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Implications of Expected Present Value Budget Balance: Application to Postwar U.S. Data

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1. Introduction

For time series on the U.S. government budget after World War II, this paper implements the test described in Section 7 of Hansen, Sargent, and Roberds [henceforth HSR], which is Chapter 5 of this book. Recall that Section 7 modifies the setup of earlier sections in two ways. First, interest rates are allowed to be time invariant *ex ante* but not *ex post*. In the notation of HSR, this requires that

$$E(\delta_t \mid J_{t-1}) = \delta \tag{1.1}$$

where δ_t is the real one period interest rate and J_{t-1} represents information available as of time $t-1$. Second, measures of the debt stock are assumed to be in the econometrician's data set. In Section 2, I summarize how these two assumptions lead to the model formulated in Section 7 of HSR. The model is then tested for postwar U.S. time series on federal government debt and deficits net of interest.

The analysis below is closely related to the work in a number of papers examining the question of *net present value budget balance* using postwar U.S. fiscal data, most notably Hamilton and Flavin (1986).[1] It is also closely related to a number of papers that test *expectational* models of the relationship between stock prices and dividends, as well as that between long rates and short rates, e.g., Campbell and Shiller (1987) and Hansen and Sargent (1981e). Differences and similarities between these papers and the present analysis are noted below.

2. Implications of Expected Net Present Value Budget Balance

As in HSR, let $\{s_t\}$ be a stochastic process of net surpluses, i.e., receipts minus expenditures net of interest. Let $\{k_t\}$ be the stochastic process representing debt at the beginning of period t, denominated in negative dollars when the government is borrowing money. Debt evolves according to the government budget constraint

$$(2.1) \qquad E(k_{t+1} \mid J_{t-1}) = (1+\delta)\,k_t + E(s_t \mid J_{t-1}) \text{ for } t = 0, 1, \ldots$$

Replicating the analysis leading up to equation (2.8) of HSR yields the *solution* for k_t

$$(2.2) \qquad k_t = -\sum_{\tau=0}^{\infty} \lambda^{\tau+1} E(s_{t+\tau} \mid J_{t-1})$$

which states that debt must be balanced by the discounted sum of expected future surpluses. Evidently restrictions imposed by (2.2) will be weaker than those implied by equation (2.8) of HSR, which does not contain an expectations operator.[2] Hence the impossibility result (Proposition 2) of HSR does not apply in the present case. To derive the restrictions implied by (2.2), suppose that as in HSR, s_t is contained in an observable vector y_t, and that HSR assumptions A1 (stationarity) and A2 (nonsingularity) hold. Let the equation corresponding to the appropriate row of the MAR for y_t be given by

$$(2.3) \qquad s_t = \sigma(L)\,w_t$$

where σ is a one-sided lag polynomial and $\{w_t\}$ is a martingale difference sequence. Applying a prediction formula of Hansen and Sargent (1980) to (2.2) yields a unique one sided representation for $\{k_{t+1}\}$

$$(2.4) \qquad k_{t+1} = \kappa(L)\,w_t \quad \text{where } \kappa(z) = \lambda[\sigma(\lambda) - \sigma(z)]/(z-\lambda)\,.$$

Because $\kappa(z)$ is one-sided, equation (2.4) translates directly into restrictions on the moving average representation (henceforth, MAR) of $\{(k_{t+1}, y_t')\}$ in terms of $\{w_t\}$. Recall that w_t represents the innovation to agents' information. So long as the dimension of w_t is greater than the dimension of y_t plus one (i.e., $n+1$ in the notation of HSR), the composite process $\{(k_{t+1}, y_t')\}$ can be nonsingular. Stochastic nonsingularity occurs when the history of the process $\{y_t\}$ generates a strictly smaller information set than does the history of $\{w_t\}$. In this instance, k_{t+1} can reveal additional information about w_t, implying stochastic nonsingularity of $\{(k_{t+1}, y_t')\}$. The analysis below assumes that such nonsingularity will in fact hold.

