DHHR. Then we map each of their productivity processes into uniform distributions within our benchmark model, a model-projection exercise that allows us in Section 4 to characterize LS and DHHR versions of our benchmark model that differ only in the widths of their uniform productivity distributions. This machinery lets us bring out implications of the disparate LS and DHHR productivity distributions for (1) effects of layoff taxes on unemployment, and (2) effects of quit turbulence on unemployment.

Table 2: Productivity distributions of LS and DHHR

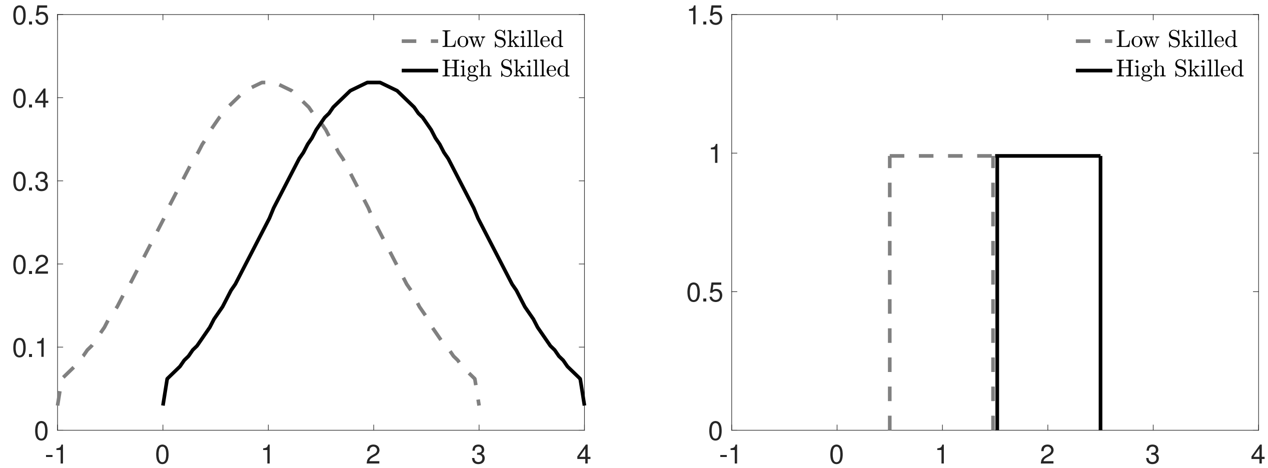
Properties	**Original model**		**Benchmark model version**	
	LS	DHHR	LS	DHHR
Functional form, $v_i(z)$	Normal	Uniform	Uniform	Uniform
Mean, low-skilled	1	1	1	1
high-skilled	2	2	2	2
Width of support	4	1	2.25	0.6
Standard deviation	1	0.289	0.650	0.173

3.1 The LS and DHHR Models

We can obtain the LS original model by simply importing the LS productivity distributions into our benchmark model. What we refer to as the DHHR model is their original framework, except for two modifications that, although they facilitate our way of mapping DHHR into our benchmark model, do

[10] LS incorrectly implemented the quadrature method at the truncation points of the normal distributions; nevertheless, the constructed distributions are still proper. Therefore, instead of recalibrating the LS model under a correct implementation of the quadrature method, we have chosen for reasons of comparability to retain the distributions presented in the published LS analysis.

Figure 1: Productivity distributions of LS and DHHR



(a) LS (Normal with wide support)

(b) DHHR (Uniform with narrow support)

not alter outcomes substantially.[11]

We verify and extend DHHR's finding that with their narrow distribution of productivities, small amounts of quit turbulence reverse the Ljungqvist-Sargent unemployment-increasing interactions between turbulence and welfare state generosity, but that this does not happen with LS's wider productivity distribution. We'll eventually see that this difference in outcomes is a tell tale sign of differences in the returns to labor mobility that come from different widths of productivity distributions.

Next, we map the LS and DHHR models into our benchmark model under the assumption of uniform distributions. For the LS. model, this is just a matter of converting LS's truncated normal distributions into uniform distributions. For DHHR, things are more complicated because their matching framework differs from our benchmark model in two ways that, for our purposes, are inconsequential.[12] In the end, mapping DHHR into our benchmark model only requires transforming DHHR's productivity distributions. So we calibrate the widths of the uniform distributions in our benchmark model to generate unemployment effects of quit turbulence like those in our analyses of the LS and DHHR models. It turns out that effects of layoff taxes on unemployment for each such calibration of the benchmark model aligns with outcomes in the corresponding analyses of the LS and DHHR model, an alignment that reflects the cross-phenomenon restriction featured in Section 4.

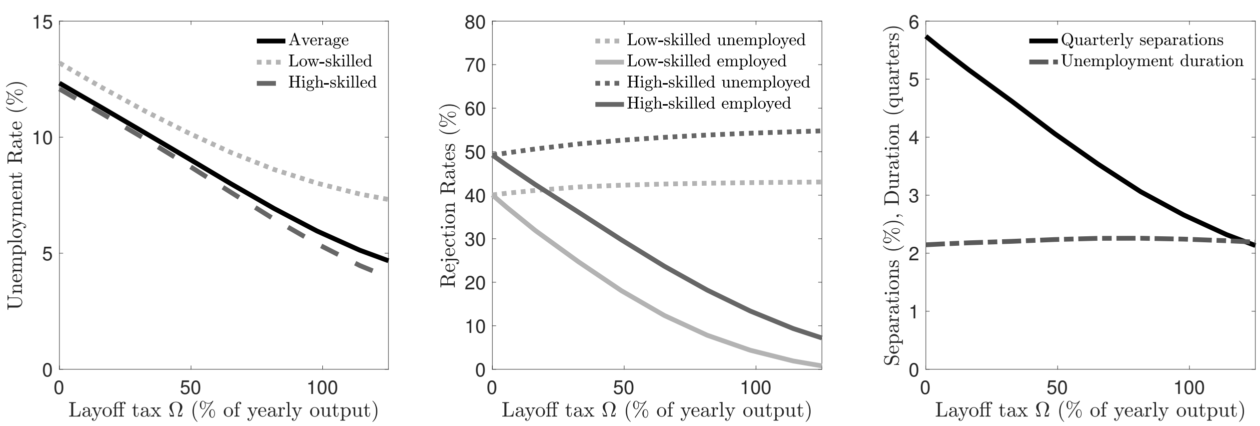
[11] Our first modification is that instead of the zero benefits that they receive in the original DHHR setup, we assume that newborn workers are eligible for the same unemployment benefits as low-skilled workers. The second modification concerns the risk of losing skills following unsuccessful job market encounters. As a "simplifying assumption," DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as she would after quitting a job; an added risk that we omit. For an assessment of these alternative assumptions, see Appendix D.

[12] As described in Appendix B, these structural differences pertain to i) how vacancies are created, and ii) how the capital gain from a skill upgrade is split between firm and worker. To show that among these two differences and the parameterization of productivity distributions it is the latter one that is the sole important source for how unemployment responds to quit turbulence, we proceed as follows. Appendix C starts with the benchmark model with LS productivity distributions and outcomes as depicted in Figure 4a below, and then successively perturbs the three potential sources one by one, to see which one brings us closest to outcomes in the DHHR model in Figure 4b. In Appendix D, we start from the DHHR model in Figure 4b and work through the perturbations in reverse. Both procedures detect productivity distributions as being the critical source for differences in outcomes.

3.2 Layoff taxes

Layoff taxes in LS In the tranquil zero-turbulence times $\gamma^{\ell} = 0$ LS model, Figure 2 shows unemployment and rejection rates of various types of workers, as well as aggregate labor flows, as functions of the layoff tax Ω expressed as a fraction of the average yearly output per worker in a $\phi = 0, \Omega = 0$ "laissez-faire" economy.[13] As the layoff tax increases, the unemployment rate falls (left panel) due primarily to a decline in endogenous separations (right panel). The rejection rates plotted in the middle panel refer to the arrival rate of new on-the-job draws of z that prompt employed workers to quit (solid lines) and the draws of z in new job offers rejected by unemployed workers (dotted lines), for both skill levels. Raising the layoff tax causes rejection rates of both high-skilled and low-skilled employed workers to fall markedly. But even at pretty substantial layoff taxes, these workers still remain mobile. Thus, if the layoff tax reaches the average annual output of a worker $\Omega = 100\%$, employed high-skilled workers reject about 12% of offers.

Figure 2: Layoff taxes in LS when $\phi = 0.7$ and $\gamma^{\ell} = 0$



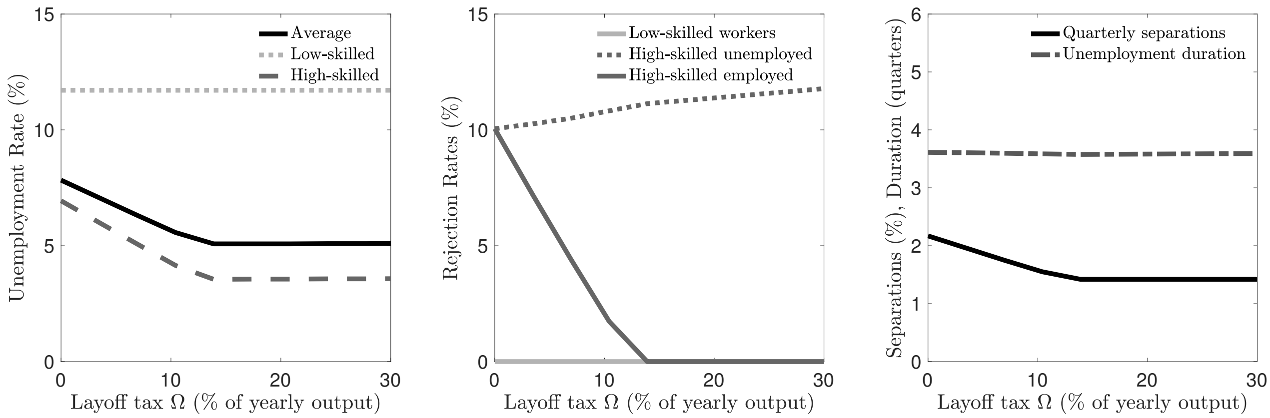
Incidentally, Figure 2 expresses forces that LS used to explain why, before the arrival of layoff turbulence, a welfare state with generous unemployment can actually have *lower* unemployment than a $\phi = 0, \Omega = 0$ laissez-faire economy (also see Mortensen and Pissarides (1999)). Thus, despite the generous Table 1 unemployment benefit replacement rate of $\phi = 0.7$, the left panel of Figure 2 shows that sufficiently high layoff taxes cause unemployment to fall below its 5% rate in the laissez-faire $\phi = 0, \Omega = 0$ economy.

For later use, we note that in the LS model, layoff taxes above 184% of the average yearly output per worker completely suppress endogenous separations. This can be discovered by extrapolating the dark solid curve in the middle panel of Figure 2; evidently, high-skilled workers are more resilient in their mobility before eventually no longer quitting. The corresponding minimum layoff tax required to close down all endogenous separations in the laissez-faire economy with no unemployment insurance is 163%. When $\phi = 0$, gains from quitting and searching for another job are smaller, requiring a smaller layoff tax to suppress endogenous separations.

[13] In the LS laissez-faire economy with $\phi = 0, \Omega = 0$, a worker's average semi-quarterly output is 2.3 goods in tranquil zero-turbulence times.

Layoff taxes in DHHR Figure 3 shows how a higher layoff tax affects equilibrium outcomes in tranquil zero-turbulence times $\gamma^\ell = 0$ DHHR model.[14] A layoff tax equivalent to 14% of the average annual output per worker in the $\phi = 0, \Omega = 0$ DHHR laissez-faire economy completely suppresses the mobility of high-skilled employed workers.[15] Above this low level of layoff taxes, the rate of which high-skilled workers reject on-the-job new draws of z becomes zero, so that job-separation rates become constant at exogenous job-termination rates. Imposing a small layoff tax devalues labor mobility. Note that at all levels of the layoff tax the rejection rate is zero for both employed and unemployed low-skilled workers with the DHHR parameterization.

Figure 3: Layoff taxes in DHHR when $\phi = 0.7$ and $\gamma^\ell = 0$



Endogenous separations occur in our DHHR model only because they are encouraged by a generous replacement rate of $\phi = 0.7$. None occur in a $\phi = 0, \Omega = 0$ laissez-faire version. This situation is symptomatic of the low returns to labor mobility in the DHHR model, a topic that we take up in Section 4.

3.3 Quit turbulence

How should a model represent that different job separators can find themselves in different situations? For example, workers with valuable skills who separate to find better-paying jobs differ from laid-off workers whose skills are no longer in demand, e.g., due to changing technologies or their types of work moving abroad to low-wage countries.

To capture such differences, the benchmark model treats involuntary separations as earlier theories did by assuming that they worsen circumstances for job separators by presenting the highest possible risks of skill losses. The benchmark model also introduces quit turbulence in the form of risk of human capital loss for workers who voluntarily separate from jobs after draws of poor job-specific productivities at their current employment. We specify that voluntary quitters are more fortunately situated

[14] In addition to the two simplifying modifications of the original DHHR framework described in footnote 11, here we assume that skill upgrades are realized immediately in the DHHR model as in the LS framework. Appendix D.2 documents a small impact on equilibrium outcomes in the DHHR model of this change in assumptions.

