Shotgun Weddings between Fiscal and Monetary Policies*

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Abstract

This paper critically surveys theories of how interactions between monetary and fiscal policies affect paths of equilibrium price levels and interest rates, including theories (a) of optimal anticipated inflation, (b) of optimal unanticipated inflation, and (c) of aspects of fiscal-monetary policies necessary to secure a "nominal anchor" in the sense of a unique price level path. We contrast theories that are specified just in terms of budget-feasible *sequences* of government issued bonds and money with those that require that monetary and fiscal policies be fully specified as sequences of functions mapping histories into time t decisions. We cite historical episodes that confirm what the government budget arithmetic implies: that lines of authority between a Treasury and a Central Bank can be obscure and ambiguous.

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The best way to defend the independence of a central bank is never to exercise it.

Anonymous Federal Reserve official

1 Introduction

In 1906 the United States had no central bank. Nevertheless, Secretary of Treasury Leslie M. Shaw adroitly deposited federal funds in temporarily distressed commercial banks and later returned them to the Treasury vaults where they belonged. In his 1906 report (United States. Department of the Treasury, 1906, page 49), Shaw wrote that "No central bank or government bank in the world can so readily influence financial conditions throughout the world as can the Secretary under the authority with which he is now clothed."[1] Secretary Shaw's Treasury report followed more than 50 years of legal, extralegal, and illegal actions by Secretaries of Treasury and compliant Congresses that by 1906 had subverted laws passed in the 1830s and 1840s that had completely separated US fiscal and monetary activities from *all* banks, public and private.

The Independent Treasury Act that governed US cash-management practices from 1844 until the founding of the Federal Reserve in 1914 banned the Secretary of Treasury from depositing federal funds in private banks, a rule gradually eroded by subterfuge, so that by Secretary Shaw's tenure, it was openly ignored. After Andrew Jackson's Democrats forced the Federal government to forego efficiencies and profits to be reaped from issuing paper notes as money before the civil war, state governments chartered state banks that issued circulating notes backed by state bonds as collateral and then shared the monopoly profits and rents with those state banks. During the Civil War, the US government nationalized those rent sharing activities by taxing state banks out of the note issuing business and establishing a National Banking system with member banks that were authorized to issue national bank notes collateralized by selected Federal bonds. In

[1] In the same report, Shaw asked Congress for even more power: "... $100,000,000 to be deposited with banks or withdrawn as he might deem expedient, ... [and] authority over the reserves of the several banks with power to contract the national-bank circulation at pleasure ...."

1862 Congress also began issuing inconvertible greenbacks that it made legal tender for almost all debts public and private and whose value soon dropped to 40 or 50 cents on a dollar vis a vis the gold dollars that continued to be used for international trades and payments of customs duties, setting off years of debate about how many greenbacks the Congress should leave outstanding and whether Congress should service interest-bearing debt in gold dollars or the depreciated greenbacks, a dispute only temporarily resolved when on January 1, 1879 the Treasury made greenbacks convertible into gold dollars one for one. From the Civil War until the founding of the Federal Reserve in 1914, monetary policy gave all U.S. Presidents and Congresses headaches and heartaches. Congresses and Presidents issued paper notes that traded as money – greenbacks, silver certificates, National Bank Notes, and against the background of a declining price level from 1865 to 1896, confronted pressures to issue more notes and to change the collateral constraints on National Bank notes which originally were cast in terms of a list of Federal bonds. Congress debated proposals to allow banks to issue notes backed by other forms of collateral. In 1889 the Farmer's Alliance called for issuing small Treasury money notes in exchange for farmers' crops as collateral that would to be stored in warehouses called subtreasuries.[2] Silver producers and advocates of measures to increase the US price level advocated coin silver or issuing federal notes collateralized backed by silver at an exchange rate two or three times above the current market price of silver. Secretary Shaw lamented in his 1906 report that his open market operations were constrained by the accumulated funds he held in independent treasury vaults. To supplement those, the "Fowler Bill" and other aborted bills authorized banks to issue notes backed by railroad bonds, municipal securities, and other assets. Clearing houses issued certificates that temporarily served as cash substitutes during financial crises. The Fowler bill and related proposals aimed to extend and nationalize those temporary arrangements, a movement that eventually led to the passage of the Aldrich-Vreeland Act in 1908 that among other things authorized emergency currency collateralized by various types of private securities.

Other stories from late 19th and early 20th century US monetary-fiscal history would also serve to illustrate the main theme of this paper that intertemporal government budget constraints

[2]See White (2017, p.830) and Malin (1944).

don't separate monetary policy from fiscal policy. Central bank independence is a fiction, a convention, or a fragile institutional artifact. Budget arithmetic forces monetary and fiscal policies to be either consolidated or coordinated. From the point of view of the sequences of bonds and money that a government chooses, only details depend on institutional arrangements that delegate decisions about components of those sequences to people working in different buildings. In this paper we mostly ignore those details and confine our attention to arithmetic that binds monetary to fiscal policy. Nevertheless, when we discuss theories of a "nominal anchor" for the price level sequence in a world with fiat paper money, we will be forced to study how a polity assigns budgets and strategies to separate decision makers called a treasury and a central bank. We also limit our discussion of another closely related fragile line separating monetary from credit policies.

Section 2 uses a baseline model to describe how a gold standard secures a nominal anchor, then tells how the addition of a paper money can improve outcomes but leaves an exchange rate between paper and gold indeterminate. That indeterminacy is our first encounter with difficulties in securing a unique nominal anchor for a paper currency. Section 3 extends our baseline model to include an intertemporal consolidated government budget constraint within which we can study theories of an optimal quantity of money under flexible prices. Here we study normative theories of both anticipated and unanticipated inflation. Section 4 uses irrelevance theorems for government open market operations in the spirit of Wallace (1981a) to frame difficulties in distinguishing between monetary and fiscal policies. While in sections 2-4 we naturally cast monetary-fiscal policies in terms of *sequences* of settings of monetary and fiscal policy variables, e.g., government taxes and purchase and bonds and money supplies, we must proceed differently when we study a "fiscal theory of the price level" in section 5 in terms of government *strategies* (i.e., sequences of functions mapping partial histories into time t actions) that are sufficient to deliver a unique price level path and that thereby serve as a *nominal anchor*. Section 6 extends our discussion of fiscal theories of a nominal anchor by describing how separate budget constraints and strategies can be assigned to a central bank and treasury. We relate the two-budget-constraints analysis to recent manifestations of "in-house fiscal policies" that have been conducted in the United States and other countries since the 2007-2008 financial crisis.

2 A Gold Standard

A fiscal/monetary authority interacts with a continuum of identical households. Time is discrete and indexed by $t = 0, 1, 2, \ldots$. At the beginning of a period each household sends one buyer and one seller to distinct decentralized markets where a seller from one household can use its labor to produce a single consumption good and a buyer from another household can purchase that consumption good for a perfectly storable asset called cash.[3] After decentralized markets close, a single centralized market opens in which all household members can trade the nonstorable good and labor for cash and interest-bearing assets. We depart from the bilateral bargaining in decentralized markets that occurs in Lagos and Wright (2005) and instead assume that enough participants attend decentralized markets to make them competitive.[4] This renders our setup equivalent to the cash-in-advance model of Lucas and Stokey (1987).[5] Agents are anonymous in decentralized markets so that trade is possible there only if a medium of exchange called cash is present. Accordingly, we refer to the consumption good exchanged in decentralized markets as the "cash good."

After producing and exchanging goods for cash in decentralized markets, households reunite and go to the centralized market where they can use additional labor to produce more of a consumption good that they can also purchase from other households. Then they consume what they have purchased in the two markets they have attended this period. In the centralized market, households interact with a monetary/fiscal authority that can tax, trade, and issue

[3]We'll use "cash" and "money" as synonyms.

[4]We assume symmetry across markets so that, while each household cannot consume the good it produces and is required to send its buyer and seller to different markets, equilibrium prices and quantities are the same in all decentralized markets.

[5]In discussing interactions between monetary and fiscal policies, it is important to describe precisely how households interact with the government. The Lagos-Wright model is more explicit about this than are many other cash-in-advance models, our reason for adopting it here. We refrain from following Bassetto (2002) in completely describing a monetary economy as a game.

paper currency. In the centralized market, three assets are traded, namely, "token money" and "bonds," and a perfectly durable costlessly storable object called "gold." As in Sargent and Smith (1997), there is a reversible linear technology that yields ϕ ounces of gold per unit of the consumption good and one unit of consumption from ϕ^{-1} ounces of gold.[6] Gold is costlessly recognizable, making it usable as cash in decentralized trade.[7]

Distinctions between fiscal and monetary policies rest partly on how differences between money and bonds are modeled. We assume that paper money can be used in the decentralized market[8] but that bonds cannot. A story that supports this outcome is that bonds can be recognized and traded only in the centralized market. Households cannot hold negative amounts of gold or money, but they can hold negative bonds by borrowing up to an exogenous real bound $\underline{B}$ denominated in units of the consumption good.

Households order consumption, labor streams $\{c_t, x_t, \ell_t\}_{t=0}^{\infty}$ by

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(c_t) + v(x_t) - \ell_t], \tag{1}$$

where c_t is consumption of the "credit good" that is traded in the centralized market, x_t is consumption of the "cash good" that is traded in the decentralized market, and ℓ_t is the sum of labor supplied in decentralized and centralized markets. We assume that u and v are strictly increasing and concave and continuously differentiable. Linearity of utility in labor helps tractability but is not essential for our results about monetary and fiscal policies.[9] E_0 denotes a mathematical expectation conditioned on time 0 information. Uncertainty can be about different things in different applications. We mostly introduce uncertainty as stochastic processes for government spending and possibly an exogenous component of tax revenues. We can also include a sunspot process that has no direct effects on preferences or technologies but only indexes multiple equilibria that emerge purely through identical expectations across households about the nominal price level.

[6]See Barro (1979) and Sargent and Wallace (1983) for related settings in which adopting a gold standard sets a nominal anchor.

[7]We ignore Sargent and Smith's tax on minting.

[8]Record-keeping facilities are not available there, making room for a role for gold or paper money.

[9]We also assume that parameter values are such that the nonnegativity constraint on labor never binds.