The restrictions induced by equation (2.4) are easily tested using a result obtained by Hansen and Sargent (1981e). This result, which applies to *exact* linear rational expectations models such as (2.4), implies that the restrictions given by (2.4) will always apply to the Wold moving average representation for $\{(k_{t+1}, y_t')\}$. Consequently one can replace the polynomials κ and σ in (2.4) with their estimable counterparts κ^+ and σ^+. The restrictions on κ^+ and σ^+ may then be tested using standard methods.

3. Application to U.S. Postwar Data

As an example of how model (2.2) can be applied to data, I estimated a simple version of this model for U.S. quarterly time series over the period 1948Q1–1986Q4. In this application, the dimension of y_t is taken to be one, so that $y_t = s_t$. The real debt series $(-k_t)$ and the real deficit net of interest series $(-s_t)$ are constructed from NIA series. Both series account for the profits of the Federal Reserve System as revenues. Details on the construction of the data series can be found in Appendix A of Miller and Roberds (1987). The series are graphed in Figures 1 and 2.

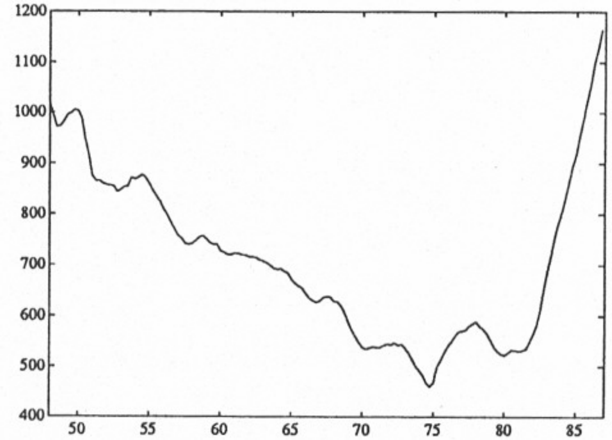

Figure 1. Real Value of Interest Bearing U.S. Federal Debt 1948:1 to 1986:4 (billions of 1982 dollars)

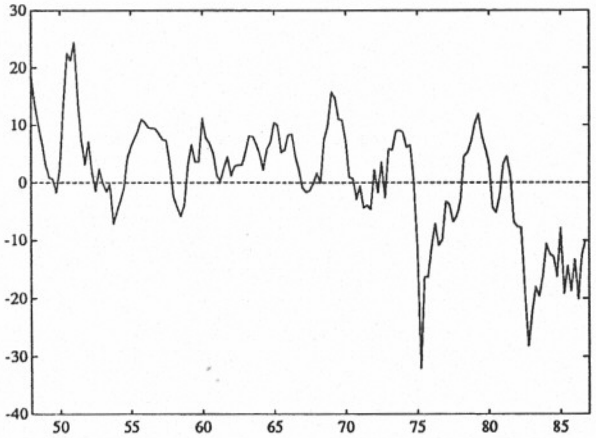

Figure 2. Net of Interest Surplus in U.S. Federal government budget,1948:1 to 1986:4 (billions of 1982 dollars)

Given that the restrictions imposed by (2.4) require stationarity of s_t and hence k_t, some pretesting for nonstationarity is appropriate. Results of standard Dickey-Fuller (henceforth DF) regressions are given in Table 1.

Table 1 shows that the Dickey-Fuller test rejects the null of a unit root for the debt series but not for deficit. These results constitute some prima facie evidence against the validity of the model of Section 2 for postwar U.S. data. If real debt evolves according to equation (2.4), and real deficits (surpluses) are stationary, then real debt cannot be nonstationary. Clearly, the budget cannot be balanced in any meaningful "expected present value" sense if debt diverges over time while deficits continue to fluctuate in a stationary fashion. On the other hand, it may be the case that such pessimistic inferences are unwarranted, because of biases inherent in the DF test. Sims (1988), Sims and Uhlig (1990), DeJong et al. (1988) and others have questioned the applicability of DF and similar classical procedures for determining the presence of a unit root. Specifically, many of the results in these papers suggest that the DF test suffers from low power against near-nonstationary alternatives, leading to a bias in favor of the unit root null.