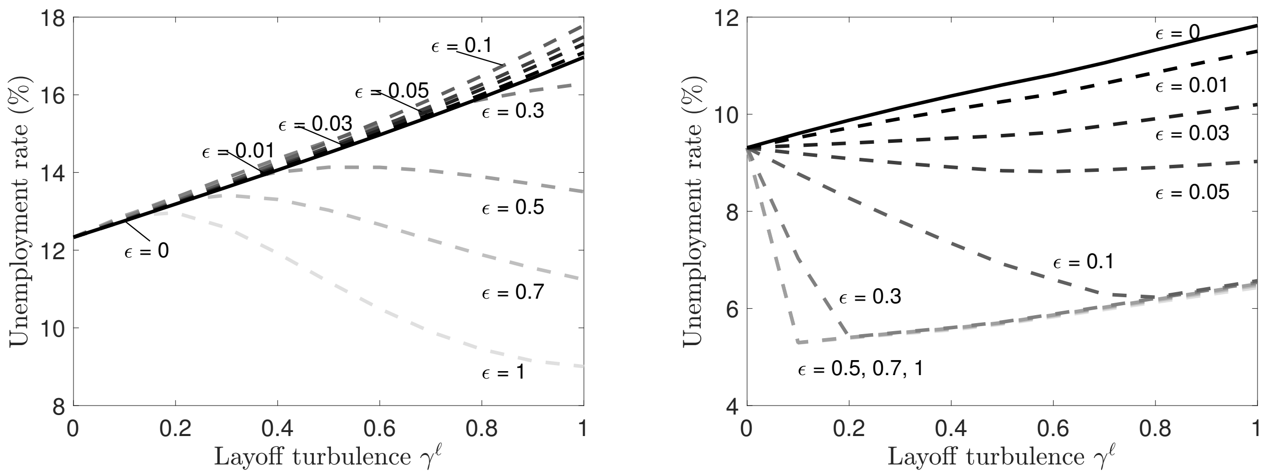
[15] In the DHHR laissez-faire economy with $\phi = 0, \Omega = 0$, a worker's average quarterly output is 1.8 goods in tranquil zero-turbulence times.

than workers who have just been laid off, both in terms of their having the option to continue working at their current jobs after receiving shocks to productivity, as well as, conditional on separating, facing a lower risk of skill loss, than are workers who suffer involuntary separations.

Like DHHR, we can study the robustness to quit turbulence of LS's attribution of high and persistent European unemployment to interactions between microeconomic turbulence and Europe's more generous welfare states. We can accomplish this by measuring how much the risk of skill loss at times of voluntary separations must be relative to the risk at times of involuntary separations to generate a negative rather than a positive turbulence-unemployment relationship. Because contending forces push for and against the LS outcome, this is a quantitative issue.

Figure 4a depicts unemployment outcomes as a function of turbulence when productivity distributions of the benchmark model are those of LS. The x-axis shows layoff turbulence γ^ℓ and the y-axis the unemployment rate in percent. Each line has its own quit turbulence γ^q represented as a fraction ϵ of layoff turbulence γ^ℓ, i.e., $\gamma^q = \epsilon\gamma^\ell$ where $\epsilon \in \{0, 0.01, 0.03, 0.05, 0.1, 0.3, 0.5, 0.7, 1\}$. In Figure 4a, we observe that the quit turbulence fraction ϵ must be *large*, about 50% of layoff turbulence, before the aggregate unemployment rate varies inversely with layoff turbulence, and even then only for relatively high levels of layoff turbulence.

Figure 4: Quit turbulence in LS and DHHR



(a) LS model

(b) DHHR model

Layoff turbulence γ^ℓ on the x-axis. Each line represents a different quit turbulence γ^q as a fraction ϵ of layoff turbulence, i.e., $\gamma^q = \epsilon\gamma^\ell$. Panel a shows the benchmark model with LS productivity distributions, i.e., the LS model with no layoff tax. Panel b is the DHHR model with our two simplifying modifications in footnote 11.

However, these consequences of adding quit turbulence to the LS model differ markedly from those in DHHR's paper. DHHR find that the turbulence-unemployment relationship already becomes negative at very *small* skill loss probabilities for voluntary separators relative to those for involuntary separators:

> "... allowing for a skill loss probability following [voluntary] separation that is only 3% of the probability following [involuntary] separation eliminates the positive turbulence-

> unemployment relationship. Increasing this proportion to 5% gives rise to a strong *negative* relationship between turbulence and unemployment." (DHHR, p. 1362)

Figure 4b reproduces DHHR's findings in our version of their model with its two modifications described in footnote 11, inconsequential though they are for the questions we are now addressing. Evidently, DHHR's assertion remains essentially intact; under our two modifications of their model, it just requires a bit more quit turbulence to recover DHHR's critical findings of a negative turbulence-unemployment relationship. Thus, as cited above for the original DHHR model, the relationship becomes markedly negative at 5% of quit turbulence ($\epsilon = 0.05$), while subject to our modifications, quit turbulence needs to be 7% ($\epsilon = 0.07$).

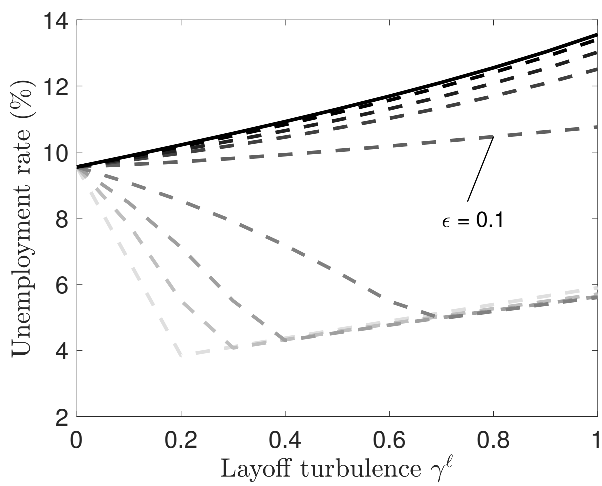
What accounts for these different outcomes emerging after adding just small amounts of quit turbulence to the LS model and the DHHR model? These forces are at work. Productivity draws on the job bring incentives for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support in Figure 1b reduce returns to labor mobility. Figure 4b shows that returns to labor mobility are so low that they fail to compensate for even small amounts of quit turbulence. Consequently, a positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative with even small amounts of quit turbulence. Notice that high-skilled workers choose to remain on the job and accept productivities at the lower end of the productivity distribution rather than quit and have to face even small probabilities of skill loss.

Figure 4b also shows that DHHR's negative turbulence-unemployment relationship can eventually turn positive, as starkly illustrated by a quit turbulence of $\epsilon = 0.3$ and higher. Those high levels of quit turbulence are initially characterized by a steep negative relationship that ends abruptly at a kink that precedes a gentler upward-sloping turbulence-unemployment relationship. At such kinks, all endogenous separations shut down. The source of unemployment suppression – reductions in quits – has vanished. What leads to a positive turbulence-unemployment relationship is that higher turbulence generates more low-skilled unemployed who are entitled to high benefits. These workers must draw relatively high productivities in order to want to join employment relationships for two reasons. First, relative to low-skilled workers who are entitled to low benefits, such workers are reluctant to give up their high benefits: a high benefit entitlement brings a stronger bargaining position. Second, a bargained wage not only must be *high* enough to induce workers to surrender their high benefits; it also must be *low* enough to induce firms to fill vacancies. As described in footnote 9, DHHR assume a fixed measure of firms, with each idle firm being endowed with a vacancy. The opportunity cost for a firm is the option value of waiting to fill the vacancy as it anticipates prospects of meeting either a high-skilled unemployed worker or a low-skilled unemployed worker who is entitled only to low benefits and who therefore has less bargaining power. Consequently, productivities drawn by low-skilled unemployed workers with high benefits have to be relatively high in order for there to exist a wage acceptable to a worker, firm pair. The resulting low hazard rate for low-skilled workers with high benefits to escape unemployment means that unemployment has to increase with layoff turbulence after all endogenous separations have shut down.

3.4 Benchmark model versions of LS and DHHR

Differences in the spreads of their assumed productivity distributions explains the markedly different implications of quit turbulence in the two models analyzed in Figure 4.[16] Indeed, by simply switching from the LS to DHHR productivity distributions in the benchmark model, outcomes in Figure 4a transform into those of Figure 5: the positive turbulence-unemployment relationship is weakened so much that we get DHHR-like outcomes. We can arrive at what we call the benchmark model version of DHHR by shrinking the width of the uniform productivity distributions from DHHR's original value of 1 to 0.6. This result in Figure 6b where the responses of unemployment to layoff and quit turbulence closely approximate those of the DHHR model in Figure 4b. The good approximation prevails while also preserving the two structural differences between the models in Figures 6b and 4b, as described in footnote 12.

Figure 5: Benchmark model with DHHR productivity distributions



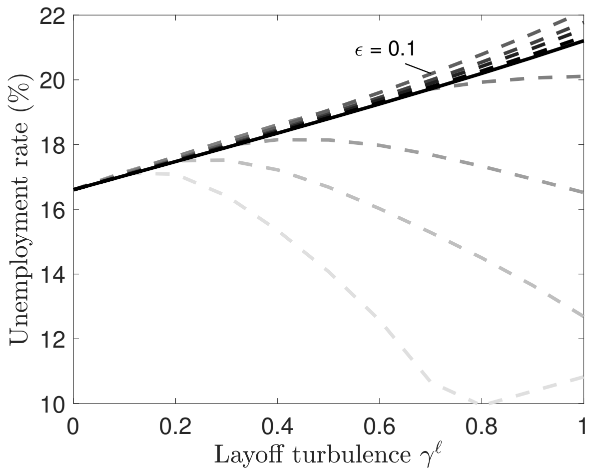
We can also construct a benchmark model version of LS with a uniform productivity distributions. The LS model's high returns to labor mobility requires a fairly big width of 2.25 for the uniform distributions. The resulting Figure 6a generates unemployment responses to turbulence that resemble those of the LS model presented in Figure 4a. Although to calibrate the benchmark model versions of LS and DHHR we target only the effects of turbulence on unemployment, a cross-phenomenon restriction should ensure that associated effects layoff taxes for unemployment survive our mappings into our benchmark model. We confirm that in the next section.

4 Cross-phenomenon restriction

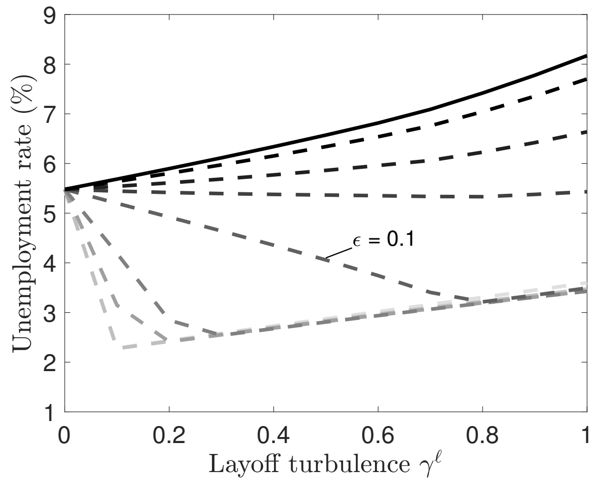
We present a cross-phenomenon restriction that emerged from our investigation of LS and DHHR by describing interrelated effects of layoff costs on unemployment and of quit turbulence on unemployment that are swept out across environments as we vary the width of the productivity processes. How much

[16]Please see footnote 12.

Figure 6: Benchmark model versions of LS and DHHR



(a) Benchmark version of LS



(b) Benchmark version of DHHR

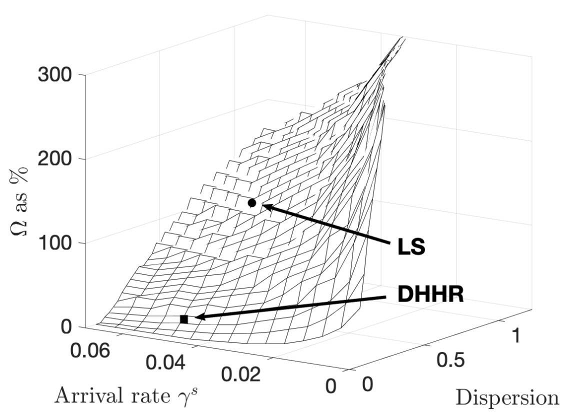
layoff costs can suppress unemployment is linked to the potency of quit turbulence risk for reversing a positive turbulence-unemployment relationship.

The strengths of both forces on unemployment are intermediated by rates of returns to labor mobility, outcomes that are vitally influenced by the widths and dynamics of the productivity process. We convey these links between the consequences of layoff costs and quit turbulence by computing two outcome criteria as functions of parameters that describe the width and dynamics of the productivity process.

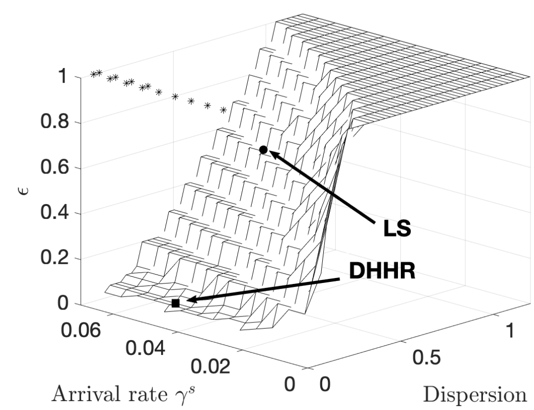
We ferret out these associations by watching two outcome criteria vary as we sweep through a set of uniform productivity processes parameterized by both their widths and their arrival rates γ^s of productivity shocks in continuing matches. We take the minimum layoff cost for which all voluntary separations shut down in $\gamma^\ell = 0$ tranquil times as our outcome criterion for the influence of layoff costs on unemployment. We express the layoff cost as a proportion of the annual output per worker in a corresponding laissez-faire $\phi = 0, \Omega = 0$ economy. We take a minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative, conditional on a magnitude of layoff turbulence γ^ℓ as our outcome criterion for the effect of quit turbulence on unemployment. We measure quit turbulence relative to the magnitude of layoff turbulence, i.e., as a fraction $\epsilon \in [0, 1]$. So conditional on a value of γ^ℓ, our quit turbulence criterion is the minimum value of ϵ that yields an inverse turbulence-unemployment relationship, i.e., that makes the unemployment rate fall with an incremental increase in layoff turbulence at the conditioned value of γ^ℓ. (When the turbulence criterion equals a maximum value of 1, indicates either a knife-edged case at an interior solution when the minimum value of ϵ that yields a negative turbulence-unemployment relationship occurs at 1 or, more often, a corner solution in which there exists no $\epsilon \in [0, 1]$ that can overturn the positive turbulence-unemployment relationship.)