2.1 Gold as Medium of Exchange

Consider a situation in which there are no taxes, government-issued bonds, or government-issued money. To carry out decentralized trade, households can use only gold as cash. In an equilibrium in which the cash good and credit good are both produced and consumed, their prices in terms of gold are equal and households are indifferent whether to supply labor in the decentralized market now in order to earn money for buying the cash good next period or to supply labor in the decentralized market to produce the credit good. Let P_t be the common price of both the cash good and the credit good in terms of gold. Since gold can be freely converted from or into the credit good at a rate ϕ ounces of gold per unit of the consumption good, we must have $P_t = \phi$. A feasible consumption/labor plan satisfies:

- Let z_{t-1} be gold carried by a household into the decentralized market. Because gold is the only cash, a buyer must start a period with enough gold to purchase the cash goods that the household wants to consume that period:[10]

$$\phi x_t \leq z_{t-1}. \tag{2}$$

- Let B_{t+1} be nominal bonds purchased by the household in period t, to be repaid in period $t+1$. The payoff of these bonds can depend on time $t+1$ shock realizations. Households trade in (dynamically) complete markets. We will often restrict the government to issue nominally risk-free bonds, in which case all state-contingent claims must be in zero net supply. Let S_{t+1} be the time-t equilibrium one-period stochastic discount factor, so that the value at time t of a portfolio B_{t+1} is $E_t S_{t+1} B_{t+1}$. Then the household budget constraint in the asset market in period 0 is

$$B_t + \phi(\ell_t - x_t - c_t) + z_{t-1} \geq z_t + E_t[S_{t+1}B_{t+1}]. \tag{3}$$

  A household starts a period owning maturing nominal bonds and gold carried over from the previous period, earns wages from working in either market, and buys consumption goods.

[10] At the beginning of a period, it is optimal for the household to allocate all of the gold to the buyer since a seller does not need it in a decentralized market.

What is not spent on consumption is allocated between gold and (possibly state-contingent) bonds to be carried into the next period.

- A "no-Ponzi condition" ensures that households cannot finance consumption by borrowing each period and forever rolling over maturing debts:[11]

$$\lim_{s \to \infty} E_t \left[q_s (B_{s+1} + z_s) \right] \geq 0, \quad t \geq 0, \tag{4}$$

where E_t is a mathematical expectation conditional on information available at time t and q_s is the cumulated stochastic discount factor between period 0 and period s:

$$q_0 = 1 \tag{5}$$

$$q_s = \prod_{t=1}^{s} S_t, \quad s > 0, \tag{6}$$

Optimal household decisions are characterized by:

- Intratemporal optimality between leisure and credit goods:

$$u'(c_t) = 1; \tag{7}$$

- Intertemporal optimality via Euler equation:[12]

$$S_{t+1} \equiv \beta : \tag{8}$$

[11] We express a no-Ponzi condition as the limit of an expectation that must hold almost surely at any time t. For a more-complete discussion of the no-Ponzi condition and the resulting transversality condition, see Weitzman (1973) for deterministic problems and Kamihigashi (2003) and Coşar and Green (2016) for stochastic problems. Because we have assumed that disutility is linear in labor, in principle a household could repay any amount of debt by working sufficiently many hours. In our applications, we implicitly assume that there is an upper bound on hours worked that is sufficiently loose that it never binds in finite time; its role is to imply that a strategy of rolling over debt indefinitely would eventually make it impossible to repay the accumulated balance.

[12] Utility is linear in leisure and the price level is constant over time, so the stochastic discount factor is constant over time.

- Optimality of cash goods:[13]

$$v'(x_t) = 1/\beta; \tag{9}$$

- Optimal gold holdings:

$$z_t = \phi x_{t-1}; \tag{10}$$

- The budget constraint (3) and the transversality condition (4) must hold as equalities.

Households are indifferent about when they work, so a household's bond position is indeterminate. In the aggregate, bonds are in zero net supply, so in a symmetric equilibrium $B_t = 0$. The labor supply is determined as a residual either from the household budget constraint or from the production function.[14] The use of gold as cash fixes the price level over time and thus provides a "nominal anchor".[15]

2.2 More Efficient Equilibrium

The equilibrium of Section 2.1 anchors the nominal price level at the social cost of requiring households to carry gold as cash. In our reference model household hold gold not because it gives them utility directly (see equation (1)) but because trading arrangements give gold utility indirectly by enabling decentralized trade. More efficient equilibria exist if it is feasible to replace gold as cash with a costlessly produced paper money whose supply society is able to limit. Then paper money can be used as cash and all gold can be converted into the consumption good. Adopting the paper dollar as the unit in which prices are denominated, the household budget

[13]Time 0 is special, as the household may inherit an exogenous level of gold that does not correspond to equations (9) and (10). For that period, we have $v'(x_0) = \max\{1, v'(z_{-1}/\phi)\}$, and the cash-in-advance constraint may be slack.

[14]These yield the same solution by Walras' law.

[15]We have fixed the relative price of gold and goods by setting the linear technology parameter ϕ to be a constant. In practice, a gold standard imperfectly stabilized the price level, as analyzed e.g. in Cogley and Sargent (2015). We could capture such observed outcomes mechanically by letting ϕ vary over time either predictably or unpredictably.

constraint now becomes

$$B_t + P_t(\ell_t - x_t) \geq M_t - M_{t-1} + \frac{(z_t - z_{t-1})P_t}{\phi} + P_t c_t + E_t[B_{t+1}S_{t+1}] + T_t, \tag{11}$$

where T_t are nominal taxes or transfers (if negative) from the government. As a simple first experiment, assume that the government distributes to each household M_0 paper dollars before the start of period 0, after which no subsequent transactions occur between the government and households, so that $T_0 = -M_0$ and $T_t = 0$ in all subsequent periods. In period 0, $B_0 = M_{-1} = 0$, and $z_{-1} > 0$ is exogenously given, as before.[16] The cash-in-advance constraint and the no-Ponzi condition now include paper money:

$$M_{t-1} + \frac{P_t}{\phi} z_{t-1} \geq P_t x_t, \tag{12}$$

$$\lim_{s \to \infty} E_t \left[q_{s+1} \left(B_{s+1} + \frac{z_s P_{s+1}}{\phi} + M_s \right) \right] \geq 0. \tag{13}$$

There exists a paper-money-only equilibrium in which equations (7), (8), and (9) are satisfied and $z_t = 0$ and $M_t/P_t = x_t$ for $t \geq 0$. In this equilibrium, consumption is the same as in the economy with only gold as cash, but households use only paper money to trade. They convert all of their gold into the consumption good at the start of period zero. That increases their utility by reducing their labor supply at $t = 0$.[17]

There exist other equilibria in which paper money and gold coexist as cash and in which the value of total cash in terms of consumption goods equals it value in the cash-must-be-gold equilibrium. In a stochastic environment, there are also sunspot equilibria in which the price level (the inverse of the value of money) fluctuates over time. The original cash-must-be-gold equilibrium also survives: if households expect paper money to be worthless, they will accept only

[16] This experiment sheds light on the end of the Hungarian hyperinflation after World War II described by Paal (2000). Hyperinflation drove real money balances nearly to zero. The government reformed fiscal-monetary policy so that it would no longer print currency to finance government expenditures. To supply new currency, the central bank extended loans to favored particular people. Many of these loans were not repaid, so that the central bank remonetized the Hungarian economy partly by gift-giving.

[17] Households reap all of the benefit in the form of leisure in period zero because we assumed linear preferences in leisure. In a richer model, the real interest rate would change, consumption in period zero would increase, and, depending on parameter values, the benefit could be spread over multiple periods.

gold as cash. If households expect paper money to be worthless tomorrow, it will be worthless now.[18] In pure paper money equilibria in which $z_t = 0$, there is no nominal price level anchor. Furthermore, even in equilibria in which $z_t > 0$ for $t \geq 0$, at time t the value of paper money has no nominal anchor: logic that underlies Kareken and Wallace (1981) exchange rate indeterminacy renders the initial value of money indeterminate.[19]

We have assumed that the government simply gives the entire money stock to the representative household. There are plenty of alternative ways for a government to spend paper money, many of which have been studied by monetary theorists and tried in practice. A government can *sell* paper money to households in exchange for either goods or for private-sector IOUs that the government can later use to finance government purchases. Furthermore, economic fundamentals don't require that a government issue the paper currency. Instead maybe a private bank could do that.

3 Government Budget Constraint and Optimal Inflation

In the Section 2.2 analysis, the government acts only at time 0 when it distributes fiat money. We now consider what happens when both the government and private households act repeatedly. In this case, there is a sequence of government budget constraints at play and interactions between monetary and fiscal policy actions over time become more complicated. To simplify (and modernize), we drop gold from the analysis and [20] also assume that the government is a monopoly provider of money.

[18]To represent this no-value equilibrium would require adjusting our notation because money cannot be taken as the numeraire when it is worthless.

[19]The evolution of the price level is also indeterminate, but we know that it must be a submartingale.

[20]When used as cash, gold puts a lower bound on the return of money, limiting a government's ability to pursue inflationary policies. Governments intent on pursuing high-inflation policies have circumvented this constraint by imposing legal restrictions that make it costly to hold gold or to use it for trades. Edwards (2018) describes and interprets actions taken by the U.S. Roosevelt administration in the 1930s to raise the US price level. Sargent and Velde (1995) describe draconian restrictions that the Jacobins imposed during the 1794 Terror in France in order to lower the price level.

The government budget constraint is thus

$$B_t^g + P_t g_t + M_{t-1}^g \leq T_t + M_t^g + E_t S_{t+1} B_{t+1}^g. \tag{14}$$

In equation (14), B_t^g are bonds *issued* by the government (i.e., liabilities on the government accounts) and g_t is government spending in goods (purchased in the centralized market). Initial holdings of money and bonds are exogenous and must satisfy the consistency conditions $B_0^g = B_0$ and $M_{-1}^g = M_{-1}$.

Market clearing requires

$$c_t + x_t + g_t = \ell_t,$$

$$B_t^g = B_t,$$

and

$$M_t^g = M_t.$$

Whether the government is also subject to a no-Ponzi condition that constrains monetary/fiscal policy sequences or whether this condition prevails only in equilibrium as a consequence of households' transversality conditions hinges on how monetary and fiscal policies are assumed to be conducted. Thus, it matters whether the Treasury and central bank together freely print money in order to service maturing bonds, or whether the paper money supply is rigidly set independently of households' preferences about rolling over their portfolios of government bonds. It also matters whether the government runs permanent primary surpluses or whether instead there are periods in which public spending temporarily exceeds tax revenues so that, in addition to rolling over preexisting claims, new loans are needed to cover temporary revenue shortfalls. Assumptions about these matters are important ingredients of a Fiscal Theory of the Price Level that we shall discuss in Section 5. For now, we only consider sequences of allocations, prices, and policies that are competitive equilibria.