As an alternative to standard DF tests of a unit root, the papers mentioned above employ Bayesian methods to obtain inferences concerning potential nonstationarity. In Table 2, some of these methods are used to analyze the real debt and deficit series. Following the approach of DeJong and Whiteman (1989 a,b) a sixth order AR model with a constant term was fit to each series. Assuming a normal likelihood function, and diffuse (normal-gamma) prior for the model parameters, Monte Carlo integration was used to obtain estimates of the posterior mean and standard deviation of the modulus of the largest root of the AR polynomial. These estimates are given in Table 2, along with the approximate posterior probability that each of the largest roots is inside the unit circle.

For both series, the Bayesian procedure places most of the posterior probability on stationarity. However, the posterior probability of nonstationarity is much higher (about 16%) for debt than for deficits (less than .1%). Sims' (1988) test of a unit root as a point null slightly favors the unit root over stationarity for debt, and vice versa for deficits. Both inferences, however, could be reversed by a relatively small change in prior odds. On balance, the evidence presented in Table 2 suggests that stationarity is the most likely inference for both series, while difference-stationarity is plausible in the case of debt but somewhat less plausible for deficits. Due to this ambiguity concerning the possible presence of unit roots, two versions of the model were fit to the data. The first was the stationary model described in Section 2. The second model assumes difference-stationarity of the deficit process.

To derive the differenced version of the model, rearrange the terms in equation (2.2) to obtain

$$\delta k_t + s_{t-1} = -\sum_{\tau=0}^{\infty} \lambda^\tau E(\Delta s_{t+\tau} \mid J_{t-1}) \tag{3.1}$$

which states that this period's expected deficit including interest payments must be balanced by the discounted sum of expected changes in all future deficits. If $\{\Delta s_t\}$ is taken to be stationary, then equation (3.1) implies that deficits including interest payments, i.e., $\{\Delta k_t\}$ will be stationary. Now assume that HSR assumptions A1 (stationarity) and A2 (nonsingularity) apply to first differences of s_t. Let the MAR for Δs_t be given by

$$s_t = \sigma(L) w_t \tag{3.2}$$

where σ is a one-sided lag polynomial and $\{w_t\}$ is a martingale difference sequence. Applying the Hansen-Sargent (1980a) prediction formula to (3.1) yields a unique one sided representation for $\{\delta k_{t+1} + s_t\}$

$$k_{t+1} = \kappa(L) w_t \quad \text{here } \kappa(z) = [\sigma(\lambda) - \sigma(z)]/(z-\lambda) \ . \tag{3.3}$$

As in the stationary case, equation (3.3) can be directly translated into restrictions on the MAR of $\{[(\delta k_{t+1} + s_t),\ \Delta s_t]\}$. The term $\delta k_{t+1} + s_t$ represents the current period's expectation of the deficit net of interest at the end of next period, i.e., $E(\Delta k_{t+1} \mid J_{t-1})$. Alternatively, this term is proportional to $k_{t+1} + \delta^{-1} s_t$, which represents the discrepancy between the value of debt and its expected net present value, assuming a constant *ex ante* real rate and that s_t follows a random walk. Similar terms appear in the price-dividend/term structure models analyzed by Hansen and Sargent (1981e) and Campbell and Shiller (1987), which are formally identical to the difference stationary model analyzed above.

To implement the tests described above, a bivariate vector autoregression with a constant term was fit to $\{k_{t+1},\ s_t\}$ for the stationary model and $\{[(\delta k_{t+1} + s_t),\ \Delta s_t]\}$ for the first differences model. Using methods described in Campbell and Shiller (1987), the restrictions implied by (2.4) and (3.3) were reduced to linear restrictions, and tested by means of likelihood ratio tests.[3] Table 3 displays results for both models, under various assumptions about lag lengths and real interest rates.