Figure 7 presents the two outcome criteria as functions of the arrival rate γ^s of new on-the-job productivity draws and the standard deviation of the uniform productivity distribution in our benchmark model, here denoted as "dispersion."[17] The layoff cost criteria in Figure 7a indicate that the minimum layoff tax required to shut down voluntary separations increases with dispersion and decreases with the arrival rate of new on-the-job productivity draws. Because a higher dispersion brings higher returns to labor mobility, a higher layoff cost is required to shut down voluntary separations. A higher arrival rate of productivity shocks in continuing matches implies a lower expected duration of a productivity draw and thereby suppresses returns to labor mobility via two forces. First, a relatively low productivity draw becomes less costly to bear when it is expected to persist for a shorter period of time. Second, the prospective gain from quitting and finding a higher productivity match becomes less attractive when the new productivity draw can be anticipated to last for less time. These considerations make the minimum layoff tax required to shut down voluntary separations decrease in the arrival rate. At the far right corner of Figure 7a that indicates high dispersion and very small arrival rates, the layoff cost criterion explodes when the graph is extended. Here the supports of the Figure 1b uniform productivity distributions extend ever further into negative territory; combined with a low arrival rate, a poor productivity draw is expected to last for a long time. Consequently, firms are willing to incur very high layoff costs to terminate exceptionally poor productivity draws.[18]

Figure 7: Cross-phenomenon restriction



(a) Layoff cost criterion: minimum layoff cost at which all voluntary separations shut down when $\gamma^\ell = 0$



(b) Quit turbulence criterion: minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^\ell = 0.3$

The Figure 7b presents the quit turbulence criterion when layoff turbulence $\gamma^\ell = 0.3$. It reveals how outcomes are linked to those revealed by the layoff cost criterion in Figure 7a. Both outcome criteria are driven by the returns to labor mobility implied by the productivity process. The interrelatedness

[17] All outcome criteria figures are drawn for dispersion greater than 0.0722 (a support of 0.25). By omitting zero dispersion, we stay clear of economies that trivially have no endogenous separations. In such degenerate economies, the layoff cost criterion is zero and all turbulence criteria equal 1 since, in the absence of quits, no force could reverse the positive turbulence-unemployment relationship.

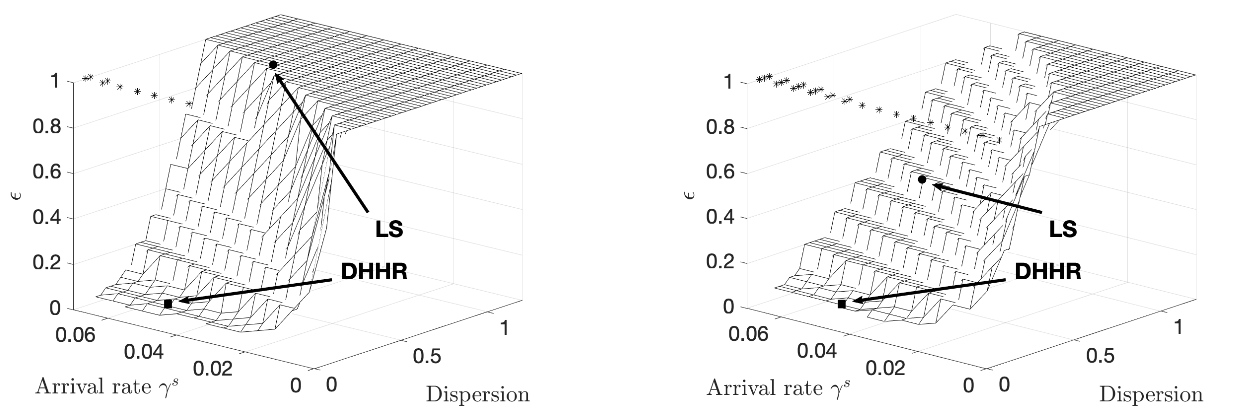
[18] As a point of reference, the axis for dispersion ends at 1.2 in the outcome criterion figures, which implies a width of just above 4 for the support of the uniform distributions. Thus, at a dispersion of 1.2, the combined productivity distributions for low- and high-skilled workers cover the entire range of the x-axis in Figure 1b.

of the effects of layoff costs on unemployment and of quit turbulence on unemployment reflects a cross-phenomenon restriction.

A notable difference between the two panels in Figure 7 is that the quit turbulence criterion plateaus at a maximum value of 1 when rates of return to labor mobility are so high that there exists no amount of quit turbulence that can reverse a positive turbulence-unemployment relationship. The stars at the front end of Figure 7b occur at very low values of dispersion and also indicate a quit turbulence criterion equal to 1. In this vicinity, for a given arrival rate, very small dispersions imply rates of return to labor mobility so low that, even without quit turbulence, no voluntary separations occur. Without voluntary separations, there is nothing to be shut down by introducing quit turbulence and hence there is no force coming from quit turbulence to reverse a positive turbulence-unemployment relationship.

Figure 8 portrays the dependence of the quit turbulence criterion on the amount of layoff turbulence γ^{ℓ}. A lower layoff turbulence $\gamma^{\ell} = 0.1$ in Figure 8a implies a steeper slope that quickens an ascent to a plateau where no amount of quit turbulence can reverse a positive turbulence-unemployment relationship. A higher layoff turbulence $\gamma^{\ell} = 0.5$ in Figure 8b slows down the ascent. At very low dispersions, the two panels show corresponding decreases and increases in the numbers of stars.

Figure 8: Quit turbulence criterion, $\gamma^{\ell} = 0.1$ and $\gamma^{\ell} = 0.5$



(a) Minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^{\ell} = 0.1$

(b) Minimum amount of quit turbulence that makes the turbulence-unemployment relationship negative when $\gamma^{\ell} = 0.5$

Figures 7 and 8 include two points denoted LS and DHHR that are our benchmark model versions of those frameworks with turbulence-unemployment outcomes as shown in Figure 6. For each framework, the arrival rate is $\gamma^s = 0.05$ as reported in Table 1, while the dispersion was chosen to target turbulence-unemployment outcomes in the appropriate framework. Recall that the width of support for the uniform distributions in the benchmark model version of DHHR is 0.6 and so that dispersion (measured as a standard deviation) equals $\sqrt{0.6^2/12} = 0.173$; corresponding numbers for the benchmark model version of LS are a width of support of 2.25 and hence a dispersion equal to 0.650. In line with Figure 6b, the quit turbulence criterion for DHHR is very low, about 0.05 for all three values of γ^{ℓ} in Figures

7b, 8a and 8b, respectively. Likewise, outcomes for LS are ones that can be inferred from Figure 6a; specifically, the quit turbulence criterion equals 0.58 at layoff turbulence $\gamma^\ell = 0.3$, 1 at the lower turbulence $\gamma^\ell = 0.1$, and 0.45 at higher turbulence $\gamma^\ell = 0.5$. These do good jobs of representing the quit turbulence outcomes in Figure 4 that we set out to explain.[19]

The cross-phenomenon restriction portrayed in Figures 7 and 8 helps assess the potential scope that quit turbulence brings for undermining LS's turbulence explanation of trans-Atlantic unemployment experiences. Starting with the DHHR analysis, its location in the space of productivity processes confirms our Section 3 conclusion that DHHR's reversal of LS relies on assuming a very compressed productivity distribution. DHHR's compressed productivity process renders their model incapable of explaining observed relationships between layoff costs and unemployment across countries.

Furthermore, DHHR's productivity process rests perilously downstream on the border of a parameter region with no voluntary separations (marked by stars). Hence, a small parameter perturbation could ironically turn DHHR's feeble positive turbulence-unemployment relationship into a strong one, as discussed above. Moving upstream to the other side of DHHR's productivity process would quickly raise the quit turbulence criterion before it reaches a parameter region consistent with observations on layoff costs and unemployment. Assuming higher values of layoff turbulence γ^ℓ provides little help to DHHR's point of view. In contrast, the LS analysis falls within a parameter region with quantitatively plausible implied returns to labor mobility, in terms of its implications for the effects of layoff costs on unemployment.

Another application We gather further insights from our parameter perturbation exercises by revisiting two celebrated macro-labor studies of layoff taxes. The first is a Mortensen and Pissarides (1999) matching model that calibrates productivity processes to unemployment statistics and outcomes in an unemployment insurance system. The second is a search-island model of Alvarez and Veracierto (2001) that enlists establishment data on firm and worker turnover to calibrate firm size dynamics. Baley, Ljungqvist and Sargent (2022) show in both frameworks how high returns to labor mobility are required to accompany empirically plausible unemployment responses to variations in layoff costs. Furthermore, they show how those high returns to labor mobility also sustain a positive turbulence-unemployment relationship even when quit turbulence is present.[20] Thus, the cross-phenomenon

[19] For the record, the layoff cost criteria in Figure 7a for the benchmark model versions of DHHR and LS are 23% and 129%, respectively, while the corresponding numbers are 14% and 186% in our layoff cost analyses in Section 3.2. The different numbers for the DHHR framework are due to the structural differences between the benchmark model version and the DHHR model described in footnote 12. In the case of LS, the difference is solely driven by the uniform productivity distributions in the benchmark model version of LS versus LS's own assumption of truncated normal distributions. Not surprisingly, it takes a higher layoff cost to shut down voluntary separations under the latter distributions with longer tails that include worse productivities than the narrower support of the uniform distributions. For our present argument, these differences are immaterial.

[20] Baley, Ljungqvist and Sargent (2022) also demonstrate that for parameterizations calibrated to fit firm size dynamics, even when parameters are perturbed, high returns to labor mobility prevail in models like Alvarez and Veracierto's (2001) in which shocks to productivity are intermediated through neo-classical production functions. But other macro-labor models that rely solely on unemployment statistics to calibrate per-worker productivity processes can have returns to labor mobility that are fragile with respect to perturbations of parameters that still fit targeted unemployment statistics. Baley *et al.* (2022) show that this is the case for Mortensen and Pissarides's (1999) calibration. Baley *et al.* conjecture that, because they focused on employment effects of layoff taxes, equilibrium outcomes probably would have prompted Mortensen and Pissarides to explore more of their parameter space if their calibration had wandered into the region with

restriction that prevails within our Section 2 benchmark model extends more broadly.

5 Concluding remarks

That the magnitude of returns to labor mobility contributes to several aggregate outcomes brings informative cross-phenomenon restrictions that can guide calibrations of productivity processes. Exploiting such restrictions adheres to the advice offered by Lucas (1980, pp. 696-697):

> "... we are interested in models because we believe they may help us to understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions."

For us, Lucas's relatively "simple question" is about how differences in layoff costs have affected labor reallocations, while the "harder question" concerns effects of quit turbulence on unemployment, about which much less is known. We recommend further studies of the role that returns to labor mobility play in macro-labor models.

Having recalibrated DHHR's model of quit turbulence to align it with a "Weinberg constraint," we rejoin the conversation with Alan Greenspan, with which DHHR began their paper. In the passage that DHHR cited, reproduced in Section 1 above, Greenspan does indeed seem to be concerned with the DHHR's quit turbulence force as well as its role in reducing job mobility that comes with DHHR's calibration. But Greenspan refrained from emphasizing such possible effects of increased turbulence more broadly. Earlier in that same paragraph, Greenspan (1998, p. 743) said that it was higher, not lower, labor mobility (i.e., "churning") that concerned him:

> "... the perception of increased churning of our workforce in the 1990s has understandably increased the sense of accelerated job-skill obsolescence among a significant segment of our workforce, especially among those most closely wedded to older technologies. The pressures are reflected in a major increase in on-the-job training and a dramatic expansion of college enrollment, especially at community colleges. As a result, the average age of full-time college students has risen dramatically in recent years as large numbers of experienced workers return to school for skill upgrading."

We read Greenspan as writing about US workers who had suffered the type of adverse human capital destruction shock that Ljungqvist and Sargent (1998, 2007, 2008) used to capture increased turbulence. Greenspan pointed out that such workers have ways of rebuilding their human capital in addition to the ways that are open to them in the Ljungqvist and Sargent models, thereby opening other ramifications of increased turbulence for outcomes studied by neither DHHR nor Ljungqvist and Sargent. It would be worthwhile to add such activities to models of trans-Atlantic unemployment experiences, while adhering to Weinberg's rules.

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extremely low returns to mobility.

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Cross-Phenomenon Restrictions:

Unemployment Effects of Layoff Costs and Quit Turbulence

Isaac Baley, Lars Ljungqvist, Thomas J. Sargent

Online Appendix

A Equilibrium computation

A.1 General algorithm structure

Here we outline the structure of the algorithm we used to compute equilibria.[21] It centers around approximating the joint continuation values $g_i(z)$ using linear projections on a productivity grid. It employs the following steps:

1. Fix a parameterization and construct productivity distributions over a grid of size N_z.

2. Guess initial values for:

   - ζ_i^k: coefficients for linear approximations $\hat{g}_i(z) = \zeta_i^0 + \zeta_i^1 z$ to $g_i(z)$
   - b_j: unemployment benefits
   - ω_{ij}^w: workers' outside values, not including current payment of benefit
   - ω^f: firms' outside value (in the benchmark model, $\omega^f = 0$)
   - τ: tax rate
   - u_{ij}, e_{ij}: masses of unemployed and employed workers

3. Given linear approximations $\hat{g}_i(z)$, use (2)–(5) to compute reservation productivities $\underline{z}_{ij}^o, \underline{z}_{ij}$.

4. Given cutoffs $\underline{z}_{ij}^o, \underline{z}_{ij}$, compute rejection probabilities ν_{ij}^o, ν_{ij} using (6) and compute E_{ij} using (7).

5. Compute the expected match surplus of a vacancy that encounters an unemployed worker:

$$\bar{s} \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}_{ij}^o}^{\infty} s_{ij}^o(y)\ dv_i^o(y).$$

6. Compute joint continuation values $g_i(z)$ using (8) and (9). Then update coefficients ζ_i^0, ζ_i^1 described in step 2 by regressing $g_i(z)$ on $[1 \ \ z]$.