Compared to the previous case of a commodity-backed standard, household optimality conditions still require (7) to hold but are otherwise modified to allow prices to vary over time:

$$S_{t+1} = \beta \frac{P_t}{P_{t+1}} \tag{15}$$

$$1 = \beta E_t \left[\frac{P_t}{P_{t+1}} v'(x_{t+1}) \right] \tag{16}$$

$$M_{t-1} \geq P_t x_t \tag{17}$$

and

$$\lim_{s \to \infty} E_t \left[q_s \left(B_s + M_{s-1} \right) \right] = 0, \quad t \geq 0. \tag{18}$$

Next, we sum the household budget constraint forward and substitute market clearing and household optimality conditions. For convenience, we also define the (gross) risk-free nominal interest rate between periods t and $t+1$ to be $R_t := (E_t S_{t+1})^{-1}$. We obtain a key equation that links monetary and fiscal policy actions:

$$\frac{B_t + M_{t-1}}{P_t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{T_s}{P_s} - g_s + \frac{M_s}{P_s} \left(1 - \frac{1}{R_s} \right) \right]. \tag{19}$$

Another representation of equation (19) is

$$\frac{B_t}{P_t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{T_s}{P_s} - g_s + \frac{M_s - M_{s-1}}{P_s} \right]. \tag{20}$$

In both versions (19) and (20), at every time t the present value of the government liabilities must equal the present value of primary surpluses $\frac{T_s}{P_s} - g_s$ augmented by revenues from "seigniorage" defined as revenues that the government raises by issuing paper money that bears a net nominal interest rate of zero.

3.1 Seigniorage

Representations (19) and (20) express *seigniorage*, i.e., government revenues from printing money, in different ways. Equation (19) emphasizes that money is a government liability, a standard accounting practice in many countries today;[21] that paper money pays zero interest leads to recording positive seigniorage revenue being earned on real money balances when $R_s > 1$ so that the net nominal interest rate exceeds its zero lower bound. In contrast, equation (20) implicitly treats only interest bearing government bonds as government liabilities and treats new issues of

[21] In practice, money is usually a liability of the central bank, which is a distinct entity from the Treasury, and seigniorage profits are a transfer from the central bank to the Treasury. We will return to this point in Section 6.

fiat money as seigniorage. Notice that the accounting scheme based on (20) records seigniorage as being negative when a government reduces its paper money supply.

Representations (19) and (20) tie together equilibrium sequences of taxes, government spending, interest bearing debt, and paper money. In models with a single government decision maker, giving names to separate monetary and fiscal policy decisions is arbitrary and without consequence. A distinction between monetary and fiscal policies comes to life only when we assign decisions about particular actions exclusively either to a Treasury or a central bank, as we do in Sections 5 and 6.

A stochastic process for inflation interacts with the government budget constraint in distinct ways: (1) an *anticipated* part of inflation acts as a tax rate on real money balances; and (2) an *unanticipated* part of inflation (a.k.a. an *innovation* in inflation) revalues the entire stock of nominal government liabilities (fiat money plus nominal interest-bearing debt).

3.1.1 Anticipated inflation

To study anticipated inflation, we shut down uncertainty. Define $\pi_{t+1} := P_{t+1}/P_t$ to be the gross inflation rate between periods $t+1$ and t. In this case, the equilibrium conditions imply that seigniorage revenues $L(\pi_{t+1})$ satisfy[22]

$$L(\pi_{t+1}) := \frac{M_s}{P_s}\left(1 - \frac{1}{R_s}\right) = v'^{-1}\left(\frac{\pi_{s+1}}{\beta}\right)\left(1 - \frac{\pi_{s+1}}{\beta}\right) \tag{21}$$

Equation (21) states that *in equilibrium* seigniorage revenues from one-period inflation inflation anticipated in period s can be represented as a function of anticipated inflation that is unambiguously increasing when inflation is negative or in a neighborhood of zero inflation. At higher inflation rates, countervailing forces contend: higher inflation increases revenues directly but then decreases seigniorage revenues by depressing households' demand for real money balances. In settings where government-issued money competes with imperfect substitutes issued by other borrowers, the second force is likely to dominate at higher levels of inflation, implying that there

[22] Note that this is true even when $R_s = 1$, so that the cash-in-advance constraint may not be binding and real money balances are not uniquely defined in equilibrium: in this case, the argument of v'^{-1} may not correctly represent real money balances, but the second factor is zero anyway.

is a maximum amount of revenues that can be raised each period.[23] To keep things simple, we assume that preferences are such that L is strictly increasing up to some inflation $\pi_{\max}$ and strictly decreasing afterwards.

A manifold of equilibria share the same sequence of real primary deficits/surpluses $\{\frac{T_s}{P_s} - g_s\}_{s=0}^{\infty}$ and the same time-0 real government obligations $(B_0^g + M_{-1}^g)/P_0$. If we consider only equilibria in which inflation stays in the range in which the function L is increasing, then equations (19) and (21) imply that all equilibria in the manifold share the same present value of revenues from seigniorage:

$$\bar{L} \equiv \sum_{s=0}^{\infty} \beta^s L(\pi_s). \tag{22}$$

The invariant object defined in equation (22) provides a concise representation of Sargent and Wallace's (1981) unpleasant monetarist arithmetic. Given initial real liabilities and a fixed profile of real primary deficits/surpluses, a lower inflation rate in some period t must necessarily be compensated by higher inflation in some other period t' in order to keep the present value of seigniorage revenues constant. Associated with equation (22) is a natural notion of an intertemporal average of future inflation rates, in particular, the unique constant $\bar{\pi}$ that satisfies[24]

$$\bar{L} \equiv \sum_{s=0}^{\infty} \beta^s L(\bar{\pi}), \tag{23}$$

which is the "Chisini (1929) mean" of the sum in (22). According to equation (23), Chisini mean inflation is uniquely pinned down by initial debt and prospective primary government surpluses.[25] If we assume an exogenous and fixed sequence of primary surpluses, then Chisini mean inflation is determined by *fiscal* policy and monetary policy determines only the distribution of inflation over time. Notice how this analysis takes exogenous sequences of fiscal actions as given and has not modeled how they have been chosen. The analysis simply derived implications from

[23]As an example, in the case of Section 2.2, gold is perfectly durable and a perfect substitute for government-issued paper, and the demand for money drops to zero as soon as $P_{t+1} > P_t$.

[24]If there exists an equilibrium that raises $\bar{L}$, our assumptions about preferences imply that $\bar{\pi}$ exists and is unique.

[25]Here we take initial *real* government liabilities $(B_0 + M_{-1})/P_0$ as exogenous. We discuss the role of initial inflation later in section 3.2.

the budget constraints that restrict monetary and fiscal policy action sequences together with inequalities describing private agents' optimal decisions that determine an equilibrium allocation and price system.

A second and equally valid use of the very same equilibrium conditions would instead take the sequence of gross inflation rates $\{\pi_t\}_{t=0}^{\infty}$ to be given exogenously, find the implied Chisini mean inflation $\bar{L}$ from (22), and then deduce a required present value of taxes net of spending from (19). A spin that can be put on this second interpretation is that it is monetary policy that "goes first" and that fiscal policy must then adjust.

These two uses of the same equilibrium conditions delineate a hypothetical irreconcilable conflict between a fiscal authority intent on reducing taxes and a monetary authority intent on reining in inflation. Thus, in our first scenario, a monetary authority is forced to adjust to choices made first (and once-and-for-all) by a fiscal authority. The second interpretation instead envisions a monetary authority as "choosing first", forcing the fiscal authority then to adjust. Sargent (1986) referred to this as "Wallace's game of chicken."[26]

While perhaps helping us to make sense of contending forces that break loose during big inflations, the hostile relationship between monetary and fiscal policy envisioned in a game of chicken differs markedly from the well aligned monetary and fiscal policies embedded in an optimal taxation theory in which coordinated monetary-fiscal policy actions implement a Friedman rule that drives the net nominal interest rate on government issued paper money to zero. In the context of our reference model, optimality of a Friedman rule follows immediately because we assumed that the government can impose lump-sum taxes. When $R_t > 0$, competitive equilibria are distorted since the marginal rate of substitution in preferences between cash goods and leisure exceeds the marginal rate of transformation in production which equal 1 in our model. There exist monetary-fiscal policies consistent with an equilibrium that undo this distortion and render the equilibrium allocation efficient by setting $R_t = 0$ and hence $P_{t+1}/P_t = \beta$. Under a policy that supports that outcome, equation (19) then requires that the present value of taxes must equal the value of initial money and debt plus the present value of government spending.

[26] For a recent attempt to formalize the game of chicken, see Barthélemy and Plantin (2018).

To deliver the required deflation, the government taxes the households and uses the proceeds to retire money over time.[27]

The Friedman rule remains optimal in some economies in which lump-sum taxes are not available. As an example, if we replaced lump-sum taxes with proportional taxes on labor income, results described by Chari, Christiano, and Kehoe (1991) imply that the Friedman rule is optimal in our simple model whenever u and v exhibit constant and equal relative risk aversions.[28] This follows from the uniform commodity taxation result in Atkinson and Stiglitz (1972): with homothetic preferences between cash and credit goods, it is optimal to tax both goods at the same rate. To do this, the production of both goods must be distorted by the same labor tax. Adopting the Friedman rule equates the unit marginal rate of substitution between cash and credit goods to the marginal rate of transformation in production. Da Costa and Werning (2008) extend Chari, Christiano and Kehoe's result to allow nonlinear income taxes.[29]

3.2 Surprise inflations

Having discussed the role of *expected* inflation in (19), we now study consequences of inflation *surprises*. Here we are concerned with responses of inflation to unanticipated shocks to government spending or tax revenues. Suppose that the government issues only nominally risk-free debt, so that B_t is predetermined and known at time $t-1$. Then (19) implies

[27]As emphasized by Cole and Kocherlakota (1998), a government has wide latitude in choosing a path of repurchases because when $R_t = 0$ the cash-in-advance constraint does not bind, so households are happy to hold excess money balances as a saving vehicle.

[28]The rule is optimal even when u and v have constant relative risk aversion, if the relative risk aversion of v is lower than that for u. In this case, it would be optimal for the government to tax cash goods less than credit goods, but this cannot be achieved when the only instrument available is the wedge induced by the inflation tax.

[29]To break this result, Albanesi (2007) explores a model where cash goods are disproportionately purchased by low-income households and inflation arises from the conflict among heterogeneous households. In her paper, low-wage households prefer greater reliance on labor taxes, while the inflation tax is favored by high-wage earners.

$$
\begin{aligned}
&(B_t + M_{t-1}) \left(\frac{1}{P_t} - E_{t-1} \frac{1}{P_t} \right) = \\
&E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{T_s}{P_s} - g_s + \frac{M_s}{P_s} \left(1 - \frac{1}{R_s} \right) \right] - E_{t-1} \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{T_s}{P_s} - g_s + \frac{M_s}{P_s} \left(1 - \frac{1}{R_s} \right) \right]
\end{aligned} \tag{24}
$$

Equation (24) links surprises in future government surpluses inclusive of seigniorage revenues to surprises in inflation. In response to an adverse fiscal shock like a war that reduces anticipated future real tax collections relative to prospective government expenditures, fiscal balance can be restored in one of three ways:[30] Either

- Taxes and spending can be adjusted to restore the present value of net of interest government surpluses $\frac{T_s}{P_s} - g_s$ to its initial value; or
- Future seigniorage revenues can be increased by raising prospective inflation rates; or
- The price level can be allowed to jump immediately in order lower the value of liabilities previously issued by the government.