The results in Table 3 show that the constant *ex ante* real rate model can be rejected at essentially arbitrary significance levels by the postwar U.S. data. Differencing seems to impact little on the significance level of the test statistics. This strong rejection of the model stands in sharp contrast to the findings of Hamilton and Flavin (1986), who conclude that equation (2.2) represents a useful approximation for the postwar U.S. case. Some possible explanations for this discrepancy are considered below.

Hamilton and Flavin's (henceforth HF) study differs from the present one in the following ways:

1. They use annual (fiscal year) data derived from the unified budget series, instead of quarterly data derived from the NIA series.
2. Their sample runs from 1960 through 1984, instead of 1948–1986.
3. They assume that the current surplus net of interest (s_t) is not Granger caused by debt (k_t).
4. As a consequence of (3), they do not formally test the restrictions implied by equation (2.4). Instead they present evidence that expected changes in future surpluses account for a substantial amount of the variation in real debt. In particular, they calculate the squared correlation between the real and implied debt series to be 0.53.

Of the differences listed above, item (3) represents the most serious distinction between the two studies. In assuming that debt does not Granger cause surpluses, HF's approach implies stochastic singularity of $\{(k_{t+1},\ s_t)\}$ and eliminates the possibility of performing tests of cross equation restrictions such as those reported in Table 3. This assumption is also strongly rejected by the data: standard tests (not reported here) of causality from debt to surpluses reject noncausality at the 1% significance level. In light of these considerations, the assumption of noncausality does not appear to be justified by the data.

To provide greater comparibility between my results and those in the HF study, the application of the *levels* models tested in Table 3 was modified to more closely resemble that of the HF paper. The data series were modified by annualizing the quarterly data (averaging debt and summing deficits) and restricting the data set to the years 1960-1984. Following HF, a constant *ex ante* real rate of 1.12 percent was assumed, and a constant term and 3 lags were included in the VAR equations. However, Granger noncausality of surpluses by debt was not assumed due to considerations mentioned above. The estimation results for this modified data set are displayed in Table 4. This table show that the results obtainable with the annualized data set are comparable to those obtained in the HF study, in the sense that the debt series implied by the model is highly correlated with the actual debt series. In fact, by dropping the unrealistic assumption that the surplus net of interest is not caused by debt, sample correlations above .9 can be easily obtained. On the other hand, the cross equation restrictions implied by the model are still strongly rejected by the modified data. These results do not qualitatively change when the sample period is expanded to the full data set (1948–86), or restricted to the years before the enactment of the Reagan tax cut (1948–80).

The fact that the model appears to be so consistently and strongly rejected by the postwar U.S. data led me to experiment with a number of different real interest rates and data subsamples, in order to see whether the model represents a reasonable approximation for some subperiod of the postwar data set. The best fit was obtained by specifying the real rate to be very close to zero and restricting the (annual) data set to the pre-oil shock period of 1948–73. For this experiment I was able to obtain $\chi^2(7)$ test statistics of approximately 22. Though this is still highly significant, applying the Schwarz correction for degrees of freedom yields values for the Schwarz criterion that are only slightly unfavorable for the restricted model.

4. Conclusion

The results of Section 3 suggest that the joint hypothesis of a one period ahead, constant *ex ante* real rate version of the government budget constraint and expected net present value budget balance is not a particularly realistic abstraction for the postwar U.S. data. Statistical rejection of the restrictions implied by this hypothesis is robust to assumptions about stationarity of the data, values of the real rate, and the choice of time unit for the model. The model is also rejected for various subsamples of the postwar data set, though the degree of rejection is substantially increased when data from the 1980s is included in the estimation period, and decreased somewhat for very low real rates when only pre-oil shock data is included.