7. Update the value of posting a vacancy, market tightness, and matching probabilities:

   - under endogenous market tightness in the benchmark model,

$$w^f = 0, \qquad \theta = \left(\frac{\beta A(1-\pi)\bar{s}}{\mu}\right)^{1/\alpha}, \qquad \lambda^w(\theta) = A\theta^{1-\alpha}, \qquad \lambda_{ij}^f(\theta) = A\theta^{-\alpha}\frac{u_{ij}}{u};$$

   - under DHHR's exogenous market tightness, compute

$$\omega^f = \frac{\beta}{1-\beta}A(1-\pi)\bar{s}, \qquad \theta = 1, \qquad \lambda^w = A, \qquad \lambda_{ij}^f = A\frac{u_{ij}}{u}.$$

[21] We are grateful to Wouter den Haan, Christian Haefke, and Garey Ramey for generously sharing their computer code. That code was augmented and modified by LS and further by us.

8. Update values ω^w_{ij} of being unemployed using (10) and (11).

9. Compute net changes in worker flows (all must be zero in a steady state)

$$\begin{aligned}\Delta u_{ll} &= \rho^r + (1-\rho^r)\left\{\rho^x + (1-\rho^x)(1-\gamma^u)\gamma^s \nu_{ll}\right\} e_{ll} \\ &- \rho^r u_{ll} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{ll})u_{ll} \end{aligned} \tag{A.1}$$

$$\begin{aligned}\Delta u_{lh} &= (1-\rho^r)\left\{\rho^x \gamma^\ell e_{hh} + (1-\rho^x)\nu_{hh}\gamma^q(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{lh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh} \end{aligned} \tag{A.2}$$

$$\begin{aligned}\Delta u_{hh} &= (1-\rho^r)\left\{\rho^x(1-\gamma^\ell) e_{hh} + (1-\rho^x)\nu_{hh}(1-\gamma^q)(\gamma^s e_{hh} + \gamma^u e_{ll})\right\} \\ &- \rho^r u_{hh} - (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \end{aligned} \tag{A.3}$$

$$\begin{aligned}\Delta e_{ll} &= (1-\rho^r)\lambda^w(\theta)\left\{(1-\nu^o_{ll})u_{ll} + (1-\nu^o_{lh})u_{lh}\right\} \\ &- \rho^r e_{ll} - (1-\rho^r)[\rho^x + (1-\rho^x)(\gamma^u + (1-\gamma^u)\gamma^s\nu_{ll}]e_{ll} \end{aligned} \tag{A.4}$$

$$\begin{aligned}\Delta e_{hh} &= (1-\rho^r)\left\{\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} + (1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}\right\} \\ &- \rho^r e_{hh} - (1-\rho^r)[\rho^x + (1-\rho^x)\gamma^s\nu_{hh}]e_{hh} \end{aligned} \tag{A.5}$$

These expressions embed the assumption of immediate realization of skill upgrades in the benchmark model. For DHHR's alternative assumption of delayed completion, see the corresponding expressions for worker flows in den Haan *et al.* (2005, Appendix A).

10. Compute average wages $\bar{p}_i$ and average productivities $\bar{z}_i$ as described in Appendix A.2, to determine government expenditures for unemployment benefits and government tax revenues using the left side and right side of (23), respectively.

11. Adjust tax rate τ in (23) to balance government budget.

12. Check convergence of a set of moments. If convergence has been achieved, stop. If convergence has not been achieved, go to 2 and use as guesses the last values computed.

A.2 Average wages and productivities

The following computations refer to the benchmark model with immediate realization of skill upgrades. For DHHR's alternative assumption of delayed completion, see den Haan *et al.* (2005, appendices A–C).

Our computation of the equilibrium measures of workers in equations (A.1)–(A.5) involve only two groups of employed workers, e_{ll} and e_{hh}, but each of these groups needs to be subdivided when we compute average wages and productivities. For employed low-skilled workers, we need to single out those who gained employment after first having belonged to group u_{lh}, i.e., low-skilled unemployed workers who received high benefits b_h. In the first period of employment, those workers will earn a higher wage $p^o_{lh}(z) > p^o_{ll}(z) \geq p_{ll}(z)$. And even afterwards, namely until their first on-the-job productivity draw, those workers will on average continue to differ from other employed low-skilled workers because of their higher reservation productivity at the time they regained employment, $\underline{z}^o_{lh} > \underline{z}^o_{ll} \geq \underline{z}_{ll}$.

Let e'_{ll} denote the measure of unemployed low-skilled workers with high benefits who gain employment in each period (they are in their first period of employment):

$$e'_{ll} = (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}.$$

Let e''_{ll} be the measure of such low-skilled workers who remain employed with job tenures greater than one period and who have not yet experienced any on-the-job productivity draw:

$$\begin{aligned} e''_{ll} &= (1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)\left[e'_{ll}+e''_{ll}\right] \\ &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^u)(1-\gamma^s)}\, e'_{ll}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed low-skilled workers and also their average productivity

$$\begin{aligned} \bar{p}_l &= \int_{\underline{z}^o_{lh}}^{\infty}\left[\frac{e'_{ll}}{e_{ll}}p^o_{lh}(y)+\frac{e''_{ll}}{e_{ll}}p_{ll}(y)\right]\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty}p_{ll}(y)\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})} \\ \bar{z}_l &= \frac{e'_{ll}+e''_{ll}}{e_{ll}}\int_{\underline{z}^o_{lh}}^{\infty} y\,\frac{dv^o_l(y)}{1-v^o_l(\underline{z}^o_{lh})} + \frac{e_{ll}-e'_{ll}-e''_{ll}}{e_{ll}}\int_{\underline{z}_{ll}}^{\infty} y\,\frac{dv_l(y)}{1-v_l(\underline{z}_{ll})}. \end{aligned}$$

For employed high-skilled workers, we need to single out those just hired from the group of unemployed high-skilled workers u_{hh} who earn a higher wage in their first period of employment, $p^o_{hh}(z) > p_{hh}(z)$. This is because they do not face the risk of quit turbulence if no wage agreement is reached and hence, no employment relationship is formed. For the same reason discussed above, we also need to keep track of such workers until their first on-the-job productivity draw (or layoff or retirement, whatever comes first). Reasoning as we did earlier, let e'_{hh} and e''_{hh} denote these respective groups of employed high-skilled workers;

$$\begin{aligned} e'_{hh} &= (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{hh})u_{hh} \\ e''_{hh} &= \frac{(1-\rho^r)(1-\rho^x)(1-\gamma^s)}{1-(1-\rho^r)(1-\rho^x)(1-\gamma^s)}\, e'_{hh}. \end{aligned}$$

Given these measures of workers, we can compute the average wage of all employed high-skilled workers and also their average productivity

$$\bar{p}_h = \int_{\underline{z}^o_{hh}}^{\infty} \left[\frac{e'_{hh}}{e_{hh}} p^o_{hh}(y) + \frac{e''_{hh}}{e_{hh}} p_{hh}(y) \right] \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} p_{hh}(y) \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}$$

$$\bar{z}_h = \frac{e'_{hh} + e''_{hh}}{e_{hh}} \int_{\underline{z}^o_{hh}}^{\infty} y \frac{dv^o_h(y)}{1 - v^o_h(\underline{z}^o_{hh})} + \frac{e_{hh} - e'_{hh} - e''_{hh}}{e_{hh}} \int_{\underline{z}_{hh}}^{\infty} y \frac{dv_h(y)}{1 - v_h(\underline{z}_{hh})}.$$

B Comparison of LS and DHHR

Our benchmark model is based on the LS model (Ljungqvist and Sargent, 2007) augmented to include quit turbulence as in the DHHR model (den Haan, Haefke and Ramey, 2005).

Besides LS having no quit turbulence, there are essentially three substantive differences between the models of LS and DHHR:

(i) how vacancies are created,

(ii) how the capital gain from a skill upgrade is split between firm and worker, and

(iii) productivity distributions.

As for vacancy creation, the LS model adopts standard assumptions of free entry of firms and an equilibrium zero-profit condition in vacancy creation, whereas DHHR assume a fixed measure of firms equal to the measure of workers so that the vacancy-unemployment ratio always equals unity under DHHR's implicit assumption of a sufficiently low vacancy posting cost that all firms without a worker post vacancies. As for skill upgrades, in the LS model an employed worker who experiences a skill upgrade can immediately choose to quit and search for employment elsewhere, whereas DHHR assume that such a worker must first work one more period with the present employer in order not to lose her skill upgrade; that has consequences for how a worker and a firm split the capital gain of a skill upgrade under Nash bargaining. Finally, the productivity distributions are assumed to be truncated normal distributions by LS and uniform distributions by DHHR, as detailed in Section 3.

Except for these differences, the remaining parameterizations of LS and DHHR are very similar.[22] A similarity that originates from an earlier exchange of views between den Haan *et al.* (2001) and Ljungqvist and Sargent (2004). Thus, Ljungqvist and Sargent (2004) advocated modifying the parameterization of den Haan *et al.* (2001) based on calibration targets in the search framework of Ljungqvist and Sargent (1998, 2008); but, as it turns out, with insufficient attention to *returns to labor mobility*. Specifically, Ljungqvist and Sargent (2004) criticized den Haan *et al.* (2001) for making low- and high-skilled workers almost indistinguishable from one another because of nearly overlapping productivity distributions for the two types of workers. As a remedy, by moving the uniform distributions apart and ending up with the disjoint supports in Figure 1b, Ljungqvist and Sargent (2004) succeeded in making low- and high-skilled workers distinct from one another; but as shown here that fails to generate returns to labor mobility consistent with historical observations. In the subsequent analysis by LS, layoff costs were introduced and productivity distributions had to be properly calibrated, as demonstrated in Section 3.2. Meanwhile, DHHR adopted Ljungqvist and Sargent's (2004) modification of den Haan et al.'s (2001) parameterization and proceeded to investigate quit turbulence.

[22]After taking into account DHHR's quarterly rather than semi-quarterly model period, their parameterization of sources of risk and labor market institutions are the same as in Table 1. Regarding the subjective discount factor $\hat{\beta}$ and the retirement probability ρ^r, DHHR set those to 0.995 and 0.005, respectively, at a quarterly frequency, which yield an adjusted discount factor β of 0.995 at a semi-quarterly frequency. We conducted a sensitivity analysis with respect to the different discount rates and found that adopting the DHHR discount rate in the benchmark model with LS productivity distributions does not substantively change our analysis.

C Perturbations of the benchmark model

As detailed in Appendix B, there are essentially three differences between the benchmark model with LS productivity distributions and the DHHR model in Figure 4: i) how vacancies are created, ii) how the capital gain from a skill upgrade is split between firm and worker, and iii) productivity distributions. To explain puzzling starkly different turbulence outcomes in Figure 4, our method is to start with the benchmark model with LS productivity distributions in Figure 4a and successively make perturbations one by one, with each perturbation addressing one of the three differences above.

To facilitate our perturbations, we renormalize the parameters (A, μ) in Table 1 so that equilibrium market tightness in tranquil times (no turbulence) becomes equal to one.[23] Recall that when calibrating a matching model to an aggregate unemployment rate, without any calibration targets for vacancy statistics, selecting the parameter pair (A, μ) is a matter of normalization.

C.1 First perturbation: Exogenous market tightness

The first perturbation concerns differences in the matching process. In the benchmark model, market tightness is endogenously determined by a typical free-entry-of-firms assumption. The equilibrium zero-profit condition in vacancy creation pins down market tightness. In contrast, DHHR assume fixed and equal masses of workers and firms so that market tightness is exogenously always equal to one.

Perturbation exercise As described in footnote 23, our renormalization of parameters (A, μ) in the benchmark model yields equilibrium market tightness equal to one at zero turbulence. Our first perturbation exercise is to keep market tightness constant at one as we turn up turbulence. We do that by subsidizing vacancy creation so that the value of a firm posting a vacancy is zero, $w^f = 0$, at market tightness equal to one for any given levels of layoff and quit turbulence. The vacancy subsidies are financed with lump-sum taxation so that government budget constraint (23) is unaffected.

In this exercise where subsidies are used to keep $w^f = 0$ at $\theta = 1$, let $\bar{S}^o(\gamma^\ell, \epsilon)$ denote the expected match surplus of a vacancy encountering an unemployed worker, given layoff turbulence γ^ℓ and quit turbulence $\gamma^q = \epsilon\gamma^\ell$:

$$\bar{S}^o(\gamma^\ell, \epsilon) \equiv \sum_{(i,j)} \frac{u_{ij}}{u} \int_{\underline{z}^o_{ij}}^{\infty} s^o_{ij}(y)\ dv^o_i(y) \tag{C.6}$$

where unemployment u_{ij}, reservation productivity $\underline{z}^o_{ij}$, and match surplus $s^o_{ij}(y)$ are understood to be equilibrium values under our particular perturbation exercise.

At zero turbulence, the operation of the subsidy scheme would not require any payments of subsidies

[23]Under the original parameterization $(A, \mu) = (0.45, 0.5)$ in Table 1, the equilibrium market tightness is equal to $\theta = 0.9618$ in tranquil times. We renormalize to attain an equilibrium market tightness of 1 and leave unchanged the probability that a worker encounters a vacancy. Let $(\hat{A}, \hat{\mu})$ be our new parameterization given by $\hat{A} = \kappa^{1-\alpha}A$ and $\hat{\mu} = \kappa\mu$. By setting κ equal to the market tightness under the old parameterization $\kappa = 0.9618$, the new parameterization, $(\hat{A}, \hat{\mu}) = (0.441, 0.481)$, achieves the desired outcomes.