When lump-sum taxes are available as they are in the model above, the first option is optimal. However, when the government can levy only distorting taxes on labor supplied, a result of Chari, Christiano, and Kehoe (1991) shows that it is optimal to use inflation as the primary shock absorber. Unanticipated inflation turns out to be a very good way of insuring the government against unfavorable fiscal shocks. Because the timing assumptions about cash holdings made by Chari, Christiano, and Kehoe as well as by Lucas and Stokey (1983; 1987) allow households to readjust their cash after observing the shock, unexpected inflation is not distortionary. So a volatility of unexpected inflation has no direct adverse welfare consquences. In contrast, our reference model adopts Svensson's (1985) timing assumption that makes cash holdings predetermined relative to the arrival of the fiscal shock. This makes unanticipated inflation depress

[30] Our discussion of inflation as a shock absorber mirrors Sims (2001). Building on Aiyagari, Marcet, Sargent, and Seppälä (2002), Sargent (2001) draws connections between precautionary saving in models with limited idiosyncratic insurance and tax smoothing when full insurance through inflation or state-contingent debt is impossible.

consumption of cash goods via constraint (17). The Svensson timing diminishes the reliance of optimal policy on unanticipated inflation.[31] An alternative way to introduce costs from unexpected inflation is to assume sticky prices, an avenue first pursued in this context by Siu (2004) and Schmitt-Grohe and Uribe (2004).

Equation (24) opens the door to a hedging theory of an optimal level of government debt that activates a new avenue by which fiscal and monetary policies interact. Thus, let the government spending process $\{g_s\}_{s=0}^{\infty}$ be given. Break taxes into two parts, an exogenous real component represented by the stochastic process $\{\tau_s\}_{s=0}^{\infty}$ and a residual component $\{T_t/P_t - \tau_t\}_{t=0}^{\infty}$ that the government adjusts to ensure that (19) is always satisfied. Also take as given the stochastic process of inflation. We can then rewrite (24) as:

$$\begin{aligned}\frac{B_t + M_{t-1}}{P_{t-1}}\left(\frac{1}{\pi_t} - E_{t-1}\frac{1}{\pi_t}\right) &= \mathrm{PV}_t(\tau) - E_{t-1}[\mathrm{PV}_t(\tau)] + \mathrm{PV}_t(T/P - \tau) \\ &\quad - E_{t-1}[\mathrm{PV}_t(T/P - \tau)] + \mathrm{PV}_t(g) - E_{t-1}[\mathrm{PV}_t(g)], \qquad (25)\end{aligned}$$

where PV represents a present-value operator that for our economy with quasilinear preferences is

$$\mathrm{PV}_t(g) := E_t \sum_{s=t}^{\infty} \beta^{s-t} g_s,$$

and similarly for the other stochastic processes. We can compute that the level of government liabilities to be issued at $t-1$ that minimizes the conditional variance of the residual component of taxes is

$$\frac{B_t + M_{t-1}}{P_{t-1}} = \frac{\mathrm{Cov}_{t-1}(\pi_t^{-1}, \mathrm{PV}_t(\tau - g))}{\mathrm{Var}_{t-1}(\pi_t^{-1})} = \mathrm{Corr}_{t-1}(\pi_t^{-1}, \mathrm{PV}_t(\tau - g))\sqrt{\frac{\mathrm{Var}_{t-1}(\mathrm{PV}_t(\tau - g))}{\mathrm{Var}_{t-1}(\pi_t^{-1})}}, \qquad (26)$$

an equation that summarizes responses to hedging motives implied by the object being minimized. If inflation and the present value of the exogenous component of surpluses are negatively correlated (that is, if $\mathrm{Corr}_{t-1}(\pi_t^{-1}, \mathrm{PV}_t(\tau - g)) > 0$), then the quantity of government debt that achieves the best hedge is larger (a) the more correlated are inflation and the present value of

[31] The Svensson timing has been used to guarantee existence of an interior solution in models in which optimal government policy is subject to time inconsistency and to moderate a temptation fully to default *ex post* on all nominal liabilities. See Albanesi, Chari, and Christiano (2003).

the surplus, and (b) the more variable is the exogenous component of the surplus, and (c) the smaller is the volatility of inflation.

In our reference model, setting nominal liabilities at the optimal hedging solution (26) is not urgent because taxes are lump sum and can be adjusted at no cost. However, in a model with only distorting labor taxes, Bhandari et al. (2016) show that an optimal fiscal policy eventually drives government debt to the value (26). Their result reflects the working out of two forces. First, that labor-tax distortions are typically convex in the tax rate creates a reason to use debt to hedge fiscal risks.[32] Second, as in Lucas and Stokey (1983), debt would remain constant in the absence of shocks. The envelope condition then implies that the cost of deviating from a constant path and tilting it slightly is of second order. In the presence of shocks, the hedging benefits of moving debt slowly toward the optimal hedging value more than compensate for losses from deviating from a constant tax rate. [33]

The next step is to consider both inflation and tax rates as jointly determined. Two aspects of this problem are worth further consideration:[34]

- If unanticipated inflation is costly, there is a reason to limit the conditional variability of inflation. However, from equation (26), hedging fiscal risk then requires higher values of debt. Depending on the initial value, this may require more costly deviations from tax smoothing. It might also imply that a level of debt that achieves an optimal hedge is above the peak of the Laffer curve.[35]

- The more correlated are inflation and government fiscal needs, the better is nominal debt

[32]We can interpret the exogenous component τ_t as the revenues associated with a constant tax rate.

[33]Bhandari et al. (2016) study a real economy in which the state-contingent payoff of debt one period ahead is exogenously given. This corresponds to our assumption of exogenous inflation since we have assumed that debt is nominally risk free.

[34]The first consideration below implicitly appeared in Siu's (2004) analysis, but he did not analyze the asymptotic level of debt in depth. Bhandari et al. (2018) analyze economies in which distortionary taxes are desirable for social-insurance purposes; since they allow for lump-sum taxes, a representative-agent version of their economy does not feature a need for the government to hedge fiscal risk.

[35]Bhandari et al. (2016) do not characterize the solution for this case. Since they consider cases in which the real payoff of debt is exogenous, they simply assume it to be such that the optimal hedging value is interior.

as a hedge. Forces ignored by our model would reduce this correlation, including shocks beyond government control that buffet nominal variables. Such shocks would have important implications for the joint conduct of monetary and fiscal policy.

Since our model includes just short-term debt, only an unanticipated jump in the price level has any effect on the real value of debt. With long-term debt, ongoing inflation depreciates the real value of future promised repayments, as discussed for instance in Cochrane (2001) and Sims (2013). That would allow spreading the costs of inflation over a longer time span. But even with long-term debt, it is only the arrival of inflation surprises after debt has been issued that affects the government budget constraint.[36]

Berndt, Lustig, and Yeltekin (2012) assessed fiscal hedging delivered by government debt in the United States after World War II. They find that innovations in defense spending were mostly absorbed by future increases in taxes, but that about 10% were absorbed by abnormally low returns on government debt. Hall and Sargent (2014) infer that during the 18th and 19th centuries the federal government of the United States earned a reputation for repaying its debts in gold and that it did not often use state-contingent inflation as a major hedge against its fiscal needs.[37] An interesting aspect of Berndt, Lustig, and Yeltekin's analysis is that low returns on government debt were realized in the years that followed the shocks, contrary to the theory described above, which stresses a *contemporaneous* correlation. This is consistent with Reinhart and Sbrancia (2015), who argue that debt erosion after World War II occurred primarily through financial repressions that delivered low real interest rates. This finding is confirmed in Hall and Sargent's (2011) decomposition of the evolution of U.S. federal debt after World War II.

[36] An empirical analysis of the degree by which inflation surprises would affect the balance sheet of the federal government in the United States is undertaken by Hilscher, Raviv, and Reis (2014).

[37] It is worth noting that convertibility of greenbacks into gold was suspended at critical junctures, and that the United States did not lose any of the wars in which it was engaged; it is thus possible that the states of nature in which the hedge would have been exercised did not materialize.

4 Modigliani-Miller Logic in Monetary Economies

Since both of these financial claims have the same zero nominal net rate of return at the Friedman rule, households regard money and nominally risk-free bonds as perfect substitutes. When real money balances exceed the satiation level $v'^{-1}(1)$, open-market operations that swap money for bonds (or vice versa) do not affect equilibrium consumption, leisure, or prices. Thus, starting from an equilibrium in which $R_t = 1$, an increase in money balances M_t^g by ΔM and a corresponding reduction in nominal bond issuance B_{t+1} by the same (uncontingent) amount ΔM affect neither household nor government budget constraints (equations (11), (18), and (14)). Furthermore, the cash-in-advance constraint (12) is slack when $R_t = 1$ and continues to be satisfied if households accept the extra money injected by the policymaker. With no change in asset or good prices, a household's optimal choice of sequences of consumption and labor also remain the same. It follows that the consumption-leisure allocation together with the asset-pricing kernel and the nominal price sequence $\{P_t\}$ that formed the original equilibrium remain an equilibrium after the open-market operation.

Such irrelevance results obscure lines between monetary and fiscal policies: the quintessential tool of *monetary* policy, namely, open-market operations, have no effects and the price level is governed exclusively by the present value of primary surpluses, an object typically viewed as being determined by *fiscal* policy. Wallace (1981b) and Chamley and Polemarchakis (1984) show that open market operations are irrelevant in many settings in which money is held purely as a store of value and provides no special transaction services. A tell-tale sign that an irrelevance theorem is at work is that money is not dominated in rate of return by other stores of value. Examples of such settings include ones in which a monetary-fiscal policy implements the Friedman rule.

Irrelevance theorems extend to operations in which the government does not simply swap one risk-free nominal liability for another one but instead issues money to buy state-contingent securities. For example, in our reference model, starting from a competitive equilibrium in which the Friedman rule applies, let the government issue money ΔM in period t and buy an arbitrary portfolio of state-contingent bonds ΔB_{t+1} of equal value at asset prices prevailing in the original equilibrium:[38]

$$\Delta M_t = -E_t[S_{t+1}\Delta B_{t+1}].$$

As of time $t+1$, this open-market operation yields profits $\Delta M_t - \Delta B_{t+1}$. If the government distributes these profits by adjusting lump-sum taxes in period $t+1$ by $\Delta T_{t+1} = \Delta B_{t+1} - \Delta M_t$, government and household budget sets remain unchanged at the original sequence of goods and asset prices, so the original equilibrium allocation continues to form a competitive equilibrium at those prices. Wallace (1981b) connects this result to the Modigliani-Miller neutrality theorem in corporate finance: the government resembles a corporation that issues nominal claims (money) in order to acquire new assets (state-contingent securities) at market prices and then increases its future dividends (reductions in lump sum taxes) by the payouts from these purchased assets, an operation that has no effects on the firm's total net present value.[39] Chamley and Polemarchakis (1984) consider instead another purchase that leaves taxes fixed and instead makes the net present value of surpluses change by the amount $\Delta M_t - \Delta B_{t+1}$ at time $t+1$ and lets prices jump just enough to restore (19).