Versions of the government budget constraint that assume a constant *ex ante* real rate are sometimes incorporated in rational expectations macroeconomic models. The results presented here suggest that models incorporating this type of constraint will fail to capture some of the dynamic features of the fiscal data for the U.S. If these data are to be consistent with the idea of expected present value budget balance, then more complex models of the budget balance relationship will have to be formulated. The addition of features such as time varying real rates and a term structure of government debt poses interesting challenges for future research in this area.

Table 1

Tests for Stationarity of Real Debt and Deficits

$$\Delta x_t = \alpha_0 + \alpha_1 \Delta x_{t-1} + \alpha_2 x_{t-1} + \varepsilon_t$$

Series (x_t)

	Debt k_{t+1}	Deficits s_t
Intercept	1.19	−.0403
	(.420)	(−.109)
Δx_{t-1}	.868	.00800
	(20.4)	(.993)
x_{t-1}	−.00104	−.133
	(−.262)	(−3.34)
Q(36)	33.5	35.4

Sample is 1948Q–1986Q4. T-Statistics are in parentheses. To reject the null of a unit root in favor of stationarity, the Dickey-fuller test requires the t statistic on x_{t-1} to be less than −1.95 at the five percent level. Q is the Ljung-Box statistic, distributed $\chi^2(36)$ under the null of white noise residuals.

Table 2

Posterior Distribution for Modulus of the Largest AR Root Λ

$$x_t = \alpha_0 + \sum_{i=1}^{6} \alpha_i x_{t-1} + \varepsilon$$

Series (x_t)

	Debt k_{t+1}	Deficits s_t
Posterior Mean	.9748	.8654
Posterior St. Deviation	.02602	.05748
$Pr(\Lambda < 1)$	.8411	.9966
App. Log Odds in favor of a unit root*	.7978	−.7873

Sample is 1949Q3–1986Q4. Calculations are based on 10,000 Monte Carlo replications. See DeJong and Whiteman (1989a) for a detailed description of the Monte Carlo technique.

*Approximate log posterior odds ratio of a unit root versus a stationary alternative. Following suggestion of Sims (1988), the log of this ratio is approximated as −log(posterior variance of Λ)−6.5, which assumes 4 to 1 prior odds in favor of the stationary alternative.

Table 3

Likelihood Ratio Tests of (2.2) and (3.3)

Assumed Constant *Ex Ante* Real Rate (Annualized)

	Lags in VAR	$r = .2\%\%$	$r = 1\%$	$r = 2\%$
Levels Model				
	4	154	144	151
	8	165	156	164
Differences Model				
	4			
	8	129	129	129

Note: Test statistics are distributed $\chi^2(2p+1)$ under the null, where p is the number of lags in the VAR model. Original sample is 1948Q1–1986Q4.

Table 4

Tests of Expected NPV Using Annual Data

Sample Period	1960–84	1948–86	1948–80
Likelihood Ratio $[\chi^2(7)]$	34.6	76.4	40.8
r^2 for Actual vs. Implied Debt Series	.901	.969	.980

Note: tests assume a constant *ex ante* real interest rate of 1.12 percent.

Notes

1. Other related papers are Wilcox (1989) and Shim (1984).
2. Inspection of equation (2.1) reveals that (2.1) is a weaker restriction than equation (2.1) of HSR. Yet a great deal of confusion exists as to whether it is necessary to assume constant real rates to obtain (2.2). Clearly, only a constant *ex ante* real rate need be assumed, since (2.2) follows from (2.1).
3. By linearizing the restrictions implied by (2.4), restrictions are in effect being imposed on the AR rather than the MA representation of the joint process for debt and deficits. Alternatively, imposing restrictions in this fashion amounts to imposing restrictions using the government budget constraint (2.1) instead of the present value relation (2.4). Also, the likelihood ratio tests of these restrictions depend crucially upon the stationarity of the debt process assumed in the derivation of (2.4). It should also be noted that the values of the likelihood ratio test statistics are invariant to the (invertible) transformations used to obtain these linear restrictions.