This renormalization will be useful below when reconciling outcomes across models. Specifically, it will facilitate a perturbation exercise in which we shall replace free entry of firms in the benchmark model with the DHHR arrangement that exogenously fixes equal masses of firms and workers and a market tightness equal to one.

because we have parameterized the matching function so that equilibrium market tightness is then $\theta = 1$, a value of θ at which the zero-profit condition in vacancy creation is satisfied, $w^f = 0$, and by equation (14):

$$\mu = \beta(1-\pi)m(1)\bar{S}^o(0,0). \tag{C.7}$$

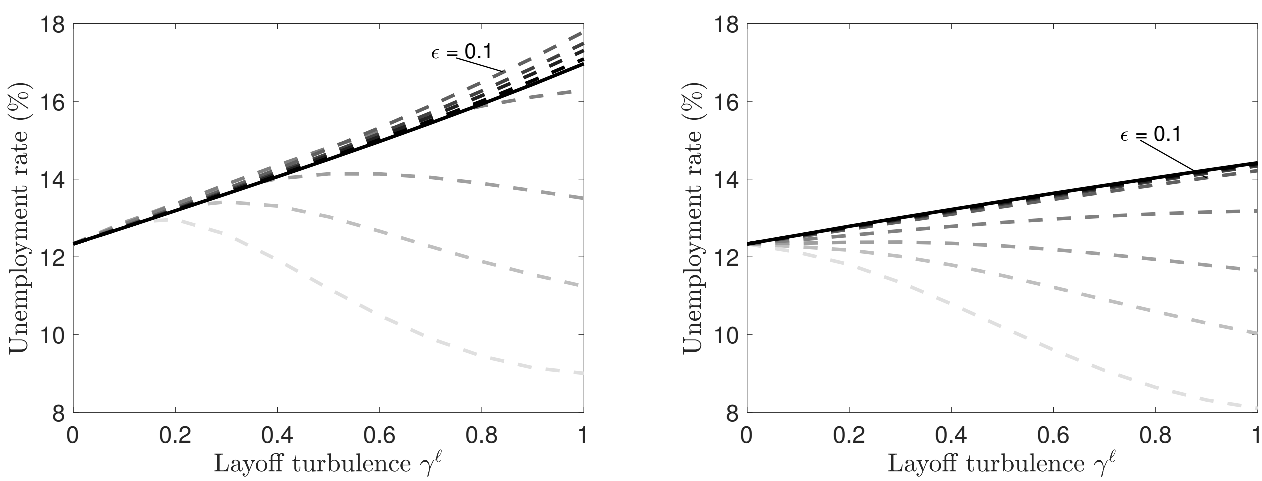
When turbulence is turned on, market tightness would have fallen if it were not for the subsidies to vacancy creation. The subsidy rate makes up for the shortfall of $\beta(1-\pi)m(1)\bar{S}^o(\gamma^\ell,\epsilon)$ when compared to the investment of incurring vacancy posting cost μ:

$$1 - subsidy(\gamma^\ell,\epsilon) = \frac{\beta(1-\pi)m(1)\bar{S}^o(\gamma^\ell,\epsilon)}{\mu} = \frac{\bar{S}^o(\gamma^\ell,\epsilon)}{\bar{S}^o(0,0)} \tag{C.8}$$

where the second equality invokes expression (C.7).

Results We observe an overall suppression of unemployment rates in Figure C.1b as compared to Figure C.1a. However, the underlying pattern of unemployment dynamics remains intact, so exogenous market tightness does not explain the puzzle.

Figure C.1: Endog. vs. exog. market tightness in benchmark with LS prod.



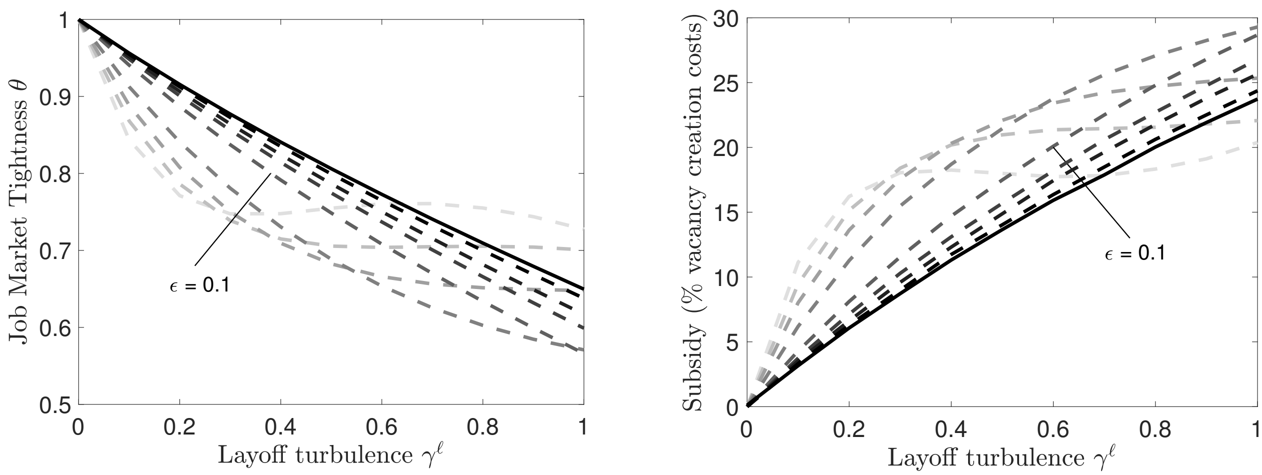
(a) Endogenous market tightness

(b) Exogenous market tightness

Discussion: Disarming the invisible hand With endogenous market tightness, there is a dramatic decline in market tightness in response to turbulence in Figure C.2a. This outcome reflects how an "invisible hand" restores firm profitability so that vacancy creation breaks even. Lower market tightness decreases the probability that a worker encounters a vacancy, which tends to increase unemployment.

Our perturbation exercise disarms those forces by exogenously freezing market tightness at one. Hence, the profitability of vacancies plummets in response to turbulence. Figure C.2b plots the subsidy rate for vacancy costs needed to incentivize firms to post enough vacancies to keep market tightness constant at one. At higher levels of turbulence, the subsidy rate becomes quite substantial. The subsidies to vacancy creation contribute to lower unemployment rates. These considerations seem to enhance a suspicion that exogenous market tightness could be the culprit behind the puzzle, so the above vindication was not a foregone conclusion.

Figure C.2: Falling market tightness vs. subsidies for vacancy creation



(a) Endogenous market tightness

(b) Exogenous market tightness

C.2 Second perturbation: Timing of completion of skill upgrades

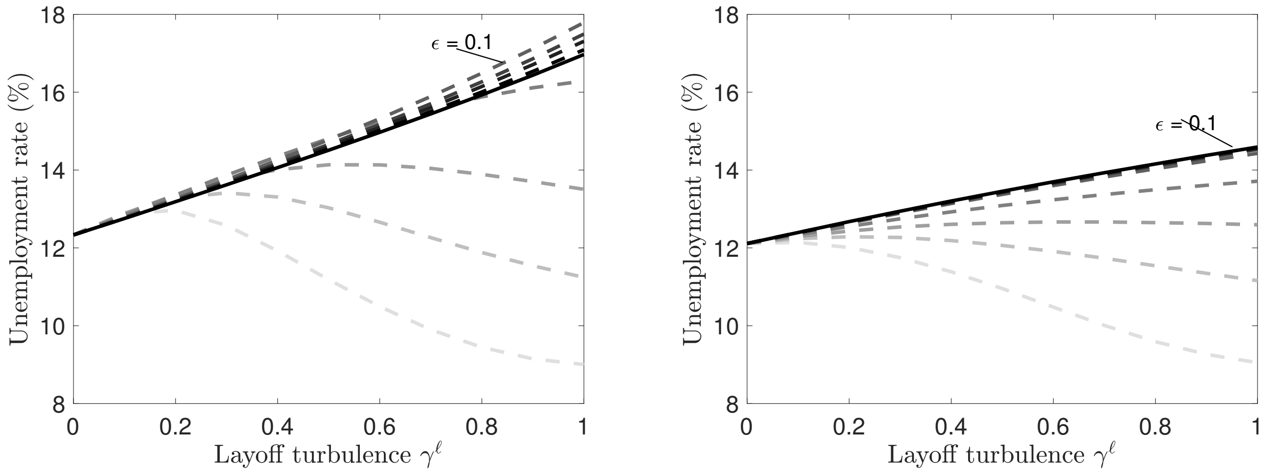
The second perturbation concerns differences in the timing of completion of skill upgrades. In the benchmark model, skill upgrades are immediately realized. In contrast, DHHR assume that a worker who receives a skill upgrade must remain with the present employer for one period in order to complete the higher skill level.

Perturbation exercise We replace immediate realization of skill upgrades in the benchmark model with delayed completion as in the DHHR model. The change in timing substantially alters the relative bargaining strengths of a worker and a firm.

Results The quantitative outcome in Figure C.3b is similar to that of the preceding perturbation exercise in Figure C.1b, i.e., it leads to an overall suppression in unemployment rates but without altering the underlying pattern of unemployment dynamics and hence, different timing of completion of skill upgrades does not explain the puzzle.

Discussion: Delayed completion requires "ransoms" Firms under DHHR's timing assumption are able to "rip off" workers whenever they transition from low to high skill at work. This is possible because the realization of that higher skill level is conditional upon a worker remaining with the present employer for at least one more period, during which the worker can be assessed a "ransom" to secure her human capital gain.

Figure C.3: Timing of completion of skill upgrade in benchmark with LS prod.

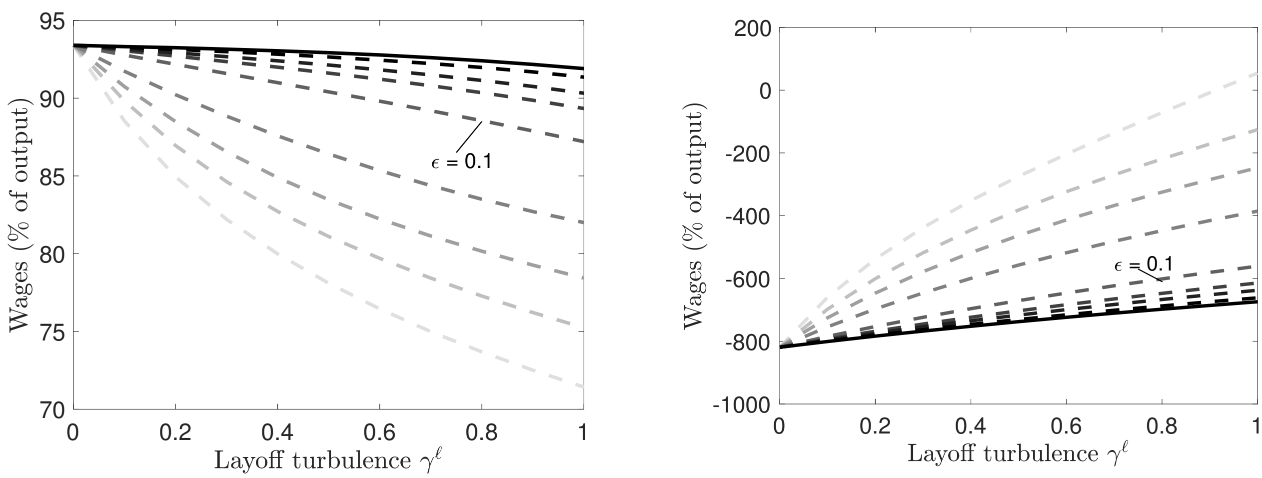


(a) Immediate upgrade

(b) Delayed upgrade

We compare average wages at skill upgrades under immediate completion (Figure C.4a) and delayed completion (Figure C.4b), expressed in terms of average output per worker in the laissez-faire economy at zero turbulence.[24] In Figure C.4b, a worker pays the "ransom" in terms of a negative semi-quarterly wage in the period of a skill upgrade, equivalent to the average annual output of a worker.

Figure C.4: Average wage in period of skill upgrade



(a) Immediate upgrade

(b) Delayed upgrade

The "ransom" becomes smaller with higher turbulence since the capital value of a skill upgrade is worth less when it is not expected to last long, as well as when quit turbulence locks high-skilled workers into employment relationships and thereby causes a less efficient allocation: fearing skill loss

[24]In the laissez-faire economy of the benchmark model with LS productivity distributions, a worker's average semi-quarterly output is 2.3 goods when $q^\ell = 0$.

at separations, high-skilled workers accept lower reservation productivities and hence, work on average at lower productivities as compared to an economy in tranquil times with higher labor mobility.

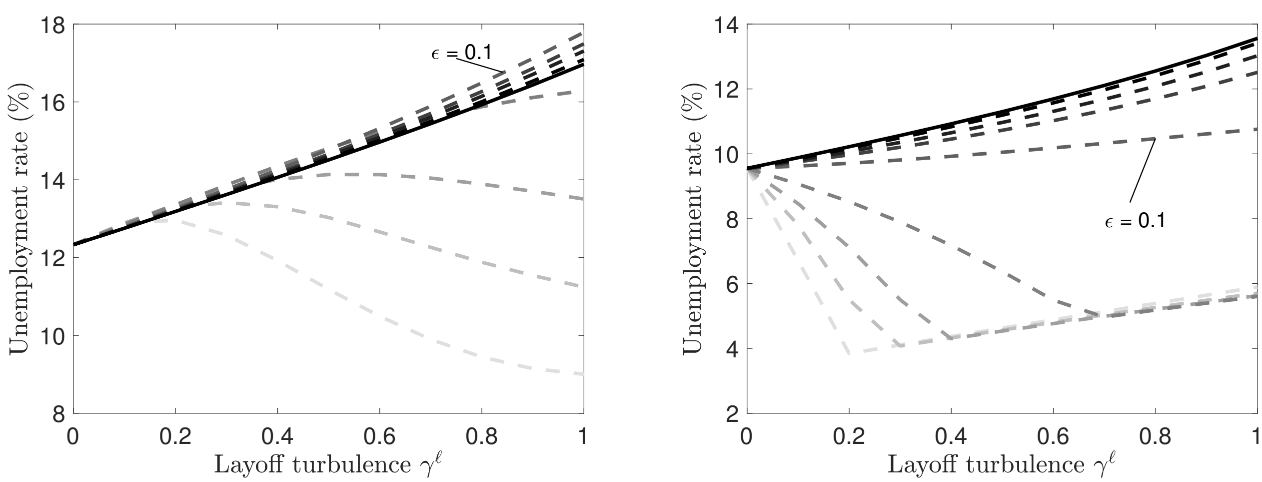
C.3 Third perturbation: Productivity distributions

The third perturbation concerns differences in productivity distributions. The benchmark model adopts the truncated normal distributions of LS with wide support. In contrast, DHHR assume uniform distributions with narrow support.