A general lesson from studying irrelevance theorems is that alterations in the central bank's balance sheet that indicate the same open-market exchange can be associated with markedly different equilibrium outcomes depending on subsequent fiscal policy adjustments triggered by that exchange.

So far, we have associated the Friedman rule with a zero nominal interest rate in models in which the nominal interest rate is the relevant cost of holding cash. However, the last decade has witnessed large increases in fractions of the monetary base being held as bank reserves that sometimes pay interest, as in the United States today. We can easily adjust our reference model to let money pay a nominal interest rate $\tilde{R}_t$. A Friedman rule then calls for supplying sufficient real balances to eradicate any gap between rates of return on money and debt so that $\tilde{R}_t = R_t$.[40] An

[38]Our convention is that B_{t+1} is a liability for the government, so a purchase of assets implies that $E_t[S_{t+1}\Delta B_{t+1}] < 0$.

[39]In the case of the government, the net present value is the present value of future primary surpluses, which is unchanged in this experiment as of time t, since, at the Friedman rule, $E_t[S_{t+1}(\Delta M_t - \Delta B_{t+1})] = 0$.

[40]Without uncertainty, equation (15) implies $P_{t+1}/P_t = \beta R_t$. By setting $\tilde{R}_t = R_t > 0$, a monetary authority

irrelevance result under this more general Friedman rule provides a useful tool for understanding the "quantitative easing" (QE) of the last decade. Under such an interest-on-reserves Friedman rule, QE will have real effects only if frictions in addition to those underlying a cash-in-advance constraint are present. Indeed, in this spirit, recall that Wallace (1981b) mentioned having some agents be credit constrained as a way to overturn his Modigliani-Miller theorem for open market operations.

Papers have used various devices to disarm irrelevance for open market operations. A common approach recently has been to assume that heterogeneous households trade financial assets with each other as well as with the government. Cúrdia and Woodford (2011) force all private transactions to go through intermediaries that bear a cost to transfer funds from savers to borrowers. Gertler and Kiyotaki (2010) and Gertler and Karadi (2013) introduce agency costs that limit intermediaries' sizes. Cui and Sterk (2018) follow Kaplan and Violante (2014) by assuming that households are subject to idiosyncratic shocks and that they face costs when rebalancing their portfolio between liquid assets (in our case, money) and illiquid assets (bonds). In all of these settings, open-market operations that swap money for bonds relax some agents' credit constraints. That affects their consumption decisions and thereby sets off other general-equilibrium effects. But even in such settings, the line between monetary and fiscal policy is still doubtful. Sargent and Smith (1987) showed that even when some agents' asset holdings are constrained, open-market operations can be neutral when type-dependent fiscal transfers are made. To reinstate irrelevance of open-market operations they adjust the timing and amounts of type-specific transfers so that the budget-feasible set of each type of agent remains unchanged at the pre-intervention prices when the mix of government-issued assets is altered. With such a policy, private agents' original choices remain optimal after the open-market operation,so no

can decouple the Friedman rule prescription from a requirement to generate a deflation. Doing that would be desirable in models with nominal frictions that make deflations costly, or in new Keynesian models where a zero lower bound on nominal interest rates prevents implementing an optimal policy. These observations are relevant for assessing papers that have argued that persistently low-interest rate policies are responsible for inflation recently undershooting central bank's target in many developing countries; Williamson (2019) reviews the argument.

general-equilibrium effects are triggered. Thus, even when enough of these frictions are present to render open market operations "relevant", the boundary between monetary and fiscal policies remains fuzzy: open market exchanges by the central bank can be regarded as having real effects only when they serve as substitutes for alternative fiscal actions.[41]

4.1 Other means of exchange

Up to now, we have distinguished sharply between "bonds" that are held purely as stores of value and "money" that also serves as a means of payment. But Treasury bonds have occasionally served as transactions vehicles, for example, in the aftermath of the financial crisis of 2008. Interest rates on Treasury bonds are below rates on bank reserves (and below zero rates of return on cash) in a number of countries today. It would be straightforward to extend our reference model to account for such a situation. We could add two types of anonymous transactions, one in which central-bank money (currency) is required, and another in which Treasury bonds can be used, leaving private bonds in zero net supply as a residual category that can be held as a pure store of value. Williamson (2012) analyzed open-market operations in a model in which money and bonds both have transaction roles.[42] In this context, open-market operations affect relative scarcities of the two payment instruments. To implement a Friedman rule, the real value of money and bonds must both be sufficiently high that they do not constrain opportunities to trade that would be present if perfect credit markets were to exist. However, the government's present-value budget constraint inextricably links the real value of total government liabilities to the present value of fiscal surpluses:[43] Once again, a government's ability to facilitate private-sector transactions cannot be isolated from its choices of taxes, transfers, and spending.

[41]In addition to the assumption of missing fiscal instruments, this avenue of research needs a story for why it is not optimal for the government to flood the market with money by purchasing enough public or private bonds, thereby restoring the irrelevance theorem, as in Sargent and Wallace (1982).

[42]In Williamson (2012), bonds are not used directly in decentralized trade, but rather they are needed to back bank deposits that are then used by households for payments.

[43]Note however that the present value is now taken with respect to an interest rate that is below the interest rate for private credit transactions.

4.2 Real Bills Doctrine and Manufactured Liquid Assets Scarcities

In Section 3.1 we discussed the optimality of the Friedman rule when money plays a special transaction role relative to either government or private bonds and the Friedman rule calls for equalizing rates of return on money and bonds. As discussed by Sims (2019), a Friedman rule prescription requires there bo be room for the government to lower a wedge between returns on money and bonds by exchanging money for bonds. In the setting studied by Sims, it is optimal for the government to inflate away all government bonds outstanding at the beginning of time 0, all of which are dominated in dollars. But at time 0, the government is not allowed to accumulate private assets. This renders inapplicable the Modigliani-Miller equivalence discussed above because there are no outstanding government bonds for the government to purchase with currency issues after time 0. In this setting, it turns out that driving the opportunity cost of money to zero is not optimal.[44] But most governments today have outstanding (large) stocks of nominal bonds, so for them the option of reducing the bond supply and increasing the money supply remains open.

When other government issued liabilities – say both money *and* bonds – play special roles in providing transaction services, a rule in the spirit of Friedman's calls for equalizing returns of both money and government-issued bonds to that paid by privately-issued nominally risk-free assets.[45] To attain this outcome, a government may have to purchase private assets; Sims's analysis is especially pertinent here. As an example, in Sargent and Wallace (1982) the Friedman rule is a requirement for Pareto optimality; in their model, the Friedman rule can be implemented by having the monetary authority open a "discount window" and offer freely to exchange money for private bonds. After a sufficient quantity of private securities have been discounted by the monetary authority to equalize returns on money and private risk-free bonds (or to eliminate other rate-of-return dominations), any further discounting of private indebtedness has no effect

[44] In Sims (2019), driving the opportunity cost of money to zero is impossible because money demand becomes infinite. This is not the case if a satiation point exists, as in our reference model.

[45] More precisely, a Friedman rule would require that government-issued liabilities and private securities are priced with by the same stochastic discount factor. This is relevant because in practice only the government has ultimate control of the money printing press and is thus able to issue nominally risk-free assets.

on either asset or goods prices, an outcome that qualifies as an instance of a “real bills doctrine” stating that central bank purchases of high-quality private indebtedness have no effects on an equilibrium price level path. This interpretation of the real bills doctrine stands as another manifestation of a Modigliani-Miller irrelevance result. Sargent and Wallace (1982) also remark that, in their environment, equilibria resulting from policies that restrict the provision of liquidity need not be Pareto inferior to those arising from the Friedman rule: while less desirable from a pure efficiency perspective, those equilibria can achieve redistribution that favors some groups at the expense of others. In the absence of direct, nondistorting fiscal instruments to achieve the same redistribution aims, it may be optimal for the government to rely on policies that artificially drive wedges between the prices of different classes of assets, even when buying private assets is possible.

With heterogeneous endowments, owners with similar endowments want to collude to drive up the relative prices of their wares. Government tax policy can substitute for such collusion. Because this insight applies to intertemporal prices as well as it does for apples or milk, different agents will have incentives to lobby for an intertemporal distribution of taxes that distorts asset prices in their favor, as was emphasized by Bassetto (2014). Several recent papers have rationalized policies that restrict supplies of government debt on those grounds. Common to these papers is the idea that some agents are constrained in their abilities to borrow, and (as in Woodford, 1990), the government can relax their borrowing constraint by lowering current taxes and issuing government debt, then raising them later. Such a policy effectively allows the government to borrow on behalf of some private agents. In such an economy, a characteristic of borrowers that are to be helped is that they are well endowed in the future relative to their preference for future consumption. To act as a cartel, borrowers would need collectively to restrict their demands for loans enough to lower the equilibrium interest rate. Borrowers do not favor a policy under which the government issues so much public debt that private borrowing constraints become slack. Rather, the optimal level of public debt from a borrowers’ perspective is such that the borrowing constraints remain binding. This common idea is then coupled with different reasons why the government may wish to favor borrowers over savers. Bhandari, Evans, Golosov, and Sargent (2017) make this point for a general case. In Azzimonti and Yared (2017, 2019), borrowers are poorer than savers and a government that is utilitarian and wishes to redistribute is prevented from doing so using type-specific taxes. Yared (2013) considers a Diamond and Dybvig (1983) setup in which (in the absence of private insurance markets) it is optimal for the government to redistribute in favor of some agents who ex post are revealed to be impatient and become borrowers. Bhandari, Evans, Golosov, and Sargent (2017) push these forces one step further: rather than take borrowing constraints as given, a government can deliberately limit enforcement of private contracts in order to restrict the demand for loans and thereby move the interest rate. Such a policy can remain optimal even when a government has access to nonlinear income taxes so long as information frictions prevent full redistribution by fiscal policy alone.[46]

In the competitive equilibria of our simple reference model, the present value of household consumption is

$$E_t \sum_{s=0}^{\infty} q_s P_s (c_s + x_s) = E_t \beta^s (c_s + x_s) \tag{27}$$

and is always well defined. If the economy is deterministic, this equation means that the real interest rate is asymptotically greater than the growth rate of consumption. The present value of household consumption continues to be well defined in more general models so long as households trade in complete markets and borrowing is constrained only by a no-Ponzi condition like (13). However, the situation changes when households have finite planning horizons. This can happen either because their lifetimes are finite, or because they might eventually face binding borrowing

[46] It is worth noting that the transaction role of government debt in Sargent and Wallace (1982) and Azzimonti and Yared (2017, 2019) differs from the role of money in our reference model. In our model, money permits trade in anonymous transactions, in which record keeping is impossible. In Azzimonti and Yared (2017, 2019) public debt alleviates a limited commitment problem that arises when agents are tempted to default on previous private obligations *ex post*. The force arising here remains valid in alternative environments in which public debt plays a money-like role that private debt cannot serve so long as government debt is disproportionately purchased by agents from whom the government wishes to redistribute. The key difference between cash and government bonds is that poor agents' portfolios are comparatively concentrated in cash and away from bonds. It is for this reason that a utilitarian government with no direct instruments for redistribution would find it optimal to pursue the Friedman rule for cash, but not for bonds.

constraints that (at the margin) will make choices of variables at the date of the binding borrowing constraint insensitive to what will happen in subsequent periods.

Darby (1984) argues that Sargent and Wallace's (1981) unpleasant monetarist arithmetic does not apply when the economy's growth rate exceeds the interest rate on government debt. Similarly, Blanchard (2019) refines arguments from Diamond (1965) and shows that the cost of issuing extra government debt may be limited or none if his historical estimates of U.S. risk-free rates of return relative to growth rates hold up in the future. By way of assessing Darby's and Blanchard's analysis, and inspired by Sargent and Wallace (1982), Yared (2013), and especially Azzimonti and Yared (2017), we now present a simple example that shows that budget balance still imposes limits on fiscal policy, and also how the line between monetary and fiscal policy becomes potentially even more tenuous in an environment in which the growth rate exceeds the interest rate. We consider a deterministic economy populated by overlapping generations of two period-lived people with constant size total population. We abstract from cash and credit goods but impose that no household can borrow. But in this model, government debt and transfers can at least partially substitute for missing credit markets.

We depart from Blanchard (2019) by introducing equal numbers of two types of agents for each cohort. Savers receive endowments $(e_y^S, e_o^S) = (\alpha, \epsilon)$ when young and old, respectively, while borrowers receive $(e_y^B, e_o^B) = (\epsilon, \gamma)$; we abstract from labor supply. We assume that ϵ is small and study the limit of equilibria as $\epsilon \to 0$.[47] Households of type $i = B, S$ born in period t have utility given by

$$\log c_{yt}^i + \log c_{o\,t+1}^i,$$

where c_{yt}^i is their consumption when young, and $c_{o\,t+1}^i$ is consumption when old. We assume that no durable good exists.[48]

The government is not allowed to redistribute directly between generations or between borrowers and savers; rather, it is restricted to apply equal lump-sum taxes and transfers to every household alive. Since young borrowers have an $\epsilon \approx 0$ endowment, the government is thus ef-

[47]Having $\epsilon > 0$ ensures that household utility is well defined even in autarky.

[48]Our results can survive introduction of a durable reproducible good such as capital, but depending on the production function may no longer apply if there is a factor in fixed supply. See Rhee (1991).

fectively prevented from taxing, but it can implement transfers. The government can also issue debt and/or money, which are equivalent here since there is no separate transaction role for one instrument relative to the other. We consider two alternative specifications of policy, one which more closely resembles "monetary" policy, and another that looks more "fiscal." We will then show that the two policies implement the same allocations; but the contrast between two implementations of the same allocations will bring insights.

In the monetary interpretation, at the beginning of each period t there is a stock of government-issued money M_{t-1} per capita, and the government makes a transfer (a negative tax)

$$-T_t = M_t - M_{t-1} \geq 0. \tag{28}$$

The budget constraints for a household of type i born in period t are

$$m_t^i = P_t(e_{yt}^i - c_{yt}^i) - T_t, \tag{29}$$

$$P_{t+1}(c_{o\,t+1}^i - e_{o\,t+1}^i) = m_t^i - T_{t+1}, \tag{30}$$

where m_t^i is the choice of money holdings for a household of type i born in period t, and the borrowing constraint imposes $m_t^i \geq 0$. The initial old cohort in period 0 faces just equation (30), taking m_{-1}^i as exogenously given. In each period, market clearing for money balances requires[49]

$$m_t^S + m_t^B = 4M_t. \tag{31}$$

In the fiscal interpretation, we use real quantities as numeraire. The government starts a period owing b_t units of goods per capita. It then auctions debt b_{t+1} payable in period $t+1$ at a real gross interest rate ρ_{t+1}, and uses the proceeds to repay existing debt obligations and make real transfers $-\tau_t$. Government policy must be such that

$$-\tau_t = b_{t+1}/\rho_{t+1} - b_t \geq 0. \tag{32}$$

The budget constraint for a household of type i born in period t is

$$\frac{b_{t+1}^i}{\rho_{t+1}} = e_{yt}^i - c_{yt}^i - \tau_t, \tag{33}$$

[49]The number 4 in equation (31) arises because in each period there are an equal number of young savers, old savers, young borrowers, and old borrowers.

$$c^i_{o\,t+1} - e^i_{o\,t+1} = b^i_{t+1} - \tau_{t+1}, \tag{34}$$

with b^i_t being real debt purchased in period t, which must be nonnegative due to the borrowing constraint. As before, the initial old cohort starts with an exogenous amount b^i_0. Market clearing for debt requires that

$$b^S_{t+1} + b^B_{t+1} = 4b_{t+1}. \tag{35}$$

To verify that our "monetary" and "fiscal" implementations correspond to the same economic fundamentals, take any allocation $(c^i_{yt}, c^i_{ot}, m^i_{t-1})^\infty_{t=0}$, price system $\{P_t\}^\infty_{t=0}$, and monetary policy $(T_t, M_{t-1})^\infty_{t=0}$ that satisfies (28), (29), (30), and (31). We then construct an allocation $(c^i_{yt}, c^i_{ot}, b^i_t)^\infty_{t=0}$, asset price system $\{\rho_{t+1}\}^\infty_{t=0}$, and fiscal policy $(\tau_t, b_t)^\infty_{t=0}$ that satisfies (32), (33), (34), and (35) as follows: take the same consumption allocation, set $b^i_t = M^i_{t-1}P_t$, $b_t = M_{t-1}P_t$, $\rho_{t+1} = P_t/P_{t+1}$, and $\tau_t = T_tP_t$. The same process works in reverse, starting from a "fiscal policy" specification and going back to a "monetary policy" specification. Since the households and the government face the same budget constraints in the two specifications (after prices have been appropriately transformed as above), the set of consumption allocations that are part of a competitive equilibrium is the same in the two economies. Yet again, a distinction between monetary and fiscal policy is tenuous and arbitrary.

We now study which equilibria are preferred by different households. For the sake of brevity, we restrict the question by considering only consumption allocations that converge to a steady state and by focusing on the welfare of generations born far into the future, at dates at which allocations have nearly converged.

As a first step, we show that in a steady state, borrowers always consume their post-transfer endowments, while savers buy all government bonds: borrowing and saving are intermediated through the government. We use our fiscal notation, which is expressed in real terms, and denote by subscript SS a steady-state value. In a steady state, since $\tau_{SS} \leq 0$, the government budget constraint (32) requires either $b_{SS} = 0$ or $\rho_{SS} \leq 1$. If $b_{SS} = 0$, no intertemporal trade can take place among households: young borrowers would indeed like to borrow from savers, but are prevented from doing so by the borrowing constraint, while savers have no other people to whom they can lend. Since bonds are in zero net supply, the interest rate must be such that savers find it optimal not to demand any bonds, which requires $\rho_{SS} = \epsilon/\alpha < 1$.[50] When $b_{SS} > 0$, the after-transfer endowment of the borrowers is $(\epsilon + \tau_{SS}, \alpha + \tau_{SS})$. With $\rho_{SS} < 1$, their optimal choice is not to save, but rather to consume their endowments, verifying our claim.

Having established that all bonds will be acquired by young savers, we characterize a steady state. In the limit as $\epsilon \to 0$, savers' bond demand is

$$b^S_{SS} = \frac{1}{2}\left[\alpha - \tau_{SS}(\rho_{SS} - 1)\right]. \tag{36}$$

Combining equations (36) and (32), along with the market-clearing condition $b^S_{SS} = 4b_{SS}$ yields a quadratic equation in τ_{SS} and b^S_{SS}. Keeping in mind that positive taxes are ruled out by the borrowing constraints, the quadratic expression yields

$$\tau_{SS} = b^S_{SS} - \frac{\sqrt{b^S_{SS}(\alpha + 2b^S_{SS})}}{2}. \tag{37}$$

The range of debt that yields a negative tax (a positive transfer) is $(0, \alpha/2)$ per each saver (or $(0, \alpha/8)$ in per capita terms). In this range, the government earns seigniorage revenues by providing an asset that savers are willing to hold to smooth consumption over time even at a negative net real interest rate. With its seigniorage revenues, the government transfers resources to all households alive, benefiting the borrowers as well.[51] Over this range, τ_{SS} is a convex function with a unique minimum $b_{\min}$. As pointed out by Miller and Sargent (1984) in their reply to Darby (1984), when the quantity of government debt affects interest rates, the observation that the interest rate is below the growth rate of the economy for the levels of debt that we historically observed does not imply that an arbitrary net-of-interest deficit is sustainable.

Since borrowers consume their after-transfer endowments, when the government acts in their interest, it chooses the level of debt that maximizes transfers, namely, $b_{\min}$. At this point, the negative net real interest rate is below the growth rate of the economy, which is zero in our case. In contrast, savers' welfare is strictly increasing in b all the way to $\alpha/2$, the point at which the

[50] Any value of $\rho_{SS} \leq \epsilon/\alpha$ would also work, but the same consumption allocation would arise.

[51] As emphasized by Bhandari, Evans, Golosov, and Sargent (2017), a similar allocation could be achieved if the government enforced a limited amount of private borrowing. This is the sense that the government borrows on behalf of private agents.

real interest rate becomes zero. We obtain results similar to Diamond (1965) and Blanchard (2019) only for the savers: so long as $b_t^S < \alpha/2$, cutting taxes and issuing extra debt allows making some transfers to an initial generation without making future savers worse off. This is because, unlike borrowers, future savers benefit from the higher interest rates more than they suffer from smaller transfers.

The lesson from this example is that it would be misleading to conclude that the government budget constraint is disarmed whenever the government pays interest rates that fall short of the growth rate. First, even when the interest rate is less than the growth rate, higher debt and the correspondingly higher interest rates may reduce the government funds that in our example finance redistributive transfers. Second, to the extent that direct redistribution channels are limited, an increase in interest rates benefits some groups at the expense of others. It is thus possible that a government could choose to limit is debt in order to accomplish a desired redistribution.