Perturbation exercise We replace the LS productivity distributions in the benchmark model with the DHHR productivity distributions.

Results The perturbation weakens the positive turbulence-unemployment relationship so much that we get DHHR-like outcomes in Figure C.5b. Thus, we conclude that differences in productivity distributions explain the different outcomes with respect to quit turbulence in Figure 4.

Figure C.5: LS vs. DHHR productivity distributions in benchmark model



(a) LS productivity distributions

(b) DHHR productivity distributions

Discussion: Meager returns to labor mobility Productivity draws on the job bring incentives for workers to change employers in search of higher productivities. The small dispersion of productivities under DHHR's uniform distributions with narrow support make returns to labor mobility be very low. As can be seen in Figure C.5b, those low returns do not compensate for even small amounts of quit turbulence and hence the initially positive turbulence-unemployment relationship at zero quit turbulence ($\epsilon = 0$) turns negative at relatively small levels of quit turbulence.

To confirm that the small dispersion of productivities explains the different outcomes with respect to quit turbulence in Figure 4, we do an additional perturbation exercise that shrinks the support of the uniform distribution further. Figure 6b in Section 3.4 shows outcomes in the benchmark model when the support of the uniform distribution has width 0.60 instead of 1. Such a shrinkage of the support takes us very close to the outcomes in the DHHR model in Figure 4b. Hence, in Section 3.4, we refer to the representation in Figure 6b as the benchmark model version of DHHR.

D Perturbations of the DHHR model

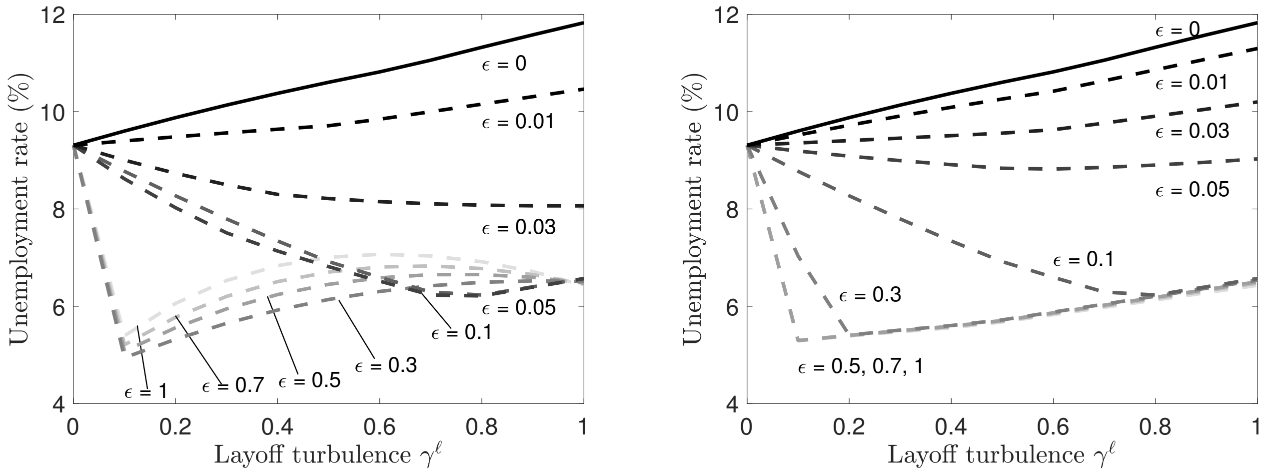
We now reverse the analysis of Appendix C by starting from the DHHR model and investigating the consequences of three perturbations. The features in the DHHR model to be perturbed are (i) exogenous labor market tightness, (ii) delayed completion of skill upgrade, and (iii) uniform productivity distributions with narrow support. But before that, we eliminate two auxiliary assumptions in the DHHR analysis.

Eliminate auxiliary assumption of zero benefits for newborn workers Instead of DHHR's assumption of no benefits during the initial unemployment spells of newborn workers, we assume that they are eligible for unemployment benefits equivalent to those of low-skilled workers. This modification reduces the number of worker types while having hardly any effect on aggregate outcomes.

Eliminate auxiliary assumption of turbulence for unemployed DHHR assume that after an encounter between a firm and an unemployed worker that does not result in an employment relationship, the worker faces the same risk of losing skills as if she had instead quit a job. DHHR describe this as an auxiliary assumption that they justify in terms of its computational tractability, but we find that it has noticeable quantitative consequences. Thus, Figure D.1 presents outcomes for the original DHHR framework with turbulence for unemployed workers and our modified DHHR model without that kind of turbulence. While the outcomes are not as stark in latter model, the underlying pattern of unemployment dynamics remains intact – it just takes some more quit turbulence to generate DHHR's key findings of a negative turbulence-unemployment relationship. From hereon, we refer to the modified model in Figure D.1b as the DHHR model.

An assumption that mere encounters between vacancies and unemployed workers are associated with risks of losing skills unless employment relationships are formed directly suppresses returns to labor mobility. But as can be inferred from Figure D.1, whether or not there is such an exposure of job seekers to skill loss does not matter much for DHHR's argumentation since, as Appendix D.3 will teach us, compressed productivity distributions in DHHR already reduce returns to labor mobility. However, the substantial incentives for labor mobility in the benchmark model with LS productivity distributions are significantly affected and suppressed by that auxiliary assumption of DHHR. Appendix E discusses this in detail.

Figure D.1: With vs. without turbulence for unemployed in DHHR



(a) With turbulence for unemployed

(b) Without turbulence for unemployed

D.1 First perturbation: Exogenous market tightness

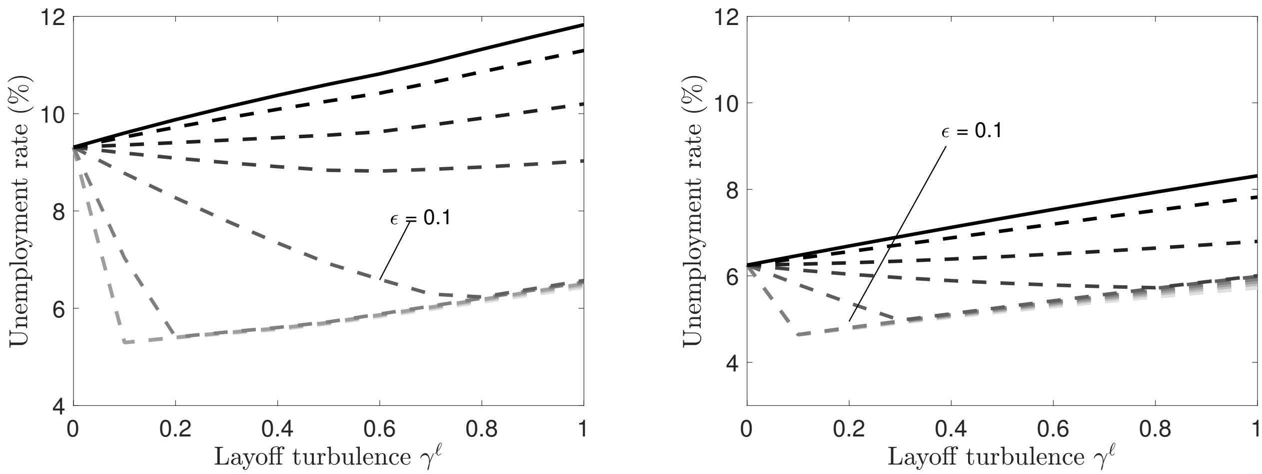
Perturbation exercise In the DHHR framework, there is an exogenous mass of firms and there are no costs for posting vacancies. Hence the value w^f of a firm posting a vacancy is trivially positive. We now perturb DHHR to feature free entry of firms, $w^f = 0$ in equilibrium, and an endogenous market tightness determined by (14). In order to implement that perturbation, we must introduce and assign values to two additional parameters, α and μ. Following the benchmark model, we assume that the elasticity of the matching function with respect to unemployment equals $\alpha = 0.5$, a fairly common parameterization.

Lacking an obvious way to parameterize the vacancy posting cost μ in this perturbation, we solve the model for different values of $\mu > 0$.[25] We find that for values of μ above 0.7, all voluntary quits vanish. Therefore, since DHHR's challenge to a Ljungqvist-Sargent positive turbulence-unemployment relationship is based on changes in the incidence of quits, we consider $\mu \in (0, 0.7)$ to be the permissible range. As an illustration, Figure D.2b depicts equilibrium outcomes for the midpoint of that parameter range, $\mu = 0.35$.

Results Except for the very top end of the parameter range $\mu \in (0, 0.7)$, the qualitative pattern of Figure D.2 represents the unemployment-turbulence relationship for the DHHR framework under the two alternative matching assumptions. In both cases, rather small amounts of quit turbulence reduce unemployment. Therefore, exogenous versus endogenous market tightness does not explain the puzzle.

[25] The vacancy posting cost μ must be positive to have an equilibrium with free entry of firms. The discrete model period and the Cobb-Douglas matching function call for an additional caveat. As the value of μ approaches zero, the equilibrium probability of filling a vacancy goes to zero. That creates a problem when the associated probability of a worker encountering a vacancy exceeds the permissible value of unity. Therefore, we only compute equilibria for μ greater than 0.0063. If one would like to compute equilibria for lower values of μ, it could be done by augmenting the match technology to allow for corner solutions at which the short end of the market determines the number of matches; e.g., in the present case, by freezing the job finding probability at unity while randomly allocating the unemployed across all vacancies that draw an "encounter." (See Ljungqvist and Sargent (2007, section 7.2).)

Figure D.2: Exogenous vs. endogenous market tightness in DHHR model



(a) Exogeneous market tightness

(b) Endogenous market tightness

In the vicinity of parameter value $\mu = 0.7$, the curve for $\epsilon = 0.1$ in the corresponding version of Figure D.2b (not shown here) takes on a positive slope, i.e., outcomes become LS-like with a positive turbulence-unemployment relationship. This might have been anticipated. As mentioned above, $\mu = 0.7$ is also the parameterization at which all voluntary quits vanish, which would seem to disarm the DHHR quit turbulence argument.[26]

Incidentally, as we will learn in Appendix D.3, the raw fact that voluntary quits vanish at a relatively low value of the vacancy posting cost $\mu = 0.7$ is indicative of low returns to labor mobility in the DHHR model that come from compressed productivity distributions.

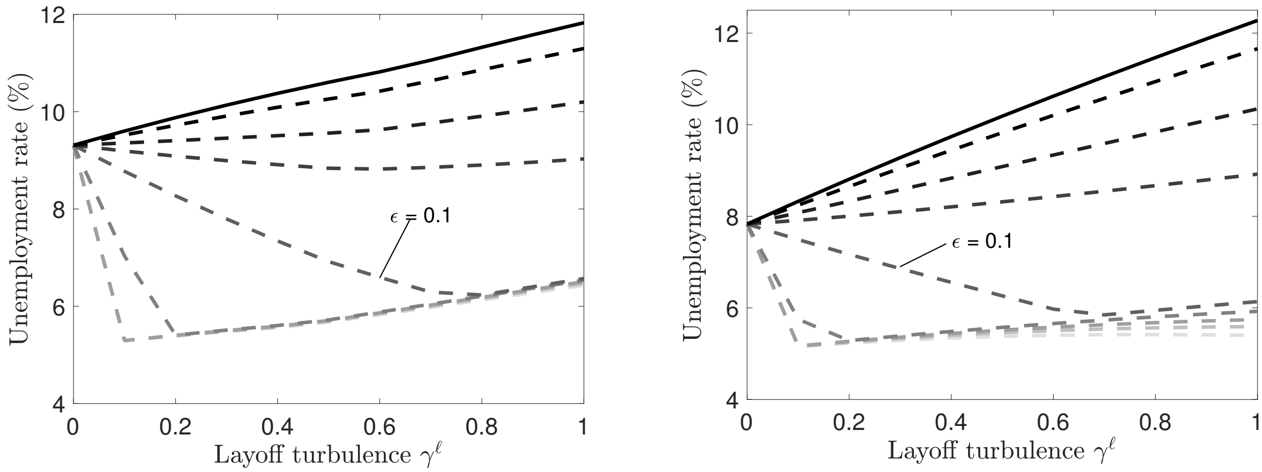
D.2 Second perturbation: Timing of completion of skill upgrades

Perturbation exercise DHHR assume that after a skill upgrade a worker must remain with the present employer for one period to complete the higher skill level. In this section, we introduce immediate completion of skill upgrades as in the benchmark model.

Results Figure D.3 shows that there is no substantial difference in the turbulence-unemployment relationship for the alternative timings in the DHHR model. Hence, delayed versus immediate completion of skill upgrades does not explain the puzzle.

[26]For a more nuanced reasoning about the equilibrium forces at work under the threat of losing skills in a matching model, see the discussion of an "allocation channel" and a "bargaining channel" in section E.2. While that section pertains to the introduction of turbulence facing unemployed workers in terms of a risk of losing skills after an encounter between a firm and a worker that does not result in employment, similar reasoning can be applied to quit turbulence for employed workers.

Figure D.3: Timing of completion of skill upgrade in DHHR model



(a) Delayed upgrade

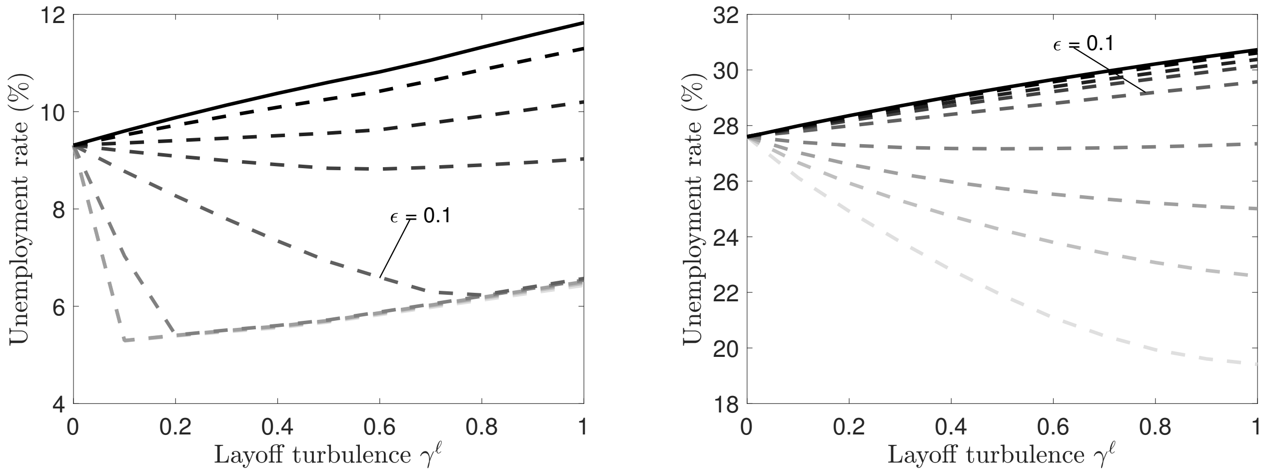
(b) Immediate upgrade

D.3 Third perturbation: Productivity distributions

Perturbation exercise DHHR assume uniform distributions with narrow support. In this section we replace those distributions in the DHHR model with the truncated normal distributions assumed by LS.

Results Figure D.4 shows how the turbulence-unemployment relationship is altered in the DHHR model when we switch from DHHR's productivity distributions to those of LS. First, the larger variances of the LS distributions exert upward pressures on reservation productivities and labor reallocation rates, but DHHR's assumption that an exogenously given market tightness equals one means that the relative number of vacancies cannot expand, so overall unemployment rates become higher. Second, and critical to our inquiry, the inference to be drawn from Figure D.4 agrees with what we inferred after studying the obverse perturbation of the benchmark model in Figure C.5; namely, differences in productivity distributions are key to explaining the puzzle. When we import the LS distributions into the DHHR model, small amounts of quit turbulence no longer unduly dissuade high-skilled workers with poor productivity draws to quit and seek better employment opportunities. Hence, the present perturbation disarms DHHR's argument for suppressed quit rates and allows the Ljungqvist-Sargent turbulence force to operate unimpeded. Figure D.4b shows how turbulence and unemployment are positively related until quit turbulence reaches about 30% of layoff turbulence after which the relationship becomes negative.

Figure D.4: DHHR vs. LS productivity distributions in DHHR model



(a) DHHR productivity distributions

(b) LS productivity distributions

E Turbulence affecting job market encounters

DHHR assume that after an encounter between a firm and an unemployed worker that does not result in employment, the worker faces the same risk of losing skills as if she had quit from a job. They justify this assumption only for its tractability in allowing them to reduce the number of worker types that they must track. In Figure D.1 of Appendix D, we confirm that the assumption does not make much of a difference for DHHR's inference about the turbulence-unemployment relationship in their model. But when we pursue a parallel analysis in the benchmark model with LS productivity distributions as we do here, we find that DHHR's simplifying assumption has a large impact. We show this in subsection E.1. To shed light on the forces at work, subsection E.2 undertakes yet another perturbation exercise that limits the exposure to such risk to the first $\bar{k}$ periods of an unemployment spell, after which there is no risk of skill loss during the rest of an unemployment spell.

To allow for a more general formulation, we assume a distinct probability γ^e of skill loss after an unsuccessful job market encounter, while γ^q continues to denote the probability of skill loss when quitting from an employment relationship.

E.1 Introducing turbulence for unemployed in benchmark model

When unemployed high-skilled workers face a probability γ^e of losing skills after unsuccesful job market encounters, the match surplus in (3) of a new job with a high-skilled worker changes to

$$s^o_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^e)\omega_{hh} + \gamma^e \omega_{lh}], \tag{E.9}$$

where the outside value in brackets reflects the risk of skill loss if the firm and worker do not enter an employment relationship. The net change of the mass of low-skilled unemployed with high benefits in

(24) changes to

$$\Delta u_{lh} = (1-\rho^r)\Bigg\{ \underbrace{\rho^x \gamma^\ell e_{hh}}_{\text{1. layoff turbulence}} + \underbrace{(1-\rho^x)\gamma^q \nu_{hh}[\gamma^s e_{hh} + \gamma^u e_{ll}]}_{\text{2. quit turbulence}} - \underbrace{\lambda^w(\theta)(1-\nu^o_{lh})u_{lh}}_{\text{3. successful matches}} + \underbrace{\lambda^w(\theta)\gamma^e \nu^o_{hh} u_{hh}}_{\text{4. turbulence unempl.}} \Bigg\} - \rho^r u_{lh}, \tag{E.10}$$

where the new term numbered 4 is the inflow of unemployed high-skilled workers who have just lost their skills after job market encounters that did not lead to employment.

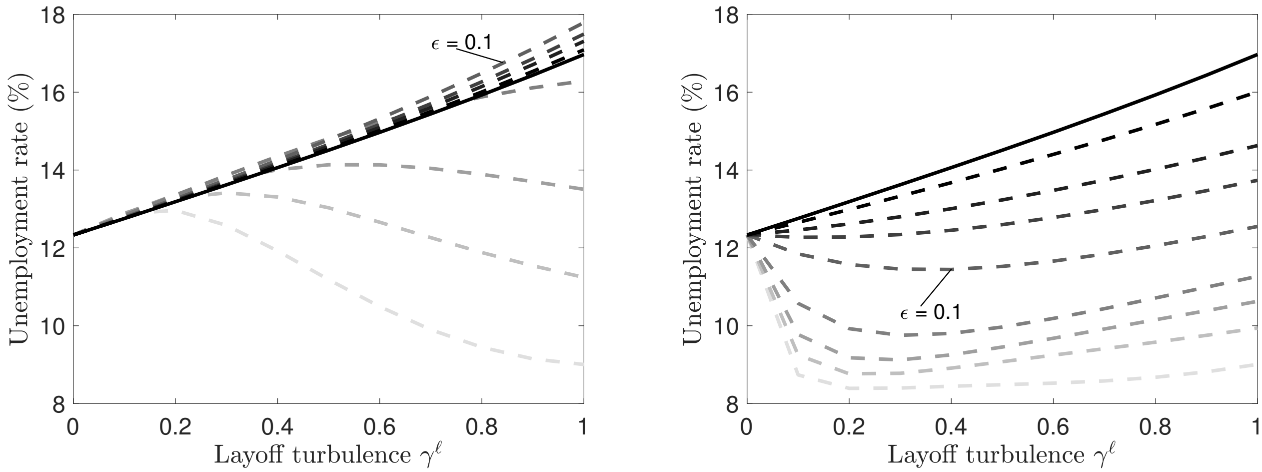
Turning to a quantitative assessment of turbulence for unemployed workers in the benchmark model with LS productivity distributions, we must take a stand on different lengths of a model period that were used in parameterizations of that model and DHHR. In the case of the exogenously given layoff risk, the probability of a layoff at the semi-quarterly frequency in the benchmark model is half of the probability at the quarterly frequency in DHHR's model, as discussed in footnote 22. Analogously, but less obviously, for the risk of skill loss after endogenously determined unsuccessful job market encounters we assume that $\gamma^e = 0.5\gamma^q$ in the semi-quarterly model as compared to DHHR's assumption that $\gamma^e = \gamma^q$ in their quarterly model. However, for the record, our conclusion from Figure E.1 remains the same with or without the latter adjustment. That is, with or without this adjustment, adding exposure of unemployed workers to risks of skill loss after unsuccessful job market encounters has sizeable effects on the turbulence-unemployment relationship in the benchmark model with LS productivity distributions.

As mentioned in footnote 11, risk of skill loss after unsuccessful job market encounters was not part of DHHR's use of quit turbulence to challenge a Ljungqvist-Sargent positive turbulence-unemployment relationship. Rather, they adopted it for computational tractability. Hence, we feel justified in discarding this auxiliary feature of DHHR's original analysis in order to focus more sharply on the key explanation to the puzzle – different productivity distributions. But it is nevertheless tempting to turn on and off their auxiliary assumption in order to shed further light on the mechanics of our particular matching model, and matching frameworks more generally. Therefore, we offer the following suggestive decomposition of forces at work.

E.2 Decomposition of forces at work

We seek to isolate two interrelated forces acting when job seekers are exposed to risk of skill loss after unsuccessful job market encounters in a matching model. First, the mere risk of losing skills when turning down job opportunities suppresses the return to labor mobility in many frictional models of labor markets, including the basic McCall (1970) search model where wages are drawn from an exogenous offer distribution. Such risks would render job seekers more prone to accept employment opportunities. We call this the "allocation channel." Second, the matching framework contains yet another force when risk of skill loss after an unsuccessful job market encounter weakens the bargaining position of a worker vis-à-vis a firm and accordingly affects match surpluses received by firms. That

Figure E.1: Turbulence for unemployed in benchmark with LS productivity



(a) Without turbulence for unemployed

(b) With turbulence for unemployed

in turn affects vacancy creation via the equilibrium condition that vacancy posting must break even. We call this the "bargaining channel."

It presents a challenge to isolate these two channels because everything is related to everything else in an equilibrium. Here we study how equilibrium outcomes change as we vary the horizon over which the risk of skill loss prevails during an unemployment spell. Thus, after an unsuccessful job market encounter, let an unemployed worker be exposed to risks of skill losses for the first $\bar{k}$ periods of being unemployed and thereafter to suffer no risk of skill loss for the remainder of that unemployment spell. To illustrate the allocation channel, consider the basic McCall search model. Starting from $\bar{k} = 0$, equilibrium unemployment would initially be significantly suppressed for each successive increase in the parameter $\bar{k}$ because workers anticipate ever longer periods of effective exposure to risk of skill loss when unemployed; but eventually, the value of $\bar{k}$ is so high that it is most unlikely that a worker remains unemployed for such an extended period of time and hence, a worker's calculation of the payoff from quitting a job would hardly be affected by any additional increase in $\bar{k}$. Thus, in a McCall search model, via the allocation channel, equilibrium unemployment would hardly change for higher values of $\bar{k}$. In contrast, we will find in the matching model that unemployment suppression effects that occur in response to increases in $\bar{k}$ don't die out beyond such high values of $\bar{k}$. We then argue that those equilibrium outcome effects can be attributed to the bargaining channel.

Notation Let u_{hh}^0 denote the mass of high-skilled workers who become unemployed in each period without losing skills, and let u_{hh}^k be the mass of those workers who remain high-skilled and unemployed after an unemployment duration of $k = 1, \ldots, \bar{k} - 1$ periods. A final category $u_{hh}^{\bar{k}}$ includes all workers who remain high-skilled and unemployed after unemployment spells of *at least* $\bar{k}$ periods, i.e., $u_{hh}^{\bar{k}}$ is the mass of unemployed high-skilled workers who no longer face any risk of skill loss in their current unemployment spells.

Using the same superscript convention, let $\omega_{hh}^{w,k}$ for $k = 0, \ldots, \bar{k}$ be the future value of unemployment of an unemployed high-skilled worker in category u^k_{hh}, with $\underline{z}^k_{hh}$ and ν^k_{hh} denoting the worker's reservation productivity and rejection probability next period, and for any match accepted next period, the match surplus is $s^k_{hh}(z)$ and the initial wage is $p^k_{hh}(z)$.

Laws of motion The laws of motion for worker categories u^k_{hh}, for $k = 0, \ldots, \bar{k} - 1$, have in common that all workers leave the category next period. The inflow to the initial category u^0_{hh} consists of employed high-skilled workers who experience non-turbulent layoffs or quits, including low-skilled employed workers who have just received a skill upgrade. Each successive category u^k_{hh}, for $k = 1, \ldots, \bar{k} - 1$, receives its inflow from not retired workers in the preceding category u^{k-1}_{hh}, those who did not match or experienced non-turbulent rejections of matches:

$$\Delta u^k_{hh} = \begin{cases} (1-\rho^r)\Big[\underbrace{\rho^x(1-\gamma^\ell)e_{hh}}_{\text{non-turbulent layoff}} + \underbrace{(1-\rho^x)\nu_{hh}(1-\gamma^q)(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{non-turbulent quit}}\Big] - u^k_{hh} & \text{if } k = 0 \\ (1-\rho^r)\Big[\underbrace{(1-\lambda^w(\theta))}_{\text{no match}} + \underbrace{\lambda^w(\theta)\nu^{k-1}_{hh}(1-\gamma^e)}_{\text{non-turbulent rejected match}}\Big]u^{k-1}_{hh} - u^k_{hh} & \text{if } 0 < k < \bar{k}. \end{cases}$$

The final category $u^{\bar{k}}_{hh}$ also receives inflows from the preceding category $u^{\bar{k}-1}_{hh}$, but now outflows are only partial. The workers who leave are the retirees and those with accepted matches (those with rejected matches are no longer affected by turbulence and thus always remain):

$$\Delta u^{\bar{k}}_{hh} = (1-\rho^r)\Big[(1-\lambda^w(\theta)) + \lambda^w(\theta)\nu^{\bar{k}-1}_{hh}(1-\gamma^e)\Big]u^{\bar{k}-1}_{hh} - \Big[\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu^{\bar{k}}_{hh})\Big]u^{\bar{k}}_{hh}.$$

The law of motion for u_{lh} workers is modified to receive the inflow from the different u^k_{hh} categories that suffered turbulent rejections in their first $\bar{k}$ periods of unemployment:

$$\begin{aligned} \Delta u_{lh} \;&=\; (1-\rho^r)\Big[\underbrace{\rho^x\gamma^\ell e_{hh} + (1-\rho^x)\nu_{hh}\gamma^q(\gamma^s e_{hh} + \gamma^u e_{ll})}_{\text{turbulent separations}} + \underbrace{\lambda^w(\theta)\gamma^e \sum_{k=0}^{\bar{k}-1}\nu^k_{hh}u^k_{hh}}_{\text{turbulent rejections}}\Big] \\ &\quad - \; [\rho^r + (1-\rho^r)\lambda^w(\theta)(1-\nu^o_{lh})]\,u_{lh}. \end{aligned}$$