In the monetary interpretation of our overlapping-generations economy, a constant real rate of return is achieved by setting money growth and therefore inflation to the common constant rate $1/\rho_{SS}$. The equivalence that we proved earlier lets us still use (37) to establish the policies preferred by different groups among cohorts that are born after the economy has (nearly) converged to steady state. Borrowers prefer the rate of money growth which maximizes the real value of the monetary transfers, the rate that prevails when real balances per young saver are at $b_{\min}$. We can derive the implied (positive) rate of money growth by solving (32) and (37) for the steady-state value of ρ_{SS}. Not surprisingly, savers' utility is decreasing in money growth;[52] they prefer to forgo transfers in exchange for being able to save with money that retains a constant value.

The forces underlying our results prevail in environments with richer sources of household heterogeneity. For example, it would be tempting to carry out similar experiments in Bewley models of idiosyncratic uninsurable income shocks. It would also be worthwhile to analyze what

[52] Savers would want to push money growth to an interior negative value; however, in our environment money growth cannot go below zero because young borrowers would be unable to pay the taxes required to reduce the money supply.

happens when a low rate of return on government debt coexists with higher returns on private assets driven by differences in different assets' transaction roles like the differences that arise between money and debt in the cash-in-advance model discussed above. In those models, an increase in debt implies a lower convenience yield, i.e., a lower spread between government debt and private assets, that reduces seigniorage revenues in ways similar to those analyzed here. Interactions between household heterogeneity, convenience yields, and rates of return on private assets add extra dimensions that are worth exploring.[53]

Our model is silent about whether monetary or fiscal authorities should be in charge of managing public liabilities. It is tempting to relate our model's silence to the section 1 troubles that nineteenth century US statesmen confronted in designing institutions that could draw sustainable lines between monetary and fiscal policies.

5 Fiscal Policy as Nominal Anchor

Up to now, we have studied consequences of alternative arbitrary *sequences* of monetary-fiscal policy *actions* and how disputes about what are properly thought of as fiscal versus monetary policy actions can boil over into conflicts among institutions that have somehow been assigned responsibility for setting different components of a government's intertemporal budget constraint. In this section, we turn to questions about price level determinacy that force us to cast monetary and fiscal policies in terms of mechanical *rules* that relate actions to past and present outcomes. This will naturally lead us to discuss versions of what is widely termed a "fiscal theory of the price level (FTPL)" that generates a determinate price level (a.k.a. a nominal anchor) by fostering complementarities between particular monetary and fiscal policy rules.

Section 2.2 told how the absence of a nominal anchor for unbacked paper money leads to a multiplicity of equilibrium price levels at time 0 and indeterminacies in the form of sunspot equi-

[53] A recent literature emphasizes how heterogeneity across households shapes macroeconomic responses to monetary and fiscal policy shocks. Kaplan, Moll, and Violante (2018) show that the effects of interest rate movements depend on how taxes and spending react to these shocks. Other examples are Auclert (2019) and Auclert, Rognlie, and Straub (2018).

libria in all subsequent periods. As an example, going back to the reference model of Section 2.2 without gold, consider a monetary-fiscal policy in which the government trades no bonds and spends nothing, but at each period $t \geq 0$ makes a proportional transfer of money to households in the amount μM_{t-1}^S so that money grows at the rate μ. We consider settings in which $\mu > 0$ and equilibria in which the cash-in-advance constraint binds.[54] Combining equations (16) and (17) and imposing market clearing, we conclude that in any equilibrium the following difference equation for consumption of the cash good x_t is satisfied:[55]

$$1 = \frac{\beta}{\mu} \left[\frac{x_{t+1} v'(x_{t+1})}{x_t} \right] \tag{38}$$

Woodford (1994) analyzes a similar difference equation and shows that for a large number of interesting preference specifications it has multiple solutions.[56] The lack of a nominal anchor manifests itself in the lack of a boundary condition for equation (38). The only candidate condition that could give rise to such a boundary condition is the government budget balance; however, for the rule that we just specified, (20) always holds, since in each period $B_t = g_t \equiv 0$ and $T_t \equiv M_s - M_{s-1}$.

To rule out equilibria with $x_t \to 0$, Wallace (1981a) and Obstfeld and Rogoff (1983) add a "fiscal backstop" in which at every time t the government offers to exchange money for goods at a price $\mu^t \bar{P}$ and raises revenues required to do so by levying lump-sum taxes on households. The government sets $\bar{P}$ high enough that it need not use the fiscal backstop along an equilibrium path so that real tax revenues required by the policy would be small even off an equilibrium path. This policy puts a floor under the real value of money and rules out self-fulfilling high-inflation equilibria in which the household expects real money balances to approach zero as time passes without limit.

Although Wallace's and Obstfeld and Rogoff's backstop excludes some equilibria, it does does not necessarily assure a unique equilibrium. It excludes equilibria having real money balances that approach zero asymptotically but possibly leaves multiple equilibria with strictly positive

[54]Woodford (1994) considers other assumptions about money growth rates, as well as more general preferences.

[55]The equilibrium is then completely characterized by obtaining S_{t+1} from (15) and the price level from the cash-in-advance constraint.

[56]Our equation differs slightly from Woodford's because of the timing of centralized and decentralized markets.

real balances that remain bounded both from above and from below. An example is provided by Sargent and Wallace (1975) who analyze a set of equilibria in an economy in which the monetary/fiscal authority sets a fixed nominal interest rate R_t at which it is willing to exchange money and one-period risk-free nominal bonds maturing in period $t+1$, appropriately adjusting taxes and transfers to ensure that the present-value budget balance holds. In our reference model, taking conditional expectations of equation (15), we obtain

$$\frac{1}{R_t} = \beta E_t \left[\frac{P_t}{P_{t+1}} \right]. \tag{39}$$

Starting from any P_t, there exist multiple stochastic solutions that make P_{t+1} and x_{t+1} functions of a sunspot and that satisfy (16) and (39).[57] There are infinitely many solutions arbitrarily close to the non-stochastic solution given by

$$P_{t+1} = \beta R_t P_t, \quad x_{t+1} = v'^{-1}(\beta R_t). \tag{40}$$

Thus, for a given price P_t, a Wallace-Obstfeld-Rogoff back-stop policy higher than $\beta R_t P_t$ fails to rule out multiple equilibria.

Leeper (1991), Sims (1994), and Woodford (1994) describe a policy that does guarantee a unique equilibrium. Shift equation (19) forward one period. Since the government trades only one-period risk-free nominal bonds, both B_{t+1} and M_t on the left-hand side are predetermined. A candidate sunspot equilibrium would imply a change in P_{t+1} in the denominator of the left-hand side that would exactly offset movements in the seigniorage term on the right-hand side, which would require that either taxes or government spending be adjusted to ensure that equilibrium condition (19) holds. Leeper, Sims, and Woodford consider fiscal policies that prohibit any such adjustment and that thereby deliver a unique equilibrium price level path. An example of such a policy is one in which government spending and nominal interest rates are constant at $\bar{g}$ and $\bar{R} > 1$ respectively and in which taxes in period t satisfy[58]

$$T_t = \bar{\tau} P_{t-1} - (M_t - M_{t-1}), \tag{41}$$

[57] When an allocation, policy, and price system satisfy (16) and (39), (17) will also hold, since the government is supplying any amount of real balances demanded by households, and (19) holds by the assumption that transfers are adjusted to make it hold.

[58] The tax policy in (41) is written so that the government need not observe P_t when it sets T_t.

with $\bar{\tau}$ exogenous.[59] When fiscal policy obeys this rule, substituting (15) and (41) into (20) implies

$$\frac{B_t - \bar{\tau} P_{t-1}}{P_t} = \frac{1}{1-\beta}\left(\frac{\bar{\tau}}{\bar{\bar{R}}} - \bar{g}\right). \tag{42}$$

In period 0, the only endogenous variable in equation (42) is the price level P_0. It follows that in any equilibrium the initial price level P_0 is uniquely determined by equation (42). In all subsequent periods, since the government only trades nominally risk-free bonds, B_t is predetermined as of time t. Equation (42) then implies then that P_t must also be predetermined and so cannot depend on the realization of a time t sunspot variable. It is then straightforward to prove that there exists a unique allocation, price sequence, and stochastic discount factor that satisfies (15), (16), (17), and (42). Here a fiscal policy provides the missing nominal anchor in the sense that it delivers and shapes a unique price level path.

Uniqueness is attained by mechanics explained well by Cochrane (2005). Our nominal anchoring fiscal policy effectively turns holders of nominal government claims into residual claimants to the stream of real payments described by the right-hand side of (42).[60] The price level is thus determined by an equilibrium condition that the real value of nominal claims issued by the government equals the present value of fiscal primary surpluses.

The preceding discussion forgets that the government issues two claims, namely, bonds and money, and leaves open how total claims are divided between these two components and doesn't explain where a relative price between the two would affect the analysis. To remedy this deficiency, Buiter (2002) includes a "debt revaluation factor" that allows a dollar of maturing nominal bonds to be different from a dollar of paper money. A justification for ignoring Buiter's revaluation factor could be that maturing bonds will be redeemed for paper money at par, a justification that automatically implies that the FTPL substantially limits the independence of whatever "monetary authority" is assigned to control the money supply and also rules out monetary-fiscal rules in which the government sets an unconditional supply of money independent

[59] In period 0, we can equivalently specify an arbitrary initial condition P_{-1}, or set $T_0 = \bar{T}_0 - (M_0 - M_{-1})$. We take the first option in the text for simplicity.

[60] Because P_{-1} is given and from the details of our timing protocols, the first-period tax receives a special treatment on the left-hand side.

of the amount of bonds that households might wish to redeem.[61]

Credibility of a FTPL hinges on plausibility attached to the particular fiscal-monetary *rules* on which it rests. An FTPL analysis is not just about competitive equilibrium sequences of allocations, prices, and monetary-fiscal policy actions but about an equilibrium collection of *strategies* – sequences of functions mapping information at time t into actions at time t – that deliver a theory not just about household and government behavior along an equilibrium path but about how everyone would behave under a vast number of alternative scenarios, most of which would never materialize. The behavior especially of the government during such "off-equilibrium path" scenarios decisively shapes outcomes along an equilibrium path. Modeling an equilibrium collection of stragegies in the rigorous fashion expected in game theory is challenging to carry out in the type of environment with competitive markets and large numbers of anonymous private decision makers who take prices as given because the presence of a single large player called the government prevents us from treating prices as being set by that convenient *deus ex machina* named the "Walrasian auctioneer". Instead, we are forced to follow Shapley and Shubik (1969) and explicitly model how prices are set as the government and the private sector interact, and to specify exactly what actions and what information the government has when it sets its taxes and its money supply. Bassetto (2002, 2005) studied these issues with the FTPL being one focus.[62] Two important conclusions emerge from Bassetto (2002), which described an economy as a game in which equilibria emerge from fully-specified strategies for households and a government.

- There exist government strategies that defend a unique price level even when the government (or central bank) pursues a policy of a constant interest rate like one that led to indeterminacy in Sargent and Wallace (1975). This "nominal anchor" outcome substantiates a FTPL.

[61]Regime changes in interactions between monetary and fiscal policies are the subject of a literature that promises to improve accounts of the relationship between output and inflation in New Keynesian models. Examples are Davig and Leeper (2007), Chung, Davig, and Leeper (2007), and Bianchi and Melosi (2014; 2018; Forthcoming).

[62]Other papers that have grappled with this question are Kocherlakota and Phelan (1999), Christiano and Fitzgerald (2000), Niepelt (2004), and Daniel (2007).

- The strategies that do the job are associated with outcomes that confront the government with "confidence crises" triggered by private households' occasional refusals to make new loans to the government. At times when government spending exceeds planned tax revenues, these crises can require the government strategy to adjust tax plans to guarantee that real tax revenues are at least as large as government spending. Details about how the government makes this adjustment determine whether fiscal policy indeed acts as a nominal anchor.

Even when monetary and fiscal policy coordinate as the theory instructs, the FTPL cannot cure all indeterminacies. Thus, when it provides additional liquidity services, the interest rate on government debt can be below the economy's growth rate. Then even though, as required for the FTPL, the intertemporal balance in the government budget continues to matter as we discussed in Section 4.2, Bassetto and Cui (2018) and Berentsen and Waller (2018) show that the FTPL is only able to select a *range* of possible equilibrium price levels, not a unique one.

6 Two Government Budget Constraints

"Unconventional monetary policies" deployed after the 2007-2008 financial crises have attracted attention to relationships between monetary and fiscal policies as events have taught policy makers that setting interest-rates in the fashion codified by Woodford (1995) or Clarida, Galí and Gertler (1999) may not suffice to attain inflation targets. Sims (2004, 2005) set the stage for analyzing such policies when he studied consequences of endowing a Treasury and a central bank with separate budget constraints.[63] To the extent that we only are interested in describing the set of equilibria consistent with a given sequence of monetary/fiscal actions, there is no reason

[63]While it is natural to think that the two main agencies in charge are the government (especially the "Treasury") and the central bank, in reality things can be more complicated, as our introduction attests. The distinction between a treasury and central bank itself can be obscure and decision makers from other agencies may enter the scene. Good examples are Fannie Mae and Freddy Mac in the United States, private corporations that were able to issue government-backed claims. We have limited our distinctions among government agencies to the Treasury and the central bank in order to emphasize obscure boundaries between monetary and fiscal policy.

to distinguish between different government agencies sharing a common budget constraint. But recall how in section 5 we moved forward from descriptions just of sequences of actions and to study the government *strategies* that generate those sequences. In that context, if the government is not made of a single entity, but rather of separate decision makers, how these decision makers manage their own budgets can affect all outcomes, including the monetary-fiscal policy sequence. For these reasons, we now split equation (14) into one equation for the Treasury and another for the central bank. To keep things simple, we assume that only the Treasury is allowed to levy taxes on the households and to spend resources, while only the central bank is allowed to issue non-interest-bearing claims in the form of paper money. While previous sections highlight how arbitrary this distinction is from the point of view of much macroeconomic theory, the division that we make does a good job of approximating institutions observed in many countries.

Define B_t^T as (potentially state-contingent) nominal bonds that the Treasury issues in period $t-1$ and repay in t, and define H_t as nominal transfers from the central bank to the Treasury. The Treasury's budget constraint at time t is

$$B_t^T + P_t g_t \leq T_t + E_t S_{t+1} B_{t+1}^T + H_t. \tag{43}$$

Define B_t^{CB} as net holdings of nominal state-contingent bonds held by the central bank; B_t^{CB} counts as positive the central bank's holdings of Treasury-issued securities as well as nominal claims issued by the private sector while it subtracts interest-bearing claims issued by the central bank, which in practice are comprised of reserves held by banks and short-term borrowing effected through repurchase agreements (repos). We also reintroduce gold as a real asset, but, unlike Section 2, we assume that gold's only role is as a store of value and that it cannot be used as a medium of exchange. Letting z_t^{CB} be central bank holdings of gold at the beginning of period t, it follows that

$$B_t^{CB} - M_{t-1}^g \geq H_t - M_t^g + E_t S_{t+1} B_{t+1}^{CB} + \frac{P_t}{\phi}(z_t^{CB} - z_{t-1}^{CB}). \tag{44}$$

If we aggregate (43) and (44), we obtain (19), except that gold now appears as an additional government asset.[64]

[64]In the aggregation, $B_t^g = B_t^T - B_t^{CB}$.

If we solve these difference equations forward, we obtain two intertemporal balance equations, namely, the Treasury's

$$\frac{B_t^T}{P_t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{T_s + H_s}{P_s} - g_s \right), \tag{45}$$

and the central bank's[65]

$$\frac{B_t^{CB} - M_{t-1}}{P_t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{H_s}{P_s} - \frac{M_s}{P_s} \left(1 - \frac{1}{R_s} \right) - \frac{z_t^{CB} - z_{t-1}^{CB}}{\phi} \right]. \tag{46}$$

Notice that gold counts as an asset only to the extent that it is sold and converted into goods. Since we assume gold not to be productive, if it forever remains in central bank vaults it is considered wasted from the perspective present-value budget balance.

Sims (2004) distinguishes two models of central bank operations: "model F" and "model E". A model F central bank resembles more closely the U.S. Federal Reserve System or the Bank of England, while a model E central bank looks more like the European Central Bank or the Hong Kong Monetary Authority. In model F, the central bank invests exclusively in government debt. In normal times (at least in the past), the nominal interest rate R_s is always positive. This implies that the seigniorage term on the right-hand side of (46) is also positive: revenues accrue to the central bank from issuing paper money bearing no interest and from investing the proceeds in bonds paying positive interest. If we assume in addition that the central bank only invests in short-term securities and that it does not pay to the Treasury more than its net interest income, then (46) ensures that a central bank that starts with positive net worth on the left-hand side will continue to have positive net worth into the indefinite future. It is easy to imagine that, if and when the central bank and the Treasury bargain over policy, possibly period by period, the central bank will be in a stronger position if its balance sheet is such that a recapitalization in the form of $H_s < 0$ is never required, as is the case here. But while a central bank in this position may look to be "independent" in the short run, the fact that its assets are purely nominal implies that the central bank alone cannot provide a nominal anchor for the price level. From equation (46), the central bank can easily control nominal transfers H_s, but it cannot control *real* transfers.[66] If money supply and/or interest rate rules potentially lead to multiple

[65] We use market clearing to substitute $M_s^g = M_s$ in equilibrium.

[66] To make this argument precise requires describing an economy more in detail along lines in Bassetto (2002).

equilibria, only the Treasury can ultimately provide fiscal backstop by levying appropriate real taxes.

In extensions of his model to long-term debt discussed in Sims (2005), if a model F central bank invests in long-term securities, then the mismatch in the duration of its asset (long-term debt) and money (a zero-maturity claim) can lead to fluctuations in the central bank's net worth, even becoming negative. Nonetheless, we can rewrite (46) as

$$\frac{B_t^{CB}}{P_t} = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{H_s}{P_s} - \frac{M_s - M_{s-1}}{P_s} - \frac{z_t^{CB} - z_{t-1}^{CB}}{\phi} \right]. \tag{47}$$

In a richer model with long-term debt, the left-hand side term represents the real value of the central bank's portfolio of bonds. Bassetto and Messer (2013) remark that, as long as all of the central bank's liabilities are in the form of paper money and the bank pursues a policy of always expanding the money supply, fluctuations in the real value of bonds do not threaten the central bank's ability to deliver $H_s > 0$ every period. But the central bank's ability to do this deteriorates markedly if the central bank issues reserves that are held by banks uniquely as a store of value and that are remunerated at the market interest rate. In this case, the mismatch between short-term interest-bearing liabilities and long-term bonds can imply that the left-hand side of (47) turns negative in an adverse scenario, forcing the central bank either to ask for fiscal help from the Treasury or to pursue more monetary expansion, with its inflationary implications.[67]

In contrast to his model F central bank, Sims's model E central bank holds substantial reserves in the form of real assets. In our equation above, these assets are represented by gold, but in

The reason the central bank cannot commit to a sequence of real transfer payments to the Treasury is that the real value of its profits is driven by the willingness of households to accept its money and its bonds. But the Treasury can force households to pay a quantity of real goods through taxes.

[67]Carpenter, Ihrig, Klee, Quinn, and Boote (2013) and Greenlaw, Hamilton, Hooper, and Mishkin (2013) analyzed in detail the magnitude of the interest-rate risk created by quantitative easing policies undertaken by the U.S. Federal Reserve after 2008, and they concluded that it would take extreme swings in long-term rates to turn the net value of interest-bearing assets and liabilities into a negative number. A similar conclusion arises in the complete dynamic stochastic general equilibrium model analyzed by Del Negro and Sims (2015). Hall and Reis (2015) extend the discussion to other sources of risk for the balance sheet of the central bank, such as exchange risk and, in the case of the Eurozone, outright default by national Treasuries. In contrast, the position of the Bank of Japan is more precarious.

practice they could be other real claims such as hard-currency reserves.[68] In this case, the required fiscal backstop that puts a lower bound on the real value of money can happen "in-house:" at any point in time t, the central bank can offer to redeem all of the money balances for its current gold holdings at a price given by M_{t-1}/z_{t-1}^{CB}. This is a natural model for the European Central Bank, since its fiscal counterpart is represented by a heterogeneous collection of national governments, with a correspondingly weaker link between fiscal and monetary authorities. However, as Sims points out, the risk arising from fluctuations in the value of the central bank portfolio may be of greater consequence in a world of weak links, because ultimate help from the fiscal authorities may not be forthcoming.[69]

7 Concluding Remarks

We began this paper by recalling examples of challenges that statesmen have faced in separating monetary from fiscal policy. Collections of U.S. coins and bank notes attest to how the promises written on the faces of the varieties of paper and metal monies reflected fiscal and and monetary policy decisions that the Congress and President had often delegated to the Treasury Department and Comptroller of the Currency before 1914, and then divided more or less ambiguously between the Treasury and the Federal Reserve System after 1914. Government budget constraint arithmetic explains why, one way or another, monetary and fiscal policies must be coordinated, if not consolidated.

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[68] Of course, with foreign-currency reserves, questions about ultimate fiscal backing are shifted to asking what ensures the real value of the foreign currency.

[69] Our equations rule out fluctuations by imposing a constant real value of gold. Also, note that, as was the case for a model F central bank, fluctuations in the price of central bank assets require no outside intervention so long as the desired policy path involves an ever-increasing money supply, as discussed in Schmitt-Grohé (2005).

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