The law of motion for high-skilled employed workers e_{hh} is adjusted to include those gaining employment from the different u^k_{hh} categories:

$$\begin{aligned} \Delta e_{hh} \;&=\; (1-\rho^r)\Big[\underbrace{\lambda^w(\theta)\sum_{k=0}^{\bar{k}}(1-\nu^k_{hh})u^k_{hh}}_{\text{accepted new matches}} + \underbrace{(1-\rho^x)\gamma^u(1-\nu_{hh})e_{ll}}_{\text{accepted upgrades}}\Big] \\ &\quad - \; [\rho^r + (1-\rho^r)(\rho^x + (1-\rho^x)\gamma^s\nu_{hh})]\,e_{hh}. \end{aligned}$$

High-skilled unemployed: match surplus, initial wage, and value of unemployment For a high-skilled worker who remains unemployed after $k < \bar{k}$ periods, the match surplus of any job opportunity next period reflects an outside option with risk γ^e of losing skills if the employment relationship is not formed; but after $\bar{k}$ periods there is no such risk:

$$s^k_{hh}(z) = \begin{cases} (1-\tau)z + g_h(z) - \Big[b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e \omega^w_{lh} + \omega^f\Big] & \text{if } k < \bar{k} \\ \\ (1-\tau)z + g_h(z) - \Big[b_h + \omega^{w,k}_{hh} + \omega^f\Big] & \text{if } k = \bar{k}. \end{cases}$$

Reservation productivities and rejection probabilities satisfy

$$s^k_{hh}(\underline{z}^k_{hh}) = 0\,, \qquad \nu^k_{hh} = \int_{-\infty}^{\underline{z}^k_{hh}} dv^o_h(y)\,.$$

The wage in the first period of employment of such a high-skilled worker is

$$\begin{array}{ll} p^k_{hh}(z) + g^w_h(z) = \pi s^k_{hh}(z) + b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e \omega^w_{lh} & \text{if } k < \bar{k} \\ p^k_{hh}(z) + g^w_h(z) = \pi s^k_{hh}(z) + b_h + \omega^{w,k}_{hh} & \text{if } k = \bar{k}. \end{array}$$

The value of unemployment for a high-skilled worker in her k:th period of unemployment is equal to $b_h + \omega^{w,k}_{hh}$, where

$$\omega^{w,k}_{hh} = \begin{cases} \beta \underbrace{\Big[\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty} \pi s^k_{hh}(y)\, dv^o_h(y)}_{\text{match + accept}} + \underbrace{\lambda^w(\theta)(b_h + (1-\gamma^e)\omega^{w,k+1}_{hh} + \gamma^e\omega^w_{lh})}_{\text{outside value with match}} \\ \qquad + \underbrace{(1-\lambda^w(\theta))(b_h + \omega^{w,k+1}_{hh})}_{\text{outside value without match}}\Big] & \text{if } k < \bar{k} \\ \\ \beta \underbrace{\Big[\lambda^w(\theta)\int_{\underline{z}^k_{hh}}^{\infty} \pi s^k_{hh}(y)\, dv^o_h(y)}_{\text{match + accept}} + \underbrace{b_h + \omega^{w,k}_{hh}}_{\text{outside value}}\Big] & \text{if } k = \bar{k}. \end{cases}$$

High-skilled employed: match surplus, wage, and joint continuation value The match surplus for continuing employment of a high-skilled worker reflects the risk of layoffs and quits that can be affected by turbulence in the form of skill loss. A non-turbulent separation falls into the initial category of high-skilled unemployed, u^0_{hh}. We adjust match surpluses, wages, and joint continuation values of these workers to include the new outside value $\omega^{w,0}_{hh}$.

The match surplus of a continuing job with a high-skilled worker is

$$s_{hh}(z) = (1-\tau)z + g_h(z) - [b_h + (1-\gamma^q)\omega^{w,0}_{hh} + \gamma^q\omega^w_{lh} + \omega^f]$$

and the wage equals

$$p_{hh}(z) + g^w_h(z) = \pi s_{hh}(z) + b_h + (1-\gamma^q)\omega^{w,0}_{hh} + \gamma^q\omega^w_{lh}.$$

The joint continuation value of a job with a high-skilled worker is

$$
\begin{aligned}
g_h(z) &= \beta\Big[\rho^x\left(b_h+(1-\gamma^\ell)\omega_{hh}^{w,0}+\gamma^\ell\omega_{lh}^w+\omega^f\right)\\
&+ (1-\rho^x)(1-\gamma^s)((1-\tau)z+g_h(z))\\
&+ (1-\rho^x)\gamma^s\left(E_{hh}+\nu_{hh}\left(b_h+(1-\gamma^q)\omega_{hh}^{w,0}+\gamma^q\omega_{lh}^w+\omega^f\right)\right)\Big].
\end{aligned}
$$

Since a low-skilled worker faces the possibility of a skill upgrade, we also need to update the joint continue value of an employed low-skilled worker as follows:

$$
\begin{aligned}
g_l(z) &= \beta\Big[\rho^x(b_l+\omega_{ll}^w+\omega^f)\\
&+ (1-\rho^x)(1-\gamma^u)(1-\gamma^s)((1-\tau)z+g_l(z))\\
&+ (1-\rho^x)(1-\gamma^u)\gamma^s\left(E_{ll}+\nu_{ll}(b_l+\omega_{ll}^w+\omega^f)\right)\\
&+ (1-\rho^x)\gamma^u\left(E_{hh}+\nu_{hh}\left(b_h+(1-\gamma^q)\omega_{hh}^{w,0}+\gamma^q\omega_{lh}^w+\omega^f\right)\right)\Big].
\end{aligned}
$$

Vacancy creation Free entry of firms make a firm's value ω^f of entering the vacancy pool be zero. With more types of unemployed high-skilled workers, zero-profit condition (14) changes to become

$$
\mu=\beta\frac{m(\theta)}{\theta}(1-\pi)\left[\frac{u_{ll}}{u}\int_{\underline{z}_{ll}^o}^{\infty}s_{ll}^o(y)\;dv_l^o(y)+\frac{u_{lh}}{u}\int_{\underline{z}_{lh}^o}^{\infty}s_{lh}^o(y)\;dv_l^o(y)+\sum_{k=0}^{\bar{k}}\frac{u_{hh}^k}{u}\int_{\underline{z}_{hh}^k}^{\infty}s_{hh}^k(y)\;dv_h^o(y)\right],
$$

where $u=u_{ll}+u_{lh}+\sum_{k=0}^{\bar{k}}u_{hh}^k$.

High-skilled unemployment spells terminated within $\bar{k}$ periods In each period, a mass u_{hh}^0 of high-skilled workers flows into unemployment. Let $\phi^{\bar{k}}$ denote the fraction of these who will experience unemployment spells of no longer duration than $\bar{k}$ periods. To enable a recursive computation, define m_h^k as the mass of workers who remain high-skilled and unemployed after k periods, and let m_l^k be the accompanying mass that remain unemployed but who have experienced skill loss by that kth period of unemployment. Given initial conditions $m_h^0=u_{hh}^0$ and $m_l^0=0$, we compute

$$
\begin{aligned}
m_h^k &= (1-\rho^r)\Big[1-\lambda^w(\theta)+\lambda^w(\theta)\nu_{hh}^{k-1}(1-\gamma^e)\Big]m_h^{k-1}\\
m_l^k &= (1-\rho^r)\Big[(1-\lambda^w(\theta)+\lambda^w(\theta)\nu_{lh})m_l^{k-1}+\lambda^w(\theta)\nu_{hh}^{k-1}\gamma^e m_h^{k-1}\Big],
\end{aligned}
$$

for $k=1,\ldots,\bar{k}$;[27] and

$$
\phi^{\bar{k}}=\frac{u_{hh}^0-m_h^{\bar{k}}-m_l^{\bar{k}}}{u_{hh}^0}. \tag{E.11}
$$

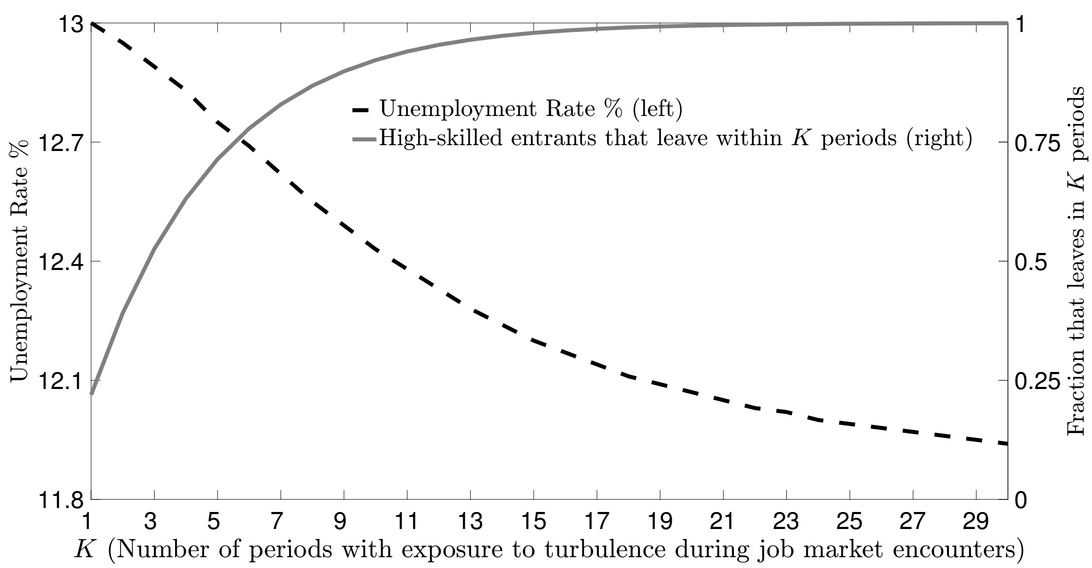
Numerical example To illustrate and decompose the forces at work, we set layoff turbulence equal to $\gamma^\ell=0.2$ and quit turbulence to $\gamma^q=\epsilon\gamma^\ell=0.1\cdot\gamma^\ell=0.02$. As discussed above, turbulence for

[27]Note that $m_h^k=u_{hh}^k$ for $k=0,\ldots,\bar{k}-1$, while $m_h^{\bar{k}}$ is merely a subset of $u_{hh}^{\bar{k}}$.

unemployed workers in the semi-quarterly benchmark model is assumed to be half of quit turbulence, i.e., $\gamma^e = 0.5\gamma^q = 0.01$.

Figure E.2 depicts two unemployment outcomes in distinct economies that differ only with respect to the parameter value of $\bar{k}$, i.e., the length of time over which an unemployed worker is exposed to the risk of losing skills due to unsuccessful job market encounters. The two outcomes are the unemployment rate u and the fraction $\phi^{\bar{k}}$ of high-skilled entrants into unemployment who will see their unemployment spells terminated within $\bar{k}$ periods by either finding employment or retiring. For each economy indexed by $\bar{k}$, the value of u can be read off from the dashed line (in percent on the left scale), and $\phi^{\bar{k}}$ from the solid line (as a fraction on the right scale).

Figure E.2: Turbulence exposure of unemployed in benchmark with LS prod.



As anticipated from our above discussion of the allocation channel, the unemployment rate in Figure E.2 is lower in economies with a higher $\bar{k}$ since longer exposure to risk of skill loss reduces the return to labor mobility. Hence, fewer high-skilled workers quit their jobs, and those who do quit will on average move back into employment more quickly. For example, when $\bar{k}$ increases from 1 to 9, the unemployment rate falls by half a percentage point. As noted earlier, the allocation channel would also be operating in the basic McCall search model, and the unemployment effects of further increases in $\bar{k}$ there should become muted when the value of $\bar{k}$ is set so high that the vast majority of unemployment spells are shorter than $\bar{k}$ in durations. But, as can be seen in Figure E.2 at $\bar{k} = 9$, 90 percent of all unemployment spells by high-skilled entrants are terminated within $\bar{k}$ periods, yet the unemployment rate falls another half a percentage point after further increases in $\bar{k}$. According to our earlier discussion of the bargaining channel, there is a force in matching models that is not present in McCall models. This other force makes it possible for skill losses at unlikely long unemployment spells to have substantial effects on equilibrium outcomes through its impact on bargaining. The reason is that even though realizations of such long unemployment spells are rare, the extended risk of skill loss will weaken the bargaining position of a worker vis-à-vis a firm throughout an unemployment spell.[28]

Figure E.3 depicts additional statistics that summarize outcomes across alternative values of $\bar{k}$. The positive relationship between $\bar{k}$ and market tightness indicates how the bargaining channel tilts match surpluses to firms when the risk of skill loss after unsuccessful job market encounters weakens the bargaining position of workers. Recall that the equilibrium zero-profit condition for vacancy posting funnels expected present values of firms' match surpluses into vacancy creation. The resulting higher market tightness implies a higher probability that an unemployed worker encounters a vacancy. Evidently, a worker's higher match probability induces low-skilled unemployed workers (as well as employed ones), both those with low and those with high benefits, to choose higher reservation productivities. The net result is still a shorter average duration of unemployment spells. And with not much change in a mildly U-shaped relationship for the job separation rate, we arrive at an unemployment rate that continues to fall over most of the range in Figure E.2. From these intricacies, we conclude that the bargaining channel already operates in tandem with the allocation channel over the first range of $\bar{k}$ in that figure, but that it operates mostly on its own over the second range where most entrants of high-skilled workers into unemployment expect to terminate their unemployment spells well before $\bar{k}$ periods.

[28]For another stark example of unlikely events having large effects on equilibrium outcomes through the bargaining channel, see Ljungqvist and Sargent's (2017) analysis of alternating-offer wage bargaining as one way to make unemployment respond sensitively to movements in productivity in matching models. A general result is that the elasticity of market tightness with respect to productivity is inversely related to a model-specific "fundamental surplus" divided by worker productivity. Under alternating-offer bargaining the fundamental surplus is approximately equal to the difference between worker productivity and the sum of the value of leisure and a firm's cost of delay in bargaining. Thus, the magnitude of the latter cost is a critical determinant of the volatility of unemployment in response to productivity shocks, even though no such cost will ever be incurred because in equilibrium there will be no delay in bargaining.

Figure E.3: More statistics pointing to the "bargaining channel"